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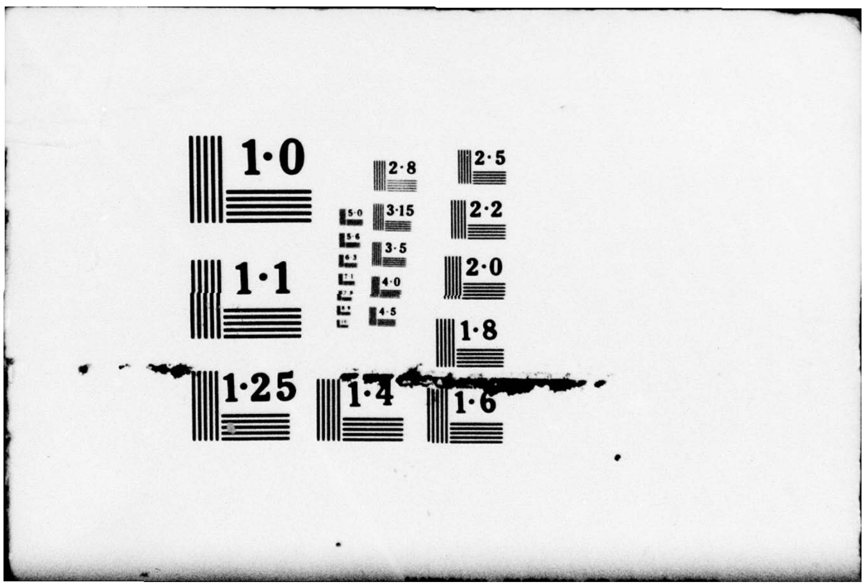
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Depolarization in Laser Probing of Inhomogeneous Magnetized Plasmas

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Plasma Physics Division

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Density gradients and magnetic fields transverse to the propagation direction can produce polarization dependent phase shifts in a laser probing beam used for Faraday rotation studies of longitudinal magnetic fields in plasmas. The resulting depolarization should be considered when probing with fast optics near the critical density.

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CONTENTS

DEPOLARIZATION IN LASER PROBING OF INHOMOGENEOUS MAGNETIZED PLASMAS

INTRODUCTION

Faraday rotation of plane polarized light has recently been used as a probe to measure magnetic fields in laser-produced plasmas.[1] It is well known, however, that optical polarization can also be affected by inhomogeneity and magnetic anisotropy in the plasma. A density gradient normal to the propagation direction introduces a phase difference between the p-polarized and s-polarized components, and the resulting depolarization can be used to estimate the scale length of the gradient near the critical region.[2] Similarly, a magnetic field component normal to the propagation direction will introduce a phase difference between the ordinary and extraordinary components.[3] In this paper, we estimate the magnitude of these effects, and relate them to the changes in the polarization state of the beam. It is shown that depolarization effects can be neglected in Faraday rotation experiments with large f-number optics (i.e., where refraction is weak); however, they may be significant when probing with fast optics near the critical density.

DENSITY GRADIENT EFFECTS

The $\vec{E}$ field wave equation for a probe beam propagating in an inhomogeneous unmagnetized plasma is

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$$\nabla^2 \vec{E} + \nabla (\vec{E} \cdot \epsilon^{-1} \nabla \epsilon) + \epsilon k_o^2 \vec{E} = 0, \tag{1}$$

where $\epsilon = 1 - \omega_p^2/\omega^2$ in the absence of damping, $\omega_p = (4\pi Ne^2/m)^{\frac{1}{2}}$ is the electron plasma frequency, and $k_o = \omega/c$. We are primarily interested in a macroscopic density gradient (scale length $\zeta \gg 1/k_o$) whose direction remains constant over a region $\gg 1/k_o$. Then for locally s-polarized light $\vec{E} \cdot \nabla \epsilon = 0$, the dispersion relation $k_s^2 = k_o^2 - \omega_p^2/c^2$ is the same as that of a homogeneous plasma. For locally p-polarized light, $\vec{E}$ is parallel to $\nabla \epsilon$, and $\nabla(\vec{E} \cdot \epsilon^{-1} \nabla \epsilon) \approx -\vec{E}/\zeta^2$. The corresponding dispersion relation is then approximately $k_p^2 \approx k_s^2 - 1/\zeta^2$; hence, $k_p \approx (k_s^2 - 1/\zeta^2)^{\frac{1}{2}} \approx k_s - 1/2k_s\zeta^2$. The relative phase retardation of the p-polarization component is, therefore, $\delta\phi_{sp} \approx c\Delta\ell/2n\omega\zeta^2$, where $\Delta\ell$ is the propagation path length and $n = \epsilon^{\frac{1}{2}}$ is the local refractive index. In the following paragraphs, we will derive a more rigorous expression [Eq. (15)] that also leads to this approximate result.

One can rewrite Eq. (1) and the corresponding $\vec{B}$ field wave equation

$$\nabla^2 \vec{B} + \epsilon^{-1} \nabla \epsilon \times \nabla \times \vec{B} + \epsilon k_o^2 \vec{B} = 0 \tag{2}$$

in terms of the complex field envelopes $\vec{\mathcal{E}}$ and $\vec{\mathcal{B}}$ defined by

$$\vec{E} = \vec{\mathcal{E}} \exp(ik_o \psi), \quad \vec{B} = \vec{\mathcal{B}} \exp(ik_o \psi), \tag{3}$$

and the eikonal equation (Ref. 4, Sec. 3.1 and Ref. 5)

$$|\nabla \psi|^2 = \epsilon(\vec{r}) = n^2(\vec{r}). \tag{4}$$

The eikonal $\psi(\vec{r})$ specifies the ray paths in the geometrical optics limit $1/k_o \to 0$. If $d\ell(\vec{r})$ is an element of the ray path whose local direction is $\hat{\ell} = n^{-1}\nabla\psi$, then $d\psi/d\ell = n$ and

$$\nabla\psi\cdot\nabla = n\partial/\partial\ell. \tag{5}$$

substituting these results into Eqs. (1) and (2), we obtain (Ref. 4, Sec. 3.1)

$$n\partial\vec{\mathcal{E}}/\partial\ell + \tfrac{1}{2}(\nabla^2\psi)\vec{\mathcal{E}} + (\vec{\mathcal{E}}\cdot\nabla\ell n\, n)\nabla\psi = -(1/ik_o)[\tfrac{1}{2}\nabla^2\vec{\mathcal{E}} + \nabla(\vec{\mathcal{E}}\cdot\nabla\ell n\, n)], \tag{6}$$

$$n\partial\vec{\mathcal{B}}/\partial\ell - n\vec{\mathcal{B}}\partial\ell n\, n/\partial\ell + \tfrac{1}{2}(\nabla^2\psi)\vec{\mathcal{B}} + (\vec{\mathcal{B}}\cdot\nabla\ell n\, n)\nabla\psi$$

$$= -(1/ik_o)[\tfrac{1}{2}\nabla^2\vec{\mathcal{B}} + (\nabla\ell n\, n)\times\nabla\times\vec{\mathcal{B}}]. \tag{7}$$

The first two terms on the left hand side of (7) can be combined, and the second term on the right hand side can be evaluated by using a standard vector identity. The result is

$$n\partial(n^{-1}\vec{\mathcal{B}})/\partial\ell + \tfrac{1}{2}(\nabla^2\psi)n^{-1}\vec{\mathcal{B}} + (n^{-1}\vec{\mathcal{B}}\cdot\nabla\ell n\, n)\nabla\psi$$

$$= -(1/ik_o)[\tfrac{1}{2}\nabla^2(n^{-1}\vec{\mathcal{B}}) + \nabla(n^{-1}\vec{\mathcal{B}}\cdot\nabla\ell n\, n) \tag{8}$$

$$- \tfrac{1}{2}n^{-1}\vec{\mathcal{B}}n\nabla^2 n^{-1} + (n^{-1}\vec{\mathcal{B}}\cdot n\nabla)\nabla n^{-1}]$$

Excepting for the last two terms, the equation for $n^{-1}\vec{\mathcal{B}}$ thus has the same form as Eq. (6). At large distances from the plasma, the gradient terms become negligible, and the incident and transmitted fields approach the usual asymptotic condition $\vec{\mathcal{E}} \rightarrow n^{-1}\vec{\mathcal{B}} \times \hat{\ell}$.

For propagation through a spherical plasma of radius $\gg \lambda$, it is reasonable to make an approximation of local cylindrical symmetry as long as the rays do not pass too close to the center. The local cylindrical $\hat{Z}$ axis is taken to be normal to the plane defined by the incident ray $\hat{\ell}$ and the plasma radius vector $\vec{r}$, and the Z dependence of n, $\vec{\mathcal{E}}$ and $\vec{\mathcal{B}}$ can be ignored. For s-polarization $[\vec{\mathcal{E}}_s = \hat{Z}\,\mathcal{E}_Z(x,y)]$, Eq. (6) reduces to

$$n\partial\mathcal{E}_Z/\partial\ell + \tfrac{1}{2}(\nabla^2\psi)\,\mathcal{E}_Z = (i/2k_o)\nabla^2\mathcal{E}_Z. \tag{9}$$

In the case of p-polarization $[\vec{\mathcal{E}}_p = (\mathcal{E}_x, \mathcal{E}_y)]$, Eq. (6) is complicated by the fact that $\vec{\mathcal{E}}_p$ changes direction in order to remain approximately normal to $\hat{\ell}$ as the rays are refracted. In this case, it is more convenient to work with $\vec{\mathcal{B}}_p(x,y)$, which remains oriented along $\hat{Z}$;[5] thus, Eq. (8) becomes

$$n\partial(n^{-1}\mathcal{B}_Z)/\partial\ell + \tfrac{1}{2}(\nabla^2\psi)\,n^{-1}\mathcal{B}_Z + i(n/2k_o)(\nabla^2 n^{-1})n^{-1}\mathcal{B}_Z$$
$$= (i/2k_o)\nabla^2(n^{-1}\mathcal{B}). \tag{10}$$

The $\tfrac{1}{2}\nabla^2\psi$ term of Eqs. (9) and (10) accounts for changes of intensity due to the focusing or defocusing effects in the plasma, while the $\nabla^2\mathcal{E}_Z$ and $\nabla^2(n^{-1}\mathcal{B}_Z)$ terms account for diffraction. The gradientinduced phase shift term $\nabla^2 n^{-1}$ appears only for p-polarization. It is useful to rewrite Eqs. (9) and (10) in terms of the functions $\alpha_s(x,y)$ and $\alpha_p(x,y)$ defined by

$$\mathcal{E}_Z = \mathcal{E}_{Zo}\exp[\alpha_s(x,y)], \quad n^{-1}\mathcal{B}_Z = \mathcal{B}_{Zo}\exp[\alpha_p(x,y)] \tag{11}$$

where $\mathcal{E}_{Zo}$ and $\mathcal{B}_{Zo}$ are constants; i.e.,

$$\begin{aligned} &\partial\alpha_s/\partial\ell + (1/2n)\nabla^2\psi \\ &= (i/2k_o n)(\nabla^2\alpha_s + |\nabla\alpha_s|^2) \end{aligned} \tag{12}$$

$$\begin{aligned} &\partial\alpha_p/\partial\ell + (1/2n)\nabla^2\psi + (i/2k_o)\nabla^2 n^{-1} \\ &= (i/2k_o n)(\nabla^2\alpha_p + |\nabla\alpha_p|^2) \end{aligned} \tag{13}$$

In general, Eqs. (19) and (10) or (12) and (13) can be solved by the use of a perturbation series in powers of i/k_o.[5] If $k_o\ell \gg 1$, and the rays passing through the region of interest remain sufficiently far from the critical region, then only the lowest order contribution due to diffraction is significant. To this approximation, the right hand sides of Eqs. (12) and (13) are thus identical, with $\alpha_p \to \alpha_s \to \alpha_o$, where

$$\alpha_o(x,y) = -\int_{-\infty}^{\ell(x,y)} \frac{\nabla^2\psi}{2n}\, d\ell \tag{14}$$

is the geometrical optics result (Ref. 4, Sec. 3.1). The relative phase retardation of the p-polarization component is therefore [subtracting Eq. (13) from Eq. (12)].

$$\delta\phi_{sp} = \mathrm{Im}(\alpha_s - \alpha_p) = \frac{c}{2\omega} \int_{-\infty}^{\infty} \nabla^2 n^{-1} d\ell, \tag{15}$$

which leads to the approximate result $\delta\phi_{sp} \approx c\Delta\ell/2n\omega\zeta^2$ suggested previously.

For a numerical estimate, consider a weakly refracted probe beam traversing a laser-produced plasma from a plane target. Typically, $n \leq 1$, $\Delta\ell \approx 100\ \mu$ and $\zeta > 10\ \mu$ in the underdense region; hence, $\delta\phi_{sp} < 2.4^{\circ}$ for a probe wavelength of $0.53\ \mu$. The magnitude of $\delta\phi_{sp}$ will therefore be significant only for those strongly refracted rays near the critical region where the density gradient can become large and the refractive index is small.

The limitations on the validity of our results can be estimated, in terms of the f-number of the probe beam collecting lens, by considering refraction in a one dimensional density gradient.[5] If β is the angle between the average ∇N and the incident beam (Fig. 1), then the refractive index at the turning point ($\hat{\ell}\cdot\nabla N = 0$) is $n_T = \sin\beta$. The density at this point is then $N_T/N_c = 1 - n_T^2 = \cos^2\beta$, where $N_c = m\omega^2/4\pi e^2$ is the critical density at the probe beam wavelength λ. In terms of the critical density N_{co} at the wavelength λ_o of the main beam, $N_T/N_{co} = (\lambda_o/\lambda)^2\cos^2\beta$. Scattered light measurements at $1.06\ \mu$ indicate that high absorption and steep density gradients ($\sim 1\ \mu$) occur only for $N \geq 0.25\ N_{co}$.[6] In order to avoid this region, one therefore requires $\cos\beta < \lambda/2\lambda_o$ and $n_T \geq (1 - \lambda^2/4\lambda_o^2)^{\frac{1}{2}}$. Now consider a collecting lens placed along the forward axis of the probe beam beyond the target. The smallest allowable f-number is then given by

$$f = \tfrac{1}{2} \cot \phi \tag{16a}$$

where

$$\phi = \pi - 2\beta = 2\sin^{-1} (\lambda/2\lambda_o) \tag{16b}$$

is the maximum scattering angle. For a second harmonic probe beam $f = 0.90$, and for the fourth harmonic $f = 2.0$.

MAGNETIC ANISOTROPY EFFECTS

The polarization of a probe beam propagating along a magnetic field $\vec{B}_o = \hat{\ell} B_o$ in a plasma will be Faraday-rotated by an amount $\Delta\theta_F = \frac{1}{2}\Delta\phi_c$, where[3]

$$\Delta\phi_c = \frac{1}{\omega^2 c} \int_{-\infty}^{\infty} \omega_p^2 \omega_e d\ell \tag{17}$$

is the phase shift between right- and left handed circular polarization components, and $\omega_e = eB_o/mc$ is the electron cyclotron frequency. For the megagauss fields measured in laser-produced plasmas, $\Delta\theta_F \approx 20^\circ$ typically[1]; hence, $\Delta\phi_c \approx 40^\circ$.

The effect is more complicated if $\vec{B}_o$ has components normal to $\hat{\ell}$. If $\vec{B}_o$ lies along the polarization direction (ordinary case), then ϵ has the usual value $1 - \omega_p^2/\omega^2$; however, if $\vec{B}_o$ lies along the optical $\vec{B}(t)$ field (extraordinary case) then[3]

$$\epsilon_{ext} \simeq 1 - \frac{\omega_p^2}{\omega^2} \frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_p^2 - \omega_e^2} . \tag{18}$$

In the case of interest here where $\omega_e \ll \omega$ and ϵ is not too close to zero, the refractive indices of the ordinary and extraordinary waves are related by

$$n - n_{ext} \simeq \omega_p^2 \omega_e^2 / 2n^3 \omega^4, \tag{19}$$

giving the relative phase retardation

$$\delta\phi_{oe} \simeq \frac{1}{2\omega^3 c} \int_{-\infty}^{\infty} \frac{\omega_p^2 \omega_e^2}{n^3} \, d\ell \tag{20}$$

Using expressions (17) and (20), one can estimate $\delta\phi_{oe}$ in comparison to $\delta\phi_c$ if $\vec{B}_o$ has components of comparable magnitude along $\hat{\ell}$ and along $\vec{B}(t)$

$$|\delta\phi_{oe}/\delta\phi_c| \approx \omega_e / 2n^3 \omega \ll 1. \tag{21}$$

Near the critical region, $n \approx 0$ and the inequality no longer applies.

DISCUSSION

In the general case where the incident beam is plane polarized at some angle θ_o to the direction of the retarded component, the transmitted beam will be elliptically polarized with its major axis at angle θ and the ratio minor axis/major axis = $\tan \chi$ (Ref. 4, Sec. 1.4)

$$\tan 2\theta = \tan 2\theta_o \cos \delta\phi \quad ,$$
$$\sin 2\chi = \sin 2\theta_o \sin \delta\phi \quad , \tag{22}$$

and $\delta\phi$ is given by expressions (15) or (20). The polarizability is then

$$P = \frac{I_{MAX} - I_{MIN}}{I_{MAX} + I_{MIN}} = (1 - \sin^2 2\theta_o \sin^2 \delta\phi)^{\frac{1}{2}} \tag{23}$$

and, as one would expect, the depolarization is largest when $\theta_o = 45^o$. It is clear from the preceding estimates that even for $\theta_o = 45^o$, the depolarization will remain small for large f-number optics where the rays remain far from the critical region.

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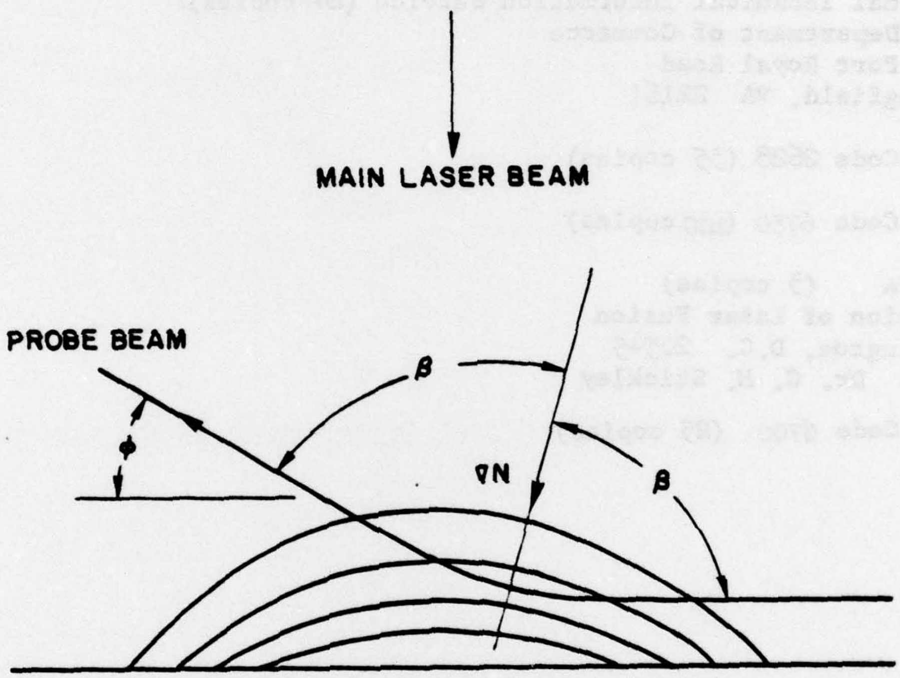

Fig. 1 - Refraction of an optical probing beam by a laser produced plasma

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