REDUCTION OF OFFSHORE PLATFORM DYNAMIC RESPONSE BY TUNED MASS DAMPER.

by

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ABSTRACT

An investigation of a tuned mass damper for reduction of offshore oil platform motions is conducted using finite element tower models disturbed by discretized wave spectra.

Vibration control principles and their application to offshore oil platforms are discussed.

Changes in system response are examined as damper parameters are varied. Response reduction and damper mass motion are found to be coequal design considerations.

An assessment is made of a tuned mass damper's effectiveness in reducing the effects of increasing the natural period of offshore platforms, fatigue in steel-jacketed platforms, and soil-degradation under gravity platforms.

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LIST OF SYMBOLS

A	Amplitude of free modal response
$\underline{B}$	Vector relating nodal displacements to element strains
c_1	Damping coefficient of main system
c_2	Damping coefficient of absorber
c_c	Critical damping
c_D	Damping coefficient of damper in structural model
C_I	Inertial coefficient
$\underline{C}_1$	Viscous damping matrix
$\underline{C}_2$	Hysteretic damping matrix
d	Depth of beam element
D	Cylinder diameter
D_f	Damage fraction
$E[y_1^2]$	Expected value of the relative motion of main mass with vibration absorber attached
$E[\eta^2]$	Mean square of wave amplitudes
f	Cyclic frequency (Hz)
f_m	Frequency at which maximum occurs in wave spectrum
g	Forced frequency ratio (ω/Ω_n)
G	9.80 m/sec^2
h	Water depth from ocean floor to still water surface
H	Wave height from tip to trough

H_{max}	Maximum wave height
H_s	Significant wave height
K	Wave number ($K = 2\pi/\lambda$)
k_1	Spring constant of main system
k_2	Spring constant of absorber
k_D	Stiffness of damper in structural model
k_L	Stiffness of translational soil springs
k_θ	Stiffness of rotational soil springs
$\underline{K}$	Stiffness matrix
$\underline{K}_e$	Equivalent stiffness matrix
L	Length of beam element
m_1	Mass of main system
m_2	Mass of absorber
m_D	Mass of damper in structural model
M	Number of DOF in structural model
$\underline{M}$	Mass matrix
$\underline{M}_e$	Equivalent mass matrix
N	Number of frequencies in discretized spectrum
N_f	Number of cycles of loading to failure
p	Ratio of damping to critical damping (c/c_c)
P	Force
$\underline{P}$	Vector of forces

$P(\omega_i)$	Force vector at frequency ω_i
r	Natural frequenty ratio (ω_a/Ω_n)
S	Stress
S_o	White noise acceleration
$S_{\eta\eta}(\omega)$	Wave amplitude spectrum
T_f	Time to failure
T_1, T_2	First two natural periods of a structure
$\dot{u}$	Fluid partical acceleration
U	Windspeed reported by Weatherships
$\underline{u}$	Vector of modal amplitudes
$\ddot{\underline{u}}$	Vector of modal accelerations
x_o	Base displacement
x_1	Displacement of mass of main system in 1 or 2-DOF model
x_2	Displacement of mass of absorber in 2-DOF model
x_D	Displacement of damper mass in structural model
x_i	Displacement of DOFi in structural model
$\dot{x}_1$	Velocity of mass of main system in 1 or 2-DOF model
$\dot{x}_2$	Velocity of mass of absorber in 2-DOF model
$\ddot{x}_1$	Acceleration of mass of main system in 1 or 2-DOF model
$\ddot{x}_2$	Acceleration of mass of absorber in 2-DOF model
X_{st}	Static deflection of main system
X_1	Steady state amplitude of main system dynamic response
X_2	Steady state amplitude of absorber dynamic response

$\underline{x}$	Vector of displacements
$\dot{\underline{x}}$	Vector of velocities
$\ddot{\underline{x}}$	Vector of accelerations
$\underline{x}(\omega_i)$	Displacement vector at frequency ω_i
$\ddot{\underline{x}}(\omega_i)$	Acceleration vector at frequency ω_i
$\underline{X}_n$	Vector of modal displacements for nth mode
y_1	Relative displacement of main mass ($y_1 = x_1 - x_o$)
z	Distance from ocean floor to any elevation on a structure
α	$4\pi[\frac{\lambda_1}{T_2} - \frac{\lambda_2}{T_1}]/[\frac{T_1}{T_2} - \frac{T_2}{T_1}]$
α_s	8.10×10^{-3}
β	0.74
β_a	Percent critical damping in mass damper
β_H	Percent critical hysteretic damping
β_n	Percent critical damping in structural model
β_v	$\frac{1}{\pi}[\lambda_2 T_1 - T_2\lambda_1]/[\frac{T_1}{T_2} - \frac{T_2}{T_1}]$
Δ	Increment of frequency
ε	Element strain
$\varepsilon(\omega_i)$	Element strain at frequency ω_i
η	Wave amplitude
$\eta(\omega_i)$	Discretized wave amplitude at frequency ω_i
θ_i	Rotation of DOFi in structural model
λ	Wavelength
λ_1, λ_2	Percent critical viscous damping assumed in the first two structural modes

μ	Mass ratio (m_2/m_1)
ρ	Density of fluid
σ_x^2	Mean square displacement
$\sigma_{\ddot{x}}^2$	Mean square acceleration
σ_y^2	Expected value of the motion of the main mass without the vibration absorber
σ_ε^2	Mean square element strain
σ_η^2	Variance of wave amplitudes
$\frac{\sigma_D}{\sigma}$	Deck displacement ratio
$\frac{\sigma_{damper}}{\sigma}$	Ratio of RMS relative damper displacement to the RMS deck displacement without the mass damper
Φ_I	Inertial force per unit of cylinder length
Φ_T	Total force per unit of cylinder length
$\Phi_T(\omega_i)$	Total force per unit of cylinder length at frequency ω_i
$\underline{\Phi}$	Modal matrix
ω	Circular frequency (rad/sec)
ω_a	Natural frequency of absorber
ω_o	G/U
Ω_1, Ω_n	Natural frequency of main system/Fundamental frequency of the structural model
Ω_2	Second natural frequency of structural model

CHAPTER 1

INTRODUCTION

1.1 General Background

Because the search for oil is becoming more intense, as the need for petroleum products grows, the oil industry is exploring offshore areas where the environment is hostile. To overcome the elements and still produce oil, the industry has adopted new platform concepts. In the North Sea, large concrete structures called gravity platforms are in use. These platforms rest on the ocean floor in 550 feet of water relying only on their own weight to keep them stable in one of the roughest ocean environments in the world. In the Gulf of California and the Gulf of Mexico, steel jacketed platforms are being constructed to bring oil from deep water. Exxon's platform is placed in 850 feet of water off the Coast of California and Shell's platform will operate in 1000 feet of water in the Gulf of Mexico.

As is usual when new concepts are employed in extreme conditions, problems have developed which demand consideration. A survey of industry people involved in the implementation of these new concepts and a review of current literature addressing these concepts reveals three problems of major concern to the industry with regard to offshore platforms.

The first problem is designing a platform for use in water deeper than 1000 feet and still having the design remain economical. Industry sources reveal that a general rule of thumb developed from experience

limits the fundamental period of offshore platforms to a maximum of five seconds. This results from the fact that the waves with the greatest energy content have frequencies greater than five seconds. Limiting the fundamental period of the structure to five seconds or less minimizes the dynamic effects of the wave loads on the structure in the frequency range of waves which produce the largest forces, thus reducing the ultimate load on the structure. If a platform is designed without reinforcement for use in water of depths greater than 1000 feet, the fundamental period is greater than five seconds resulting in a higher ultimate load on the structure and requiring more structural material adding to the cost. If a platform is designed with reinforcement so that the fundamental period is five seconds or less for water depths greater than 1000 feet, the ultimate load on the structure is reduced, but the cost of the reinforcing material is high.

The second problem is combating the low cycle - high stress fatique of deep water steel jacketed structures in severe ocean environments such as the North Sea. Nearly all industry sources surveyed agreed that dynamically amplified fatigue becomes a major consideration in design as steel jacketed platforms [with fundamental periods greater than four seconds] move into deeper water in areas such as the North Sea. Here, conditions are such that normal day-to-day waves with frequencies near the fundamental frequency of the structure contain sufficient energy to reduce significantly the life expectancy of the structure through fatigue failure. Kallaby

and Price [1] calculated the cost of overcoming fatigue effects in these structures and found that the fatigue premium was 4.5% of the cost of the structure.

The third problem concerns degradation of the soil under gravity platforms due to cyclic loading during storm conditions. As large waves strike the gravity structure, forces are transmitted to the soil in the form of stress increases or decreases between the soil and the structure. The nature of wave loading causes these stress changes to be cyclic. Increased stresses in the soil cause increases in the pore-water pressure of the soil and cyclic stress reversals cause the pore-water pressure to increase with time. As the pore pressure increases, the soil becomes weaker and finally loses all strength through liquification. Calculations on soil degradation resulting from this type of loading have been done by Bjerrum [2], Anderson [3] and Hoeg [4]. They show that over a period of storms it is possible for the strength of soil beneath a gravity platform such as those in operation in the North Sea to be reduced significantly. Tayloe [5], Watt et al [6] and Utt et al [7] showed that as the soil strength degraded in the fashion described above, the weakened foundation caused the fundamental period of the structure to increase into the range where dynamic amplification of the peak load would occur, thereby increasing the ultimate load on the structure perhaps beyond the design load. The offshore industry has no field data available at this time to assess the seriousness of this problem but steps are being taken to secure an answer in the next few years.

It has been suggested that an effective and economic means of overcoming some of the problems described above is the use of a mass damper on structures used in deep water. The mass damper is an energy absorbing system consisting of a mass-spring-dashpot apparatus placed near the deck of deep water platforms which, when properly tuned, reduces the dynamic response of the platform. The mass damper is not a new concept. In its earliest and simplest form it was known as a vibration absorber and was used in industry applications to reduce vibrations [8]. Recently, however, the mass damper has been installed in two tall buidlings to reduce their response to wind inputs. The extension of the mass damper concept to offshore platforms and the effectiveness of such a system in overcoming the problems cited above is the subject of this thesis.

1.2 Scope of Thesis

To examine the potential use of a mass damper on an offshore platform, this thesis addresses two questions: how effective is the system when used on a multi-degree of freedom offshore tower subject to frequency dependent sea states and how does system response change as damper parameters are varied?

To pursue these answers, offshore platforms are modelled with a finite element computer program developed by DuVall [9] which computes the frequency domain response of an offshore platform to designated sea states. The platform models are of two types: steel-jacketed structures with fundamental periods ranging from

5-7 seconds and a concrete gravity platform of the Condeep type with a fundamental period of about three seconds. The various sea states used as environmental input are modeled by the Pierson-Moskowitz wave amplitude spectrum which is developed in Chapter 2. DuVall's program has been modified to include the mass damper, structural damping, and soil effects.

The vibration absorber and mass damper are explained and their mathematics discussed.

An illustrative analysis is presented showing structural responses with and without the mass damper in operation. Next, a modal analysis of this same example is made to determine participation of each mode in the total structural response.

Finally the multi-modal model platform is analyzed for varying sea states to determine damper effectiveness and the variation of system response with damper parameters. Results are presented and discussed and an evaluation of overall effectiveness and feasibility is made.

CHAPTER 2

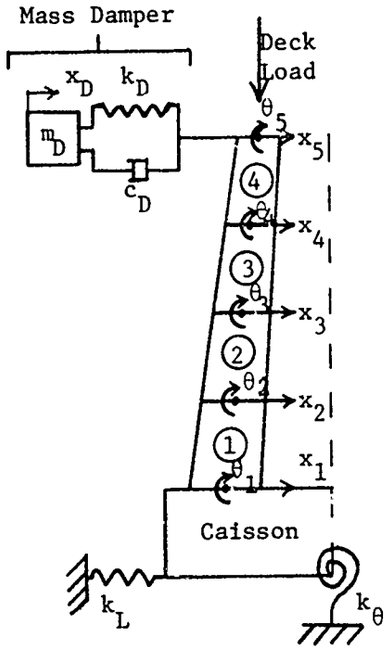
THEORETICAL FORMULATIONS

2.1 The Structural Model

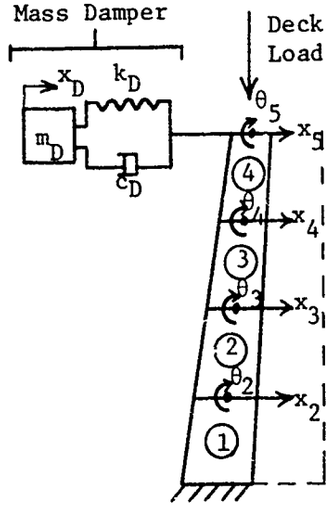
The offshore platforms analyzed in this thesis are modelled using a finite element computer program developed by DuVall [9]. This program assembles a model of an offshore platform, either gravity or steel-jacketed, based on platform dimensions input by the user. Both types of structures are modelled (see Figure 2.1) as 4 two-dimensional axisymetric beam elements supporting a load representing the deck.

The gravity structure model supports the beam elements with a caisson which rests on a flexible soil base represented by linear springs. This model has ten degrees of freedom when the mass damper is not in operation. These are x_1 through x_5 and Θ_1 through Θ_5 as shown in Figure 2.1. When the mass damper is in operation, an additional degree of freedom, x_D, is added bringing the total to eleven.

The steel-jacketed model fixes the beam elements to the ocean floor in a cantilever fashion representative of piled structures. This model has eight degrees of freedom when the mass damper is not in operation. Degrees of freedom x_1 and Θ_1 are set to zero as boundary conditions representing a fixed base and the remaining degrees of freedom are x_2 through x_5 and Θ_2 through Θ_5. An additional degree of freedom, x_D, is added when the mass damper is in operation.



Gravity Structure Model



Steel-Jacketed Structure Model

Figure 2.1

The models are loaded in their plane using a discretized wave height spectrum (see Section 2.2.1) and the model response is given as discretized frequency spectrums of horizontal displacements and accelerations, and in-plane rotations of the beam element nodes. It has been shown by Nath [10] that platform response is independent of the direction of loading so that a two-dimensional model is valid for analysis.

Three models are used in the subsequent analysis: the 1000 foot steel-jacketed platform currently being constructed by Shell Oil Corporation, a 1200 foot steel-jacketed platform of theoretical design, and the Condeep Brent B gravity platform in operation in the North Sea. Table 2.1 lists the finite element model parameters for each model.

DuVall's program has been modified to include damping effects and a mass damper in the analysis, and the output of a discretized frequency spectrum of element strains. A detailed description of the unmodified program can be found in [9]. Appendix A contains a listing of the modified program.

Two types of damping, viscous and hysteretic, are incorporated into the program. Viscous damping accounts for fluid-structure interaction and slippage and rubbing in structural joints. It is assumed to be linearly proportional to the structure's velocity and is represented by the matrix $\underline{C}_1$ in the equation of motion:

$$\underline{M}\,\ddot{\underline{x}} + \underline{C}_1\dot{\underline{x}} + \underline{Kx} = \underline{P} \qquad (2.1)$$

	1000 Ft. Steel-Jacket	1200 Ft. Steel-Jacket	Condeep Gravity
Height	366 m	427 m	173 m
Water Depth	305 m	366 m	145 m
External Radius			
Base	33.75 m	37.875 m	17.3 m
Platform	9.0 m	9.0 m	4.9 m
Internal Radius			
Base	33.6 m	37.717 m	16.0 m
Platform	8.9 m	8.9 m	4.75 m
Deck Mass	4180.0 MT-sec^2/m	7500.0 MT-sec^2/m	1580.0 MT-sec^2/m
Deck Inertia	2432760 MT-sec^2/m	2432760 MT-sec^2/m	500187 MT-sec^2/m
Total Mass	11213.0 MT-sec^2/m	22304.0 MT-sec^2/m	17141.0 MT-sec^2/m
Modulus of Elasticity	2.039 x 10^7 MT/m^2	2.039 x 10^7 MT/m^2	2.9 x 10^6 MT/m^2
Caisson Height	-	-	60.0 m
Caisson Radius	-	-	39.7 m
Caisson Mass	-	-	15330.0 MT-sec^2/m^2
Soil Density	-	-	400.0 MT-sec^2/m^4
Shear Modulus	-	-	5000.0 MT/m^2
ν Soil	-	-	.5

Table 2.1

Model Finite Element Parameters

where

$\underline{M}$ = mass matrix

$\underline{K}$ = stiffness matrix

$\underline{P}$ = vector of forces

$\ddot{\underline{x}}$ = vector of accelerations

$\dot{\underline{x}}$ = vector of velocities

$\underline{x}$ = vector of displacements

Determining the proper viscous damping for multi-degree of freedom systems is difficult because the nature of damping is not well described. In terms of the idealized model there is no "exact" way to assign values to the elements of $\underline{C}_1$. One method, described by Maddox [11], and used in this analysis, assumes Rayleigh type damping which corresponds to

$$\underline{C}_1 = \alpha\underline{M} + \beta_v \underline{K} \tag{2.2}$$

where α and β_v are scalars defined by:

$$\alpha = 4\pi \left[\frac{\lambda_1}{T_2} - \frac{\lambda_2}{T_1}\right] / \left[\frac{T_1}{T_2} - \frac{T_2}{T_1}\right]$$

$$\beta_v = \frac{1}{\pi} \left[\lambda_2 T_1 - T_2 \lambda_1\right] / \left[\frac{T_1}{T_2} - \frac{T_2}{T_1}\right] \tag{2.3}$$

and

λ_1, λ_2 = percent critical damping assumed in the first two structural modes

T_1, T_2 = first two natural periods of the structure.

With only viscous damping considered the equation of motion becomes:

$$\underline{M}\,\ddot{\underline{x}} + (\alpha\underline{M} + \beta_v\,\underline{K})\dot{\underline{x}} + \underline{K}\,\underline{x} = \underline{P} \tag{2.4}$$

Hysteretic damping is due to internal friction within the material itself and is proportional to the stiffness and the deflection of the structure. It is modelled as:

$$\underline{C}_2 = i2\beta_H\,\underline{K} \tag{2.5}$$

where

$i = \sqrt{-1}$

β_H = percent critical hysteretic damping

Including hysteretic damping, the equation of motion becomes

$$\underline{M}\,\ddot{\underline{x}} + (1 + i2\beta_H)\underline{K}\,\underline{x} = \underline{P} \tag{2.6}$$

When both viscous and hysteretic damping effects are considered the equation of motion for the structure expands to

$$\underline{M}\,\ddot{\underline{x}} + (\alpha\underline{M} + \beta_v\underline{K})\dot{\underline{x}} + (1 + i2\beta_H)\underline{K}\,\underline{x} = \underline{P} \tag{2.7}$$

There is general agreement on the representation of damping in a structure but no agreement has been reached concerning values for λ_1, λ_2, and β_H. A survey of several authors indicates that overall damping in offshore structures can vary from 1% of critical to 5% of critical damping depending on the author and type of structure. In general, most values ranged around 3%. Table 2.2 shows values of percent critical damping for different materials estimated by Zijp et al [12]. Note that a steel structure in water combines a 1% critical damping value for steel and a 1.5%

Material	Damping (% of Critical)
Soil, Translation	9
Soil, Rocking	4
Steel	1
Concrete	2
Hydrodynamic	1.5

Table 2.2

Damping Values for Various Materials [11]

Percent Critical Damping A	Standard Deviation of Response at A% Critical Damping / Standard Deviation of Response at 3% Critical Damping
1	1.7
3	1.0
5	0.8

Table 2.3

Variation of Computed Response with Damping

critical damping value for hydrodynamic effects to display 2.5% critical damping overall. Similarly, a concrete structure in water displays 3.5% critical damping overall. Again the values range around 3%.

A concern in choosing a value of overall damping is the sensitivity of the structural response to that value. To check the sensitivity of the model to changes in overall damping, the response of the 1000 foot model to wave excitation was calculated for different values of overall damping. Results of these computations, shown in Table 2.3, indicate that the response is very sensitive to damping. Table 2.3 shows that as damping is decreased from 3% to 1% of critical, the response standard deviation increases by a factor of 1.7. As th. damping is increased to 5% of critical, the response decreases to 0.8 of the 3% response. Therefore as damping is increased, the rate of change of the response is reduced, but in the range of damping found in offshore structures, the change in response can be significant for small changes in structural damping.

Because the value of damping in any structure can only be estimated and the response is dependent on the value chosen, a difficult situation arises. This analysis uses the damping value most often found in research, 3% critical damping for all cases, with viscous and hysteretic damping each accounting for one-half the damping. Therefore, $\lambda_1 = \lambda_2 = \beta_H = 1.5\%$ of critical damping. Table 2.4 shows the values of the damping parameters α, β_v and β_H

		1000 ft Steel-Jacket	1200 ft Steel-Jacket	Condeep Gravity
First Natural Frequency	Ω_1	1.244 rad/sec	.905 rad/sec	2.094 rad/sec
Second Natural Frequency	Ω_2	5.712 rad/sec	4.217 rad/sec	4.189 rad/sec
First Natural Period	$T_1 = \frac{2\pi}{\Omega_1}$	5.05 sec	6.94 sec	3.00 sec
Second Natural Period	$T_2 = \frac{2\pi}{\Omega_2}$	1.10 sec	1.49 sec	1.50 sec
	λ_1	.015	.015	.015
	λ_2	.015	.015	.015
	α	.0302	.0224	.0420
	βv	.0043	.0059	.0048
	β_H	.015	.015	.015

Table 2.4

Natural Frequencies and Damping Parameters for Structural Models

for the three models to be used in this study.

The mass damper is modelled as a simple spring-mass-dashpot system connected to the top node of the structure (see Figure 2.1).

Computation of element strains have been included in the modified program. For the beam element used in this program and displayed in Figure 2.2, the strain at any point in the element equals:

$$\varepsilon = [-x(-\frac{6}{L^2} + \frac{12y}{L^3})]x_1 + [-x(-\frac{4}{L} + \frac{6y}{L^2})]\Theta_1$$

$$+ [-x(\frac{6}{L^2} - \frac{12y}{L^3})]x_2 + [-x(-\frac{2}{L} + \frac{6y}{L^2})]\Theta_2 \qquad (2.8)$$

Midpoint strains are taken as the representative strain measures for the elements. Specializing (2.8) results in

$$\varepsilon = \frac{d}{2L}[\Theta_1 - \Theta_2] \qquad (2.9)$$

2.2 Hydrodynamic Forces

2.2.1 The Wave Amplitude Spectrum

Common models for the surface of the sea assume the water surface can be described as a stationary, ergodic Gaussian, or normal process with zero mean [13].

A stationary process is one for which the statistics of the process, or the probability law, remain constant with respect to time. An ergodic process is one in which any averages taken with respect to a fixed position, with respect to time, are equal to

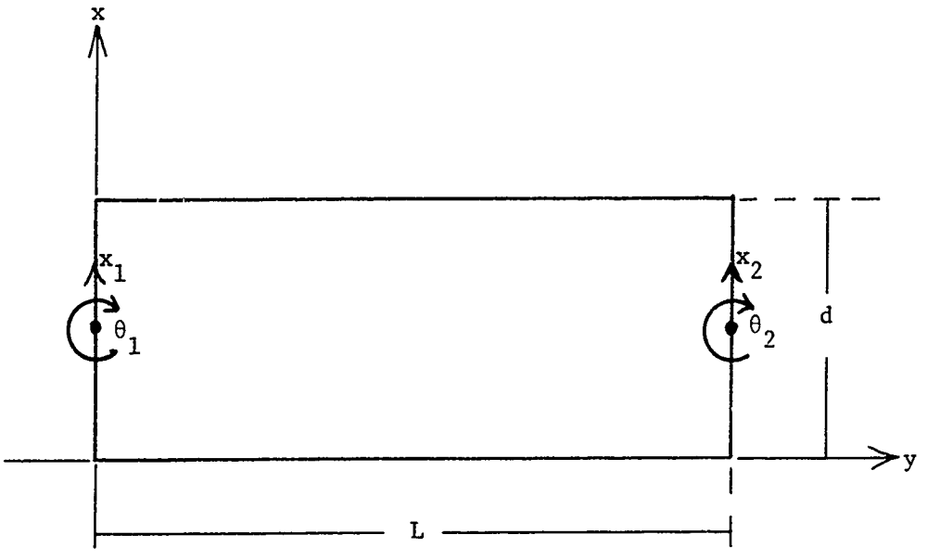


Figure 2.2
BEAM ELEMENT

averages taken at a fixed time over the ensemble, or collection, of all possible realizations of the process.

Because of these properties, the sea at any place and time can be described as a wave amplitude spectrum, $S_{\eta\eta}(\omega)$ which has the property that the variance of the wave amplitudes is equal to the integral of the wave amplitude spectrum over all positive frequencies, or

$$\sigma_\eta^2 = E(\eta^2) = 2\int_0^\infty S_{\eta\eta}(\omega)\,d\omega \tag{2.10}$$

where

$S_{\eta\eta}(\omega)$ = wave amplitude spectrum in m^2/sec

σ_η^2 = variance of wave amplitudes (in spectrum)

$E(\eta^2)$ = mean square of wave amplitudes (in spectrum)

η = wave amplitude associated with random ocean wave

Of the several empirically derived wave amplitude spectra available, the most widely used is the Pierson-Moskowitz spectrum [14] representing a fully developed sea. Its form is

$$S_{\eta\eta}(\omega) = \left(\frac{\alpha_s G^2}{\omega^5}\right) e^{-\beta\left(\frac{\omega_o}{\omega}\right)^4} \tag{2.11}$$

where

$\alpha_s = 8.10 \times 10^{-3}$

$\beta = 0.74$

$G = 9.80\ m/sec^2$

$\omega_o = G/U$

U = windspeed reported by weatherships in m/sec

Any one of four parameters can be used to specify a specific sea state. These are U, f_m, H_s, and H_{max} where

$$f_n = \frac{\omega_o}{2\pi} = \frac{G}{2\pi U}$$

$$H_s = 4\sqrt{\int_0^\infty S\eta\eta(\omega)\,d\omega} = 4.078 \times 10^{-3} \left(\frac{G}{f_m^2}\right)$$

$$H_{max} = 2H_s$$

In this analysis, three specific Pierson-Moskowitz wave amplitude spectrums are used. They correspond to seas generated by 70 mph winds, 40 mph winds and 30 mph winds and are plotted in Figure 2.3. Table 2.5 contains the parameters of these seas. A detailed development of random waves and the wave spectrum can be found in Nath [10].

The computer program of DuVall uses a condensed spectrum represented by a finite number of frequencies. This condensed spectrum is derived by evaluating the area between $\omega-\Delta$ and $\omega+\Delta$ of the P-M spectrum around a specified frequency ω, taking the square root of that area, and assigning that value to the specific frequency. This is the rms wave amplitude in the band $\omega-\Delta$ to $\omega+\Delta$. This is done for all the frequencies specified, forming a histogram of frequencies and equivalent wave amplitudes which represent the P-M spectrum, from $\omega = 0$ to $\omega = .35$ rad/sec. Although the actual spectrum does not equal zero until $\omega = \infty$, the area under the spectrum

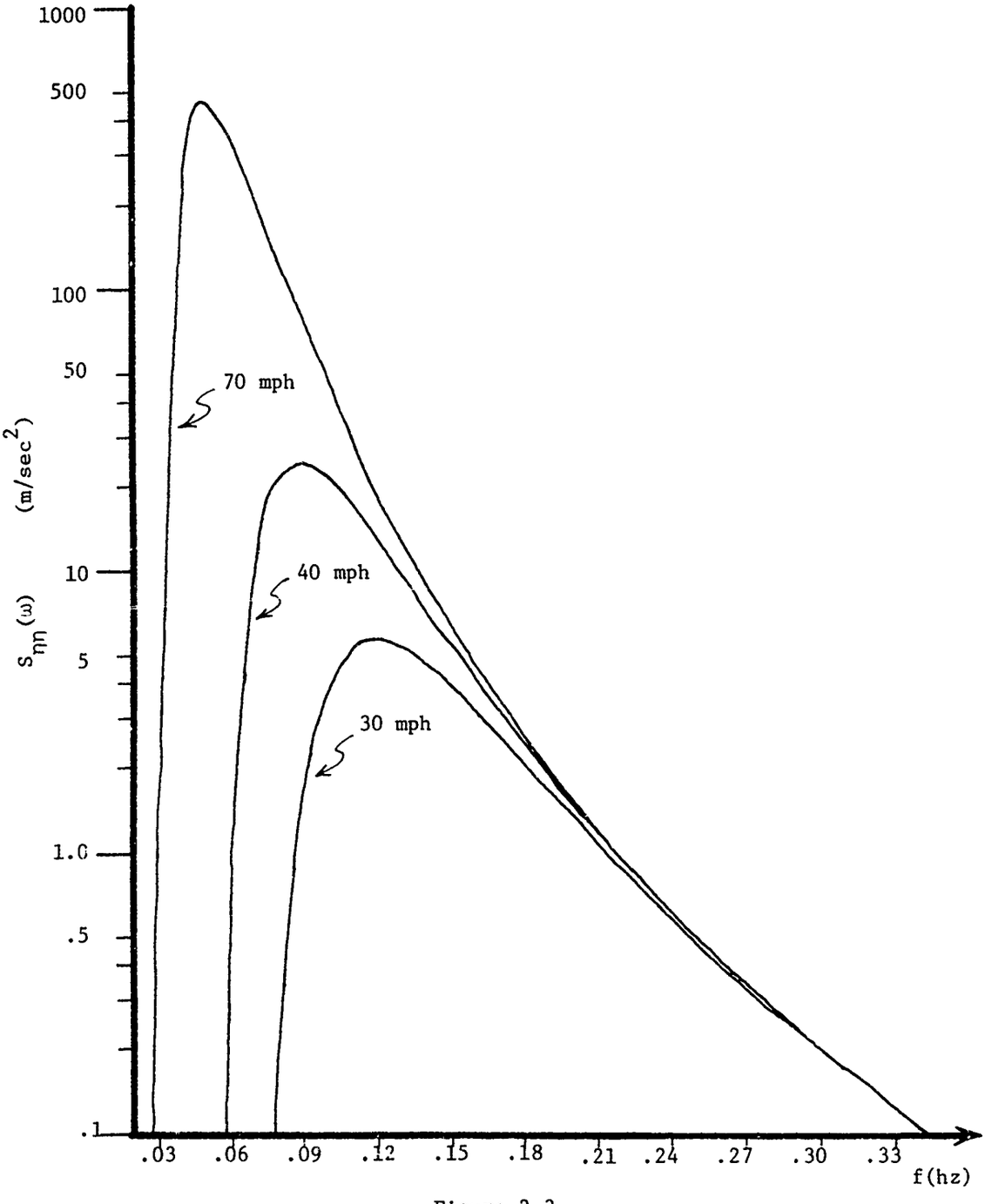


Figure 2.3

Pierson-Moskowitz Wave Amplitude Spectra Used in the Analysis

Windspeed	70 MPH	40 MPH	30 MPH
f_m	.05 hz	.09 hz	.12 hz
H_s	52.5 ft/16.0 m	16.2 ft/4.9 m	9.0 ft/2.7 m
H_{max}	105.0 ft/32.0 m	32.4 ft/9.8 m	18.0 ft/5.4 m

Table 2.5

Parameters for the P-M Wave Amplitude Spectrums
Used in the Analysis

from $\omega = .35$ to $\omega = \infty$ is only .05% of the total area from $\omega = 0$ to $\omega = \infty$, which is insignificant. An example of this process and the resulting discretized wave amplitude spectrum $\eta(\omega_i)$ is shown in Figure 2.4. The discretized wave amplitude spectra for the three sea states used in this analysis are shown in Figure 2.5.

The discretized spectrum has the property that the variance of the wave amplitudes represented by the wave-amplitude spectrum is equal to the sum of the squares of the discretized wave amplitudes, that is

$$\sigma_\eta^2 = \sum_{i=1}^{N} (\eta(\omega_i))^2 \tag{2.12}$$

2.2.2 Wave Forces on Offshore Platforms

An expression for the total wave force on a cylinder per unit of cylinder length has been developed by Morison (1950). This total force consists of two parts, a drag component and an inertial component. The drag component can be shown to be negligible for deep water structures. Therefore the total force per unit of cylinder length is assumed here to consist of only the inertial component which is expressed as

$$\Phi_T = \Phi_I = \rho C_I \frac{\pi D^2}{4} \dot{u} \tag{2.13}$$

where

Φ_T = total force per unit of cylinder length

Φ_I = inertial force per unit length

ρ = density of fluid transmitting the wave

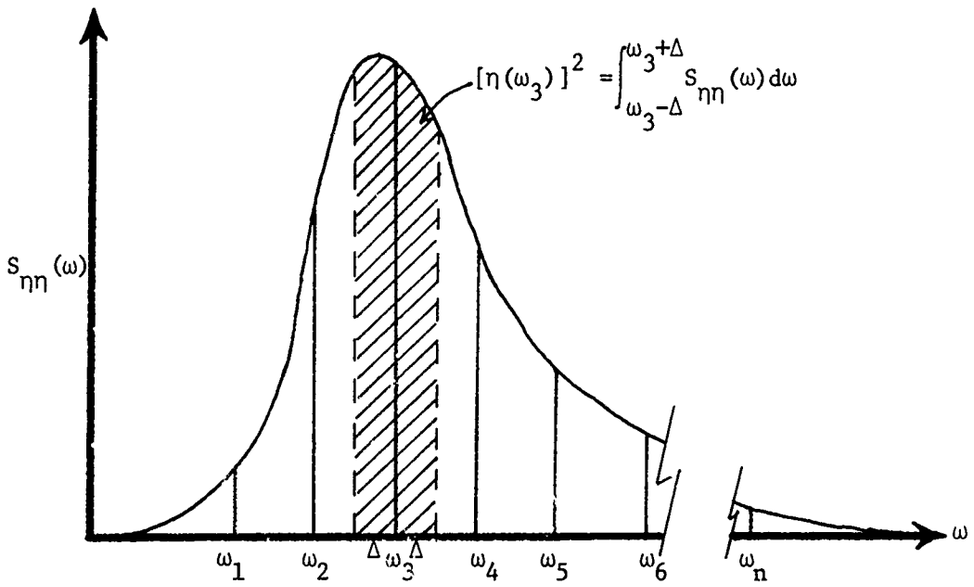


Figure 2.4a

Typical Wave Amplitude Spectrum

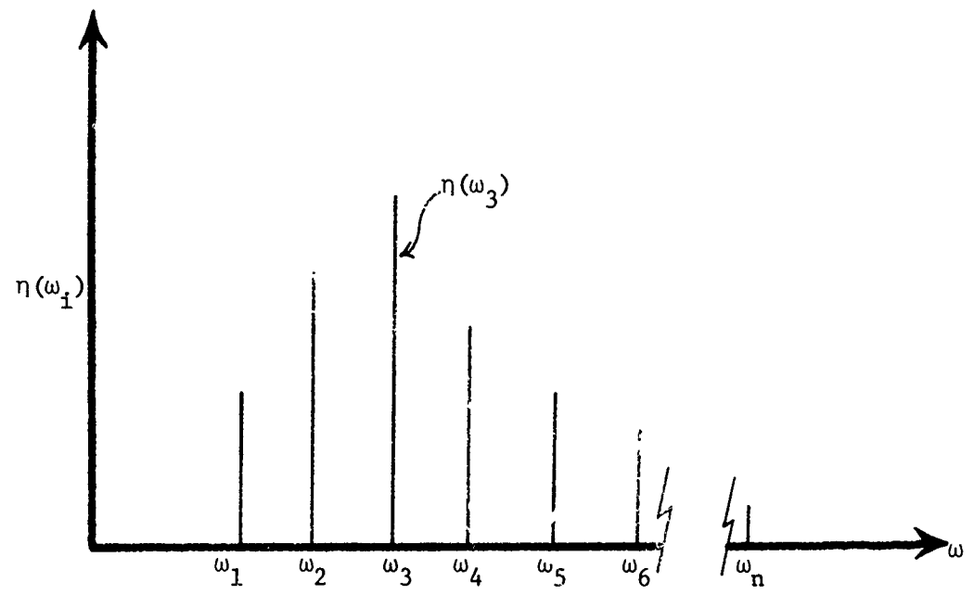


Figure 2.4b

Discretized Wave Amplitude Spectrum

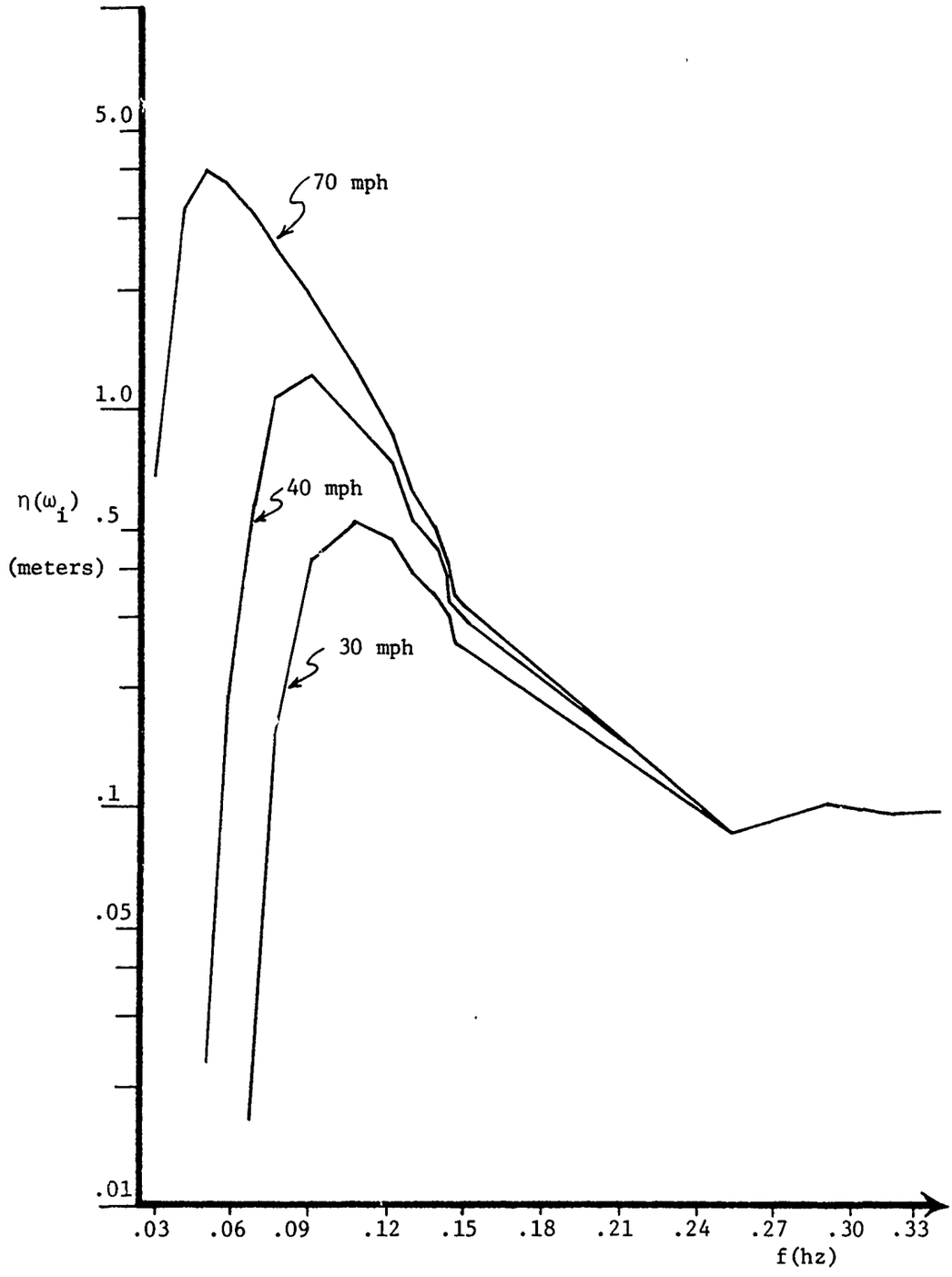


Figure 2.5

Discretized Wave Spectra Used in the Analysis (49 Frequencies)

C_I = inertial coefficient

D = cylinder diameter

$\dot{u}$ = fluid partical acceleration

Airy wave theory shows that

$$\dot{u} = \frac{H}{2}\omega^2 \frac{\cosh(Kz)}{\sinh(Kh)} \sin\omega t \tag{2.14}$$

where

H = wave height from tip to trough

K = wave number, $K = 2\pi/\lambda$

λ = wave length

h = water depth from ocean floor to still water surface

z = distance from ocean floor to any elevation on structure

$\omega^2 = KG\tanh(Kh)$

Therefore

$$\Phi_T = \frac{H}{2}\rho C_I \omega^2 \frac{\pi D^2}{4} \frac{\cosh(Kz)}{\sinh(Kh)} \sin\omega t \tag{2.15}$$

Since a single wave is described by $\eta = \frac{H}{2}\sin\omega t$, a linear relationship with respect to amplitudes exists between η and Φ_T.

This linear relationship allows the use of the discretized wave amplitude spectrum so that

$$\Phi_T(\omega_i) = (TF)\eta(\omega_i) \tag{2.16}$$

where

$$TF = C_I \rho \frac{\pi D^2}{4} \omega^2 \frac{\cosh(Kz)}{\sinh(Kh)} \tag{2.17}$$

A detailed development of this subject can be found in Nath [10].

Using the set of force distributions $\Phi_T(\omega_i)$ where i = 1,N, the computer program determines a vector of work-equivalent nodal forces $\underline{P}$ for each ω_i to be applied to the structure during the frequency domain analysis. Development of this procedure is covered by DuVall [9].

2.3 Frequency Domain Solution

Using the set of vectors $\underline{P}(\omega_i)$ determined from the set of force distributions $\Phi_T(\omega_i)$ in Section 2.2.2 to excite the structural model, the displacement response is found by the computer program to be a set of vectors $\underline{x}(\omega_i)$ such that

$$\underline{x}(\omega_i) = (\underline{TF})\ \underline{P}\ (\omega_i) \tag{2.18}$$

The model is represented by a linear system whose equation of motion is

$$\underline{M}\ \ddot{\underline{x}} + (\alpha\underline{M} + \beta_V\underline{K})\dot{\underline{x}} + (1+i2\beta_H)\underline{K}\ \underline{x} = \underline{P} \tag{2.7}$$

Assuming $\underline{P} = \underline{P}(\omega)e^{i\omega t}$ and $\underline{x} = (\underline{TF})\ \underline{P}(\omega)e^{i\omega t}$, substituting into the equation of motion yields

$$(-\omega^2\underline{M} + i\alpha\omega\underline{M} + \underline{K} + i2\beta_H\underline{K} + i\omega\beta_V\underline{K})(\underline{TF})\underline{P}(\omega)e^{i\omega t} = \underline{P}(\omega)e^{i\omega t} \tag{2.19}$$

It follows that

$$\underline{TF} = (-\omega^2\underline{M} + i\alpha\omega\underline{M} + \underline{K} + i2\beta_H\underline{K} + i\omega\beta_V\underline{K})^{-1} \tag{2.20}$$

The linearity of the system allows the transfer of the properties of the input to the response so that the variance of the response of

each degree of freedom is equal to the sum of the squares of responses of that degree of freedom at each discretized frequency.

$$(\sigma^2_{\underline{x}})_j = \sum_{i=1}^{N} (\underline{x}(\omega_i)_j)^2 \qquad j=1,M \tag{2.21}$$

where M = number of degrees of freedom in the model.

To determine acceleration response, the following is used:

$$\underline{\ddot{x}}(\omega_i) = -\omega_i^2 \, \underline{x}(\omega_i)$$

and

$$(\sigma^2_{\underline{\ddot{x}}})_j = \sum_{i=1}^{N} (\underline{\ddot{x}}(\omega_i)_j)^2 \qquad j=1,M \tag{2.22}$$

Element strains are computed using the relationship

$$\varepsilon(\omega_i)_k = \underline{B}_K [\underline{x}(\omega_i)]_K \qquad K = 1,4 \tag{2.23}$$

where, as outlined in Section 2.1,

$$\underline{B}_K = [0 \ , \ (\frac{d}{2L})_K \ , \ 0 \ , \ (\frac{d}{2L})_K]$$

$$[\underline{x}(\omega_i)]^T_K = [x_1(\omega_i)_K, \ \Theta_1(\omega_i)_K, \ x_2(\omega_i)_K, \ \Theta_2(\omega_i)_K]$$

K = element number

and finally the variance of each element strain is found to be

$$(\sigma^2_{\varepsilon})_K = \sum_{i=1}^{N} (\varepsilon(\omega_i)_K)^2 \tag{2.24}$$

CHAPTER 3

VIBRATION CONTROL SYSTEMS

3.1 The Dynamic Vibration Absorber Subject to Sinusoidal Input

The undamped dynamic vibration absorber, invented by Frahm in 1909, is used to eliminate unwanted vibrations from machinery. Consider a machine represented by a one degree of freedom system subject to an unwanted sinusoidal force $P\sin\omega t$ (Figure 3.1). Let the vibration absorber be represented by a comparitively small vibratory system k_2, m_2, attached to the main mass m_1 (Figure 3.2). If the parameters k_2, m_2 are chosen so that the natural frequency $(k_2/m_2)^{1/2}$ of the attached absorber is equal to the frequency ω of the disturbing force, then the main mass m_1 will not vibrate at all. The mathematical proof of this is contained in Den Hartog [8].

This result is most useful when the frequency of the disturbing force is near the natural frequency of the main system putting that system at or near resonance. Consider the case where the main system whose response spectrum is shown in Figure 3.3, is disturbed at its natural frequency. A vibration absorber is added whose natural frequency equals the disturbing frequency which equals the natural frequency of the main system, or:

$$\omega a = \Omega n \text{ or } \frac{k_2}{m_2} = \frac{k_1}{m_1} \text{ or } \frac{k_2}{K_1} = \frac{m_2}{m_1} = \mu$$

where

$\omega a = (k_2/m_2)^{1/2}$ = natural frequency of absorber

$\Omega n = (k_1/m_1)^{1/2}$ = natural frequency of main system

$\mu = m_2/m_1$ = mass ratio = absorber mass/main mass

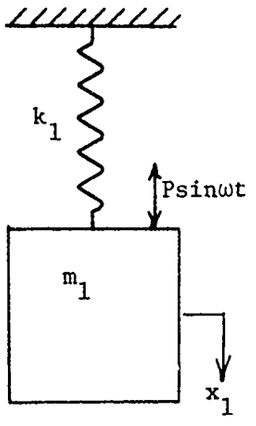


Figure 3.1

One Degree of Freedom System Disturbed by Force Psinωt

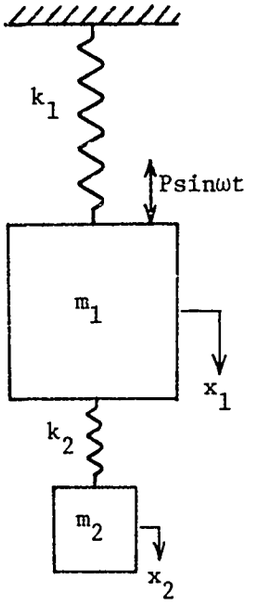


Figure 3.2

Main System m_1, k_1 with Vibration Absorber Attached

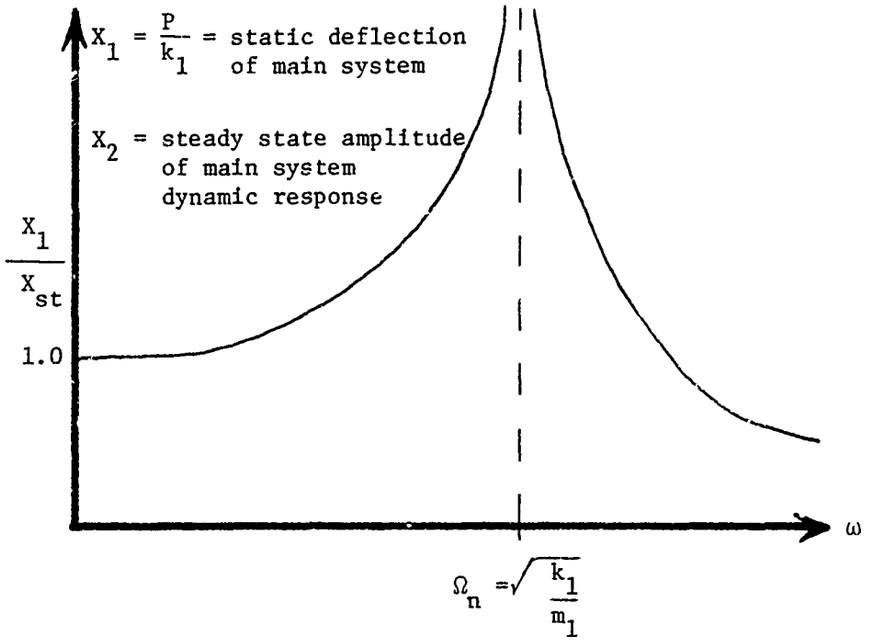


Figure 3.3

Response of One Degree of Freedom System to Disturbance Psinωt as a Function of ω

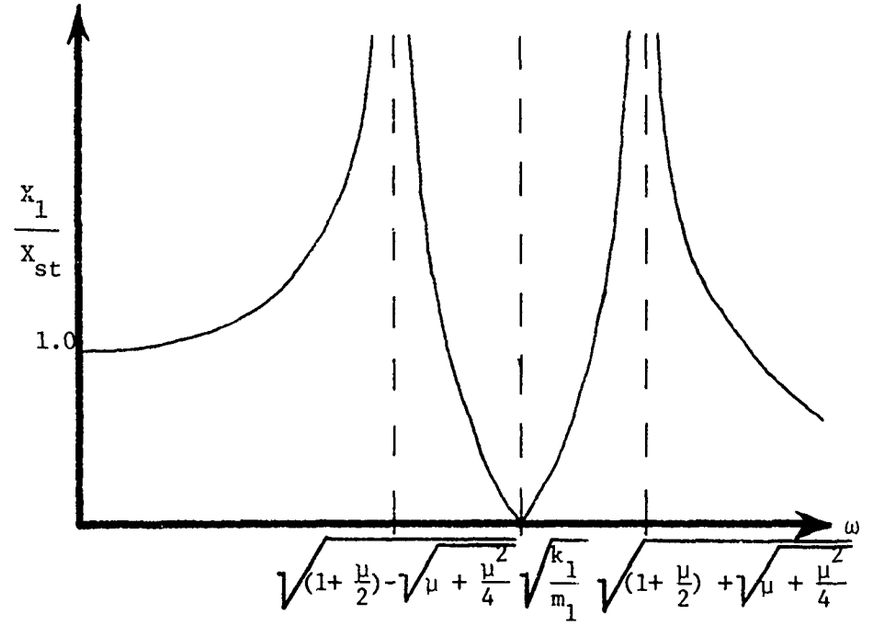


Figure 3.4

Response of Main System with Vibration Absorber Attached to Disturbance Psinωt as a Function of ω

The response spectrum for the main system with vibration absorber attached is shown in Figure 3.4. The system now has two natural frequencies but the vibration of the main mass at its old natural frequency is eliminated.

Den Hartog [8] shows that the new natural frequencies are dependent only on the mass ratio and are found with the following formula:

$$\left(\frac{\omega}{\omega a}\right)^2 = \left(1 + \frac{\mu}{2}\right) \pm \sqrt{\mu + \frac{\mu^2}{4}} \tag{3.1}$$

Consider the system of Figure 3.2 in which a dashpot is arranged parallel to the vibration absorber spring k_2, between masses m_1 and m_2. The equations of motion for this system become

$$\begin{aligned} m_1\ddot{x}_1 + k_1x_1 + k_2(x_1-x_2) + c_2(\dot{x}_1-\dot{x}_2) &= P\sin\omega t \\ m_2\ddot{x}_2 + k_2(x_2-x_1) + c_2(\dot{x}_2-\dot{x}_1) &= 0 \end{aligned} \tag{3.2}$$

Assuming the solutions

$$\begin{aligned} x_1 &= X_1e^{i\omega t} \\ x_2 &= X_2e^{i\omega t} \end{aligned} \tag{3.3}$$

and substituting these into Equation (3.2) yields

$$-m_1\omega^2 X_1 + k_1 X_1 + k_2(X_1-X_2) + i\omega C_2(X_1-X_2) = P$$

$$-m_2\omega^2 X_2 + k_2(X_2-X_1) + i\omega C_2(X_2-X_1) = 0$$

The solution of these equations involves complex arithmetic and is lengthy. Den Hartog [8] shows the solution for the displacement of the main mass to be:

$$\frac{X_1}{X_{st}} = \sqrt{\frac{(2\frac{c_2}{c_c}g)^2 + (g^2-r^2)^2}{(2\frac{c_2}{c_c}g)^2(g^2-1+\mu g^2)^2 + [\mu r^2 g^2-(g^2-1)(g^2-r^2)]^2}} \qquad (3.4)$$

where

r = $\omega a/\Omega n$ = frequency ratio (natural frequencies)

g = $\omega/\Omega n$ = forced frequency ratio

c_c = $2m_2\Omega n$ = "critical" damping

The general response spectrum for X_1/X_{st} is shown in Figure 3.5. The shape of the response spectrum, that is the response at any input frequency, varies with the change in the parameters μ, c_2/c_c, r, and g. The maximum value of X_1/X_{st} is

$$\frac{X_1}{X_{st}} = \sqrt{1+\frac{2}{\mu}} \qquad (3.5)$$

Note that X_1/X_{st} can be significant when μ is small.

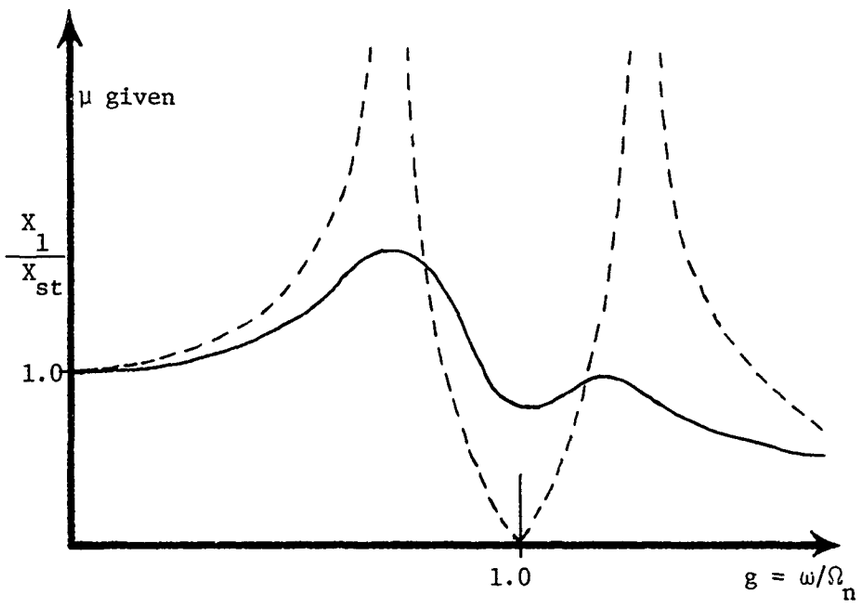


Figure 3.5

Response Curves for Motion of the Main Mass. Damped Absorber (Solid), Undamped Absorber (Dashed)

3.2 The Damped Dynamic Vibration Absorber Subject to White Noise Input

In general, mechanical systems exhibit internal damping, and often they are subject to spectral inputs rather than single sinusoids. This is the case with the offshore oil platforms under consideration. Crandall and Mark [15] carried out some parameter optimization studies with the system shown in Figure 3.6 where both the main system and the vibration absorber were damped. The system input was a white noise acceleration applied to the system foundation.

A comparison was made between the values of $E[y_1^2]/\sigma_y^2$ for various values of the system parameters where

$E[y_1^2]$ = expected value of the relative motion of m_1 with vibration absorber attached

σ_y^2 = expected value (standard deviation) of the motion of m_1 without the vibration absorber

$y_1 = x_1 - x_o$

The results were similar to those obtained by Den Hartog in his work with damped vibration absorbers. Crandall found that the response depended on the mass ratio μ, the damping ratio c/c_c and the frequency ratio r.

As μ was increased the minimum value of $E[y_1^2]/\sigma_y^2$ generally decreased.

The minimum value of $E[y_1^2]/\sigma_y^2$ for each mass occured at a frequency ratio r which followed a law such that

$$r = \frac{1}{1+\alpha\mu} \tag{3.6}$$

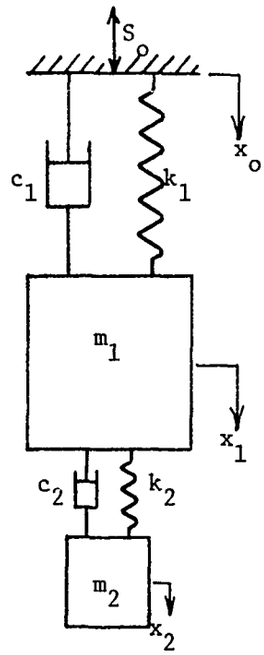


Figure 3.6

Crandall and Mark's Model Subject to

White Noise Acceleration S_o

where $\alpha > 1$ by a small amount.

The response not only depends on the damping ratio of the absorber but also the ratio of each absorber damping ratio to the main system damping ratio.

Finally, the amplitude of the absorber mass response varies inversely with the mass ratio when the system is tuned for optimum vibration absorption, and can be significant, and a limiting design factor, as the mass ratio decreases.

3.3 The Mass Damper and the Offshore Platform

The mass damper affixed to an offshore oil platform is essentially a damped vibration absorber attached to a damped vibrating system. There are two main differences between this system and those studied by Den Hartog and Crandall and Mark. First, the tower is a multi-degree of freedom system rather than a single degree of freedom system and secondly the input to the system is a varying spectral input rather than a single sinusoid or a constant white noise.

To check the feasibility of using a mass damper on an offshore tower, a modal analysis of the 1000 foot model was carried out. The details of this analysis and the complete results are in Appendix C. The results show that the model responds primarily in its fundamental mode. This indicates that the mass damper should be effective and it should be tuned to surpress the response of the first mode.

The purpose of the mass damper is to reduce the standard deviation of the response of the structure when the structure is excited by

a spectral input representing the action of the sea.

An illustrative example demonstrates the ability of the mass damper to reduce response. The model used is the 1000 foot steel jacketed tower currently being constructed by Shell Oil Corporation.

The input is the Pierson-Moskawitz wave-amplitude spectrum for a one-hundred year return sea state discretized into forty-nine frequencies as explained in Chapter 2. The one-hundred year return sea state is the sea state corresponding to a wind speed of 70 mph [16].

First the model without the mass damper was excited by the one-hundred year return sea. The results are shown for deck displacement (Figure 3.7), deck acceleration (Figure 3.8), and strain in the fourth element (Figure 3.9).

Next the mass damper was activated with the parameters in Table 3.1. The model was then excited by the same sea state. The results are shown in Figures 3.7, 3.8 and 3.9. These results are normalized to the rms response of the tower without the mass damper.

The standard deviation of each response was computed using the method outlined in Chapter 2. The standard deviations of response were reduced by the use of the damper as follows

Deck rms displacement - reduced 23.7%

Deck rms acceleration - reduced 29.2%

Top element strain- reduced 28.3%

The motion of the damper in relation to the deck was 3.42 times the motion of the deck.

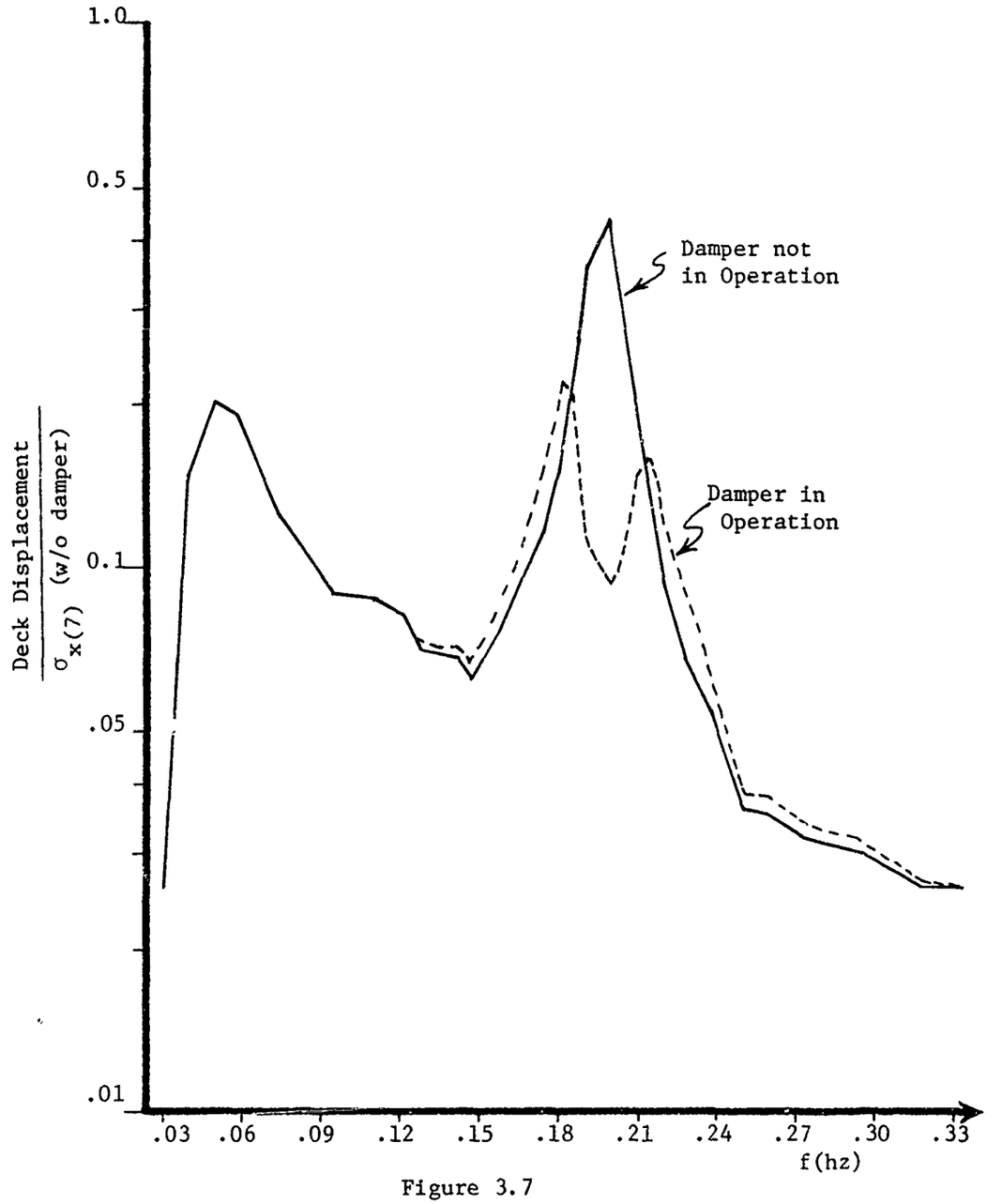


Figure 3.7

Deck Displacement vs. Frequency

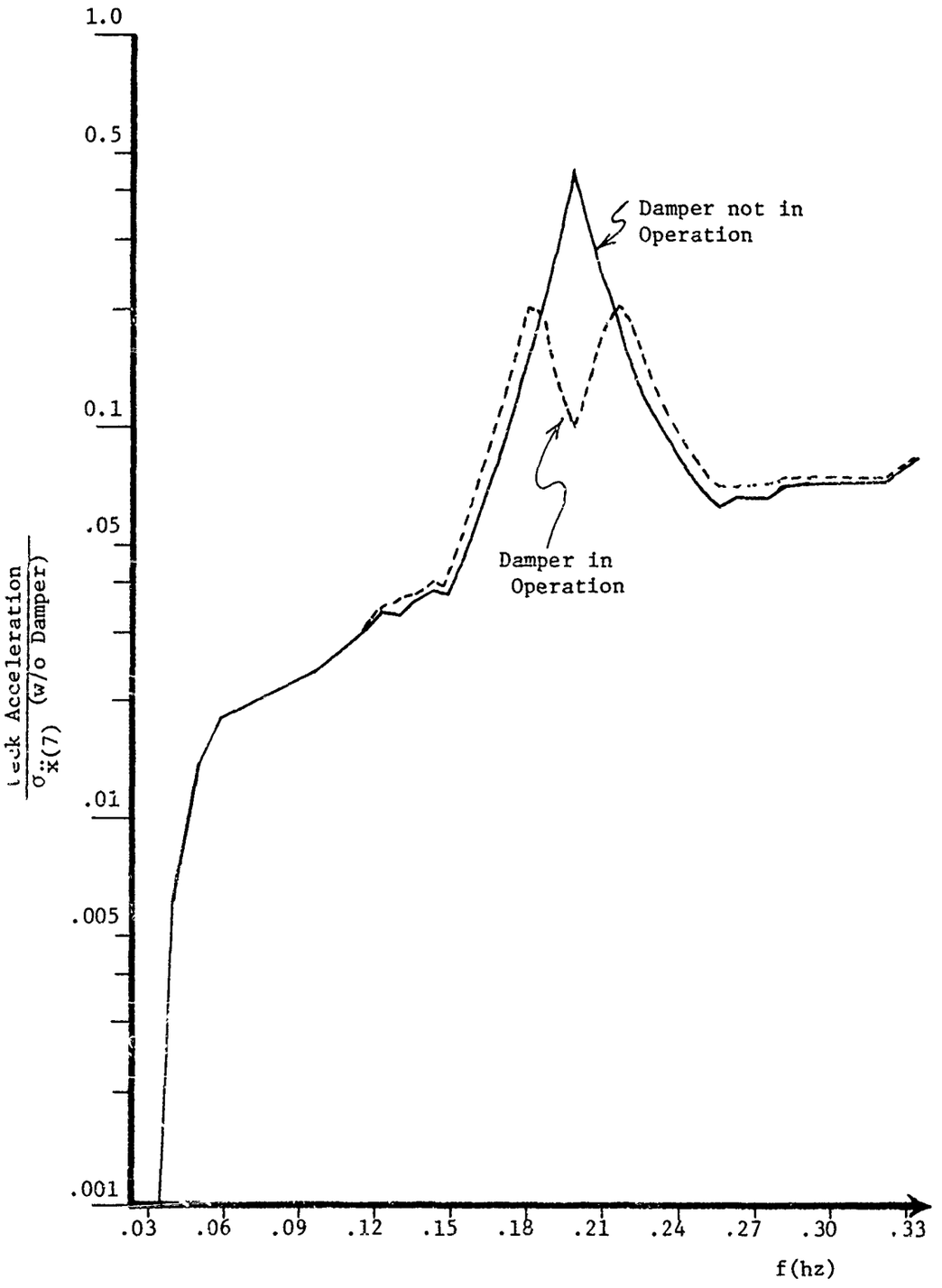


Figure 3.8

Deck Acceleration vs. Frequency

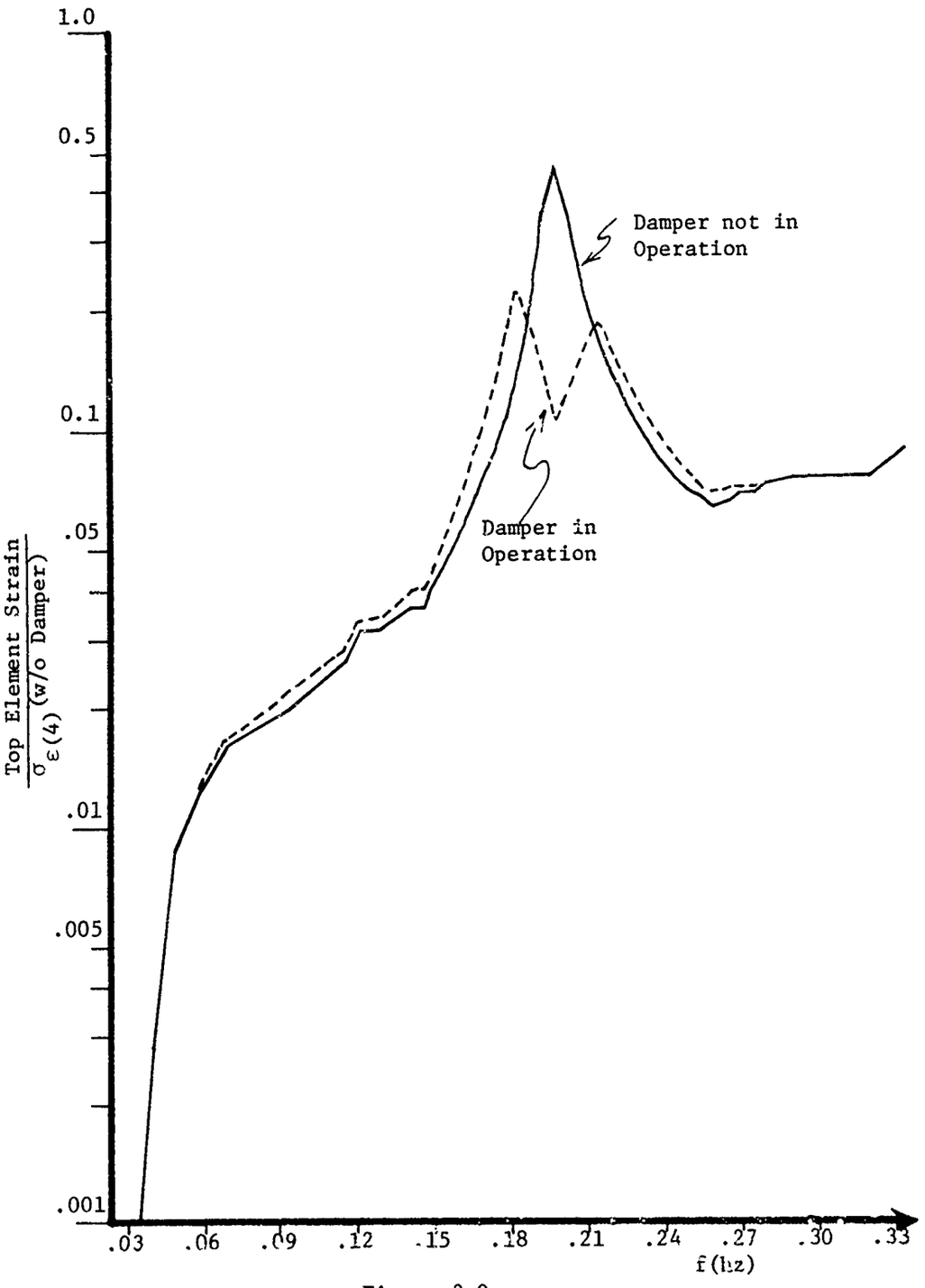


Figure 3.9

Top Element Strain vs. Frequency

μ	.01 (damper mass = 112.0 $\frac{MT\text{-}sec^2}{m}$)
ω_a/Ω_n	1.00
$k_{\text{mass damper}}$	166.7 MT/m
$\frac{c_{\text{damper}}}{c_{c\ \text{damper}}}$	5%

Table 3.1

Mass Damper Parameters

The important variables to be examined when judging the effectiveness of the damper depend on two things. First, the space available for free travel of the damper and second, the reduction in amplitude of the response variable, for example, the element strains, which are most important for fatigue and deck acceleration, which is most important for numan comfort considerations.

In this case, the damper is effective in reducing all three responses significantly, but the relative motion of the damper is probably excessive for the crowded conditions found on drilling platform decks where space is a premium.

CHAPTER 4

PARAMETER VARIATION AND SYSTEM RESPONSE

The two important factors in the design of mass dampers are the effectiveness in reducing platform response and the motion of the mass damper relative to the tower deck. The final design of a mass damper is normally a compromize between the effectiveness desired and the space allocated for movement of the damper. In this chapter the variation of these factors is studied for a range of damper parameter values. The 1000 foot tower model with eight degrees of freedom active is subjected to the 100 year return storm. Thus an extension is made from a single degree of freedom system with an attached damper studied by Den Hartog and Crandall to a mass damper in operation on a multi-degree of freedom system.

The offshore tower, while it essentially acts as a one degree of freedom system, is not, however, an idealized one degree of freedom system and it is subject to an input whose intensity varies with frequency throughout the spectrum.

There are three parameters which determine the response of the system: the mass ratio μ, the frequency ratio $\frac{\omega a}{\Omega n}$, and the ratio of percent critical damping in the absorber to percent critical damping in the tower $\beta a/\beta n$. The mass ratio μ, of the multi-degree of freedom system is the ratio of the damper mass to the total mass of the tower. To determine the variation in system response, each ratio is varied in turn with the other two constant. The

effectiveness of response reduction is measured as the deck displacement ratio σ_D/σ (the ratio of the rms deck displacement with mass damper to the rms deck displacement without the mass damper).

To determine the effect of mass ratio on response, $\omega a/\Omega n$ is held constant at 1.0 and βa is held at .05 (βn is constant at .03 as discussed in Chapter 2), and the mass ratio is set at .01, .03, and .05. These values of μ were chosen as being the limits and mid-point of the range of values that might be practical in this type of application. The results are shown in Figure 4.1. As μ increases, σ_D/σ decreases but the rate of decrease in σ_D/σ is decreasing. This result is similar to that predicted by MacDonald [17] in her work with Crandall's model and shows that the additional cost of an increment in μ is rewarded with a smaller increment in reduction of the deck displacement ratio. This fact is important in the economic analysis of the mass damper system.

Also plotted in Figure 4.1 is the damper displacement ratio $\frac{\sigma_{damper}}{\sigma}$ (the ratio of the rms relative mass damper displacement to the rms deck displacement without the mass damper). This ratio also decreases in the same manner as the deck displacement ratio. By increasing μ the room needed by the mass damper to operate is reduced, which again affects the cost and feasibility of the damper system.

The variation in deck displacement versus frequency for four values of μ is plotted in Figure 4.2. When $\mu \neq 0$, that is, the mass damper is in operation, the system has two resonant peaks and the response is reduced in the area of the natural frequency of the tower.

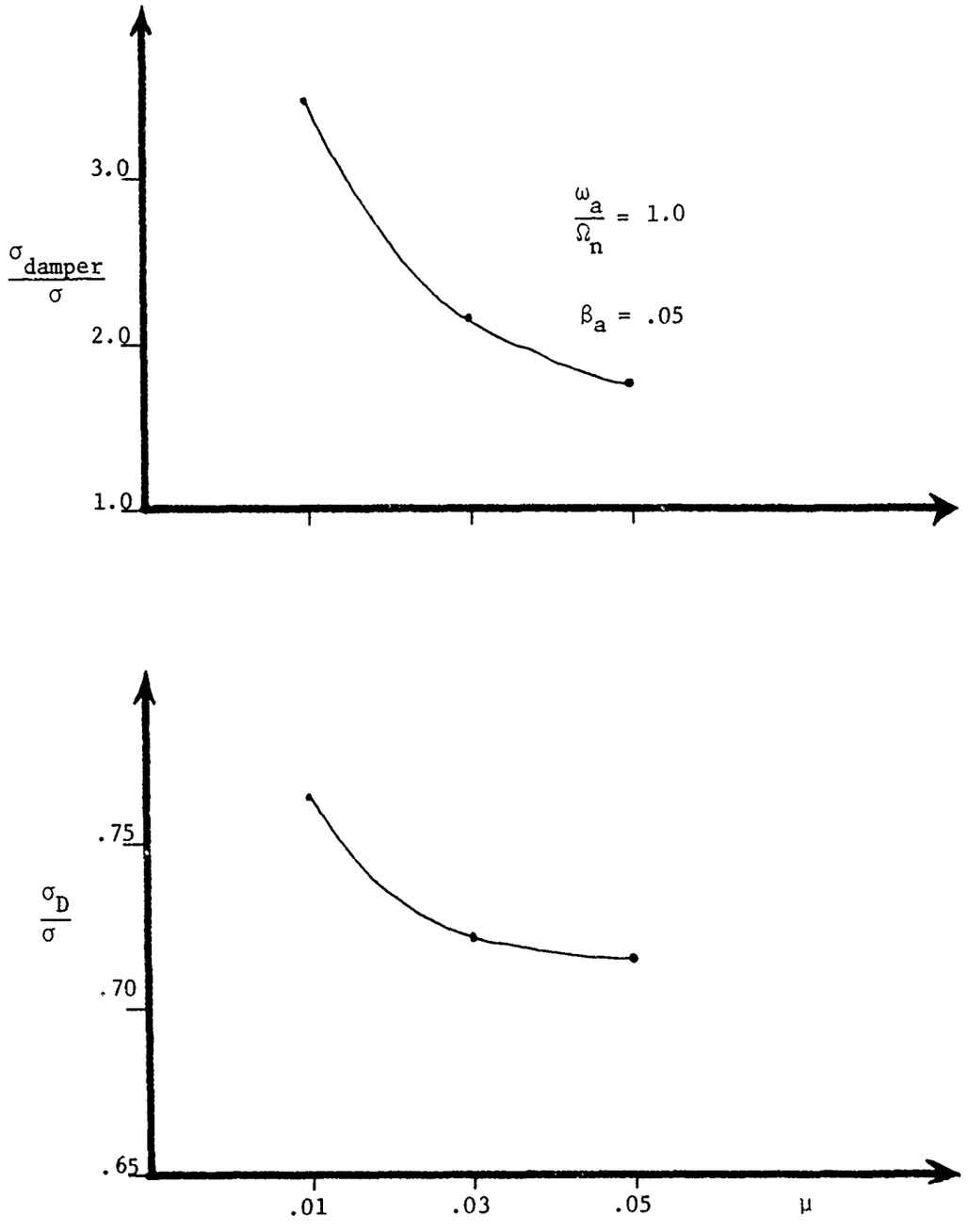


Figure 4.1

Displacement Ratios vs. Mass Ratio

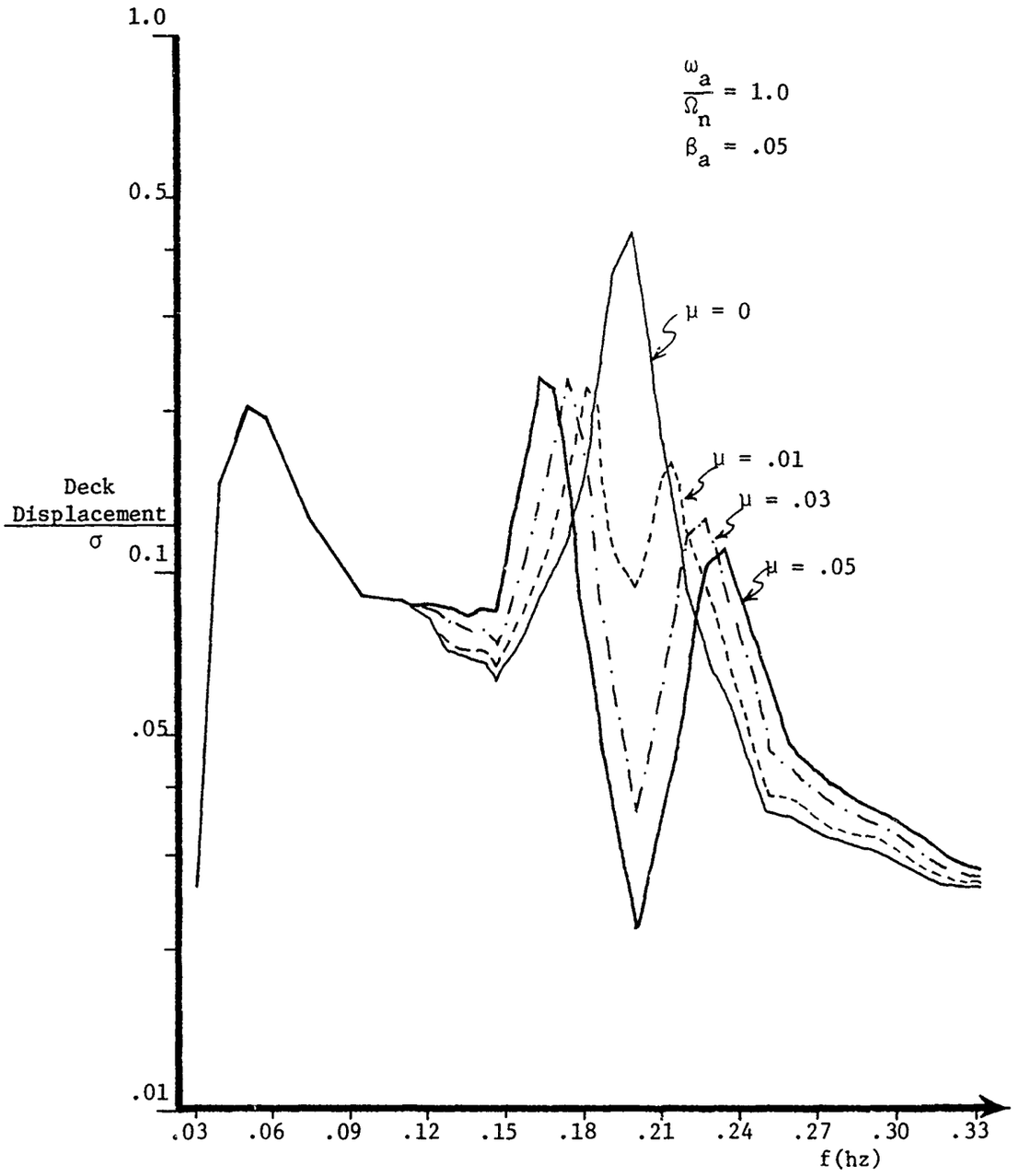


Figure 4.2

Deck Displacement vs. Frequency (μ Varying)

As μ is increased this reduction of response is increased which reduced the deck displacement ratio as shown in Figure 4.1.

To determine the impact the frequency ratio $\omega a/\Omega n$ has on response, βa was held constant at .05 and $\omega a/\Omega n$ was varied for each value of μ. Figure 4.3 shows the results. The minimum values of the deck displacement ratio were achieved at the values of $\omega a/\Omega n$ indicated by a cross on each curve.

It can be seen from Figure 4.4 where the variation in deck displacement versus frequency is plotted for values of $\omega a/\Omega n$, that the minimum response occurs when the two resonant peaks of the response are equal.

Returning to the results shown in Figure 4.3, note that as the mass ratio increases the deck displacement ratio becomes less sensitive to variations in the frequency ratio, allowing broader variations in tuning with little change in results.

The damper displacement ratio plotted in Figure 4.3 shows the damper displacement to be very insensitive to variations in the frequency ratio for all values of μ.

To determine the importance of the damping in the absorber on the structural response, the frequency ratio was held constant at 1.0 and the percent critical damping βa was varied for the three values of μ. The results are plotted in Figure 4.5. Again the minimum value of the deck displacement ratios were achieved at the values of βa indicated by a cross on each curve. Figure 4.6 shows the variation in deck displacement versus frequency for values of β_a.

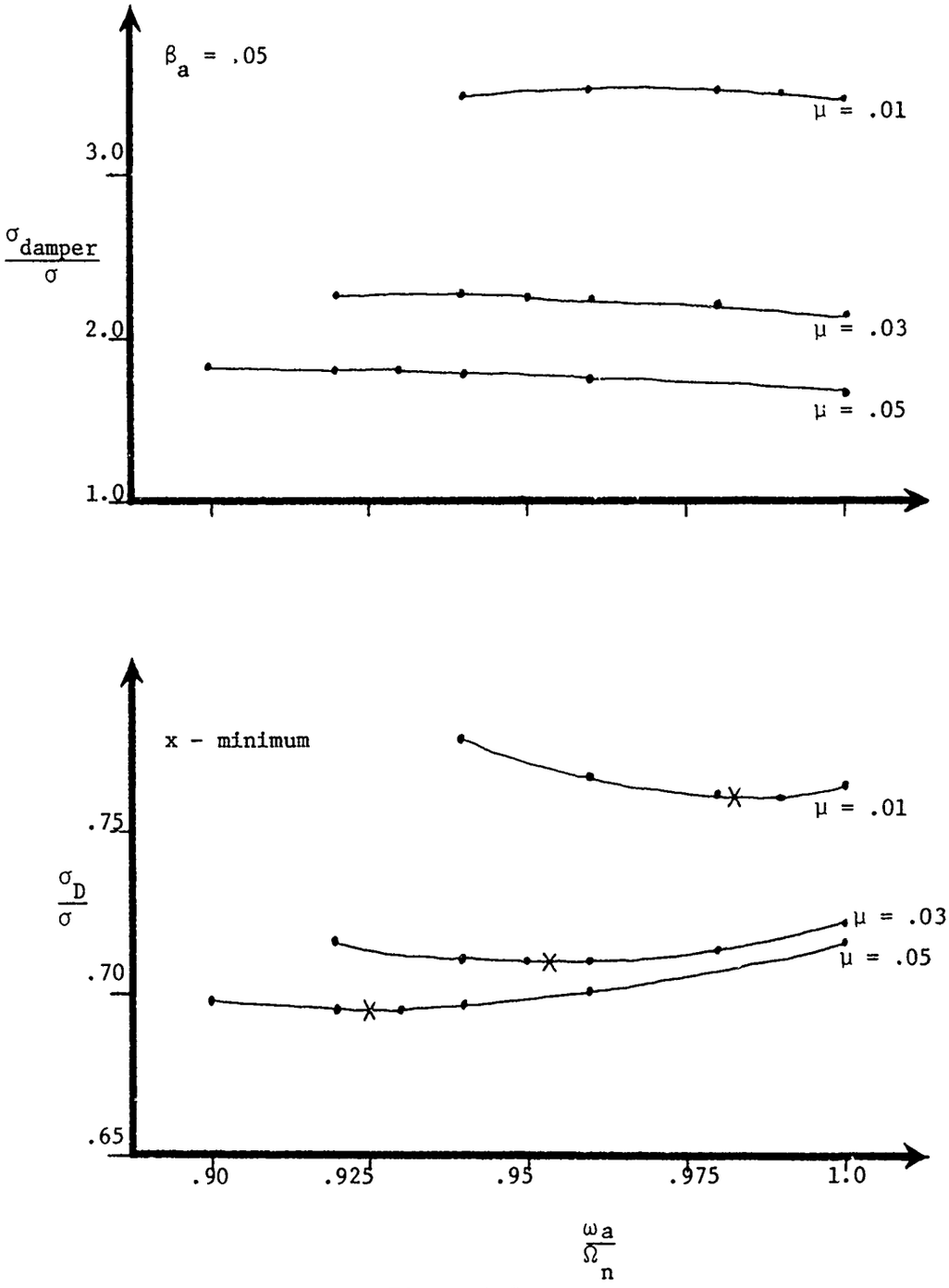


Figure 4.3

Displacement Ratios vs. Frequency Ratio

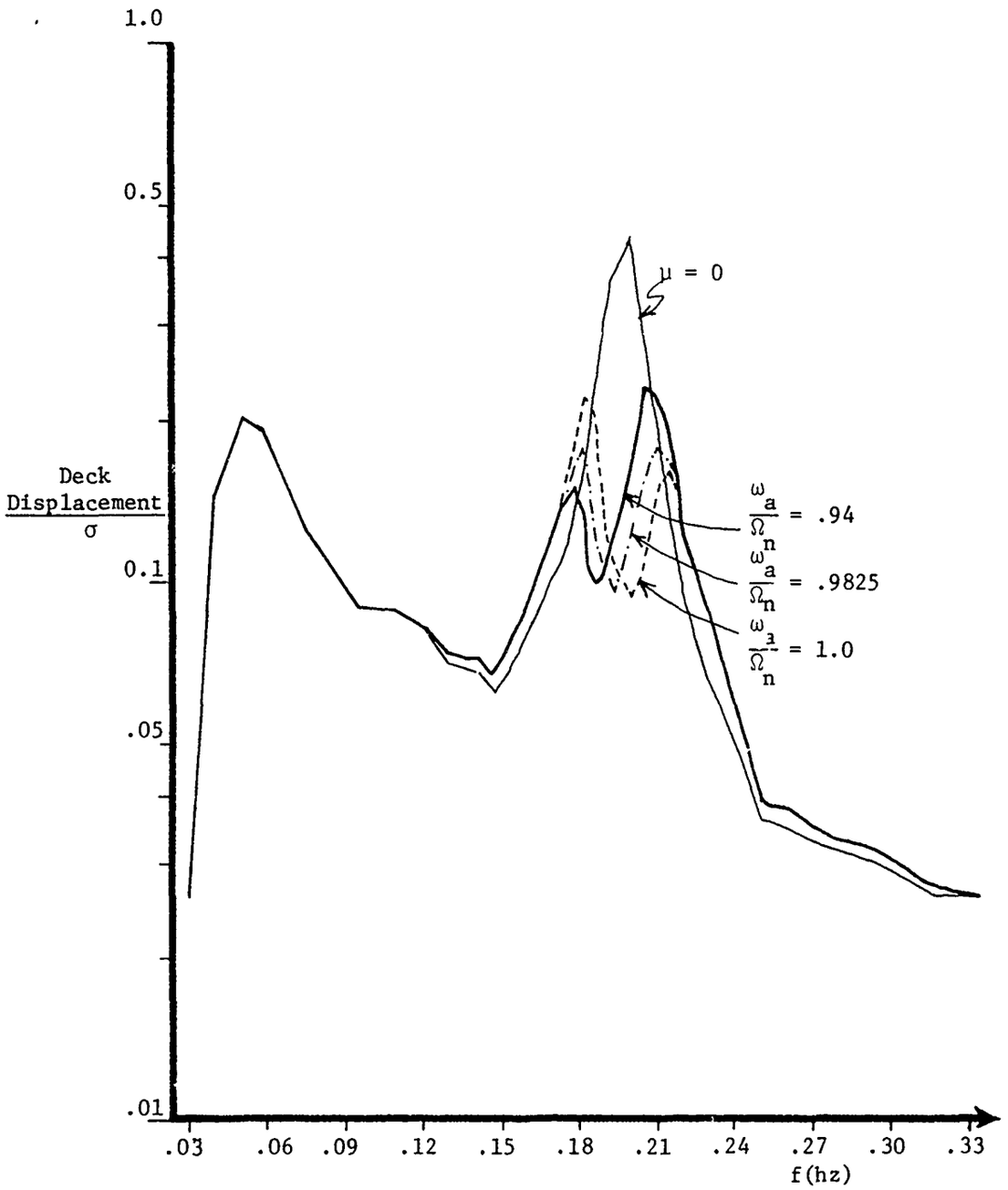


Figure 4.4

Deck Displacement vs. Frequency (ω_a/Ω_n Varying)

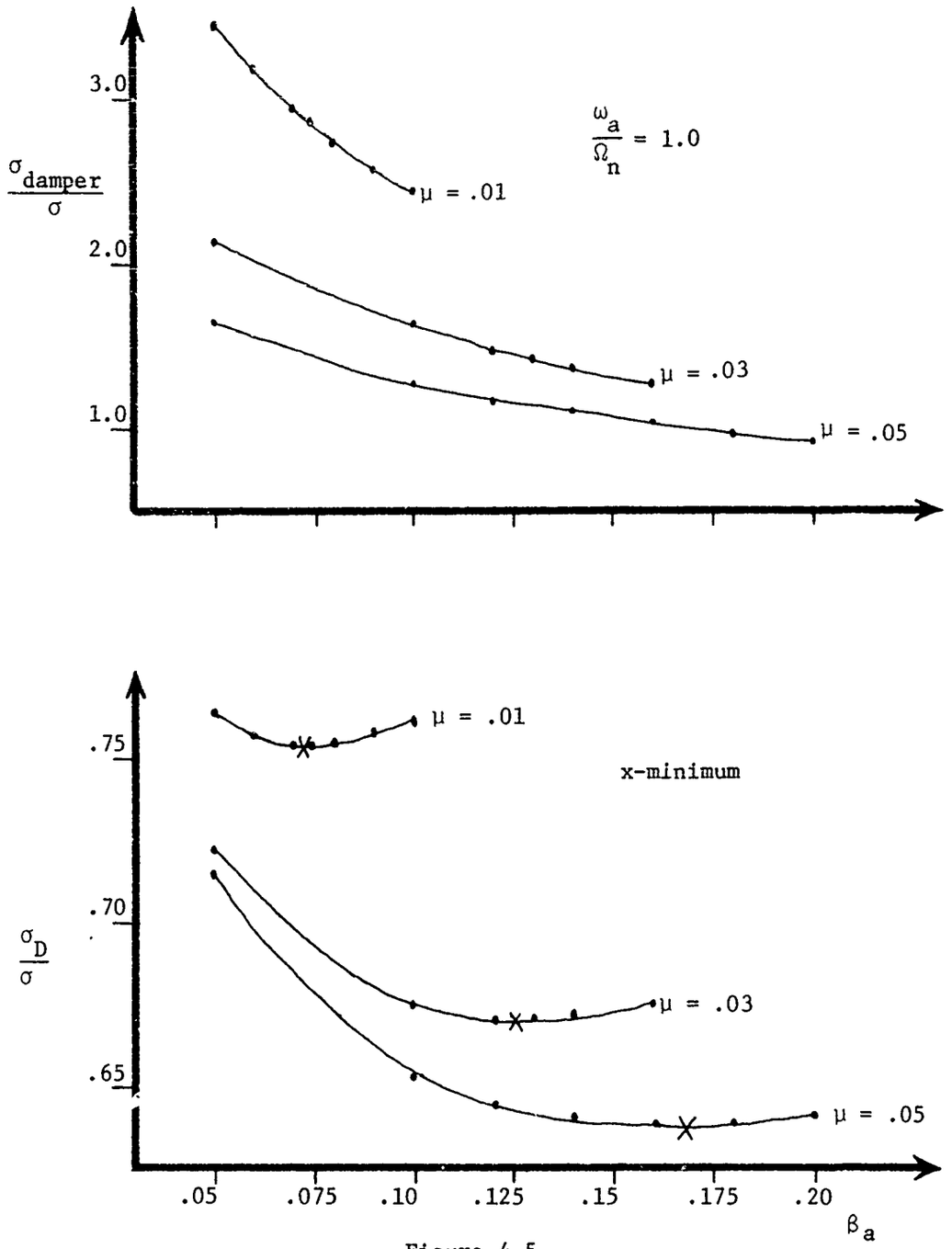


Figure 4.5

Displacement Ratios vs. β_a

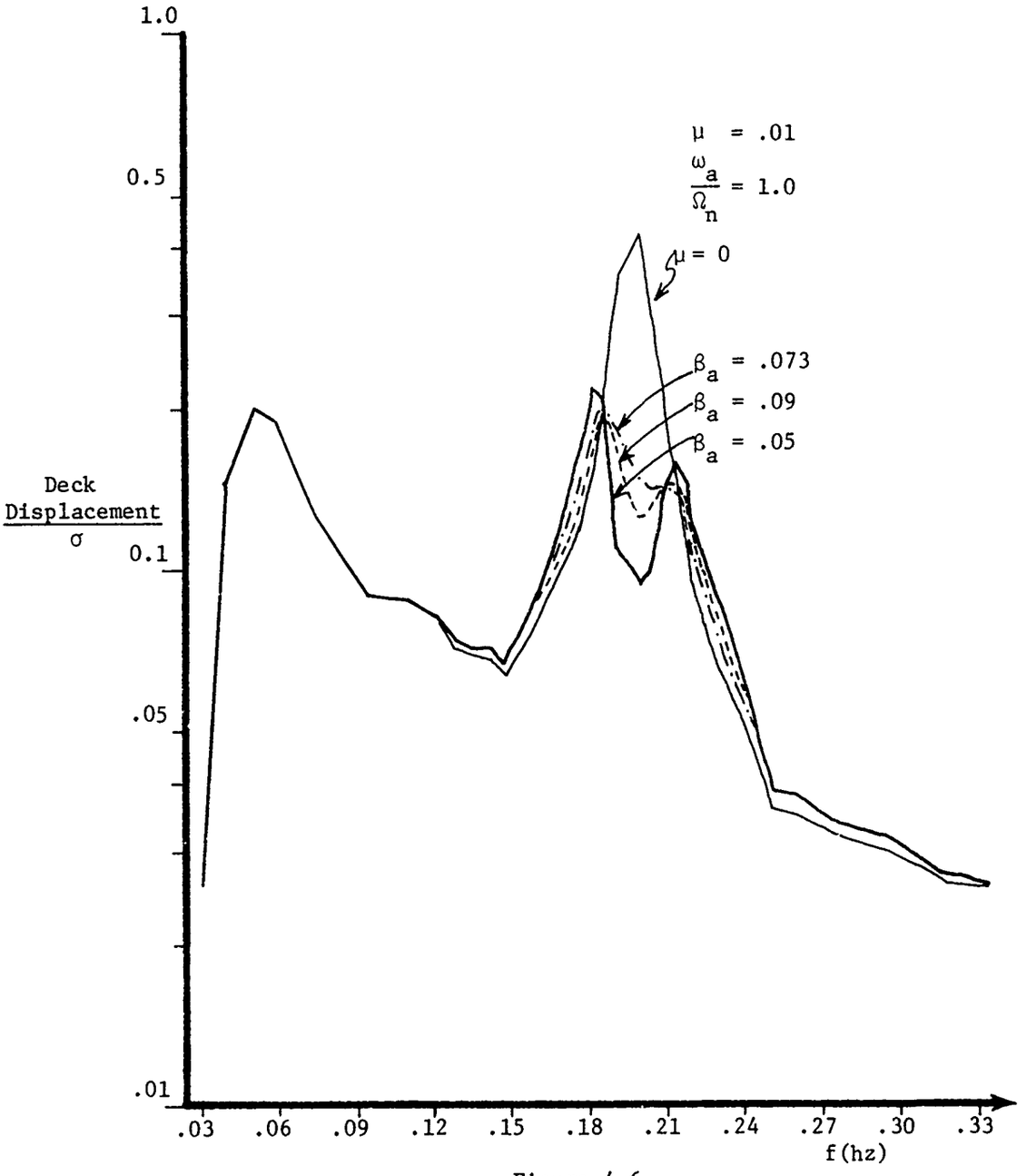


Figure 4.6

Deck Displacement vs. Frequency (β_a Varying)

The variation of damper displacement ratios is also plotted in Figure 4.5. As the value of βa increases, $\frac{\sigma_{damper}}{\sigma}$ decreases for all mass ratios. The decrease is more pronounced for $\mu = .01$. Note that although σ_D/σ varies only 1.5% in the range shown, the damper displacement ratio decreases 30%. It would seem practical, and certainly economical, to set βa at a value higher than that causing the minimum deck displacement ratio for smaller masses to reduce the damper motion without unjust penalty in deck displacement reduction.

Table 4.1 summarizes the parameter values for a mass damper utilized on offshore towers which cause a minimum in the deck displacement ratio.

μ	$\frac{\omega a}{\Omega n}$	βa
.01	.9825	.0725
.03	.9535	.1250
.05	.9260	.1675

Table 4.1
Mass Damper Parameters for Minimum Response

These results were checked to insure their validity in sea states developed from winds of 30 mph up to 100 mph. The results

remain valid without significant change because the spectra representing the various sea states are the same in the range of frequencies where resonance occurs in offshore structures.

These values cannot be considered optimum values. They do not account for the variation in relative damper displacement which must be a major factor, along with damper effectiveness, in the design of a mass damper system.

CHAPTER 5

DAMPER APPLICATION TO INDUSTRY PROBLEMS

5.1 Offshore Platforms in Water Depths Greater than 1000 Feet

As outlined in Section 1.1, offshore platforms designed for safe operation in water depths greater than 1000 feet are too expensive to be constructed in the current economic climate. This high cost results from either of two possible restrictions on the design. First, that the fundamental period of the structure be five seconds or less, or second, that if the fundamental period is allowed to be greater than five seconds, then additional reinforcing steel is required to withstand the resulting increased design loads. It appears that it might reduce the total system cost if the platform had a fundamental period greater than five seconds and the design loads were equal or less than the design loads of a platform with a fundamental period of five seconds. This second requirement permits only normal amounts of material to be used in the platform, without additional reinforcing.

To check the possible use of the tuned mass damper to help meet these two requirements, the 1200 foot steel jacketed model, with a fundamental period of 6.94 seconds, was loaded with a 100 year return sea state with and without the mass damper in operation. The damper parameters used were those listed in Table 5.1. In addition, the 1000 foot model without a mass damper was loaded with the same sea state as a basis for comparison. The responses compared

Table 5.1

Comparison of Tower Response when Subject to a
100 Year Return Sea State

Model	μ	RMS Deck Displacement*	RMS Top Element Strain**
1000 foot (T=5 sec)	0	1.000	1.000
1200 foot (T = 6.94 sec.)	0	1.158	1.012
1200 foot (T=6.94 sec.)	.01	.852	.707
1200 foot (T=6.94 sec.)	.03	.746	.596
1200 foot (T=6.94 sec.)	.05	.701	.547

*Normalized to 1000 foot model RMS deck displacement

**Normalized to 1000 foot model RMS top element strain

are RMS deck displacement and RMS top element strain. RMS deck displacement is indicative of the overall structural response and the RMS top element strain is representative of member stresses. All responses are normalized to either RMS deck displacement or RMS top element strain of the 1000 foot tower. The results are shown in Table 5.1.

The increase of tower depth to 1200 feet and fundamental period to 6.94 seconds increases the RMS deck displacement 1.158 times. The increase in RMS deck displacement is expected as the fundamental period moves to frequencies where the waves are larger. The use of the damper however reduces this response below the RMS displacement of the 1000 foot tower. The same trend is observed for RMS top element strain. When the damper is not in operation member strains in the 1200 foot model are larger than those in the 1000 foot model, thus requiring additional material to insure safety. When the damper is in operation the member strains are less than those in the 1000 foot model for all mass ratios.

Thus the two criteria are satisfied when the mass damper is applied to the 1200 foot model. The fundamental period of the structure is greater than five seconds and the member loads are less than those of a platform with a fundamental period of five seconds. Therefore the use of a tuned mass damper on a structure with a fundamental period of greater than five seconds might make the design of such a structure affordable in today's economic climate since it reduces the response.

The travel of the damper mass has not been considered in this application although it is an important design parameter. The amount of travel allowed depends on the space available, the damper mass used, and the response reduction desired. Reduced damper travel can be accomplished by increasing the damper mass or changing the damper frequency. The former, while also decreasing tower response, increases the loads on the structure caused by the additional weight. The latter increases the tower response. It is apparent in this case that there is probably a sufficient margin in response reduction at all values of μ to allow for damper motion considerations and still insure reduced costs.

5.2 Fatigue Failure in Steel Jacketed Platforms

The cyclic stresses induced in steel-jacketed deep water platforms by ocean waves in areas such as the North Sea are of sufficient magnitude on a daily basis for fatigue to be a major design consideration. As mentioned in Section 1.1, the cost of overcoming fatigue effects in such structures can be significant.

Muga and Wilson [19] present a simplified theory which can be applied to the design of ocean structures subjected to random-type forces which explains the mechanism of fatigue failure.

Assume that a fatigue curve for a structural metal has been experimentally determined under conditions similar to those of the proposed ocean structure. The results are approximately a straight

line on a plot of log S (stress) against log N_f (number of cycles of loading to failure). Figure 5.1 is a sample of such a plot. Note that the slope and intercept of the curve vary depending on material and environment.

Figure 5.1 can be used to find an estimate of the number of cycles to failure, N_{f1}, for a structural member at a given constant stress amplitude, S_1. If the constant stress amplitude is at a constant frequency, f_1, which would be the case in wave loading, then the time to failure, T_f, is:

$$T_f = \frac{N_1}{f_1} \tag{5.1}$$

When the stress is not of constant amplitude, but is random in nature, such as those stresses caused by ocean waves, the problem is more difficult. Empirical theory is not available, but an extrapolation of fixed amplitude fatigue data can be made as follows. Assume a specimen has been tested first at S_1 failing at N_{f1}, and then at S_2 failing at N_{f2} (see Figure 5.1). The fraction of the material's life which is used up for n_{f1} cycles at S_1 is assumed to be n_{f1}/N_{f1}. The fraction of life remaining is $1 - n_{f1}/N_{f1}$. This remaining life is assumed to be used up at stress level S_2 in n_{f2} cycles, where the fraction of life remaining is n_{f2}/N_{f2}. Thus failure occurs when

$$\frac{n_{f1}}{N_{f1}} + \frac{n_{f2}}{N_{f2}} = 1 \tag{5.2}$$

It follows that, if the specimen experiences n_{fi} cycles of

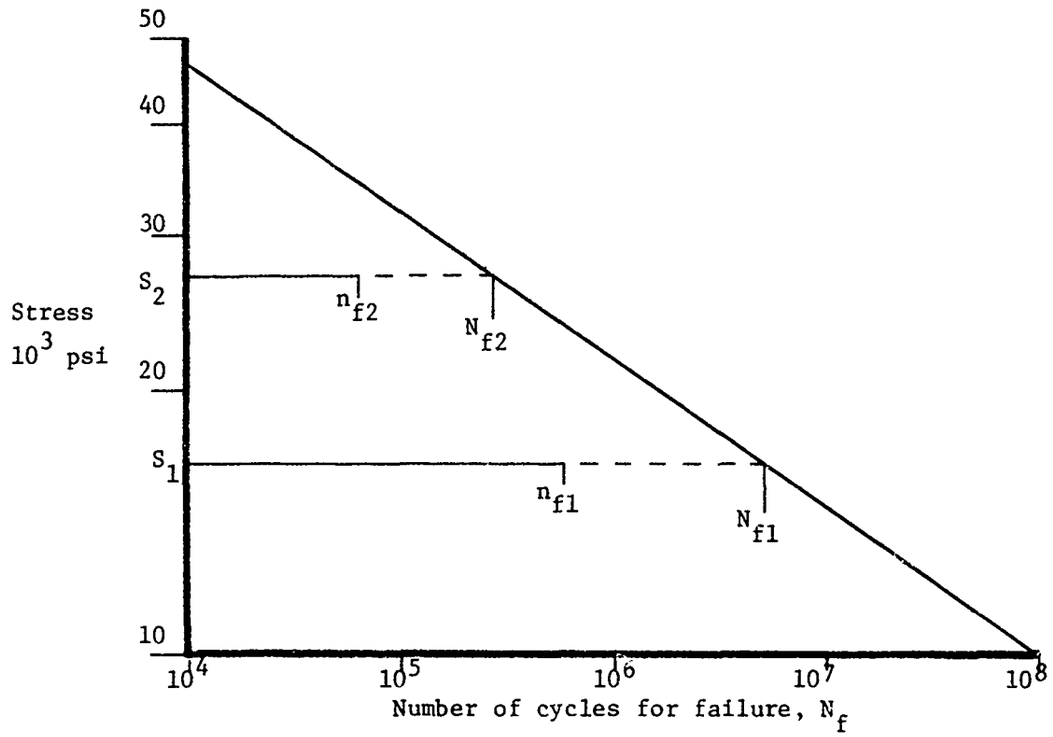


Figure 5.1

S-N_f Data in Salt Water Environment for Annealed Steel, 0.37% Carbon (21)

stress amplitude S_i for i = 1,2,..., the total cumulative damage fraction is taken to be

$$D_f = \sum_{i=1,2,\ldots} \left(\frac{n_f}{N_f}\right)_i \tag{5.3}$$

Fatigue failure occurs when $D_f = 1$.

From the above theory of fatigue failure and from Masubuchi's [19] observation that when the applied stress is lowered, the number of cycles to failure is increased, it can be concluded that when member stress levels are lowered the time to failure T_f is increased, thus increasing the life of the structure. Equation 5.1 shows this for the single stress level case. As N_{f1} increases so does T_f. For the random stress level case, Equation 5.3 shows that as the N_{fi} become larger, D_f decreases, requiring more n_{fi} before failure will occur, which in turn increases T_f.

As a corollary to the above conclusion, the time to failure can be held constant, if as stress levels are decreased, member cross sections are reduced to restore old stress levels. This allows reduction in the material used to combat fatigue affects.

The effect of a tuned mass damper on fatigue in offshore platforms was checked by loading the 1000 foot model with sea states generated by windspeeds of 30 and 40 mph (see Table 2.5). These sea states were chosen because they occur in the North Sea from 40 to 45 percent of the time [1]. RMS top element strain is the response examined as strain is a direct representation of member stresses.

The model was excited by both sea states with and without the mass damper in operation. The damper parameters used were those listed in Table 4.1. The results were normalized to the RMS strains of the structure without the mass damper and are tabulated in Table 5.2.

Note the significant reduction in RMS top element strain for all values of μ in both sea states. This reduction shows the mass damper to be very effective in reducing member stress levels which contribute to cyclic fatigue failure. Therefore it appears that the mass damper could be quite useful in increasing the fatigue life of offshore platforms or in reducing the material required in platforms to combat fatigue, thus reducing the cost of these platforms.

Again the travel of the damper mass has not been considered here, but the amount of strain reduction found allows for sufficient design margin to include allowances for damper travel.

5.3 Soil Degradation under Gravity Platforms

As outlined in Section 1.1, weakening of the soil under large gravity platforms can occur when cyclic waveloading causes pore water pressures in the soil to increase beyond a critical limit. If the loading continues, the soil becomes weaker until finally the soil liquifies and loses all bearing capacity. Several authorities [2,3,4] have done calculations in this area and they have shown that over a period of storms it is possible for the strength of the soil beneath a gravity platform such as those in operation in the

Table 5.2

Comparison of RMS Top Element Strain for 30 and 40 MPH Sea States

μ	RMS Top Element Strain*	
	30 MPH Sea State	4C MPH Sea State
0	1.0	1.0
.01	.703	.703
.03	.584	.586
.05	.529	.533

*Normalized to RMS Top Element Strain of Each Sea State without Mass Damper

North Sea to be reduced. As mentioned in Section 1.1, when this occurs the weakened foundation causes the fundamental period of the structure to increase into the range where dynamic amplification of the peak load would occur, thereby increasing the ultimate load on the structure, perhaps beyond the design load.

Analysis of soil degradation depends on three variables: the soil, the structure, and the storm loading. To determine what soil degradation could occur beneath the Condeep Brent B platform and what effect this could have on the response of the platform, preliminary analysis was carried out which used a soil model approximating that found under the Brent B platform [20], the Condeep Model described in Table 2.1, and a one-hundred year return storm described by Lee and Focht [21].

The cyclic loads on the soil were determined by loading the Condeep model with the one-hundred year return storm. These loads were then applied in a conservative manner to the soil model [22], to determine the reduction in strength of the soil after the storm. The shear modulus of the soil decreased from 5000 T/m^2 prior to the storm to 470 T/m^2 after the storm.

The shear modulus of the soil was then varied and the fundamental period of the Condeep model was determined for each value of the shear modulus. The results are plotted in Figure 5.2.

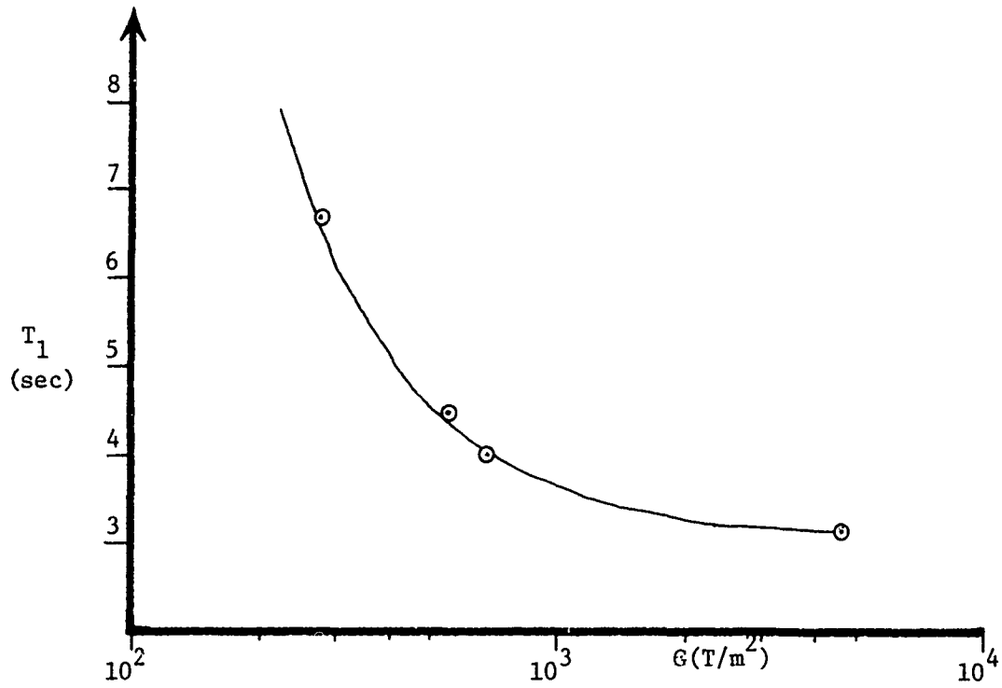


Figure 5.2

Fundamental Period vs. Shear Modulus, Condeep

To check the effectiveness of the tuned mass damper in reducing stresses caused by loads which increase as the soil degrades, the Condeep model was excited by a one hundred year return sea state after its soil had degraded to a shear modulus of 470 T/m^2 which corresponds to a fundamental period of 4.65 seconds. These results were compared to the response of the Condeep model with normal soil conditions ($G = 5000\ T/m^2$, $T_1 = 3.0$ sec.) when excited by the same sea state. The results, normalized to the RMS responses of the Condeep under normal conditions, are listed in Table 5.3.

The RMS deck displacement has increased 1.609 times and the RMS top element strains have increased 1.239 times. Assuming the platform was designed for 100 year return conditions, the response of the platform with degraded soil is greater than allowed by design. To examine the effect of the mass damper on the response, the Condeep with degraded soil was again excited with the 100 year sea state, but with the mass damper in operation. The results for three values of μ are given in Table 5.3. Note that the damper was not able to reduce RMS displacements below design value, but the RMS strains, which are more closely related to member stresses, were reduced below design levels for all values of μ. The reduction, however, for $\mu = .01$ is only slightly below design values. If damper mass travel is considered, the long travel distances which accompany small mass ratios would probably make the use of dampers with small μ prohibitive. The results therefore show the damper to be effective only if strains are of primary importance and heavier mass ratios are used.

Table 5.3

Comparison of Platform Response when Subject

to 100 Year Return Sea State

Model	μ	RMS Deck Displacement*	RMS Top Element Strain**
Condeep (G=5000, T=3.0)	0	1.0	1.0
Condeep (G=470, T=4.65)	0	1.609	1.239
Condeep (G=470, T=4.65)	.01	1.292	.983
Condeep (G=470, T=4.65)	.03	1.134	.717
Condeep (G=470, T=4.65)	.05	1.055	.603

*Normalized to Condeep (G=5000, T=3.0) RMS Deck Displacement

**Normalized to Condeep (G=5000, T=3.0) RMS Top Element Strain

CHAPTER 6

SUMMARY AND CONCLUSIONS

6.1 Summary of Findings

This thesis has examined the tuned mass damper and its potential employment on an offshore platform. This examination was conducted using finite element models of three offshore platforms and discretized frequency varying spectra of three sea states.

This analysis showed that:

a) When the mass damper was applied to the platform models, the response of the platforms was reduced. The largest reductions occurred when the damper parameters were set at $\mu = .05$, $\omega_a/\Omega n = .926$ and $\beta_a = .1675$.

b) The mass ratio μ, has the greatest effect on response reduction. As the mass ratio is increased from $\mu = .01$ to $\mu = .05$ (holding other parameters constant) the RMS deck displacement reduction changes from 23.6% to 28.6%.

c) The damper mass travel, an important design consideration, varies inversely with the mass ratio and is most affected by the damping in the mass damper.

d) The mass damper on offshore platforms in water depths greater than 1000 feet can reduce the RMS deck displacement by as much as 29.9% and the RMS strain by as much as 43.3%.

e) The mass damper can reduce the RMS strain in steel-jacketed offshore platforms subject to adverse fatigue conditions by as much as 47.1% in a 30 MPH sea and 46.7% in a 40 MPH sea.

f) When the mass damper is applied to a gravity platform subject to soil degradation the RMS strains can be reduced to levels below those for which the platform is designed.

6.2 Conclusions

The analysis in this thesis has shown the tuned mass damper on an offshore platform may be effective in reducing the high costs of platforms designed for water depths greater than 1000 feet, and fatigue in steel-jacketed platforms. It has shown the tuned mass damper to be of limited effectiveness in overcoming problems caused by soil degradation beneath gravity platforms.

While these results are based on the use of simple models, they are an initial assessment of the feasibility and effectiveness of applying a tuned mass damper to an offshore platform. The results are very encouraging. The results indicate that more detailed analyses should be undertaken, analyses which include details of specific platforms, and the economic aspects of the damper.

6.3 Areas for Further Study

An area of further study is the economics of the tuned mass damper applied to the offshore platform. A central question is - are the costs of the damper system less than the savings realized?

Another area requiring study is the effect the tuned mass damper has on reducing soil degradation and not just reducing the

effects of soil degradation.

Lastly, an area not covered in this thesis is the specific design of the mass damper for an offshore platform and the possible use and tuning of entrapped liquids in offshore platforms as damping devices.

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APPENDIX A

COMPUTER PROGRAM LISTING

```
C      *  MAIN PROGRAM  *                                                    MP 00
       COMPLEX A,B
       REAL  MASS, LENGHT
       DIMENSION  TOWER (20),  FREQ (100),          MASS (200),  STIFF(2    MP 01
      *00),   A (200),  B(50),  DIS (50),  VEL (50),  ACC (50),  R (50),    MP 01A
      *S(50),H(50),PHASE(50),Q(500),P(50)                                    *
       DATA  IR, IW  /8,5/                                                  MP 03
C      INPUT DATA                                                           MP 05
       CALL INPUT  (IR, IW, TOWER, NELEM, NFREQ,                            MP 06
      *HEIGHT,ZO,REB,RET,RIB,RIT,DEPTH,E,RO,DMASS,DINER,FREQ,               MP 06A
      *H,PHASE,DT,TIME,NRIGD,KQ,KD,KM,ALPHA,BETAV,BETAH,DK,DM,BETAD,KNW)
C                                                                           MP 07
C      STIFFNESS AND MASS MATRICES                                          MP 08
       CALL ASMBL  (NELEM, NEQ, MBW,                                        MP 09
      *HEIGHT, REB, RET, RIB, RIT, E, RO, DMASS, DINER, STIFF,MASS,KQ,
      *NRIGD,IR,IW,DEPTH,KD,DM,DK,KM,LENGHT,DA,DB)
C      TIME OR FREQUENCY DOMAIN ANALYSIS
       IF ( NFREQ.GE.0 ) GO TO 12                                           MP 12
       NFREQ = - NFREQ                                                      MP 12A
C      FREQUENCY DOMAIN ANALYSIS                                                *
       CALL TF  (IW,TOWER,NELEM,HEIGHT,ZO,RET,REB, DEPTH,                  MP 14
      *NEQ,MBW,STIFF,MASS,A,B,FREQ,KQ,NRIGD,H,BETAV,BETAH,ALPHA,
      *KD,LENGHT,DA,DB,RIT,S,BETAD,DK,DM,NFREQ,KNW)
       STOP                                                                 MP 15A
C      DYNAMIC ANALYSIS IN TIME DOMAIN                                      MP 16
   12  IF  ( NFREQ  .EQ.  0 )  STOP                                         MP 16A
       CALL TIME H  ( IW , NEQ , MBW , NFREQ , NELEM ,                      MP 16B
      *RET , REB , HEIGHT , ZO , DEPTH , DT , TIME ,                        MP 16C
      *STIFF , MASS , FREQ , A , B , H , PHASE ,                            MP 16D
      *DIS , VEL , ACC , R , S, P , Q,KQ,NRIGD          )
       STOP                                                                 MP 17
       END                                                                  MP 18
```

```
C     SUBROUTINE  INPUT                                                    SI 00
      SUBROUTINE  INPUT  (IR, IW, TOWER, NELEM, NFREQ,                     SI 01
     *HEIGHT, Z0, REB,RET,RIB,RIT, DEPTH ,E,R0,DMASS,DINER,FREQ,           SI 02
     *H,PHASE,DT,TIME,NRIGD,KQ,KD,KM,ALPHA,BETAV,BETAH,DK,DM,BETAD,KNW)
      DIMENSION  TOWER(20) , FREQ(1) , H(1) , PHASE(1)                     SI 03
C                                                                          SI 05
C     READ TOWER AND CONTROL CARD                                          SI 06
      READ (IR,1) TOWER,NELEM,NFREQ,NRIGD,KQ,KD,KM,KNW
      WRITE  (IW,2 )  TOWER,   NELEM,  NFREQ                               SI 08
      IF (KQ.NE.0)  WRITE(IW,50)
   50 FORMAT(///5X, ' EARTHQUAKE OPTION IS IN EFFECT')
      IF (KD.NE.0) WRITE(IW,51)                                                *
   51 FORMAT (//5X, ' DAMPER OPTION IS IN EFFECT')
C                                                                          SI 09
C     READ OVERALL DIMENSIONS                                              SI 10
      READ  (IR, 3 )  HEIGHT, Z0, REB, RET, RIB, RIT, DEPTH               SI 11
      WRITE (IW,4)  HEIGHT, Z0, REB, RET, RIB, RIT,  DEPTH                SI 12
C                                                                          SI 13
C     READ MATERIAL PROPERTIES                                             SI 14
      READ   (IR, 3 )  E,  R0                                              SI 15
      WRITE  (IW ,5 )  E,  R0                                              SI 16
C                                                                                *
C     READ ALPHA, BETAV, BETAH FOR DAMPING. IF NO DAMPING READ IN ZERO V       ALUE
      READ(IR,3) ALPHA, BETAV, BETAH                                           *
      WRITE (IW,21) ALPHA, BETAV, BETAH
C                                                                          SI 17
C     READ DECK LOAD                                                       SI 18
      READ  (IR,3)  DMASS , DINER                                          SI 19
      WRITE  (IW,6)   DMASS, DINER                                         SI 20
C                                                                                *
      IF(KD .EQ. 0) GO TO 22                                                   *
      READ(IR,41) DM, OMEGA, BETAD                                             *
      DK=OMEGA**2*DM
```

```
      WRITE(IW,42) DK,DM,OMEGA,BETAD                                    *
   41 FORMAT (3F8.4)
   42 FORMAT(4X, 'DAMPING PARAMETERS'/2X, 'SPRING CONSTANT =',F10.4/2X,  *
     * 'DAMPER MASS  =',F8.4/2X, 'DAMPER FREQUENCY (RAD/SEC)  =',F8.4/
     *2X, ' PERCENT CRITICAL DAMPING  =',F8.4)                          *
   22 CONTINUE                                                          *
C     READ FREQUENCIES                                                 SI 21
      IF  ( NFREQ .GE.  0 )  GO  TO  10                                SI 22
C     READ FREQUENCIES FOR STATIC RESPONSE IN FREQUENCY DOMAIN         SI 23
      N = -NFREQ                                                       SI 24
      READ (IR,3) (FREQ(I),H(I), I= 1,N )
      WRITE (IW,7) (FREQ(I),H(I), I= 1,N )                             *
      RETURN                                                           SI 27
C     READ TIME AND TIME INTERVAL PLUS CONDENSED SPECTRUM PARAMETERS   SI 28
C     FOR DYNAMIC ANALYSIS                                             SI 28A
   10 IF  ( NFREQ  .LE.  0 )  GO  TO  12                               SI 29
      READ  (  IR , 3 )  DT , TIME                                     SI 29A
      WRITE  ( IW , 8 )  DT , TIME                                     SI 29B
      DO  11  I = 1,NFREQ                                              SI 30
   11 READ  ( IR , 3 )   FREQ(I) , H(I) , PHASE(I)                     SI 30A
      WRITE  ( IW , 9 ) (FREQ(I) , H(I) , PHASE(I) ,  I = 1,NFREQ  )   SI 30B
   12 RETURN                                                           SI 31
C                                                                      SI 32
    1 FORMAT (20A4,/, 7I4)                                             *
    2 FORMAT   ('1',/,5X,  20A4,//,                                    SI 34
     * 5X, I5, ' ELEMENTS' , / , 5X, I5, ' FREQUENCIES' ,/)           SI 35
    3 FORMAT (10F8.2)
    4 FORMAT (///5X, '*OVERALL DIMENSIONS*'//                          SI 37
     *5X, 'TOWER HEIGHT      ', F8.2, ' M.'/                           SI 38
     *5X, 'CAISSON HEIGHT    ', F8.2, ' M.'//                          SI 39
     *5X, 'EXTERNAL RADIUS AT THE BOTTOM   ', F7.2, ' M.'/             SI 40
     *5X, 'EXTERNAL RADIUS AT THE TOP      ', F7.2, ' M.'/             SI 41
     *5X, 'INTERNAL RADIUS AT THE BOTTOM   ', F7.2, ' M.'/             SI 42
```

```
     *5X, 'INTERNAL RADIUS AT THE TOP      ', F7.2, ' M.'//                    SI 43
     *5X, 'DEPTH OF WATER    ', F8.2, ' M.'/)                                  SI 44
   5 FORMAT (///5X, '*MATERIAL PROPERTIES*'//                                  SI 45
     *5X, 'E  =', E10.3/5X,'RO =',  E10.3/)                                    SI 46
   6 FORMAT (///5X, 'DECK MASS       ', E12.4/                                 SI 47
     *                5X, 'DECK INERTIA    ', E12.4/)                         SI 48
   7 FORMAT(///5X, '*CONDENSED SPECTRUM PARAMETERS * '///3X,' FREQ' ,8X      ,
     *' WAVE HEIGHT'//(3X,2F15.4))                                                *
   8 FORMAT  (///5X ,  ' * TIME INTERVAL AND TOTAL TIME * '//5X ,              SI 50
     *'DELTA T =  ', F7.3/5X , 'TIME    =  ', F7.3/  )                         SI 51
   9 FORMAT  ( ///5X , ' * CONDENSED SPECTRUM PARAMETERS * '//13X ,            SI 52
     *'FREQ' , 8X , 'WAVE HEIGHT' , 3X , 'PHASE ANGLE'//(3X , 3F15.4))        SI 52A
  21 FORMAT(///5X,' * DAMPING COEFFICIENTS *'//5X,' ALPHA =  ',E10.3/5X      ,     *
     *' BETAV =  ',E10.3/5X,' BETAH =  ',E10.3/)                                  *
C                                                                              SI 53
      END                                                                      SI 54
```

```
C     SUBROUTINE  ASMBL                                                    SAS 00
      SUBROUTINE  ASMBL  (NELEM, NEQ, MBW,                                 SAS 01
     *HEICHT, REB, RET, RIB, RIT, E, RO, DMASS, DINER, STIFF,MASS,KQ,
     *NRIGD,IR,IW,DEPTH,KD,DM,DK,KM,LENGHT,DA,DB)                                *
      REAL  MASS,LENGHT,MG                                                       *
      DIMENSION STIFF(200),MASS(200),A(10),AMAS(12,12),STIF(12,12)               *
C                                                                          SAS 06
C     INITIALIZE                                                           SAS 07
      NEQ = 2*(NELEM +1 )                                                  SAS 08
      IF(KQ.NE.0) NEQ=NEQ+2
      IF (KD .NE. 0) NEQ=NEQ+1
      MBW = 4                                                              SAS 09
      LIM = NEQ * MBW                                                      SAS 10
      DO  10  I = 1,LIM                                                    SAS 11
      STIFF(I) = 0.                                                        SAS 12
   10 MASS (I) = 0.                                                        SAS 13
C                                                                          SAS 14
C     COMPUTE ELEMENT MATRICES AND ASSEMBLE                                SAS 15
      DN = NELEM                                                           SAS 16
      LENGHT = HEIGHT/DN                                                   SAS 17
      DA = (RET-REB)/DN                                                    SAS 18
      DB = (RIT-RIB) / DN                                                  SAS 19
      A1 = REB                                                             SAS 20
      B1 = RIB                                                             SAS 21
      EMASS = DMASS                                                        SAS 22
      IF (KD .NE. 0) EMASS= EMASS + DM                                            *
C                                                                          SAS 23
      DO  12   N = 1, NELEM                                                SAS 24
      A2 = A1 + DA                                                         SAS 25
      B2 = B1 + DB                                                         SAS 26
      CALL SUBK  (E,A1,B1,A2,B2,LENGHT, A  )                               SAS 27
      CALL ADD  (NEQ  ,STIFF , N , A  )                                    SAS 28
      CALL       SUBM  (RO,A1,B1,A2,B2,LENGHT, EMASS, A      )             SAS 29
```

```
      CALL ADD  (NEQ  ,MASS,   N,    A    )                              SAS 30
      A1 = A2                                                            SAS 31
   12 B1 = B2                                                            SAS 32
C                                                                        SAS 33
C     ADD DECK MASS                                                      SAS 34
      I = 2 * NELEM                                                      SAS 35
      MASS  (I+1) = MASS(I+1)  +  DMASS                                  SAS 36
      MASS (I+2) = MASS (I+2) + DINER                                    SAS 37
C                                                                        SAS 38
      A1 = REB                                                           SAS 39
      B1 = RIB                                                           SAS 40
      DO  14  M = 1, NELEM                                               SAS 41
      A2 = A1 + DA                                                       SAS 42
      B2 = B1 + DB                                                       SAS 43
      CALL SUBKG (RO, REB, RIB, A1, B1, A2, B2, LENGHT, EMASS, M,A )     SAS 44
      CALL ADD (NEQ, STIFF, M, A)                                        SAS 45
      A1 = A2                                                            SAS 46
   14 B1 = B2                                                            SAS 47
      IF(NRIGD.NE.0) GO TO 35
      READ (IR,36) GS,ROS,HH,CC,DC,PNU,BL,BR,ROC
      WRITE (IW,37) GS,PNU,ROS,HH,CC,DC,BL,BR,ROC
   36 FORMAT(9F8.4)
   37 FORMAT(4X,'SOIL PROPERTIES'/2X,'SHEAR MODULAS  =',F8.2/2X,'POI
     1SSONS RATIO  =',F8.2/2X,'DENSITY        =',F8.2//4X,'DIMENSIONS OF
     2 CAISSON'/2X,'HEIGHT       =',F8.2/2X,'WIDTH      =',F8.2/2X,'LENG
     3TH     =',F8.2//2X,'MASS RATIO, HORIZONTAL  =',F8.2/2X,'MASS RATIO
     4, ROCKING    =',F8.2/2X,'MASS RATIOS FOUND FROM GRAPH,P.351,RICHAR
     5T'/2X,'DENSITY OF CAISSON =',F8.2)
C     ***   CALCULATE KL AND KO   ***************
      SKL=2.0*(1.0+PNU)*GS*BL*(CC*DC)**0.5
      SKO=(GS/1.0-PNU)*BR*CC*(DC**2)
C     ***   MASS AND INERTIA OF CAISSON   *******
      CMASS=ROC*CC*DC*HH
```

```
      CINER=((DC*(HH/2.)**3)/12.)*ROC*CC
      FT=EMASS*9.81
      WRITE (IW,42) FT
   42 FORMAT (2X,'LOAD ON CAISSON =',E10.2/)
C     ***   EARTHQUAKE OPTION   *****************
      IQ=0
      IF(KQ.NE.0) IQ=2
      I=1+IQ
      J=2+IQ
      K=NEQ+1+IQ
C     ***   MODIFY MATRICES   ********************
      STIFF(I)=STIFF(I)+SKL
      STIFF(J)=STIFF(J)+SKL*HH*HH+SKO-FT*HH
      STIFF(K)=STIFF(K)+SKL*HH
      MASS(I)=MASS(I)+CMASS
      MASS(J)=MASS(J)+HH*HH*CMASS/2.0+2.0*(CINER+(HH/2.0)*DC*(HH/4.0)**2
     *)
      MASS(K)=MASS(K)+HH*CMASS/2.0
      WRITE (IW,39) GS,SKL,SKO,CMASS
   39 FORMAT (2X,'SHEAR MODULUS  =',E10.2/2X,'EFFECTIVE SPRING CONSTANTS
     1'/2X,'HORIZONTAL  ',4X,E10.2/2X,'ROTATIONAL   ',4X,E10.2/2X,'MASS
     2 OF CAISSON ',4X,E10.2)
   35 IF (KD .EQ. 0) GO TO 40
C     MODIFY MATRICES FOR MASS DAMPER
      I=NEQ-2
      J=NEQ
      K=3*NEQ-2
      STIFF(I)=STIFF(I)+DK
      STIFF(J)=DK
      STIFF(K)=-DK
      MASS(J)=DM
   40 CONTINUE
      IF(KQ.EQ.0) GO TO 41
```

```
      STIFF(1)=SKL
      STIFF(2)=SKO
C     ***   MODIFICATIONS FOR MASS MATRIX    *****
      REQM=((DC*CC)/3.14159)**0.5
      REQI=(DC*CC**3)/(3.0*3.14159)**0.25
      MG=(0.77*ROS*(REQM**3))/(2.0-PNU)
      YG=(1.92*ROS*(REQI**5))/(3.0*(1.0-PNU))
      MASS(1)=MG
      MASS(2)=YG
   41 IF(KM.EQ.0) GO TO 43
C     PRINT OUT MASS AND STIFFNESS MATRICES
      WRITE(IW,29)
   29 FORMAT(//10X, ' MASS MATRIX'//)
      DO 100  I=1,NEQ
      DO 75 J=1,NEQ
   75 AMAS(I,J)=0.
  100 CONTINUE
      DO 101  I=1,NEQ
      K=I+3
      DO  76   J=I,K
      IJ=(J-I)*NEQ+I
   76 AMAS(I,J)=MASS(IJ)
  101 CONTINUE
      DO 102  I=1,NEQ
      K=I+1
      L=I+3
      DO 77 J=K,L
   77 AMAS(J,I)=AMAS(I,J)
  102 CONTINUE
      DO 30  I=1,NEQ
      WRITE (IW,31)(AMAS(I,J),J=1,NEQ)
   31 FORMAT(/11(1X,F10.2))
   30 CONTINUE
```

```
      WRITE (IW,32)
   32 FORMAT(//10X, ' STIFFNESS MATRIX'//)
      DO 103  I=1,NEQ
      DO 78 J=1,NEQ
   78 STIF(I,J)=0.
  103 CONTINUE
      DO 104  I=1,NEQ
      K=I+3
      DO 79 J=I,K
      IJ=(J-I)*NEQ+I
   79 STIF(I,J)=STIFF(IJ)
  104 CONTINUE
      DO 105  I=1,NEQ
      K=I+1
      L=I+3
      DO 80 J=K,L
   80 STIF(J,I)=STIF(I,J)
  105 CONTINUE
      DO 28 I=1,NEQ
      WRITE (IW,27)(STIF(I,J),J=1,NEQ)
   27 FORMAT(/11E10.3)
   28 CONTINUE
   43 RETURN
      END                                                    SAS 49
```

```
C       SUBROUTINE  SUBK                                                    SK 00
        SUBROUTINE  SUBK  (E,A1,B1,A2,B2,LENGHT,STIFF)                      SK 01
        REAL  LENGHT
        DIMENSION  A(5), W(5),  F(4),  STIFF(10)                            SK 03
        DATA A,W / 0., -.538469,  .538469,  -.906180,  .906180,  .568889,   SK 04
       *2*.478629,  2*.236927            /                                  SK 04A
        DO  1  I = 1,10                                                     SK 05
      1 STIFF(I) = 0.                                                       SK 06
        DO  2    N = 1,5                                                    SK 07
        X = A(N)                                                            SK 08
        X = .5*(X+1.)                                                       SK 09
        F(1) = 12.*X-6.                                                     SK 10
        F(3) = -F(1)                                                        SK 11
        F(2) = (-4.+ 6.*X)*LENGHT                                           SK 12
        F(4) = (-2.+6.*X)*LENGHT                                            SK 13
        AA = A1*(1.-X)+A2*X                                                 SK 14
        B = B1*(1.-X)+B2*X                                                  SK 15
        C = 0.7853981 * (AA**4-B**4)/LENGHT**3/2.*W(N)*E                    SK 16
        IJ = 0                                                              SK 17
        DO  2  J = 1,4                                                      SK 18
        DO  2  I = 1,J                                                      SK 19
        IJ = IJ+1                                                           SK 20
      2 STIFF(IJ) = STIFF(IJ) + C*F(I)*F(J)                                 SK 21
        RETURN                                                              SK 22
        END                                                                 SK 23
```

```
C       SUBROUTINE ADD                                                  SA 00
        SUBROUTINE  ADD  ( NEQ , A , N , B  )                           SA 01
        DIMENSION  A(1) , B(1)                                          SA 02
        NN = 2*(N-1)                                                    SA 03
        KL = 0                                                          SA 04
        DO  10  J=1,4                                                   SA 05
        DO  10  I=1,J                                                   SA 06
        II =  J-I                                                       SA 07
        IJ = NEQ*II+NN+I                                                SA 08
        KL = KL+1                                                       SA 09
   10   A(IJ) = A(IJ) + B(KL)                                           SA 10
        RETURN                                                          SA 11
        END                                                             SA 12
```

```
C     SUBROUTINE SUBM                                                    SM 00
      SUBROUTINE SUBM  (RO,A1,B1,A2,B2,LENGHT,EMASS,MASS)                SM 01
      REAL   LENGHT, MASS                                                    *
      DIMENSION  A(5), W(5),  F(4),    MASS(10)                          SM 03
      DATA A,W / 0., -.538469,  .538469,  -.906180,  .906180,  .568889,  SM 04
     *2*.478629 ,    2*.236927        /                                  SM 04A
      DO  1  I = 1,10                                                    SM 05
    1 MASS (I) = 0.                                                      SM 06
      DO  2    N = 1,5                                                   SM 07
      X = A(N)                                                           SM 08
      X = .5*(X+1.)                                                      SM 09
      F(3) = X*X*(3.-2.*X)                                               SM 10
      F(1) = 1.-F(3  )                                                   SM 11
      F(2) = X*LENGHT*(1.-2.*X+X*X)                                      SM 12
      F(4) = X*X*LENGHT*(X-1.)                                           SM 13
      AA = A1*(1.-X)+A2*X                                                SM 14
      B = B1*(1.-X)+B2*X                                                 SM 15
      C = 3.14159265*(AA*AA-B*B)*RO*LENGHT/2.*W(N)                       SM 16
      EMASS = EMASS+C                                                    SM 16A
      IJ = 0                                                             SM 17
      DO  2  J = 1,4                                                     SM 18
      DO  2  I = 1,J                                                     SM 19
      IJ = IJ+1                                                          SM 20
    2 MASS (IJ) = MASS(IJ) + C* F(I)  * F(J)                             SM 21
      RETURN                                                             SM 22
      END                                                                SM 23
```

```
C
C       SUBROUTINE  SUBKG                                                       SKG 00
        SUBROUTINE SUBKG (RO, REB, RIB, A1, B1, A2, B2, LENGHT, EMASS, M,       SKG 01
       *                                  STIFG)                               SKG 02
        REAL   LENGHT                                                           SKG 03
        DIMENSION  A(5),W(5),F(4),STIFG(10)                                     *
        DATA A, W / -.906180, -.538469, 0. , .538469, .906180, .236927,         SKG 05
       *.478629, .568889, .478629, .236927  /                                   SKG 06
        DM = M   -1                                                             SKG 07
        DO  1   I = 1,10                                                        SKG 08
      1 STIFG(I) = 0.                                                           SKG 09
        DO  2  N = 1,5                                                          SKG 10
        X = A(N)                                                                SKG 11
        X = .5*(X+1.)                                                           SKG 12
        F(1) = X*6.*(X-1.)                                                      SKG 13
        F(2) = (1.-4.*X+3.*X*X)*LENGHT                                          SKG 14
        F(3) =  -F(1)                                                           SKG 15
        F(4) = (3.*X-2.)*LENGHT*X                                               SKG 16
        A3 = (A2-A1)*X + A1                                                     SKG 17
        B3 = (B2-B1)*X + B1                                                     SKG 18
        QMASS = RO * 3.14159265 * ( DM + X ) * LENGHT/ 3. * (( REB * REB        SKG 19
       *+ A3*REB + A3*A3 ) - (RIB * RIB + B3*RIB + B3*B3))                      SKG 20
        P = 9.81 *  ( EMASS - QMASS )           /LENGHT * W(N) / 2.              SKG 21
        IJ = 0                                                                  SKG 22
        DO  2  J = 1,4                                                          SKG 23
        DO  2  I = 1,J                                                          SKG 25
        IJ = IJ + 1                                                             SKG 26
      2 STIFG(IJ) = STIFG(IJ) + P*F(I)*F(J)                                     SKG 27
        RETURN                                                                  SKG 28
        END                                                                     SKG 29
                                                                                SKG 30
```

```
C                                                                       TF 00
C       SUBROUTINE  TF                                                  TF 01
        SUBROUTINE TF  (IW,TOWER,NELEM,HEIGHT,ZO, RET,REB,DEPTH,        TF 02
       *NEQ,MBW,STIFF,MASS,A,B,FREQ,KQ,NPIGD,H,BETAV,BETAH,ALPHA,
       *KD,LENGHT,DA,DB,RIT,S,BETAD,DK,DM,NFREQ,KNW)                        *
        COMPLEX A,B,C,G,F,T,R,Q                                             *
        REAL    MASS,LENGHT                                                 *
        DIMENSION TOWER(20),STIFF(1),MASS(1),A(1),B(10),P(4),S(1),US(15),
       *VS(15),HS(15),SS(5),FREQ(1),H(1)                                    *
        J=NELEM+1                                                           *
        DO 100 I=1,J                                                        *
        US(I)=0.0                                                           *
        VS(I)=0.0                                                           *
        HS(I)=0.0                                                           *
  100   SS(I)=0.0                                                           *
        USD=0.0                                                             *
        HSD=0.0                                                             *
        DO 99 L=1,NFREQ                                                     *
        W=6.28318531*FREQ(L)
        W2=W*W
C                                                                       TF 07
C       FORM COEFFICIENT MATRIX                                         TF 08
        LIM = NEQ*MBW                                                   TF 09
        D=W*BETAV+2.*BETAH
        G=CMPLX(1.,D)
        F=CMPLX(-W2,W*ALPHA)
        DO  1  I = 1,LIM                                                TF 10
      1 A(I)=G*STIFF(I)+F*MASS(I)
C                                                                       TF 12
        IF(KD.EQ.0) GO TO 15
        CD=W*2.0*BETAD*SQRT(DK)*SQRT(DM)
        T=CMPLX(0.,CD-D*DK)
        R=CMPLX(0.,-CD+D*DK)
```

```
      X=D*DK+W*ALPHA*DM
      Q=CMPLX(0.,CD-X)
      A(NEQ-2)=A(NEQ-2)+T
      A(NEQ)=A(NEQ)+Q
      A(3*NEQ-2)=A(3*NEQ-2)+R
   15 CONTINUE
C
C     FORM LOAD VECTOR                                                    TF 13
      DO  2  I = 1,NEQ                                                    TF 14
    2 B(I) = 0.                                                           TF 15
      DN = NELEM                                                          TF 16
      LENGHT = HEIGHT/DN                                                  TF 17
      DA = ( RET-REB)/DN                                                  TF 19
      A1 = REB                                                            TF 21
      DO  4  N = 1,NELEM                                                  TF 23
      A2 = A1 + DA                                                        TF 24
      CALL SUBP  (N, A1, A2, LENGHT, ZO, DEPTH, W2,   P )                 TF 25
      J = 2*(N-1)                                                         TF 26
      DO  3  I = 1,4                                                      TF 27
      J = J+1                                                             TF 28
      C=P(I)*H(L)                                                         TF 29
    3 B(J)=B(J)+C
    4 A1 = A2
C                                                                         TF 31
C     IMPOSE DISPLACEMENT BOUNDARY CONDITIONS                             TF 32
      IF(NPIGD.EQ.0) GO TO 9                                              TF 33
      DO  5  I = 1, LIM, NEQ
      A(I) = 0.                                                           TF 34
    5 A(I+1) = 0.                                                         TF 35
      B(1) = 0.                                                           TF 36
      B(2) = 0.                                                           TF 37
C                                                                         TF 38
C     SOLVE SYSTEM OF EQUATIONS                                           TF 39
                                                                          TF 40
```

```
      9 CALL SOLVE (0,IW,NEQ,MBW,1,A,B,LIM,NELEM,RET,RIT,LENGHT,DA,DB,S)
C                                                                       TF 42
C     PRINT OUT STEADY - STATE RESPONSE                                 TF 43
      IF (KNW.NE.0) GO TO 103
      FREQ=FREQ(L)
      WRITE(IW,6) TOWER,FREQ
  103 CONTINUE
      I2 = NEQ                                                          TF 45
      IF (KD.EQ.0) GO TO 10
      HACC=-(CABS(B(I2))*W2)
      Y=CABS(B(I2)-B(I2-2))
      USD=USD+Y**2
      HSD=HSD+HACC**2
      IF (KNW.NE.0) GO TO 101
      WRITE(IW,11) Y,HACC
   11 FORMAT (13X,E15.5,21X,E15.5)
  101 CONTINUE
      I2=I2-1
   10 CONTINUE
      NNODE = NELEM + 1                                                 TF 46
      DO  7  I = 1,NNODE                                                TF 47
      J = NNODE-I+1                                                     TF 48
      HACC=-(CABS(B(I2-1))*W2)
      U=CABS(B(I2-1))
      V=CABS(B(I2))
      US(J)=US(J)+U**2
      VS(J)=VS(J)+V**2
      HS(J)=HS(J)+HACC**2
      IF (KNW.NE.0) GO TO 102
      WRITE (IW,8) J,U,V,HACC
  102 CONTINUE
    7 I2 = I2-2                                                         TF 50
    6 FORMAT ('1'/5X,20A4//5X,'*STEADY-STATE RESPONSE AT ',F8.3, ' CPS*' TF 51
```

```
     *//5X,'NODE',6X, 'DISPLACEMENT',10X, 'ROTATION',10X, 'ACCELORATION'
     */)
    8 FORMAT(5X,I4,3(3X,E15.5))
      IF (KNW.NE.0) GO TO 104
      WRITE(IW,12) REQ
  104 CONTINUE
   12 FORMAT(/////5X,'*ELEMENT STRAINS AT ',F8.3, ' CPS*'//5X,'ELEMENT',
     *6X,'STRAIN'/)
      DO 13 I=1,NELEM
      SS(I)=SS(I)+S(I)**2
      IF (KNW.NE.0) GO TO 105
      WRITE(IW,14) I, S(I)
   14 FORMAT(5X,I4,3X,E15.5)
  105 CONTINUE
   13 CONTINUE
   99 CONTINUE
C     COMPUTE AND PRINT RESPONSE STANDARD DEVIATIONS
      WRITE(IW,107)
  107 FORMAT('1'/5X, 'RESPONSE STANDARD DEVIATIONS'//5X, 'NODE',6X,
     *'DISPLACEMENT',10X, 'ROTATION',10X, 'ACCELERATION'/)
      IF (KD.EQ.0) GO TO 108
      USD=SQRT(USD)
      HSD=SQRT(HSD)
      WRITE(IW,109) USD,HSD
  109 FORMAT(5X, 'DAMPER',2X,E15.5,21X,E15.5)
  108 CONTINUE
      J=NELEM+1
      DO 110  I=1,J
      US(I)=SQRT(US(I))
      VS(I)=SQRT(VS(I))
      HS(I)=SQRT(HS(I))
  110 WRITE(IW,8) I,US(I),VS(I),HS(I)
      WRITE(IW,111)
```

```
111 FORMAT(/////5X, 'STRAIN STANDARD DEVIATIONS'//5X, 'ELEMENT',5X,
   *'STRAIN'/)
    DO 112   I=1,NELEM
    SS(I)=SQRT(SS(I))
112 WRITE(IW,14) I,SS(I)
    RETURN                                                          TF 54
    END                                                             TF 55
```

```
C                                                                       RK 00
C     FUNCTION RK                                                       RK 01
      FUNCTION RK (W2,G,DEPTH)                                                *
      RK = W2/G                                                         RK 03
      IF  ( W2  .EQ.  0. )  RETURN                                      RK 04
      IF  ( RK * DEPTH  .LT.  10. )  GO  TO  8                          RK 05
      RK = 10. / DEPTH                                                  RK 05A
      RETURN                                                            RK 05B
    8 A2 = 0.                                                           RK 06
      DK = ( 2. - RK ) / 10.                                            RK 07
   10 A1 = A2                                                           RK 08
      B = RK * DEPTH                                                    RK 09
      A2 = W2 - RK * G *DSINH (B) /DCOSH(B)                             RK 10
      IF  ( A2  .EQ.  0.  )  RETURN                                     RK 11
      RK = RK + DK                                                      RK 12
      IF  ( A1/A2  .GE.  0. )  GO  TO  10                               RK 13
      RK = RK - 1.5 * DK                                                RK 14
      DK = DK / 2.                                                      RK 15
   12 IF  ( DK/RK  .LT.  1.E-4 )  RETURN                                RK 16
      B = RK * DEPTH                                                    RK 17
      A3 = W2 - RK * G *DSINH(B) /DCOSH(B)                              RK 18
      IF  ( A3  .EQ.  0. )  RETURN                                      RK 19
      DK = DK / 2.                                                      RK 20
      IF  ( A3 / A1  .GT.  0. )  GO  TO  14                             RK 21
      RK = RK - DK                                                      RK 22
      A2 = A3                                                           RK 23
      GO  TO  12                                                        RK 24
   14 RK = RK + DK                                                      RK 25
      A1 = A3                                                           RK 26
      GO  TO  12                                                        RK 27
      END                                                               RK 28
```

```
      FUNCTION DCOSH (A)                                              SP 02
      B =DEXP(A)                                                      SP 03
      DCOSH   =   .5*(B+1./B)                                         SP 04
      RETURN                                                          SP 05
      END
```

```
      FUNCTION DSINH (A)                                              *
      B =DEXP (A)                                                  SP 07
      DSINH   =  .5*(B-1./B )                                      SP 08
      RETURN                                                       SP 09
      END                                                          SP 10
```

```
C       SUBROUTINE SUBP                                                    SP 00
        SUBROUTINE SUBP ( N, A1, A2, LENGHT, Z0, DEPTH, W2,    P)          SP 11
        REAL    LENGHT,K                                                          *
        DIMENSION  A(5), W(5),                                    P(4)     SP 13
        DATA A,W / 0., -.538469,  .538469,  -.906180,  .906180,  .568889,  SP 14
       * 2*.478629,  2*.236927       /                                    SP 15
        DATA  CI, RO,  G /2.,  1. , 9.81  /                               SP 16
        K = RK  ( W2 , G, DEPTH )                                         SP 16A
C                                                                         SP 17
        DO  1  I = 1,4                                                    SP 18
      1 P(I) = 0.                                                         SP 19
        Z1 = Z0 + LENGHT *DFLOAT (N-1 )                                   SP 20
        C=3.1415926*CI*RO*W2/DSINH(K*DEPTH)                               SP 22
        DO  2  I = 1,5                                                    SP 23
        X = A(I)                                                          SP 24
        X = .5 * (X + 1. )                                                SP 25
        RADIUS = A1 * (1.-X) + A2 * X                                     SP 26
        Z = Z1 + LENGHT * X                                               SP 27
        IF  ( Z.  GT.  DEPTH  )  RETURN                                   SP 28
        F = C * RADIUS ** 2 *DCOSH (K*Z)    * W(I) / 2.*LENGHT            SP 29
        H = X*X*(3.-2.*X)                                                 SP 30
        P(1) = P(1) + (1.-H) * F                                          SP 31
        P(2) = P(2) + (X*LENGHT*(1.-2.*X + X*X))*F                        SP 32
        P(3) = P(3) + H * F                                               SP 33
        P(4) = P(4) + X*X*LENGHT*(X-1.) * F                               SP 34
      2 CONTINUE                                                          SP 35
        RETURN                                                            SP 36
        END                                                               SP 37
```

```
C     SUBROUTINE SOLVE                                                  SS 00
      SUBROUTINE SOLVE(IO,IW,NEQ,MBW,NLS,A,B,LIM,NELEM,RET,RIT,LENGHT,
     *DA,DR,S)                                                                *
      REAL   LENGHT
      COMPLEX A(1),B(1),C,D
      DIMENSION  S(1)
C                                                                       SS 04
C     * SOLVES SYMMETRIC SYSTEM OF EQUATIONS USING GAUSSIAN REDUCTION *  SS 05
C     IO     OPERATION INDICATOR   0 SOLVES SYSTEM A*X=B                SS 06
C                                  1 REDUCES MATRIX A                   SS 07
C                                  2 REDUCES AND BACKSUBSTITUTES B      SS 08
C     NEQ    NUMBER OF EQUATIONS                                        SS 09
C     MBW    MAXIMUM BANDWIDTH                                          SS 10
C     NLS    NUMBER OF SYSTEMS                                          SS 11
C     A      SUARE BANDED SYMMETRIC POSITIVE DEFINITE MATRIX            SS 12
C            STORED AS MONODIMENSIONAL ARRAY   NEQ*MBW                  SS 13
C            ELEMENT A(I,J) IS STORED IN A(I+(J-I)*NEQ)                 SS 14
C     B      RECTANGULAR ARRAY, STORED COLUMNWISE                       SS 15
C*************************************************************************
C                                                                       SS 16
C     REDUCTION OF A. ORIGINAL ARRAY IS DESTROYED                       SS 17
C                                                                       SS 18
      IF (IO.EQ.2) GO TO 20                                             SS 19
      NRD = NEQ-1                                                       SS 20
      DO  18  I = 1,NRD                                                 SS 21
      D = A(I)                                                          SS 22
      IF (CABS(D).EQ.0.) GO TO 18
      IJ=I                                                              SS 24
      DO 16 J=2,MBW                                                     SS 25
      IJ=IJ+NEQ                                                         SS 26
      IF (CABS(A(IJ)).EQ.0.) GO TO 16                                          *
      C=A(IJ)/D                                                         SS 28
      IK=IJ                                                             SS 29
```

```
      JK=I+J-1                                                          SS 30
      DO 14 K=J,MBW                                                     SS 31
      A(JK)=A(JK)-C*A(IK)                                               SS 32
      IK=IK+NEQ                                                         SS 33
   14 JK=JK+NEQ                                                         SS 34
   16 CONTINUE                                                          SS 35
   18 CONTINUE                                                          SS 36
      IF (ID.EQ.1) RETURN                                               SS 37
C                                                                       SS 38
C     REDUCTION OF B. ORIGINAL ARRAY IS DESTROYED                       SS 39
C                                                                       SS 40
   20 NRE=NEQ-1                                                         SS 41
      DO 26 I=1,NRE                                                     SS 42
      D=A(I)                                                            SS 43
      IF (CABS(D).EQ.0.) GO TO 26                                                *
      IJ=I                                                              SS 45
      DO 24 J=2,MBW                                                     SS 46
      IJ=IJ+NEQ                                                         SS 47
      IF (CABS(A(IJ)).EQ.0.) GO TO 24
      C=A(IJ)/D                                                         SS 49
      IK=I                                                              SS 50
      JK=I+J-1                                                          SS 51
      DO 22 K=1,NLS                                                     SS 52
      B(JK)=B(JK)-C*B(IK)                                               SS 53
      IK=IK+NEQ                                                         SS 54
   22 JK=JK+NEQ                                                         SS 55
   24 CONTINUE                                                          SS 56
   26 CONTINUE                                                          SS 55A
C                                                                       SS 57
C     BACKSUBSTITUTION. RESULTS STORED IN B.                            SS 58
C                                                                       SS 59
      I=NEQ                                                             SS 60
   30 IF (CABS(A(I)).EQ.0.) GO TO 34
```

```
      IK=I                                                              SS 62
      DO 32 K=1,NLS                                                     SS 63
      B(IK)=B(IK)/A(I)                                                  SS 64
   32 IK=IK+NEQ                                                         SS 65
   34 I=I-1                                                             SS 66
      IF (I.EQ.0) GO TO 40                                                  *
      IJ=I                                                              SS 68
      DO 38 J=2,MBW                                                     SS 69
      IJ=IJ+NEQ                                                         SS 70
      IF (CABS(A(IJ)).EQ.0.) GO TO 38                                       *
      IK=I                                                              SS 72
      JK=I+J-1                                                          SS 73
      DO 36 K=1,NLS                                                     SS 75
      B(IK)=B(IK)-A(IJ)*B(JK)                                           SS 76
      IK=IK+NEQ                                                         SS 77
   36 JK=JK+NEQ                                                         SS 78
   38 CONTINUE                                                          SS 79
      GO TO 30                                                          SS 80
C     SOLVE FOR STRAINS                                                     *
   40 I=(NELEM+1)*2
      DO 39  K=1,NELEM
      J=NELEM-K+1
      N=K-1
      D=RET+(FLOAT(N)*DA)+(DA/2.0)-RIT-(FLOAT(N)*DR)-(DR/2.0)
      S(J)=CABS((B(I)-B(I-2)))*D/(2.0*LENGHT)
      I=I-2
   39 CONTINUE
C
      RETURN                                                            SS 91
      END                                                               SS 92
```

```
C     SUBROUTINE TIME H                                                    TH 01
      SUBROUTINE TIME H  ( IW , NEQ , MBW , NFREQ , NELEM ,                TH 02
     *RET , REB , HEIGHT , ZO , DEPTH , DT , TIME ,                        TH 03
     *STIFF , MASS , FREQ , A , B , H , PHASE ,                            TH 04
     *DIS , VEL , ACC , R , S, P , Q,KQ,NRIGD                    )         MP 16E
      IMPLICIT  REAL  * 8 ( A-H , O-Z  )                                   TH 06
      REAL*8   MASS , LENGHT                                               TH 07
      DIMENSION  DIS( 1) , VEL ( 1) , ACC ( 2) , MASS ( 1) , STIFF ( 1),   TH 08
     *A(1) , P(1) , S(1) , B(2) , FREQ(1) , H(1) , PHASE(1) ,              TH 09
     *Q( NEQ , NFREQ )          , P(1)                                     TH 10
C                                                                          TH 11
      NTIME = TIME / DT                                                    TH 12
C                                                                          TH 13
C     FORM MATRIX OF COEFFICIENTS                                          TH 14
      C2 = DT * DT / 6.                                                    TH 15
      LIM = NEQ * MBW                                                      TH 16
      DO  50  I = 1,LIM                                                    TH 17
   50 A(I) = MASS(I) + C2 * STIFF(I)                                       TH 18
C                                                                          TH 19
C     PRESCRIBE FIRST TWO DOF AS ZERO                                      TH 20
      IF(NRIGD.EQ.0) GO TO 10
      DO  51  I = 1, LIM , NEQ                                            TH 21
      A(I) = 0.                                                            TH 22
   51 A(I+1) = 0.                                                          TH 23
C                                                                          TH 24
C     TRIANGULARIZE MATRIX                                                 TH 25
   10 CALL SOLVE (1,IW,NEQ,MBW,1,A,B)
C                                                                          TH 27
C     FORM VECTOR OF APPLIED FORCES                                        TH 28
      DO  5  M = 1,NFREQ                                                   TH 29
      DO  5  N = 1 , NEQ                                                   TH 30
    5 Q(N,M) = 0.                                                          TH 31
      DN = NELEM                                                           TH 32
```

```
      LENGHT = HEIGHT / DN                                                    TH 33
      DA =  ( RET - REB ) / DN                                                TH 34
      DO  7  N = 1,NFREQ                                                      TH 35
      A1 = REB                                                                TH 36
      W2 =(6.28318531 * FREQ(N) )  **  2                                      TH 37
      DO  7  I = 1,NELEM                                                      TH 38
      A2 = A1 + DA                                                            TH 39
      CALL SUBP  ( I , A1 , A2 , LENGHT , ZO , DEPTH , W2 , P          )     TH 40
      J = 2 * (I-1)                                                           TH 41
      DO  8  M = 1,4                                                          TH 42
      J = J + 1                                                               TH 43
    8 Q( J , N ) =   Q( J , N )  +  P(M)    * H(N)                            TH 44
    7 A1 = A2                                                                 TH 45
C                                                                             TH 46
C     INITIALIZE DISPLACEMENTS, VELOCITIES, AND ACCELERATIONS                 TH 47
      DO  52  I = 1,NEQ                                                       TH 48
      DIS (I) = 0.                                                            TH 49
      VEL (I) = 0.                                                            TH 50
   52 ACC (I) = 0.                                                            TH 51
      TIME = 0.                                                               TH 52
C                                                                             TH 53
      C1 = DT / 2.                                                            TH 54
      C2 = DT * DT / 3.                                                       TH 55
      C3 = C2 / 2.                                                            TH 56
C                                                                             TH 57
C     TIME INTEGRATION (LINEAR ACCELERATION METHOD). NO DAMPING               TH 58
      DO  62  N = 1,NTIME                                                     TH 59
      TIME = TIME + DT                                                        TH 60
C                                                                             TH 61
      DO  9  I = 1,NEQ                                                        TH 62
    9 B(I) = 0.                                                               TH 63
C                                                                             TH 64
      DO  11  J = 1,NFREQ                                          N          TH 65
```

```
      C =DCOS  ( 6.28318531 * FREQ(J) * TIME - PHASE(J) )                TH 66
      DO  11  I = 1,NEQ                                                  TH 67
   11 R(I) = R(I) + C * Q( I,J )                                          TH 68
C                                                                        TH 69
      DO  55  I = 1,NEQ                                                  TH 70
      R(I) = VEL(I) + C1 * ACC(I)                                        TH 71
      S(I) =  DIS(I) + DT * VEL(I) + C2 * ACC(I)                         TH 72
   55 ACC(I) = R(I)                                                      TH 73
C                                                                        TH 74
      CALL AMBC  (NEQ, MBW, ACC, STIFF, S)                               TH 75
      IF(NRIGD.EQ.0) GO TO 70
      ACC(1) = 0.                                                        TH 76
      ACC(2) = 0.                                                        TH 77
   70 CALL SOLVE  ( 2, IW, NEQ, MBW, 1, A, ACC)                          TH 78
C                                                                        TH 79
      DO  56  I = 1,NEQ                                                  TH 80
      VEL (I) = R(I) + C1 * ACC(I)                                       TH 81
   56 DIS(I) = S(I) + C3 * ACC(I)                                        TH 82
C                                                                        TH 83
      IF(N/50*50-N.NE.0) GO TO 52
C                                                                        TH 84
      WRITE ( IW, 2)      TIME                                           TH 85
      DO  60  I = 1,NEQ                                                  TH 86
   60 WRITE (IW, 3)   I, R(I), ACC(I), VEL(I), DIS(I)                    TH 87
   52 CONTINUE                                                           TH 88
C                                                                        TH 89
    2 FORMAT  (///5X , ' TIME =' , F8.2// , 9X , 'I' , 8X ,              TH 90
     *'FORCE' , 8X , 'ACCELERATION' , 6X , 'VELOCITY' , 5X ,             TH 91
     *'DISPLACEMENT'/  )                                                 TH 92
    3 FORMAT  ( 6X , I4 , 4E16.4  )                                      TH 93
      RETURN                                                             TH 95
      END                                                                TH 96
```

```
C       SUBROUTINE AMBC                                                 AM 00
        SUBROUTINE AMBC  (NEQ, MBW, A, B, C  )                          AM 01
        IMPLICIT  REAL  * 8 ( A-H , O-Z  )                              AM 01A
        DIMENSION  A(1), B(1), C(1)                                     AM 02
        LIM = MBW - 1                                                   AM 03
        DO  1  I = 1,NEQ                                                AM 04
        A(I) = A(I) - B(I) * C(I)                                       AM 05
        M = I                                                           AM 06
        DO 7 J=1,LIM
        K = I + J                                                       AM 08
        M = M + NEQ                                                     AM 09
        IF  ( K  .LE.  NEQ )    A(I) = A(I)  - B(M) *C(K)               AM 10
        K = I-J                                                         AM 11
        L = M-J                                                         AM 12
        IF  ( K  .GT. 0    )    A(I) = A(I)  - B(L) *C(K)               AM 13
      7 CONTINUE
      1 CONTINUE                                                        AM 14
        RETURN                                                          AM 15
        END                                                             AM 16
```

APPENDIX B

MODEL MATRICES

1.0	0	0	0	0	0	0	0	0	0
	1.0	0	0	0	0	0	0	0	0
		2959.	-3831.	433.6	-9678.	0	0	0	0
			6.364E5	9422.	-2016E5	0	0	0	0
				2087.	-3334	293.	-6564.	0	0
					4.491E5	6344.	-1.353E5	0	0
			SYMMETRIC			1335.	-2839.	173.4	-3911.
							2.877E5	3726.	-8060E4
								4598.	-5733.
									2.532E6

Figure B-1

Mass Matrix for 1000 Foot Tower

1.0	0	0	0	0	0	0	0	0	0
	1.0	0	0	0	0	0	0	0	0
		6.23E6	-6.83E7	-1.99E6	7.85E7	0	0	0	0
			1.65E10	-1.04E8	2.80E9	0	0	0	0
				2.76E6	-3.70E7	-7.71E5	2.90E7	0	0
					7.09E9	-4.15E7	1.09E9	0	0
						9.85E5	-1.68E7	-2.14E5	7.36E6
							2.37E9	-1.22E7	3.06E8
								2.14E5	-7.36E6
									3.68E8

SYMMETRIC

Figure B-2

Stiffness Matrix for 1000 Foot Tower

1.0	0	0	0	0	0	0	0	0	0
	1.0	0	0	0	0	0	0	0	0
		2.69E4	-2.94E5	-8.54E3	3.38E5	0	0	0	0
			7.10E7	-4.47E5	1.20E7	0	0	0	0
				1.19E4	-1.59E5	-3.31E3	1.25E5	0	0
					3.05E7	-1.78E5	4.69E6	0	0
						4.28E3	-7.23E4	-9.15E2	3.15E4
							1.02E7	-5.23E4	1.31E6
								1.06E3	-3.18E4
									1.66E6

SYMMETRIC

Figure B-3

$\underline{C}_1$, Viscous Damping Matrix for 1000 Foot Tower

1.0	0	0	0	0	0	0	0	0	0
	1.0	0	0	0	0	0	0	0	0
		i1.87E5	-i2.05E6	-i5.97E4	i2.36E6	0	0	0	0
			i4.95E8	-i3.12E6	i8.40E7	0	0	0	0
				i8.28E4	-i1.11E6	-i2.31E4	i8.70E5	0	0
					i2.13E8	-i1.24E6	i3.27E7	0	0
						i2.95E4	-i5.04E5	-6.42E3	i2.21E5
							i7.11E7	-i3.66E5	i9.18E6
								i6.42E3	-i2.21E5
									i1.10E7

SYMMETRIC

Figure B-4

$\underline{C}_2$, Hysteretic Damping Matrix for 1000 Foot Tower

1.0	0	0	0	0	0	0	0	0	0
	1.0	0	0	0	0	0	0	0	0
		4009.	-6455.	581.	-1.514E4	0	0	0	0
			1.174E6	1.471E4	-3.675E5	0	0	0	0
				2757.	-5545.	380.	-9954.	0	0
					8.079E5	9591.	-2.407E5	0	0
						1695.	-4630.	213.	-5622.
							4.956E5	5324.	-1.348E5
								8001.	-8091.
									2.597E6

SYMMETRIC

Figure B-5

Mass Matrix for 1200 Foot Tower

1.0	0	0	0	0	0	0	0	0	0
	1.0	0	0	0	0	0	0	0	0
		5.69E6	-7.63E7	-1.77E6	8.06E7	0	0	0	0
			2.03E10	-1.08E8	3.39E9	0	0	0	0
				2.41E6	-3.98E7	-6.43E5	2.78E7	0	0
					8.32E9	-4.07E7	1.24E9	0	0
						8.02E5	-1.70E7	-1.60E5	6.21E6
							2.56E9	-1.08E7	3.12E8
								1.60E5	-6.21E6
									3.51E8

SYMMETRIC

Figure B-6

Stiffness Matrix for 1200 Foot Tower

1.0	0	0	0	0	0	0	0	0	0
	1.0	0	0	0	0	0	0	0	0
		3.37E4	-4.50E5	-1.04E4	4.75E5	0	0	0	0
			1.20E8	-6.37E5	2.00E7	0	0	0	0
				1.43E4	-2.35E5	-3.78E3	1.64E5	0	0
					4.91E7	-2.40E5	7.31E6	0	0
						4.77E3	-1.00E5	-9.39E2	3.65E4
							1.51E7	-6.36E4	1.84E6
								1.60E5	-3.68E4
									2.13E6

SYMMETRIC

Figure B-7

$\underline{C}_1$, Viscous Damping Matrix for 1200 Foot Tower

1.0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0
		i1.71E5	-i2.29E6	-i5.31E4	i2.42E5	0	0	0	0
			i6.09E8	-i3.24E6	i1.02E8	0	0	0	0
				i7.23E4	-i1.19E6	-i1.93E4	i8.34E5	0	0
					i2.50E8	-i1.22E6	i3.72E7	0	0
						i2.41E4	-i5.10E5	-i4.80E3	i1.86E5
							i7.68E7	-i3.24E5	i9.36E6
								i4.80E3	-i1.87E5
									i1.05E7

SYMMETRIC

Figure B-8

$\underline{C}_2$, Hysteretic Damping Matrix For 1200 Foot Tower

1.522E4	4.592E5	32.	-342.	0	0	0	0	0	0
	1.532E6	331.	-3361.	0	0	0	0	0	0
		148.	-143.	19.	-208.	0	0	0	0
			7147.	198.	-2035.	0	0	0	0
				83.	-108.	10.	-107.	0	0
					4044.	100.	-1034.	0	0
						37.	-73.	3.	-38.
							1809.	34.	-359.
								1586.	-46.
									5.006E5

SYMMETRIC

Figure B-9

Mass Matrix for Condeep

6.87E6	2.11E8	-5.68E6	1.06E8	0	0	0	0	0	0
	9.77E9	-1.39E8	1.78E9	0	0	0	0	0	0
		7.96E6	-4.88E7	-2.28E6	4.10E7	0	0	0	0
			4.58E9	-5.75E7	7.19E8	0	0	0	0
				2.98E6	-2.26E7	-6.98E5	1.18E7	0	0
					1.63E9	-1.84E7	2.23E8	0	0
						8.28E5	-8.04E6	-1.31E5	1.92E6
							4.04E8	-3.71E6	4.27E7
								1.31E5	-1.92E6
									4.04E7

SYMMETRIC

Figure B-10

Stiffness Matrix for Condeep

3.36E4	1.03E5	-2.81E4	5.09E5	0	0	0	0	0	0
	4.20E7	-6.67E5	8.54E6	0	0	0	0	0	0
		3.82E4	-2.34E5	-1.09E5	1.97E5	0	0	0	0
			2.20E7	-2.76E5	3.45E6	0	0	0	0
				1.43E4	-1.08E5	-3.35E3	5.66E4	0	0
					1.82E6	-8.83E4	1.07E6	0	0
						3.98E3	-3.86E4	-6.29E2	9.21E3
							1.94E6	-1.78E4	2.05E5
								6.95E2	-9.22E3
									2.15E5

SYMMETRIC

Figure B-11

$\underline{C}_1$, Viscous Damping Matrix for Condeep

i2.06E5	i6.33E6	-i1.76E5	i3.18E6	0	0	0	0	0	0
	i2.93E8	-i4.17E6	i5.34E7	0	0	0	0	0	0
		i2.39E5	-i1.46E6	-i6.84E4	i1.23E6	0	0	0	0
			i1.37E8	i1.72E6	i2.16E7	0	0	0	0
				i8.94E4	-i6.78E5	-i2.09E4	i3.54E5	0	0
					i4.89E7	-i5.53E5	i6.67E6	0	0
						i2.48E4	-i2.41E5	-i3.93E3	i5.76E4
							i1.21E7	-i1.11E5	i1.2[illegible]E6
								i3.95E3	-i5.76E4
									i1.21E6

SYMMETRIC

Figure B-12

$\underline{C}_2$, Hysteretic Damping Matrix for Condeep

APPENDIX C

MODAL ANALYSIS OF THE WAVE EXCITED PLATFORM

It is a characteristic of mechanical systems that a normal mode exists for each degree of freedom. Associated with each normal mode is a natural frequency and a characteristic shape. A normal mode is distinguished by the fact that the system, when properly excited, could vibrate freely in that mode alone. When this occurs the ratio of the displacements of any two degrees of freedom is constant with time. These ratios define the characteristic shape of the mode. Most important, the complete motion of the system may be obtained by superimposing the independent motions of the individual modes.

By examining the motion in the normal modes and the contribution of each mode to the total motion of the excited system, it is possible to determine the nature of the motion in terms of how much each mode participates in the total motion.

The modal analysis will be carried out on the 1000 foot platform used as an example in Chapter 3. It will be excited by the same 100 year return sea state discretized into twenty-five frequencies.

C.1 Natural Frequencies and Characteristic Shapes [23]

The equations of motion for an undamped multi-degree of freedom system may be written as

$$\underline{M}\,\ddot{\underline{x}} + \underline{K}\,\underline{x} = \underline{P} \qquad \text{(C.1)}$$

where $\underline{M}$ and $\underline{K}$ are symmetric.

If the system is vibrating in a normal mode, which must be harmonic with time, then:

$$\underline{x} = \underline{X}_n \sin\omega_n t$$

$$\ddot{\underline{x}} = -\omega_n^2 \underline{X}_n \sin\omega_n t \tag{C.2}$$

$$\underline{P} = 0$$

Substituting these equations into the equation of motion yields:

$$-\omega_n^2 \underline{M}\,\underline{X}_n + \underline{K}\,\underline{X}_n = 0$$

or

$$(\underline{K} - \omega_n^2 \underline{M})\underline{X}_n = 0 \tag{C.3}$$

where $\underline{X}_n$ is the vector of modal displacements for the n^{th} mode. Since $\underline{X}_n$ can not be zero, then $(\underline{K} - \omega_n^2 \underline{M}) = 0$ which by Cramer's Rule becomes

$$|\underline{K} - \omega_n^2 \underline{M}| = 0 \tag{C.4}$$

This is a characteristic value problem and the roots of Equation (C.4) are the characteristic numbers, or eigenvalues, which are equal to the squares of the natural frequencies of the modes. For each root there is a characteristic vector solution $\underline{X}_n$ having an arbitrary magnitude and representing the characteristic shape of that mode.

There are various schemes for solving the characteristic value problem above and there is normally one available as a program

package at any computer facility. The one used for this work was Access II, available at the Joint Computer Facility, M.I.T.

The vectors $\underline{X}_n$ are arranged as columns of the matrix $\underline{\Phi}$, the model matrix of the system, such that:

$$\underline{\Phi} = [\{\underline{X}_1\}\{\underline{X}_2\}\ldots\{\underline{X}_n\}]$$

C.2 Orthogonality of Modes

The orthogonality condition which will be established in this section is important to the development of the modal equations. For any two roots corresponding to the n^{th} and m^{th} modes, Equation C.3 is

$$\omega_n^2 \underline{M}\,\underline{X}_n = \underline{K}\,\underline{X}_n \tag{C.5a}$$

$$\omega_m^2 \underline{M}\,\underline{X}_m = \underline{K}\,\underline{X}_m \tag{C.5b}$$

Postmultiply the transpose of (C.5a) by $\underline{X}_m$, thus

$$(\omega_n^2 \underline{M}\,\underline{X}_n)^T \underline{X}_m = (\underline{K}\,\underline{X}_n)^T\,\underline{X}_m$$

or

$$\omega_n^2 \underline{X}_n^T\,\underline{M}^T = \underline{X}_n\,\underline{K}^T\,\underline{X}_m \tag{C.6}$$

Premultiply Equation (C.5b) by $\underline{X}_n^T$, thus

$$\omega_m^2 \underline{X}_n^T\,\underline{M}\,\underline{X}_m = \underline{X}_n^T\,\underline{K}\,\underline{X}_m \tag{C.7}$$

Since $\underline{M}^T = \underline{M}$ and $\underline{K}^T = \underline{K}$ because both are symmetric, then the right

hand sides of Equations (C.6) and (C.7) are equal. Subtracting Equation (C.7) from Equation (C.6) yields

$$(\omega_n^2 - \omega_m^2)\ \underline{X}_n^T\ \underline{M}\ \underline{X}_m = 0 \tag{C.8}$$

Since $\omega_n^2 \neq \omega_m^2$

$$\underline{X}_n^T\ \underline{M}\ \underline{X}_m = 0$$

which is the orthogonality condition. From Equation (C.6) it can also be seen that

$$\underline{X}_n^T\ \underline{K}\ \underline{X}_m = 0$$

C.3 Modal Equations

Starting with the system equation of motion:

$$\underline{M}\ \ddot{\underline{x}} + \underline{K}\ \underline{x} = P \tag{C.1}$$

Let $\underline{x} = \underline{\Phi}\ \underline{u}$, where $\underline{u}$ is the vector of modal amplitudes. This states that the displacement vector $\underline{x}$ is a linear combination of modal values. Substitution into the equation of motion yields:

$$\underline{M}\ \underline{\Phi}\ \ddot{\underline{u}} + \underline{K}\ \underline{\Phi}\ \underline{u} = \underline{P} \tag{C.9}$$

Premultiply by $\underline{\Phi}^T$:

$$\underline{\Phi}^T\underline{M}\ \underline{\Phi}\ \ddot{\underline{u}} + \underline{\Phi}^T\ \underline{K}\ \underline{\Phi}\ \underline{u} = \underline{\Phi}^T\ \underline{P} \tag{C.10}$$

Consider $\underline{\Phi}^T\underline{M}\ \underline{\Phi}$. Each element in the resultant matrix equals

$$M_{ij} = \underline{X}_i^T\ \underline{M}\ \underline{X}_j$$

This equals zero for all $i \neq j$ because of orthogonality. Therefore the resultant matrix is a diagonal matrix called the equivalent mass matrix $\underline{M}_e$ whose elements are:

$$M_i = \underline{X}_i^T \underline{M} \underline{X}_i$$

The same result is obtained from $\underline{\Phi}^T \underline{K} \underline{\Phi}$ which becomes $\underline{K}_e$ whose elements are

$$K_i = \underline{X}_i^T \underline{K} \underline{X}_i$$

Returning to Equation (C.10) which is now

$$\underline{M}_e \ddot{\underline{u}} + \underline{K}_e \underline{u} = \underline{\Phi}^T \underline{P}$$

This is a series of n uncoupled differential equations

$$M_1 \ddot{u}_1 + K_1 u_1 = (\underline{\Phi}^T \underline{P})_1$$

$$M_2 \ddot{u}_2 + K_2 u_2 = (\underline{\Phi}^T \underline{P})_2$$

$$\vdots$$

$$M_n \ddot{u}_n + K_n u_n = (\underline{\Psi}^T \underline{P})_n$$

which can be solved individually such that

$$u_i = \frac{(\underline{\Phi}^T \underline{P})_i}{K_i - M_i \omega^2} \sin\omega t + A_i \sin\left(\sqrt{\frac{K_i}{m_i}}\, t + \Theta_i\right) \qquad \text{(C.11)}$$

where u_i is the sum of the forced and free modal responses.

Up to this point damping has been ignored in the analysis. If damping had been included, the original equation of motion would

have been

$$\underline{M}\,\ddot{\underline{x}} + \underline{K}\,\underline{x} + \underline{C}\,\dot{\underline{x}} = \underline{P}$$

Carrying out the same analysis as above and ignoring the free vibration portion of the solution; then

$$u_i = \frac{(\underline{\Phi}^T \underline{P})_i}{\sqrt{(K_i - M_i\omega^2)^2 + c_i^2\omega^2}} \qquad (C.12)$$

Critical damping, c_c, is defined as the damping which eliminates any harnomic vibration and is equal to $2\sqrt{KM}$ for a one degree of freedom system.

The ratio of damping to critical damping is the percent of damping in the system, which is labeled p.

Therefore

$$p = \frac{c}{c_c} = \frac{c}{2\sqrt{KM}}$$

Then

$$c = (p)(2\sqrt{KM})$$

Substituting for c in Equation (C.12) yields

$$u_i = \frac{(\underline{\Phi}^T \underline{P})_i}{\sqrt{(K_i - M_i\omega^2)^2 + 4p^2K_iM_i\omega^2}}$$

As was pointed out in Chapter 2, damping in the computer model was of two types which varied with each frequency and therefore with each mode. For simplification, damping will be assumed to be a

total of 3% of critical damping in all modes for this modal analysis. This is conservative for this procedure because it will tend to reduce the suppression of higher modes which occurs in real structures.

The point has been reached where n steady-state solutions for u_i exist where u_i includes damping. The $\underline{u}$ vector has been formed.

Recalling that $\underline{x} = \underline{\Phi}\,\underline{u}$ the solution for $\underline{x}$ in terms of modal participation has been reached where

$$x_1 = \phi_{11}u_1 + \phi_{12}u_2 + \ldots + \phi_{1n}u_n$$

$$x_2 = \phi_{21}u_1 + \phi_{22}u_2 + \ldots + \phi_{2n}u_n$$

$$\vdots$$

$$x_n = \phi_{n1}u_1 + \phi_{n2}u_2 + \ldots + \phi_{nn}u_n$$

The displacement of each degree of freedom is expressed as a linear combination of modal displacements. The percentage contribution of any mode to the displacement of any degree of freedom can be found by

% contribution of ith node to jth DOF =

$$\frac{|\phi_{ji}u_i|}{|\phi_{j1}u_1| + |\phi_{j2}u_2| + \ldots + |\phi_{jn}u_n|} \qquad \text{(C.13)}$$

The above discussion is concerned with only one input $\underline{P}$. In the analysis of a tower subject to excitation by a wave spectrum which has been discretized into N frequencies, the input vector $\underline{P}$ becomes a function of frequency. Evaluating the modal solution at

each frequency yields N modal solutions. To find the rms value of the total response of each degree of freedom the solution of each degree of freedom at each frequency was used in the following formula

$$(\sigma_x)_i = \sqrt{\sum_{j=1}^{N} (x_i)_j^2} \tag{C.14}$$

where

i = ith DOF

j = jth frequency

If the same formula is used to evaluate the contribution of each mode to the rms value of the total response of each degree of freedom, the result is

$$(\sigma_x)_i = \sqrt{\sum_{j=1}^{N} [(\underline{\Phi}\ \underline{u})_i]_j^2} \tag{C.15}$$

where

i = ith DOF

j = jth frequency

When the term $(\underline{\Phi}\ \underline{u})_i$ is squared the separation of each modal contribution is lost and the solution $(\sigma_x)_i$ cannot be found in terms of modal contribution.

In order to evaluate the modal contribution to the rms value of the response, the following procedures are used.

At each frequency j, solve the equation of motion

$$\underline{M}_e \ddot{\underline{u}}_j + \underline{K}_e \underline{u}_j = \underline{\Phi}^T \underline{P}_j \qquad j = 1,N$$

with

$$u_{ji} = \frac{(\underline{\Phi}^T \underline{P}_j)_i}{\sqrt{(K_i - M_i\omega_j^2)^2 + 4p^2 K_i M_i \omega_j^2}} \qquad \begin{matrix} j=1,N \\ i=1,M(\text{number of modes}) \end{matrix}$$

so that

$$\underline{x}_j = \underline{\Phi}\, \underline{u}_j$$

Procedure A

At each frequency j evaluate the contribution of each mode to the total response. Then sum the contributions of the mode of interest for all frequencies j = 1,N and find the average contribution of that mode.

Procedure B

Assuming that the sum of contributions to the total response of all higher modes at any frequency is small compared to the contribution of the first mode at that frequency, evaluate the following formula

% Contribution of first mode to RMS value of total response for DOF i =

$$\frac{\sqrt{\sum_{j=1}^{N} (\phi_{i1} u_1)_j^2}}{\sqrt{\sum_{j=1}^{N} \left[\sum_{K=1}^{N} |\phi_{iK} u_K| \right]_j^2}} \times 100 \qquad \text{(C.16)}$$

This formula states that the percent contribution of the first mode to the numerical sum of the modal components for a particular degree of freedom equals the square root of the sum of the squares of the first mode contribution at that DOF for each frequency divided by the square root of the sum of the squares of the numerical sum of the modal components at that DOF for each frequency times 100.

After the results of these two procedures are obtained, chose the most conservative figure for evaluation.

C.4 Platform Analysis and Results

The mass and stiffness matrices for the tower were obtained and the natural frequencies and modal matrix $\underline{\Phi}$ were determined. The next step is to evaluate the equivalent mass and stiffness matrices where

$$\underline{M}_e = \underline{\Phi}^T \underline{M} \underline{\Phi}$$

$$\underline{K}_e = \underline{\Phi}^T \underline{K} \underline{\Phi}$$

The input vectors $\underline{P}$ were obtained for each of twenty-five frequencies in the discretized wave spectrum by evaluation of the work equivalent forces by DuVall's finite element program.

At each frequency the vector $\underline{\Phi}^T \underline{P}$ was evaluated and the vector of modal amplitudes $\underline{u}$ was evaluated. The percent modal contribution will be evaluated for the deck displacement which is DOF number 9 or x_7. Evaluating $\underline{x} = \underline{\Phi}\ \underline{u}$ at each frequency and examining x_7 yields the following results.

Table C.1 shows that the first mode varies for percent contribution to the total response from a low of 96.0% to a high of 99.8%. The second mode generally provides the remainder of the response but only 3.0% at the highest.

Using Procedure A and averaging the first mode contribution the result shows the first mode participates in the response with an average of 98.3%.

Using Procedure B and evaluating Equation C.16 yields

$$\frac{\sqrt{\sum_{j=1}^{25} (\phi_{71} u_1)_j^2}}{\sqrt{\sum_{j=1}^{25} \left[\sum_{K=1}^{8} |\phi_{7K} u_K| \right]_j^2}} \times 100 = 99.2\%$$

The most conservative of these results indicates that the response is that of a one degree of freedom system and that contributions from higher modes can be ignored.

Two other factors must be considered. First, how does the modal participation change if the sea state is less intense, causing the distribution of wave heights versus frequency in the sea spectrum to shift. Secondly, how does the modal participation change if the structure is much stiffer, placing the fundamental frequency of the structure in the range of frequencies where the sea spectrum is more level. This results in all the higher modes being excited by waves of nearly equal intensity.

Malhotra and Penzien [24] did a study which answers the

Freq.	% Mode 1	% Mode 2	$(\phi_{71}u_1)^2$	$(\sum_{K=1}^{n} \lvert\phi_{7K}u_K\rvert)^2$
.03	96.6	2.7	3.96E-2	4.24E-2
.042	96.0	2.4	6.02E-1	6.54E-1
.046	97.1	2.4	5.68E-1	6.02E-1
.5	97.3	2.3	6.45E-1	6.82E-1
.054	97.4	2.2	7.79E-1	8.21E-1
.06	97.5	2.1	7.35E-1	7.72E-1
.066	97.7	2.0	5.72E-1	5.99E-1
.072	96.5	1.8	3.92E-1	4.21E-1
.078	97.0	1.8	3.60E-1	3.75E-1
.09	98.2	1.6	2.14E-1	2.22E-1
.102	98.3	1.5	2.26E-1	2.34E-1
.114	98.5	1.3	2.47E-1	2.55E-1
.126	98.7	1.2	2.96E-1	3.05E-1
.138	98.8	1.0	3.71E-1	3.80E-1
.15	99.0	.8	7.02E-1	7.15E-1
.175	99.5	.4	3.06	3.09
.186	99.7	.2	4.37	4.39
.192	99.8	.2	4.78	4.80
.196	99.8	.2	3.90	3.92
.2	99.8	.2	2.47	2.48
.204	99.7	.3	1.36	1.37
.208	99.6	.3	1.85	1.87
.275	99.2	.7	8.24E-1	8.38E-1
.25	98.4	1.4	3.51E-1	3.62E-1
.3	96.6	3.0	1.26E-1	1.34E-1

Table C.1

Response Statistics of DOF 9 (x_7) of 1000 Foot Tower

first question. They showed that varying the input sea states from one based on a wind speed of 60 miles per hour (H_s = 36 feet) to one based on a wind speed of 30 miles per hour (H_s = 9 feet) reduced the participation of the first mode only 3%. When the sea state was reduced below those based on a wind speed of 30 miles per hour, the decrease in first mode participation became more pronounced, but the total response in these seas is negligible.

The second question was tackled by Millman [25] in his analysis of an offshore platform with a fundamental frequency of .754 cycles per second (T = 1.3 seconds). His modal analysis of this structure excited by a strong sea state (H_s = 38 feet) showed the first mode participation in the total response to be on the order of 97%, clearly still dominant.

The result is that independent of sea state or fundamental frequency of this type of tower, the first mode participation in the total response should not fall out of the extremely dominant range.

There are two conclusions which can be drawn from this result. First that higher modes can be ignored when considering methods of reducing the dynamic response of the tower and second that results obtained by DenHartog and Crandall and Mark in their work with vibration absorbers on one degree of freedom systems should be applicable to the mass damper applied to this tower.

Therefore the mass damper should be effective and it should be tuned to surpress the response of the first mode.