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LABORATORY AND NUMERICAL MODELS OF ROTATING STRATIFIED FLOWS.(U)

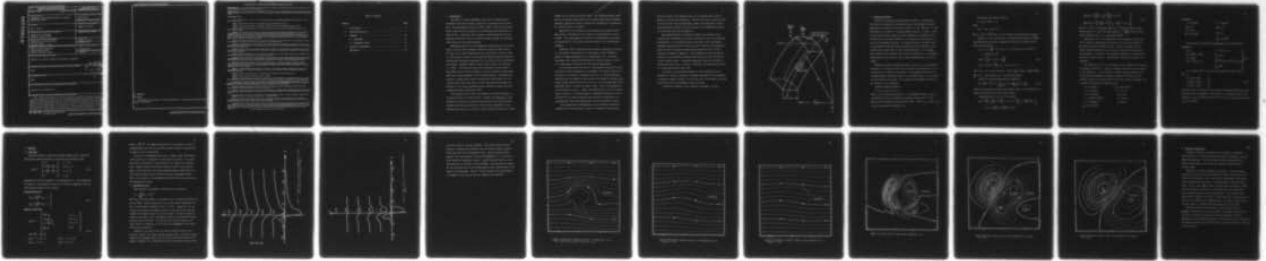
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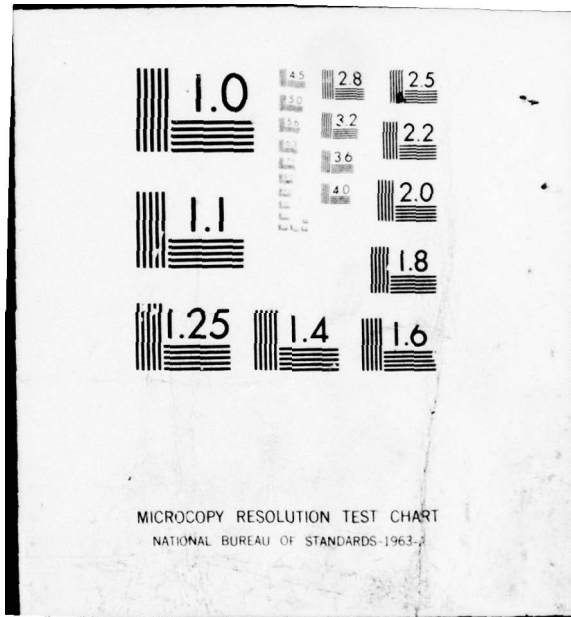
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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) LABORATORY AND NUMERICAL MODELS OF ROTATING STRATIFIED FLOWS		5. TYPE OF REPORT & PERIOD COVERED Final Report
7. AUTHOR(s) Don L. Boyer		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Wyoming Laramie, Wyoming 82071		8. CONTRACT OR GRANT NUMBER(s) N00014-76-C-0272
11. CONTROLLING OFFICE NAME AND ADDRESS Contracts and Grants Office P.O. Box 3314 Laramie, Wyoming 82071		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Office of Naval Research 1914 Hampden Avenue Denver, Colorado 80202		12. REPORT DATE April 21, 1978
		13. NUMBER OF PAGES
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		15. SECURITY CLASS. (of this report) Unclassified
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) None		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
18. SUPPLEMENTARY NOTES None		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Rotating, stratified flow.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report considers the flow of a stably stratified rotating flow over shallow topographies located on a horizontal plane surface; the system is bounded above by a horizontal plane surface. The theoretical formulation is restricted by the following parameter restrictions $E \ll 1$, $Ro \gg E^{1/2}$, $B^2 \gg 0(1)$, $H/L \gg 0(1)$ and $h/L \ll 1$ where E is the Ekman number, Ro the Rossby number, B^2 a stratification parameter, H the distance between the plane surfaces, L the topography width and h its height. Solutions for a long ridge of constant cross-section and an axisymmetric feature		

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20. ABSTRACT

are given.

Preliminary information on the development of a two-layer stratified water tunnel is advanced.

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1. Introduction

The effect of bottom topography on the flow of rotating and/or stratified fluids has received a great deal of attention in the recent past. The experiments by Sir G.I. Taylor (1921) in which the so-called Taylor column was first observed in the laboratory were the genesis of these efforts. McCartney (1975) presents a table delineating the most important theoretical studies on Taylor-columns including the parameter restrictions for the various analyses.

Theoretical studies for the homogeneous incompressible case include those of Jacobs (1964), Ingersoll (1969) and Vaziri and Boyer (1971). The first two of these investigations were restricted to inviscid flows: i.e. $E^{\frac{1}{2}} \ll Ro$ where E and Ro are the Ekman and Rossby numbers respectively. Unfortunately laboratory experiments on this class of flows are generally restricted to parameter ranges in which viscous effects are important; i.e., $E^{\frac{1}{2}} \sim Ro$. The Vaziri and Boyer study developed a viscous model in which $E^{\frac{1}{2}} \sim Ro$ and in addition presented a series of laboratory experiments which were in good agreement with the theory advanced. The experiments utilized a rotating water tunnel (analogy with a wind tunnel in a non-rotating frame) which was found to provide improved observational capabilities to the towing experiments first conducted by Taylor and later considered by Hide and Ibbetson (1966).

More recently Davies (1972) has extended the towing experiments to include the effects of stratification. Whereas, the homogeneous incompressible experiments were generally restricted by the Taylor-Proudman theorem which implies a horizontal velocity field independent of the distance along the axis of rotation, these stratified experiments allowed

changes in the velocity field with depth. The Davies' experiments again were in the general range $Ro\sqrt{E}^{\frac{1}{2}}$ so that viscous effects were presumably important; specifically Davies utilized the following set of parameters: $Ro = 8.8(10)^{-3}$ and $E = 4.0(10)^{-4}$.

Hogg (1973) in a theoretical study followed up the Davies' experiments with an investigation of the effect of stratification on Taylor columns. This study neglected viscosity and investigated three ranges of stratification: $S \ll Ro$; $S \sim Ro$; and $S \sim O(1)$ where S is a stratification parameter.

McCartney (1975), neglecting viscous effects, presented an analysis of a two-layered rotating flow on a beta-plane; he also conducted a series of towing experiments for the single-layer case. In another beta-plane study Janowitz (1975) discussed an inviscid solution for flow of a continuously stratified fluid over shallow topographies.

In the present study the flow of a continuously stratified rotating fluid over shallow topographies is considered. The initial objective of the study is to develop a theoretical model for a rotating stratified flow which can be examined in the laboratory utilizing the rotating water tunnel concept advanced earlier by one of us; e.g. see Boyer-Ridge (1971), Boyer-Step (1971), or Vaziri and Boyer (1971). That is an experiment is to be developed in which a stably stratified fluid is to be in uniform motion relative to a rotating observer. Topographic features are then to be placed in the moving stream with the objective of determining the resulting flow field and comparing same with the theoretical studies.

In the homogeneous incompressible case the laboratory experiment consisted of the flow through a channel of rectangular cross-section.

The flow outside of the boundary layers on the channel walls could be shown to be uniform and geostrophic. Because there were no free surfaces, it could be shown that the centrifugal terms in the equations of motion were unimportant and thus, that the location of the channel with respect to the rotation axis was not important.

When stratification is introduced, however, the position of the rotation axis with regard to the tunnel test-section will be an important consideration. In the analysis given in Sections 2 and 3 we assume an idealized physical system in which the flow is forced to move azimuthally about the axis of rotation in an annulus whose upper and lower bounding surfaces are parabolic (Figure 1). These parabolas are the equilibrium surfaces of a given stably-stratified fluid system at rest with respect to the rotating system. A specified topographic feature is located on the lower surface and it is desired to determine the resulting flow field for a specified upstream shear flow.

In section 2 the governing equations for the resulting flow are derived for a range of parameters considered attainable in the laboratory. In section 3 specific solutions for the flow over a long ridge of constant cross-section and an axisymmetric "bump" are given.

In Section 4 progress on the laboratory experiments is given.

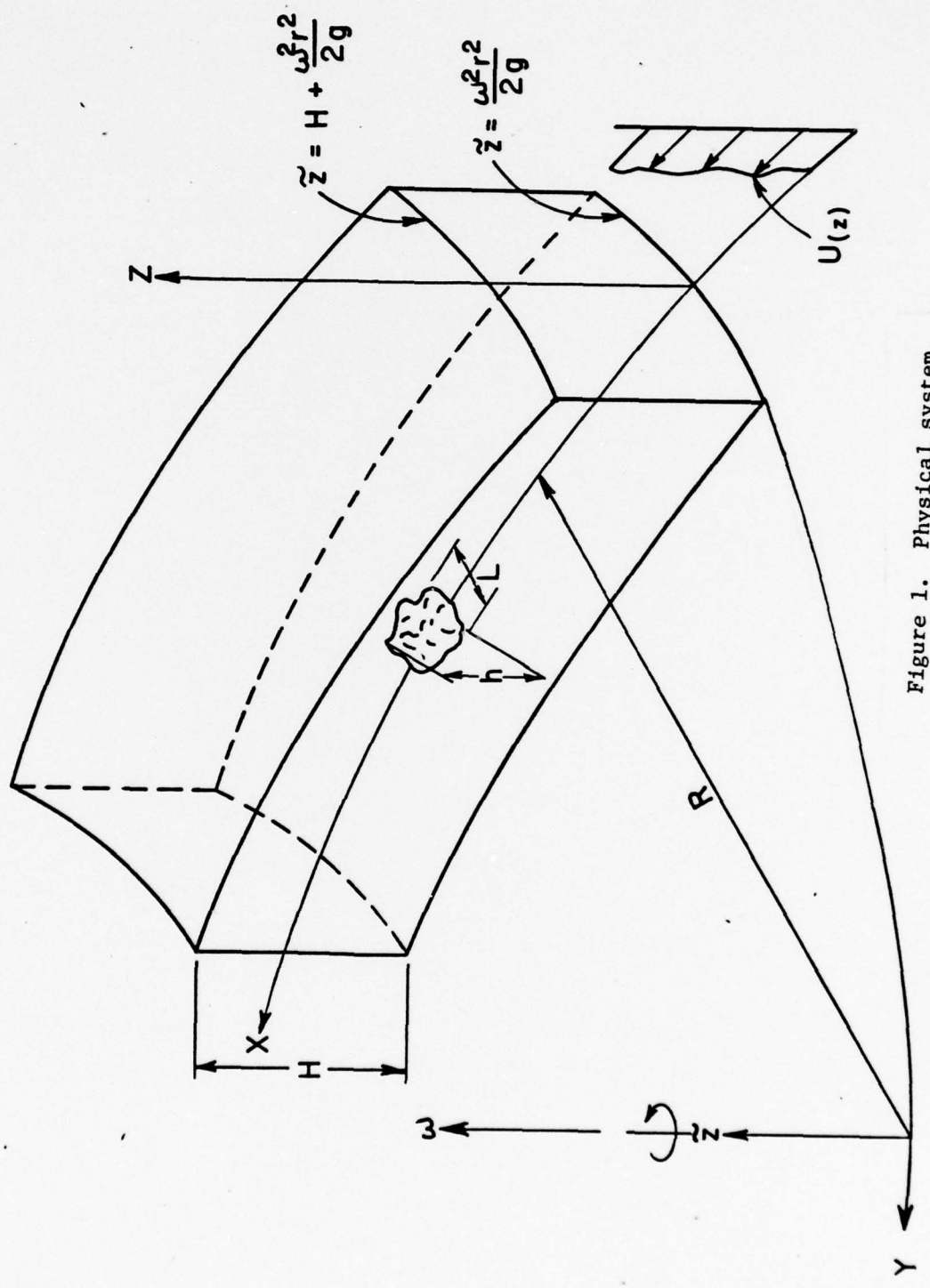


Figure 1. Physical system.

2. Governing Equations

Consider the physical system sketched in Figure 1 a stably stratified fluid is confined in an annulus defined by two parabolic surfaces separated by a distance H and vertical walls at $r = R - \frac{L_0}{2}$ and $r = R + \frac{L_0}{2}$. The annulus is rotating at a constant angular velocity, ω , about a vertical axis and the fluid, away from solid boundaries, is assumed to be in uniform motion with speed dependent on the distance from the lower surface in the azimuthal direction. As discussed earlier the Ekman layers on the parabolic bounding surfaces combined with the constraints of the vertical walls would tend to alter the stratification. It is assumed here, however, that the Ekman layers can be controlled i.e., by removing fluid from the corner regions so that a stable stratification can be maintained.

Define an intrinsic coordinate system in which x is in the azimuthal direction, y is along the surface with positive being toward the axis of rotation and z is normal to the surface. A topographic feature $h(x, y)$ of characteristic horizontal length, L , and vertical scale, h , is located in the vicinity of the origin. It is desired to determine the motion in the vicinity of the topography.

Define the density field as

$$\rho = \rho_0 + \rho_s(z) + \hat{\rho}(x, y, z)$$

where ρ_0 is the specified density at $z = 0$ assuming no motion, $\rho_s(z)$ is the specified vertical distribution assuming no motion and $\hat{\rho}$ is the density field compatible with the motion field. Define $\Delta\rho_s = \rho_s(H) - \rho_s(0)$ as the characteristic magnitude of ρ_s .

Also define the pressure field as

$$p = p_0(z) + \hat{p}(x, y, z)$$

where

$$\nabla p_0 \equiv -(\rho_0 + \rho_s(z)) G \hat{k}$$

Here $G = (g^2 + \omega^4 R^2)^{\frac{1}{2}}$ and p_0 is the pressure associated with no motion.

Taking H to be of the order of L and restricting to $\frac{L}{R} \ll 1$ and $\frac{\omega^2 L}{g} \ll 1$, the steady-state equations of momentum, diffusion and conservation of mass can be written as

$$\begin{aligned} (\rho_0 + \rho_s + \hat{\rho}) \frac{D\bar{v}}{Dt} &= -\nabla \hat{p} - 2(\rho_0 + \rho_s + \hat{\rho})(\bar{\omega} \times \bar{v}) - \hat{\rho}(G + O(\omega^2 L)) \hat{k} \\ &\quad + \hat{\rho} O(\omega^2 L) \hat{j} + \mu \nabla^2 \bar{v} \\ (\bar{v} \cdot \nabla) \hat{\rho} + \frac{d\rho_s}{dz} w &= \kappa \nabla^2 \hat{\rho} + \kappa \frac{d^2 \rho_s}{dz^2} \end{aligned} \quad (2.1)$$

$$\nabla \cdot \rho \bar{v} = (\bar{v} \cdot \nabla) \hat{\rho} + w \frac{d\rho_s}{dz} + (\rho_0 + \rho_s + \hat{\rho}) \nabla \cdot \bar{v} = 0$$

where $\bar{v}(u, v, w)$ is the velocity; $\nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} + O(\frac{L}{R})$ and $\frac{\omega^2 L}{g}$;

$\frac{D}{Dt} = \bar{v} \cdot \nabla$; μ the viscosity and κ the mass diffusivity.

Define the following dimensionless quantities:

$$\bar{x}^* = \frac{\bar{x}}{L}; \bar{v}^* = \frac{\bar{v}}{U}; \hat{\rho}^* = \frac{\hat{\rho}}{2\omega U \rho_0 G^{-1}}; \hat{p}^* = \frac{\hat{p}}{2\omega U \rho_0 L}; \rho_s^* = \frac{\rho_s}{\Delta \rho_s} \quad (2.2)$$

where we anticipate the density and pressure field to be determined by a balance of gravitational and coriolis effects.

Introducing (2.2) into (2.1) one obtains:

$$\left. \begin{aligned} \text{Ro} \left(1 + \frac{\Delta \rho_s}{\rho_0} \rho_s\right) \frac{D\bar{v}^*}{Dt} + \text{Fr}^2 \frac{D\bar{v}^*}{Dt} &= -\nabla \hat{p}^* - \left(1 + \frac{\Delta \rho_s}{\rho_0} \rho_s + \frac{2\omega U}{G}\right) (\hat{\omega} \times \bar{v}^*) \\ &\quad - \hat{\rho}^* \left(1 + O\left(\frac{\omega^2 L}{G}\right)\right) \hat{k} + \hat{\rho}^* O\left(\frac{\omega^2 L}{G}\right) \hat{j} + \text{Ev} \nabla^2 \bar{v}^* \end{aligned} \right\}$$

$$\left. \begin{aligned} R_o(\bar{v} \cdot \nabla) \hat{\rho} + B^2 \frac{d\rho_s}{dz} w &= \frac{\kappa}{UL} B^2 \frac{d^2 \rho_s}{dz^2} + E \sigma^{-1} \nabla^2 \hat{\rho} \\ \frac{2\omega U}{G} (\bar{v} \cdot \nabla) \hat{\rho} + \frac{\Delta \rho_s}{\rho_o} \frac{d\rho_s}{dz} w + (1 + \frac{\Delta \rho_s}{\rho_o} \rho_s + \frac{2\omega U}{G} \hat{\rho}) \nabla \cdot \bar{v} &= 0 \end{aligned} \right\} \quad (2.3)$$

where $R_o = U/2\omega L$ is the Rossby number; $Fr^2 = U^2/GL$ the square of the Froude number; $E = \nu/2\omega L^2$ the Ekman number; $B^2 = \frac{\Delta \rho_s G}{\rho_o L} / 4\omega^2$ the stratification parameter; and $\sigma = \nu/\kappa$ the Prandtl number.

Relations (2.3) are five equations in the unknowns $(u, v, \omega, \hat{p}, \hat{\rho})$ and will be solved in approximate form subject to the no slip condition on the upper and lower bounding surfaces. It will be assumed that the lateral bounding surfaces will not effect the motion in the vicinity of the topography. Upstream of the topographic feature it is assumed that the motion is given by $U(z)$. Downstream we require the velocity to be azimuthal.

The reduction of (2.3) to a tractable form is guided by the objective of deriving solutions in a region of parameter space that can be examined in the laboratory. In this regard we consider typical parameters used in the similar homogeneous experiments conducted by Vaziri and Boyer (1971) and further use stratification values that could be attainable from a saline stratified water experiment; e.g.

$$\begin{aligned} U &= 0.1 \text{ cm/sec} & G \approx g &= 980 \text{ cm/sec}^2 \\ \omega &= 0.1 \text{ rad/sec} & L &= 2.5 \text{ cm} \\ \nu &= 0.01 \text{ cm}^2/\text{sec} & H &= 2.5 \text{ cm} \\ \kappa &= 1.6(10)^{-5} \text{ cm}^2/\text{sec} & h &= 0.25 \text{ cm} \\ \rho_o &= 1.01 \text{ gm/cm}^3 & R &= 100 \text{ cm} \\ \Delta \rho_s &= 0.01 \text{ gm/cm}^3 \end{aligned}$$

from whence

$$\begin{aligned}
 E &= 8.0(10)^{-3} & \sigma^{-1} &= 1.6(10)^{-3} \\
 Ro &= 0.2 & \frac{H}{L} &= 1.0 \\
 B^2 &= 97.0 & \frac{h}{L} &= 0.1 \\
 Fr^2 &= 4.1(10)^{-6} & \frac{L}{R} &= 2.5(10)^{-2}
 \end{aligned}$$

We now assume the following relationships for the dimensionless parameters:

$$\left. \begin{aligned}
 E \ll 1 & & \sigma^{-1} &\sim E^\gamma \quad (\gamma > 0) \\
 Ro = kE^{\frac{1}{2}} \quad (k \sim \text{unity}) & & \frac{H}{L} &\sim \text{unity} \\
 B^2 \sim \text{unity} & & \frac{h(x,y)}{L} &= h_0(x,y) E^{\frac{1}{2}} \quad (h_0 \sim \text{unity}) \\
 Fr^2 \sim E^{2+\alpha} \quad (\alpha > 0) & & \frac{L}{R} &\sim E^{\frac{1}{2}+\alpha}
 \end{aligned} \right\} (2.4)$$

It is assumed that \bar{v} , $\hat{\rho}$ and \hat{p} can be expanded in power series in $E^{\frac{1}{2}}$: i.e.

$$\left. \begin{aligned}
 \bar{v} &= \bar{v}_0 E^0 + \bar{v}_1 E^{\frac{1}{2}} + \dots \\
 \hat{\rho} &= S_0 E^0 + S_1 E^{\frac{1}{2}} + \dots \\
 \hat{p} &= P_0 E^0 + P_1 E^{\frac{1}{2}} + \dots
 \end{aligned} \right\} (2.5)$$

The zeroth order is dictated by the method of non-dimensionalization while the first order is suggested by the character of the Ekman layers on the parabolic surfaces. Substituting (2.5) into (2.3) and utilizing restrictions (2.4) one obtains:

First Order

$$\left. \begin{aligned} 0 &= -P_{ox} + V_o \\ 0 &= -P_{oy} = U_o \\ 0 &= -P_{oz} - S_o \\ W_o &= 0 \\ U_{ox} + V_{oy} + W_{oz} &= 0 \end{aligned} \right\} \quad (2.6)$$

Second Order

$$\left. \begin{aligned} -k(U_o U_{ox} + V_o U_{oy}) &= -P_{1x} + V_1 \\ k(U_o V_{ox} + V_o V_{oy}) &= -P_{1y} - U_1 \\ 0 &= -P_{1z} - \frac{1}{4} \frac{R}{L} \frac{Fr^2}{Ro} U_o - S_1 \\ k(U_o S_{ox} + V_o S_{oy}) + B^2 \frac{d\rho_s}{dz} W_1 &= 0 \\ U_{1x} + V_{1y} + W_{1z} &= 0 \end{aligned} \right\} \quad (2.7)$$

Cross-differentiating the first two of (2.7) and utilizing the last of (2.7) gives an expression for W_{1z} in terms of zeroth order terms. This can then be equated to W_{1z} as determined from the fourth of (2.7). Utilizing the third of (2.6) then one determines the following relation for P_o

$$J(P_o, \nabla_2^2 P_o) = -\frac{1}{B^2} \frac{\partial}{\partial z} \left(\frac{P_{oy} P_{oxz} - P_{ox} P_{oyz}}{\frac{d\rho_s}{dz}} \right) \quad (2.8)$$

If we assume a linear profile for ρ_s (i.e., $\rho_s = -\frac{L}{H} z$) and introduce a new vertical coordinate ζ define by $\zeta = B \left(\frac{L}{H}\right)^{\frac{1}{2}} z$, (2.8) reduces to

$$J(P_o, \nabla_3^2 P_o) = 0 \quad (2.9)$$

where

$$\nabla_3^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}$$

Under the above scaling, the boundary layers on the parabolic surfaces are, to lowest order, Ekman layers. The boundary conditions on these surfaces are the same as those given in Vaziri and Boyer (1971).

Along the lower surface the Ekman layer solutions specify the vertical velocity at the outer edge of the layer (i.e. $z = 0$ of the interior) as

$$W_1 = \frac{1}{\sqrt{2}} \nabla^2 P_o + J(P_o, h_o) \quad (2.10)$$

while along the upper surface ($z = \frac{H}{L}$)

$$W_1 = -\frac{1}{2} \nabla^2 P_o \quad (2.11)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. These vertical components must match the interior values specified by the fourth of (2.7) from whence one obtains

$$\frac{1}{\sqrt{2}} \nabla^2 P_o + J(P_o, h_o) + \frac{H}{L} \frac{k}{B^2} J(P_o, P_{oz}) = 0 \quad \text{at } z = 0 \quad (2.12)$$

and

$$\frac{1}{\sqrt{2}} \nabla^2 P_o - \frac{H}{L} \frac{k}{B^2} J(P_o, P_{oz}) = 0 \quad \text{at } z = \frac{H}{L} \quad (2.13)$$

respectively. The boundary conditions far upstream and downstream are

$$P_{oy} = -1, \quad x \rightarrow -\infty; \quad \text{and} \quad P_{ox} = 0, \quad x \rightarrow +\infty \quad (2.14)$$

respectively. At large lateral distances from the topographic feature

$$P_{oy} = -1, \quad y \rightarrow \pm \infty \quad (2.15)$$

The task is thus, to solve (2.9) subject to (2.12) through (2.15).

From (2.9) we note that

$$\nabla^2 P_o = f(P_o) \quad (2.16)$$

Then from (2.14) one notes that $\nabla^2 P_o$ far upstream is zero for all P_o and hence $f(P_o)$ is identically zero. The analytical problem thus reduces to a solution of Laplace's equation subject to the non-linear set of boundary conditions (2.12) to (2.15). Let us now consider some specific examples.

3. Examples

A. Long Ridge

We first consider a ridge with infinite extent in the y-direction and having a uniform triangular cross-section; specifically take

$$h_0(x) = \begin{cases} 0 & x \leq -\frac{1}{2} \\ E^{-\frac{1}{2}} \left(\frac{h}{L} + \frac{2h}{L} x \right) & -\frac{1}{2} \leq x \leq 0 \\ E^{-\frac{1}{2}} \left(\frac{h}{L} - \frac{2h}{L} x \right) & 0 \leq x \leq \frac{1}{2} \\ 0 & x \geq \frac{1}{2} \end{cases} \quad (3.1)$$

Assuming the velocity components to be independent of y, the formulation of Section 2, now expressed in terms of the velocity components, leads to the following boundary value problem:

Governing Equations

$$\left. \begin{aligned} v_{0xx} + \frac{1}{B^2} \left(\frac{H}{L} \right) v_{0zz} &= 0 \\ w_{1xx} + \frac{1}{B^2} \left(\frac{H}{L} \right) w_{1zz} &= 0 \end{aligned} \right\} \quad (3.2)$$

Boundary Conditions

$$W_1(x, 0) = \begin{cases} \frac{1}{\sqrt{2}} v_{0x} & x \leq -\frac{1}{2} \\ \alpha + \frac{1}{\sqrt{2}} v_{0x} & -\frac{1}{2} \leq x \leq 0 \\ -\alpha + \frac{1}{\sqrt{2}} v_{0x} & 0 \leq x \leq \frac{1}{2} \\ \frac{1}{\sqrt{2}} v_{0x} & x \geq \frac{1}{2} \end{cases} \quad (3.3)$$

$$W_1(x, \frac{H}{L}) = -\frac{1}{\sqrt{2}} v_{0x}$$

$$W_1(x \rightarrow -\infty, z) = 0$$

$$W_1(x \rightarrow \infty, z) = 0$$

$$V_0(x \rightarrow -\infty, z) = 0$$

$$V_0(x \rightarrow \infty, z) = 0$$

where $\alpha = \frac{2h}{L} E^{-\frac{1}{2}}$. The numerical solution of (3.2) subject to (3.3) is straightforward and leads to V_0 and W_1 velocity profiles as exemplified by Figures 2 and 3 respectively.

For flow of a homogeneous fluid over a similar ridge, Boyer-Ridge (1971) found V_0 to be negative for the entire flow field. It is noted here that V_0 is positive in the vicinity of the ridge on its upstream side. Similar results were found by Janowitz (1975) in his inviscid study. One should also note the decreased horizontal deflection of the flow field at large vertical distances from the topographic feature. This is in contrast to the homogeneous case for which these deflections are independent of depth.

B. Axisymmetric Bump

Next consider an axisymmetric bump given by the relation

$$h_0 = \frac{h_d}{2RE^{\frac{1}{2}}} \{1 - 4r^2\}^2$$

where h_d is the bump height, R its radius, and r the radial coordinate in the x - y plane. Laplace's equation in P_0 was thus solved numerically subject to the boundary conditions (2.12) to (2.15). Figures 4A, 4B, and 4C give the horizontal streamline pattern for the flow at the "bottom", "middle" and "upper" surfaces of the flow field. We note the rapid decrease of the effect of the topographic feature as the vertical distance from the feature increases. We also note that the flow field at the lower levels is reminiscent of the flow in the homogeneous case considered by Boyer and Vaziri.

Figures 5A, 5B, and 5C give the vertical velocity field at the "bottom", "middle" and "upper" surfaces respectively. The "low" pressure regions on the diagram depict areas in which the vertical velocity component is negative (i.e. downward) while the high pressure regions depict

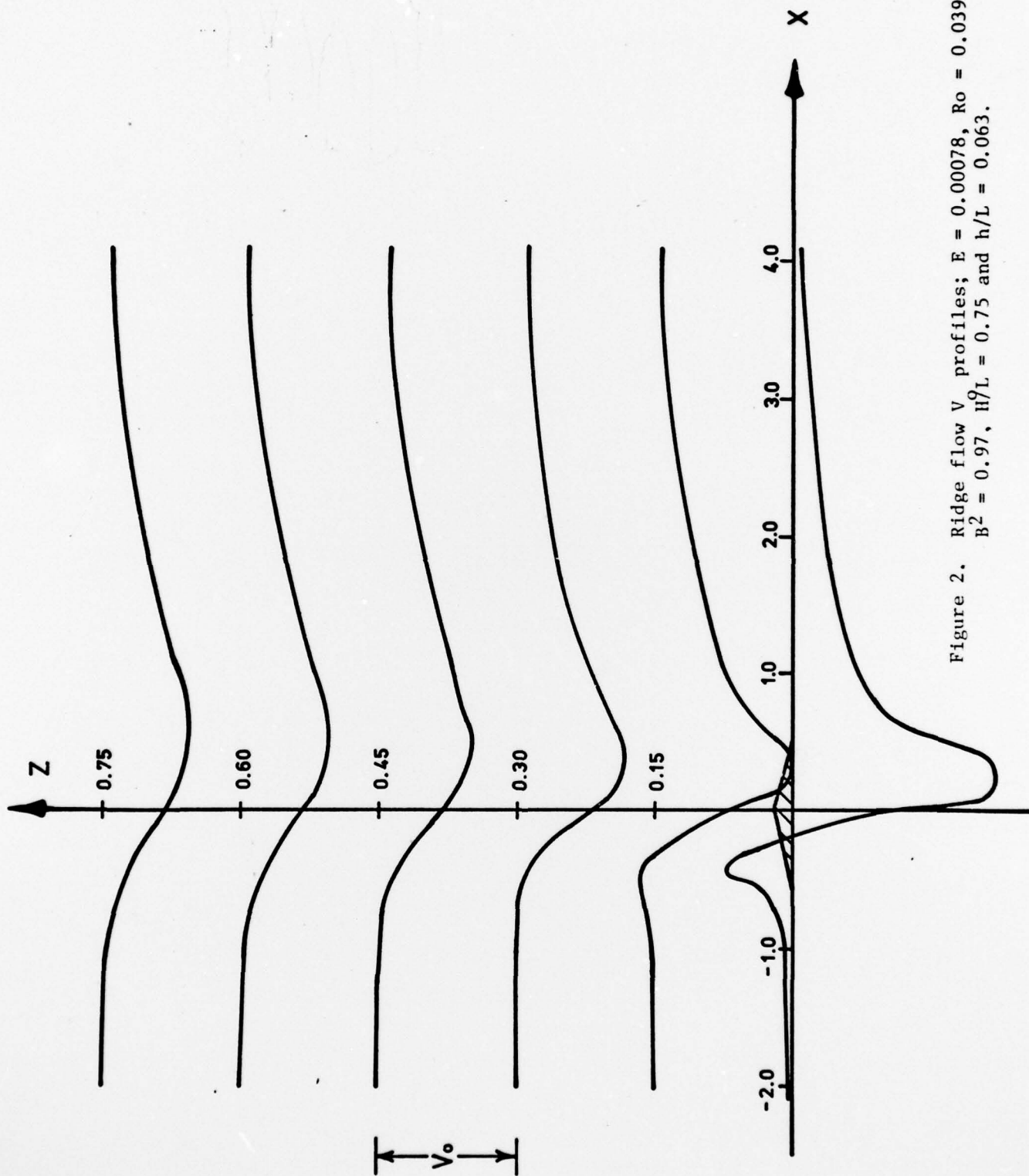


Figure 2. Ridge flow V profiles; $E = 0.00078$, $Ro = 0.039$,
 $B^2 = 0.97$, $H^2/L = 0.75$ and $h/L = 0.063$.

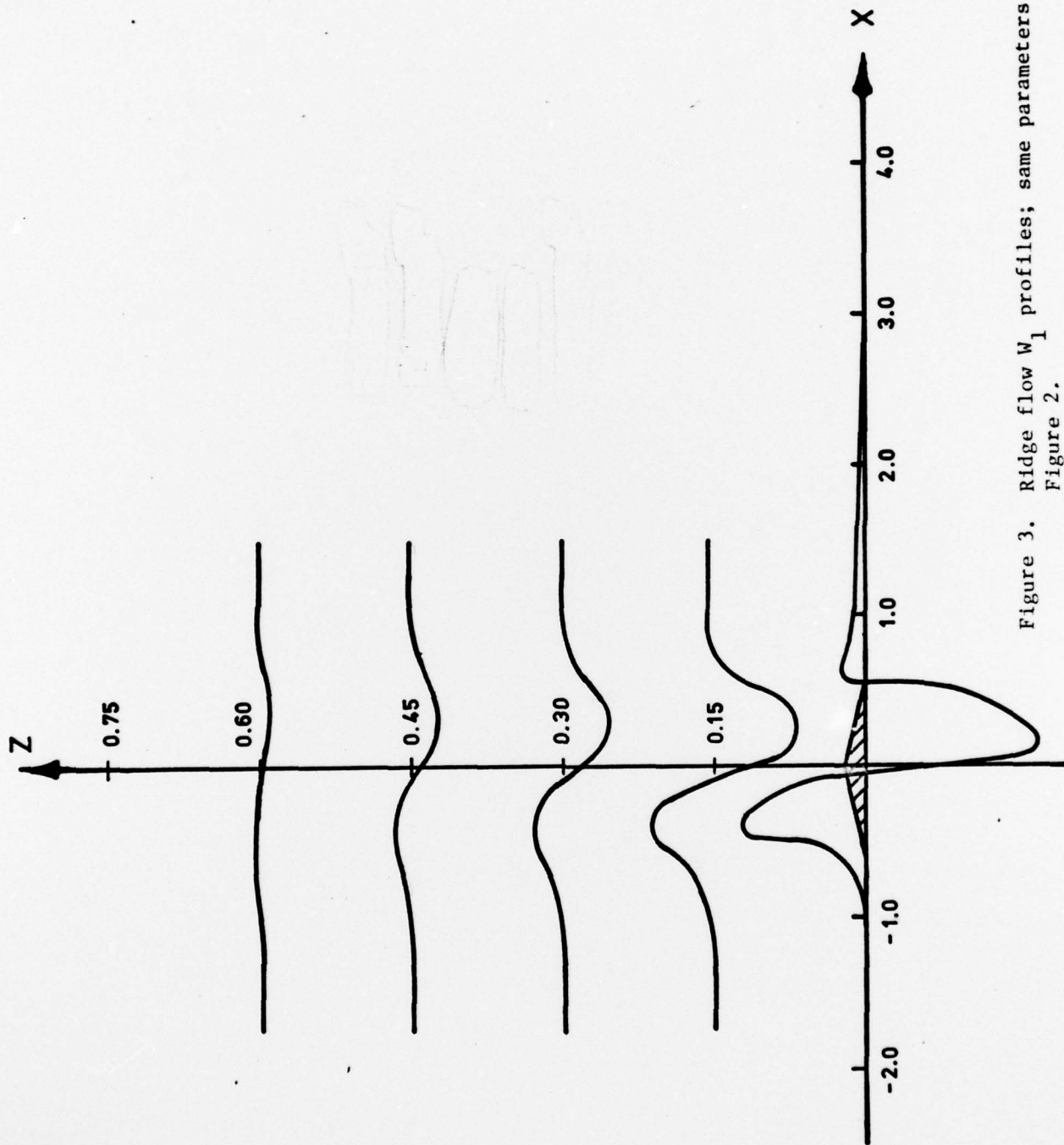


Figure 3. Ridge flow W_1 profiles; same parameters as Figure 2.

a positive vertical velocity component. The profile along the lower surfaces is substantially different than the two-cell pattern found by Vaziri and Boyer for the homogeneous case. Vaziri and Boyer found a negative cell that generally covered the topography; i.e., a suction of fluid toward the topographic feature. Upward flow was found in a cell downstream and to the right of the topography. The principal new aspect for the stratified case is the strong positive cell, upstream and to the right of the topography. Figures 5B and 5C depict the rapid decrease in strength of the vertical velocity component with elevation.

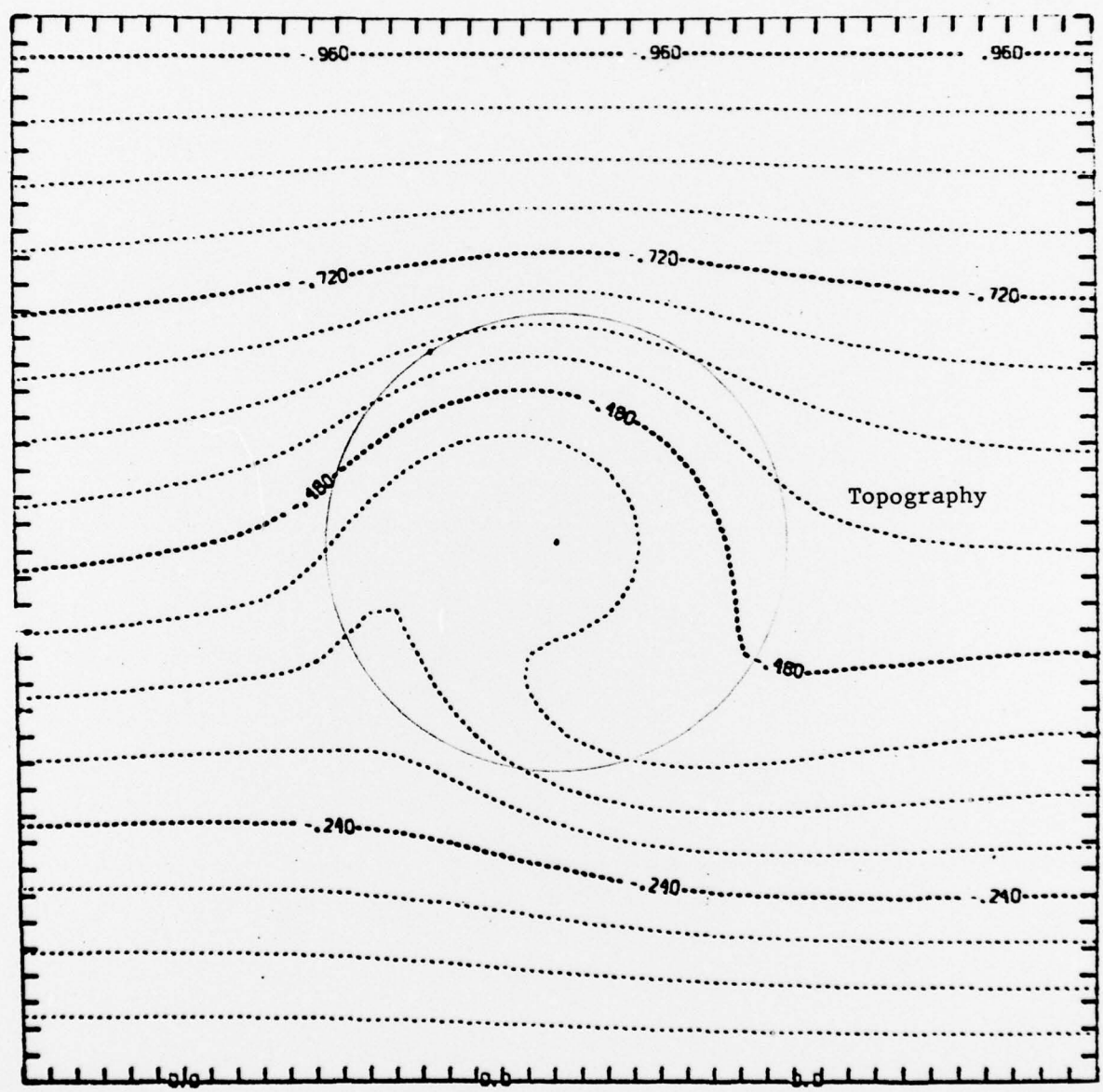


Figure 4A Horizontal streamline pattern; $E = 0.0004$, $R_o = 0.04$,
 $B^2 = 2.0$, $H/L = 0.5$, $h/L = 0.1$, and $z = 0.0$.

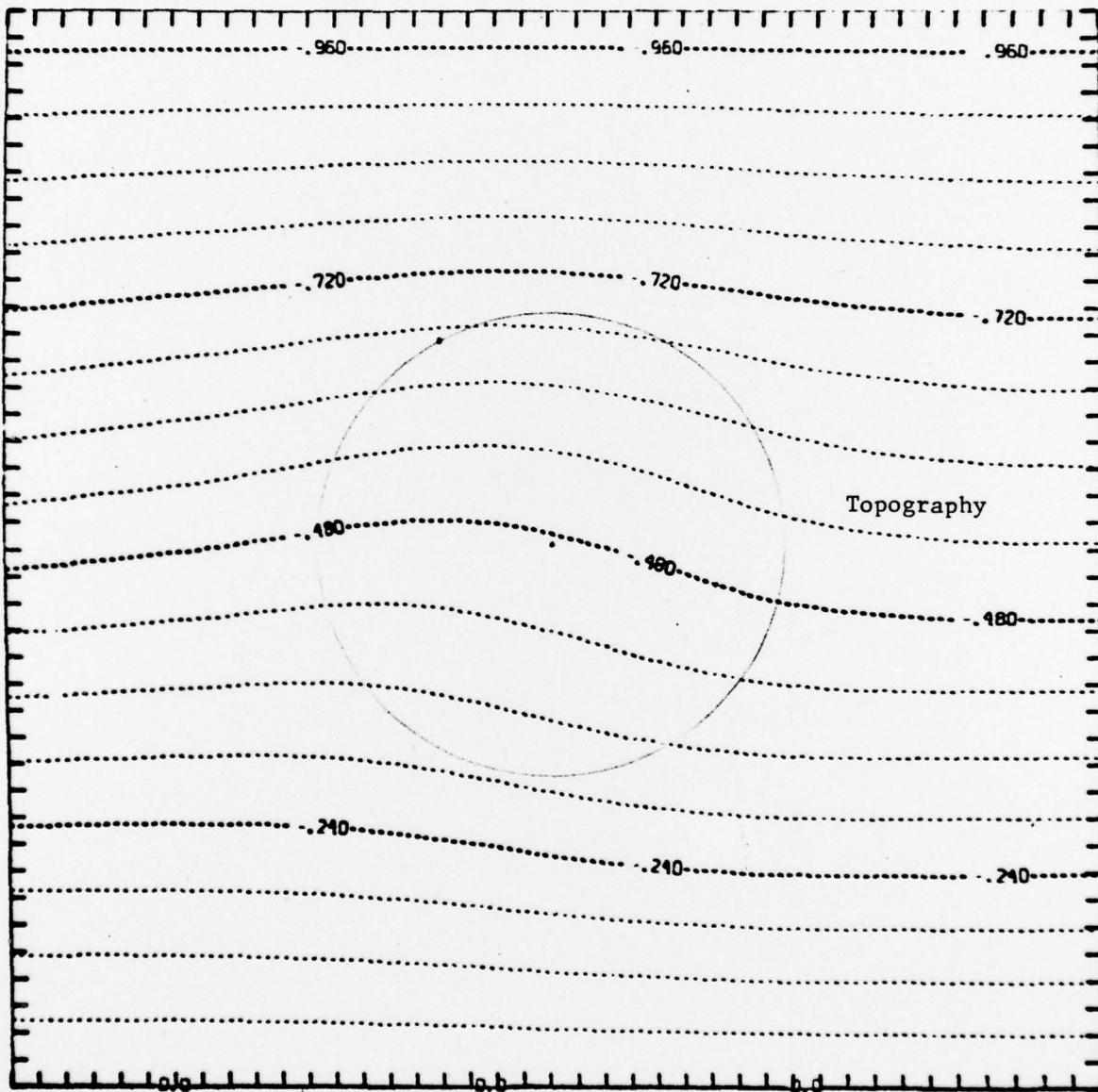


Figure 4B Horizontal streamline pattern; same parameters as 4A
except $z = 0.25$.

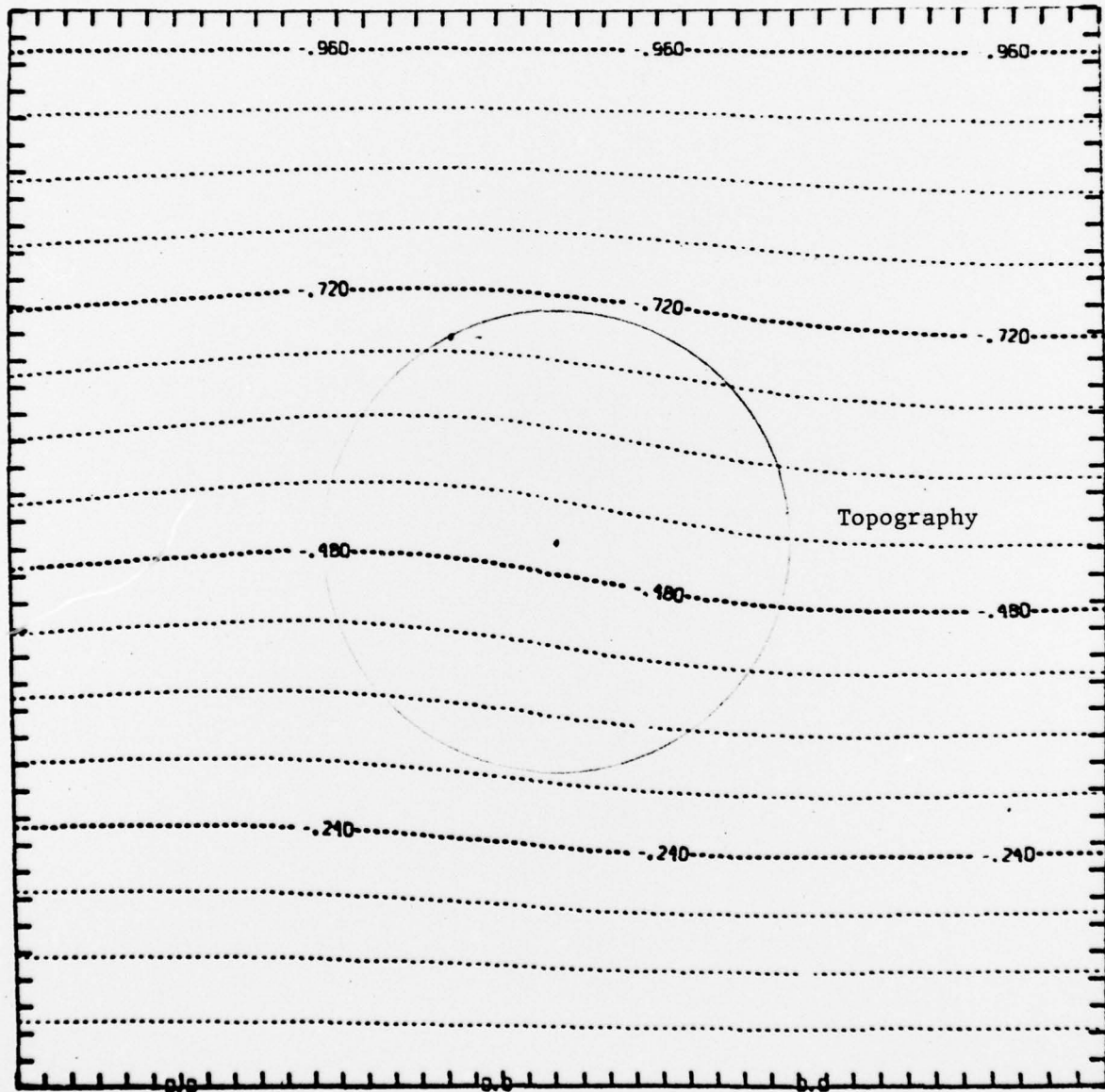


Figure 4C Horizontal streamline pattern; same parameters as 4A except $z = 0.50$.

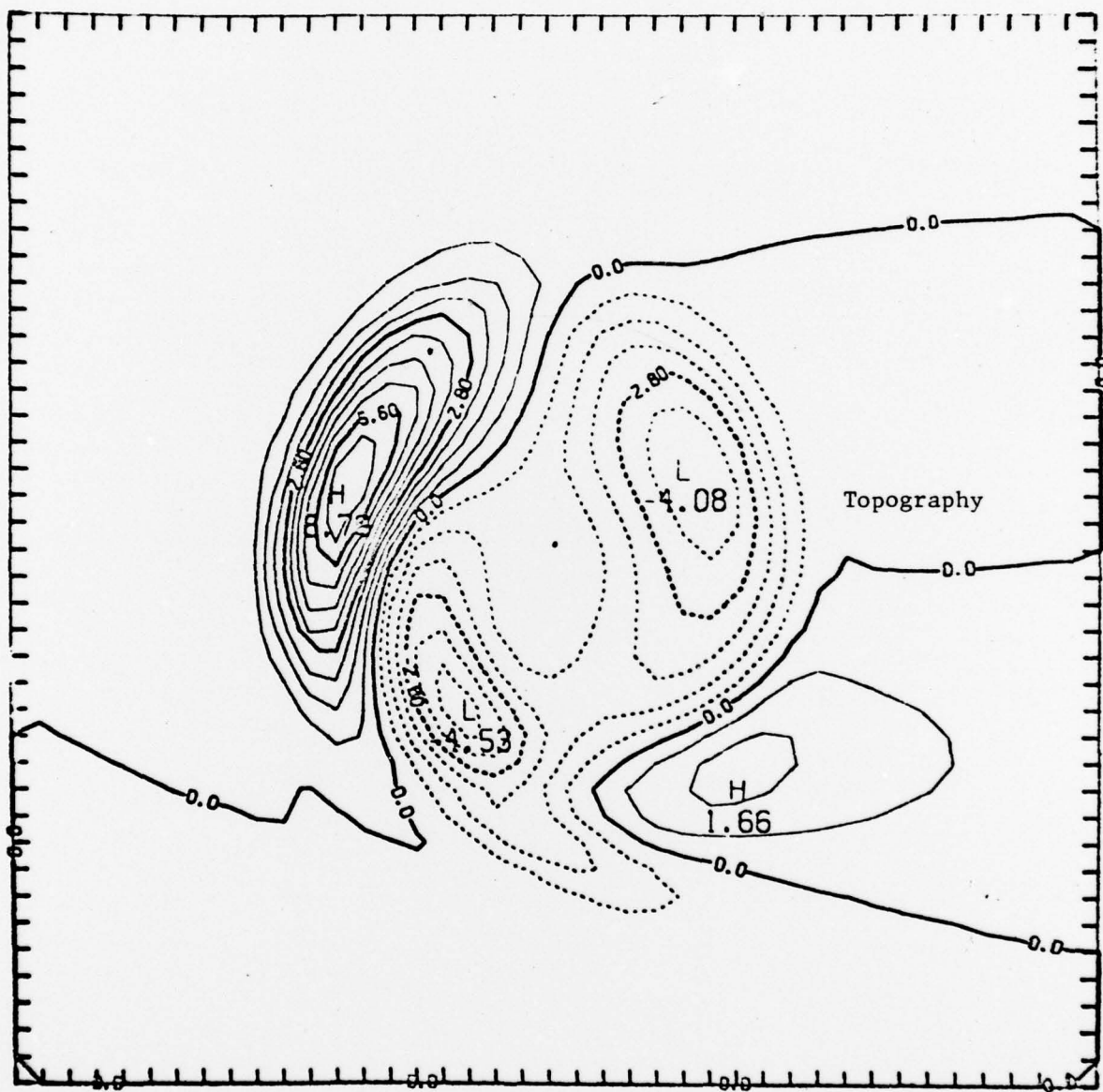


Figure 5A Vertical velocity field; same parameters as 4A.

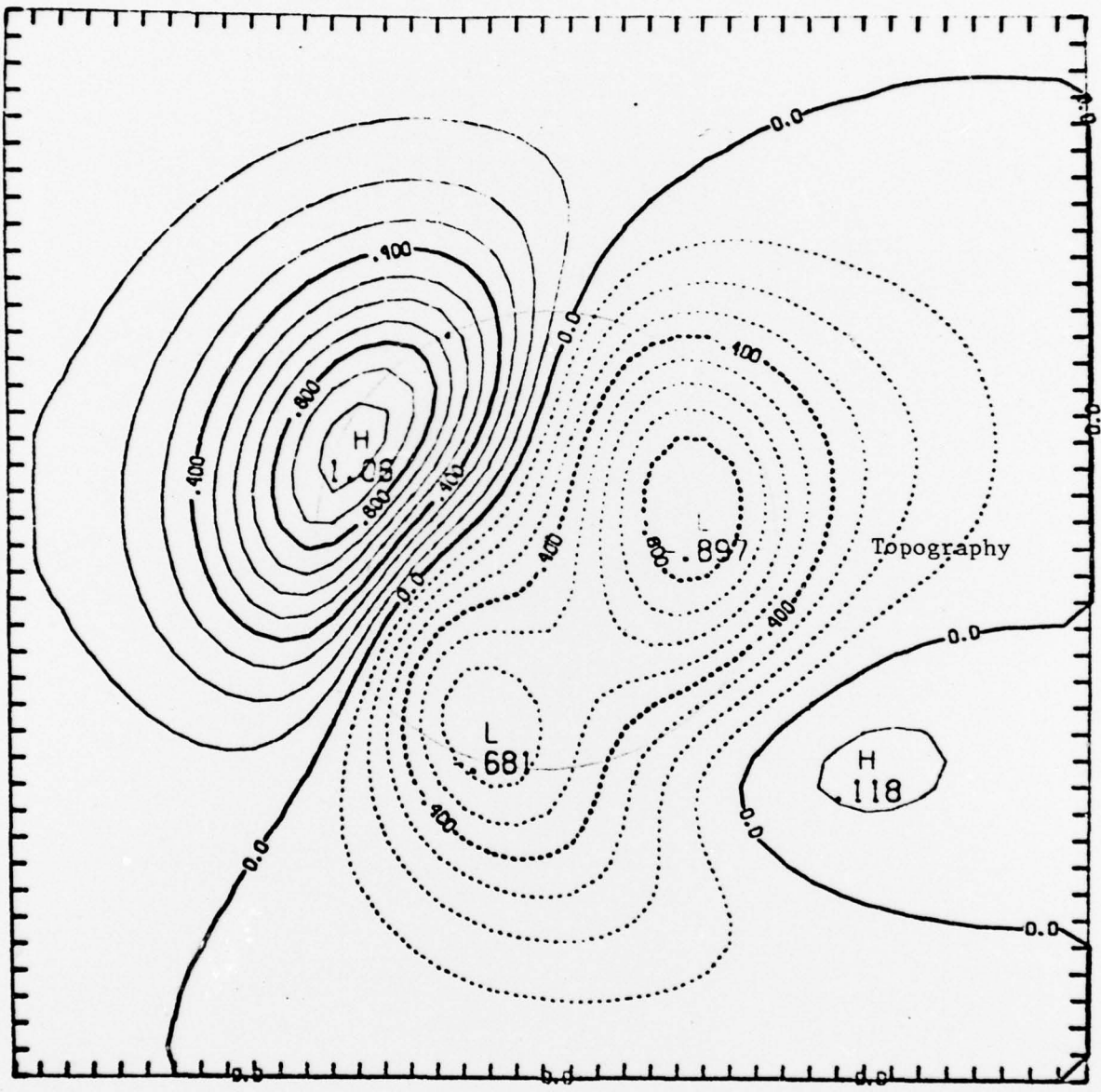


Figure 5B Vertical velocity field; same parameters as 4A except $z = 0.25$.

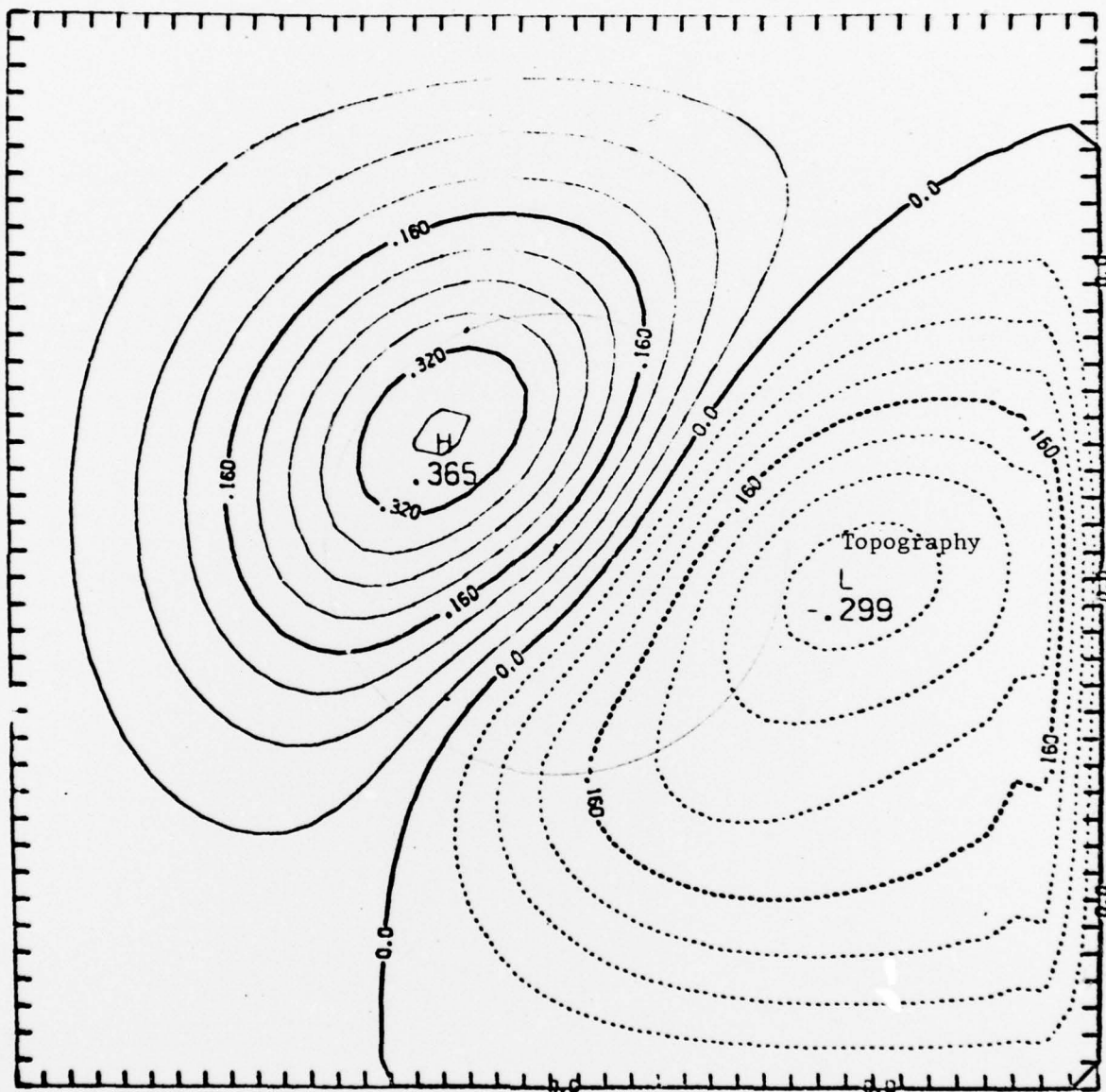


Figure 5C Vertical velocity field; same parameters as 4A except
 $z = 0.50$.

4. Laboratory Experiments

A series of laboratory experiments was initiated to establish a stratified flow in a rotating system which could be utilized to test the theory advanced above. To date a system has been developed which is capable of providing a uniform flow of a two-layer fluid system in a rotating frame.

Figure 6 is a schematic diagram of the system. The basic system consists of simultaneously moving a lighter fluid (Tank A) and a heavier fluid (Tank B-salt stratified) from a non-rotating frame through a test section in the rotating frame and hence to a disposal tank in the non-rotating frame. Fluid A, for example starts in Supply Tank A and thence is piped by a pump through a block valve, a throttling valve and a flow meter, into a reservoir mounted on the rotating table. The fluid then flows under gravity through a flow meter, a block valve and a throttling valve into the test section of the rotating tunnel. The fluid is then removed from the tunnel to a disposal tank located on the rotating table.

The above system is now undergoing a final series of tests to establish a uniform flow in the test section. The system has worked well in that the fluids move through the tunnel with a distinct interface. Refinement in the horizontal excursions of the flow from that of a uniform rectilinear free-stream are required before tests with particular obstacles can be conducted.

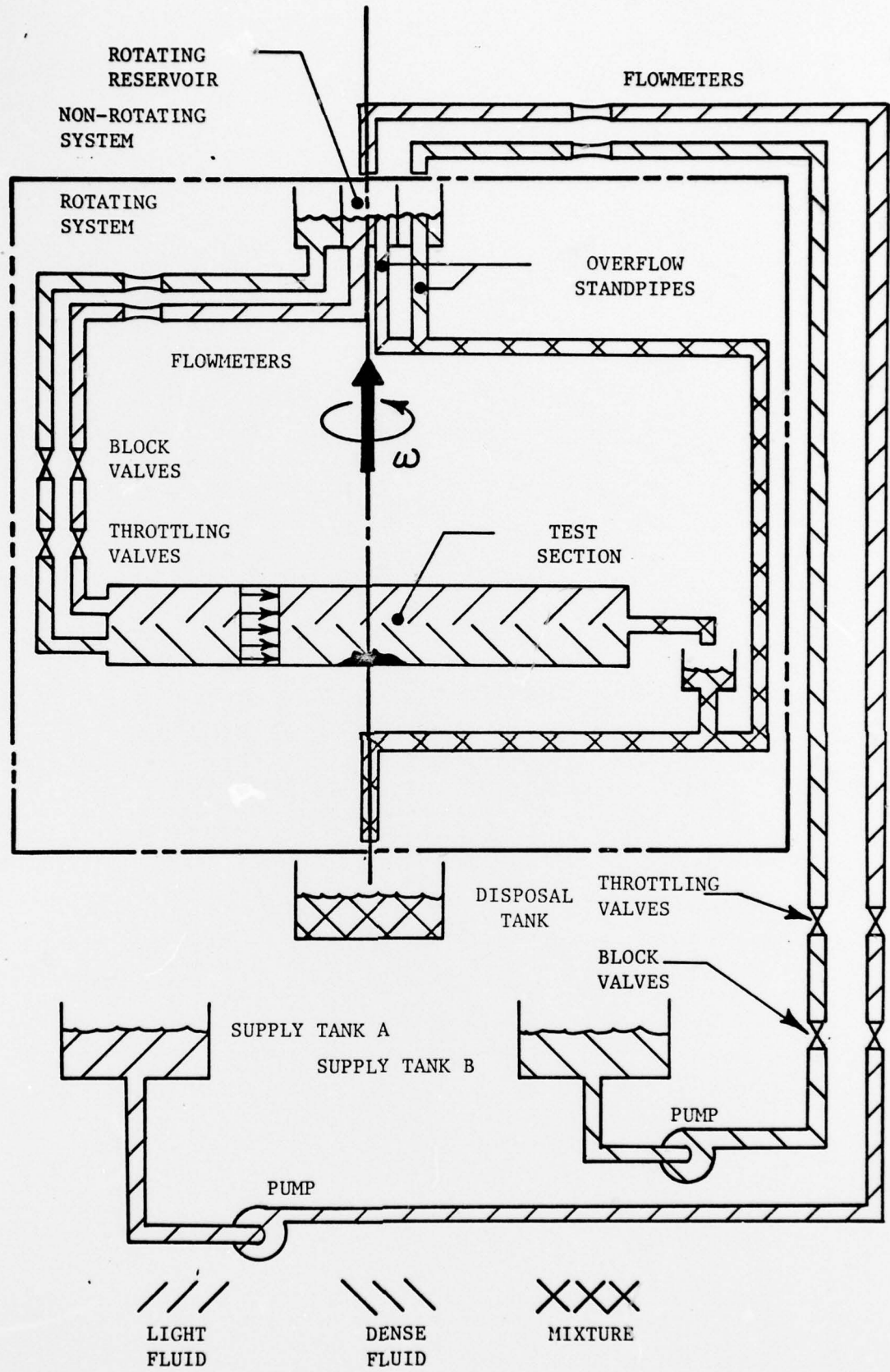


Figure 6. Stratified Rotating Water Tunnel

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