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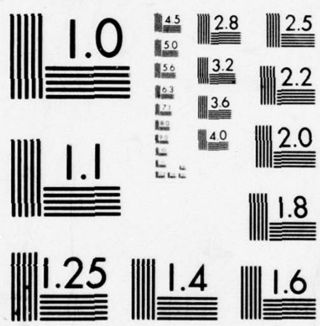

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THE MULTI-PRODUCT PRODUCTION CYCLING PROBLEM: TESTING OF HEURISTICS

by

STEPHEN C. GRAVES

Technical Report No. 147

Work Performed Under
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FOREWORD

The Operations Research Center at the Massachusetts Institute of Technology is an interdepartmental activity devoted to graduate education and research in the field of operations research. The work of the Center is supported, in part, by government contracts and grants. The work reported herein was supported by the Office of Naval Research under Contract N00014-75-C-0556.

Richard C. Larson
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Abstract

The multi-product production cycling problem is concerned with the determination of a production/inventory policy for a single capacitated production facility which is dedicated to producing a family of products. In an earlier paper, heuristic policies were proposed for various versions of the multi-product problem. This paper tests the effectiveness of these heuristics by simulation. It is seen that the proposed heuristic utilizing composite products and the lead-time adjustment, is the most effective of the heuristics considered over the set of test problems for the identical-product problem. Likewise, the composite product heuristic is the best heuristic for the identical-cost problem and the correlated demand problem.

1. Introduction

In this paper the multi-product production cycling problem (MPCP) is studied by means of simulation. The MPCP is concerned with determining production and inventory control policies for a group of products, all of which are produced on a single production facility. An earlier paper [3] has addressed this problem and has proposed heuristic decision policies for two special classes of the MPCP: the identical-product problem, and the identical-cost problem. This paper extends the previous work by testing the proposed heuristics by means of simulation.

The paper is organized as follows: In section 2 the simulation program is described in general. In section 3 the simulation is used to test the effectiveness of the proposed heuristics for the identical-product problem. This set of heuristics is compared against some alternative heuristics which are proposed in this section. Sections 4 and 5 consider the identical-cost problem and the correlated-demand problem, respectively. In both instances, the simulation is used to compare the proposed composite-product heuristic against a set of alternative heuristics. The last section gives a summary of the results.

2. Description of Simulation

A computer program has been developed to simulate a multi-product single-machine system. This simulation program is characterized by the following set of specifications for an n-product problem:

1. Demand for each product is an independent Poisson process where λ_i is the demand rate for product i, i=1,2,...,n.
2. The production rate for each product is one unit per period. This implies that expected machine utilization is $U = \sum_{i=1}^{n} \lambda_i$.
3. Inventory holding and backorder costs are linear with respect to the net inventory level at the end of the period.
4. All products have identical cost structures. In particular, the inventory holding cost rate is 1 cost unit per unit of inventory per period, while the backorder cost rate is 5 cost units per backorder per period. The setup cost K is an input parameter.
5. The production lead time is ten periods.
6. All simulation runs are for 20,200 periods. The first 200 periods are used to initialize the system. System costs are accumulated over the remaining 20,000 periods.

Given this description of the simulation, a test problem is specified by setting the number of products n, the demand rates for the products (λ_i), and the setup cost K. To run the simulation, a heuristic policy must be specified for making production decisions. For each test problem, the generated demand realization is identical for every heuristic that is considered for that test problem. Hence, any variation among the performances of the heuristics on the test problem is due to the heuristics, and is not a consequence of differences in the generated demand. At the start of the simulation, each product is given an initial inventory of three times the product's economic order quantity.

3. The Identical-Product Problem

The identical-product problem was defined and analyzed in [3]; essentially, for this problem, all products have identical costs, identical production rates, and identical, but independent, demand distributions. In [3], a heuristic decision procedure was developed which relied on the individual one-product analysis, the notion of composite products and the lead-time adjustment. To test the effectiveness of this heuristic and its components, the following five heuristics are defined.

H1: The heuristic H1 is the decision procedure relying solely on the one-product analysis of the indvidual identical products. Each product is governed by a two-critical-number policy (I^*, I^{**}). A product has triggered whenever it was produced in the previous period and its inventory is currently less than I^{**}, or it was not produced in the previous period and its inventory is now below I^*. If no products trigger, then the machine is idle in the current period, while if exactly one product triggers, then that product is produced. If more than one product triggers, then the product to produce is found by comparing value functions, as proposed in [3].

H2: The heuristic H2 extends H1 by using composite products (as developed in [3]) in the decision procedure. Again, value functions are used to resolve conflicts when more than one product, or more than one composite product, triggers in a period.

H3: The heuristic H3 adds the lead-time adjustment [3] to H2.

H4: Heuristic H4 is identical to H2 except that the comparison of relative inventory levels [3] rather than value functions is used to resolve conflicts when more than one product triggers.

H5: Heuristic H5 is identical to H3 except that relative inventory levels rather than value functions are compared to resolve conflicts.

These five heuristics were selected so that both the effectiveness of the composite product (H1 vs. H2 or H1 vs. H4), and the incremental effectiveness of the lead-time adjustment (H2 vs. H3 or H4 vs. H5) could be studied. In addition, we can contrast the comparison of value functions against the use of relative inventory levels. To test the effectiveness of these heuristics compared with possible alternative heuristic policies, three additional heuristics have been defined.

H6: The heuristic H6 is characterized by two parameters (I^*,N^*). Whenever a product is chosen to be produced, it will be produced for N^* consecutive periods, regardless of anything else. After N^* periods of production, a decision is made either to shutdown the facility or begin production on the least-inventory product. The decision will be to shutdown if the least-inventory product has inventory greater than I^*; otherwise, the decision is to produce the current least-inventory product for N^* periods.

H7: The heuristic H7 parameterized by (I^*,I^{**}). The product currently setup on the machine will continue to be produced until its inventory reaches or exceeds I^{**}. At that time, the machine is shutdown only if the product with least inventory has inventory greater than I^*; if not, then the least-inventory product is setup.

H8: The heuristic H8 is parameterized by (I^*,I^{**}). It is identical to H7 except in that it allows for the machine to be switched from one product to another product prior to the first product's inventory having reached I^{**}. Let I_1 and I_2 be the inventory levels of the product with least inventory, and of the product being produced, respectively. Then this heuristic will switch to the least-inventory product whenever

$I^{*} - I_1 \geq I^{**} - I_2$.

These three heuristics are based on standard notions of inventory theory. The first heuristic H6 corresponds to a (Q,R) policy. Orders are triggered when inventory drops below a reorder point (R or I^{*}); the order size is fixed equal to Q units or N^{*} periods of production. The second heuristic H7 represents an (s,S) policy. Again there is a reorder point (I^{*} or s), but now the order quantity is such to bring inventory up to some predetermined level (I^{**} or S). Heuristic H8 is a modification of H7 which allows for switching of products prior to the setup product reaching the order-up-to level (I^{**}).

No theory exists concerning how to choose the parameters for these three heuristics. The inventory theory developed to find (Q,R) or (s,S) policies is not useful since that theory is restricted to single-product uncapacitated problems. Therefore, for each of the three heuristics H6, H7, and H8, to estimate the best set of parameters, a grid search is done over a restricted policy space. To determine the expected cost for each policy in the grid, the system is simulated for 600 periods where the first 100 periods are for the initialization, while costs are recorded only over the last 500 periods. The policy with least cost is then chosen to be the best choice of parameters.*

* An examination was made of the sensitivity of this search procedure to the length of the simulation run; a run size of 600 periods was found to be sufficient for finding an optimal or near-optimal parameter choice.

These eight heuristics are compared on eighteen identical-product problems. These test problems vary with respect to the number of products, the machine utilization and the setup cost. The number of products n ranges over three values: n=2, n=5, or n=10. Machine utilization U also takes on three values: U = 70%, U = 90%, or U = 95%. The setup cost may be either "high" or "low". The high setup cost is set so that the economic order quantity (EOQ) for an individual product is equal to five periods of production (since the production rate is one for all test problems, this is EOQ = 5); the low setup cost is such that the individual product's EOQ is two units. By taking all possible combinations, eighteen test problems are constructed with each test problem being specified by the triplet n/U/EOQ.

For the test problems with n=2 and n=5, the production lead time is ten periods. For n=10, the lead time is set to twenty periods; it was observed that for n=10 there is not sufficient demand variation over a lead time of ten periods to allow for the heuristics to delineate themselves.

The results from the simulation for the eighteen test problems are given in Figure 1. Each entry in the figure is the observed average total cost per period for a given test problem using a given heuristic scheduling procedure. In studying this figure, the following observations may be made on the relative performances of the heuristics tested:

1. Heuristic H2, which uses the composite product seems to be an improvement over H1. This is especially true at the higher machine utilization levels (i.e. U = 90, 95) where H2 dominates H1. At low machine utilization (U=70), however, the benefit of the composite product is not clearcut. It is unclear why the composite product may be counter-productive in these instances. Since the composite

Test Problem	Heuristic Procedure H1	H2	H3	H4	H5	H6	H7	H8
2/70/2	8.06	7.93*	7.93*	8.09	7.93*	9.51	8.69	8.13
2/70/5	11.01	11.00	10.93*	11.18	11.09	14.98	11.39	11.07
2/90/2	13.52	11.21	11.05*	11.14	11.17	15.69	15.54	13.49
2/90/5	16.37	14.17	13.97	13.78	13.70*	21.57	17.44	13.77
2/95/2	26.77	16.68	16.43	16.66	16.42*	27.47	27.41	17.91
2/95/5	26.44	18.17	18.73	17.89*	17.89*	26.60	26.99	22.68
5/70/2	14.63	14.56	14.63	14.55*	14.55*	16.28	15.75	14.63
5/70/5	25.59*	25.89	25.86	25.89	25.72	26.25	26.00	25.63
5/90/2	20.71	17.93	17.12	17.79	17.08*	19.05	19.52	17.16
5/90/5	33.62	28.59	27.72	27.99	27.30*	30.02	29.64	27.98
5/95/2	30.13	19.80	19.57	19.94	19.26*	20.30	24.39	19.76
5/95/5	35.33	29.68	29.64	29.80	28.84*	31.32	33.09	30.04
10/70/2	29.18*	29.19	29.21	29.53	29.53	29.64	31.19	30.16
10/70/5	52.20*	53.91	54.12	54.26	54.26	53.40	52.74	54.81
10/90/2	38.64	36.00	32.99*	36.14	33.35	34.31	35.45	33.90
10/90/5	62.96	57.17	56.78	57.47	55.93	56.67	57.81	55.73*
10/95/2	51.94	40.82	41.34	39.05	38.95*	49.74	39.79	41.80
10/95/5	64.08	60.24	62.93	61.53	57.21*	64.58	80.62	57.96

Figure 1: Simulation Results for Identical-Product Problem

product heuristic will not overproduce, it seems that the only explanation for the observed behavior at low utilization is that the composite product heuristic occasionally chooses the wrong product to produce. For instance, H2 might continue to product the currently-setup product, which in fact it should switch to the least-inventory product. Otherwise, the composite

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* Indicates best performing heuristic for test problem.

product appears to be very beneficial.

2. The heuristic H3, which incorporates the lead-time adjustment into its analysis, performs slightly better than H2, though H3 does not dominate H2. Hence, here the usefulness of the lead-time adjustment is unclear.
3. It is difficult to distinguish between the overall performance of H2 versus H4. On this basis no conclusions may be made on the merit of the value function over relative inventory levels.
4. The performance of H5 seems to dominate that of the other heuristics. This heuristic is the best in eleven out of eighteen problems while performing close to best in the remaining seven problems. In particular, H5 performs best in all six problems with highest machine utilization (U = 95). These high utilization problems are typically the most difficult to schedule due to the reduction in flexibility caused by the tight capacity constraint. Consequently, these problems are the most sensitive to the scheduling policy used, and should exhibit the largest reward from improvements in the scheduling policy.
5. The alternative heuristics H6, H7, and H8 were proposed as reasonable heuristics derived from traditional practices in inventory theory. The performances of H6 and H7 are by far the worst of any of the heuristics. H8 performs reasonably well, but is outperformed by H5. Note again for these three heuristics, there is no theory on determining their policy parameters; rather, this must be done by means of a simulation search procedure.

To support the above observations, statistical tests have been performed on the data in Figure 1. To test the null hypothesis that the eight heuristic policies are indistinguishable, a two-way analysis of variance is done; using an F-statistic the null hypothesis may be rejected at a significance level of .005. To test the dominance of H5, pairwise comparisons are made between H5 and each of the other heuristics. To test the null hypothesis that there is no difference between H5 and another heuristic, against the alternative heuristic that H5 performs better, a one-tailed test is made using a student's t statistic; the student's t statistic is formed from the differences between H5 and the other heuristic on the eighteen test problems. The results of these tests are presented in Figure 2. Clearly, in all instances, the null hypothesis may be rejected, and thus the heuristic H5 is significantly better than each of the other heuristics. Note that for H3, though, the significance level is only .10; this is to be expected since H3 and H5 are nearly identical, differing only in the use by H3 of the value function.

heuristic	t-statistic*	significance level
H1	3.01	.005
H2	2.49	.025
H3	1.64	.100
H4	2.19	.025
H5	-	-
H6	2.98	.005
H7	2.46	.025
H8	2.47	.025

Figure 2: Pairwise Comparison of Heuristics with H5

In addition to these tests, nonparametric tests were applied to the data in Figure 1 to test the same hypotheses. These tests are appropriate if the normality assumptions necessary for the F-statistic and the t-statistics are violated. To test whether the heuristics are different, a Friedman two-way analysis of variance by ranks is used (see [4]). To

* All t-statistics have m-1 = 17 degrees of freedom.

determine if H5 is the best heuristic, again pairwise comparisons are made with each other heuristic. Here, the Wilcoxon matched-pairs signed-ranks test is used (see [4]). The results of these non-parametric tests are consistent with those for the parametric tests; in fact, the non-parametric results have higher significance levels. Details of the application of these tests are given in [2].

Thus, it has been shown that

1. The eight heuristics perform differently over the eighteen test problems;
2. Heuristic H5 outperforms the other heuristics based on a pairwise comparison of H5 with each of the other heuristics.

Note that H5 uses both composite products and the lead-time adjustment; hence, these notions appear helpful in scheduling the multi-product problem. However, H5 does not use the value function to distinguish between triggered products; rather, it simply uses differences in inventory levels. Thus for the multi-product problem, the value function from the one-product analysis does not accurately reflect the value of the machine to a particular product, and its use can be outperformed by simply comparing inventory levels. Note, however, that whereas H5 is an improvement over H3, H4 performs slightly worse than H2. Since H3 (H5) differs from H2 (H4) only in the use of the lead-time adjustment, it seems that the merit of the value function may depend on the other components in the heuristic policy.

4. The Identical-Cost Problem

The identical-cost problem assumes that all products have the same cost structure and the same production rate; however, the demand rates may vary across products. In [3] it is shown how heuristics H1 and H2, which were originally proposed for the identical-product problem, may be extended to the identical-cost problem. In this section, three additional heuristics are proposed for the identical-cost problem. These five heuristics are then simulated on four test problems to determine the best heuristic for this problem.

Heuristic H1 relies entirely on the one-product analysis of the individual products. Heuristic H2 improves H1 by introducing the use of composite products. Note that the lead-time adjustment cannot be applied to the identical-cost problem. In addition to these two heuristics, heuristics H6, H7, and H8 from the previous section may be modified for the identical-cost problem. The modification is a rescaling of inventory for decision purposes. For the identical-product problem, for each of these three heuristics, decisions are made by comparing inventory levels to the policy parameters of the heuristic. To implement these heuristics for the identical-cost problem, comparisons are made with respect to periods-of-supply rather than with inventory levels. The periods-of-supply for a product is the inventory level divided by the demand rate per period. By dividing by the demand rate, this modification allows for comparisons across products of differing demand rates. Again, the parameters for these heuristics must be determined by a simulation search.

These five heuristics are tested on four identical-cost problems. Each of the four test problems has five products and demand for each product i is Poisson with rate λ_i. The machine utilization is 90% for each problem. The four test problems are as follows:

a) $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = .1, \lambda_5 = .5;$ EOQ = 2.

b) $\lambda_1 = \lambda_2 = \lambda_2 = \lambda_4 = .1, \lambda_5 = .5;$ EOQ = 5.

c) $\lambda_1 = \lambda_2 = \lambda_3 = .1, \lambda_4 = \lambda_5 = .3;$ EOQ = 2.

d) $\lambda_1 = \lambda_2 = \lambda_3 = .1, \lambda_4 = \lambda_5 = .3;$ EOQ = 5.

Thus the problems are characterized by the demand rates for the five products, and by the setup cost. The setup cost is set to obtain the chosen economic order quantity (EOQ) for the product with the smallest demand rate (i.e. $\lambda_1 = .1$).

The results from the simulation of the test problems are given in Figure 3. In comparing H1 with H2, it is clear here that the introduction of the composite product in H2 provides a substantial improvement. Heuristics H6 and H7, the analogs to inventory theory's (Q,R) and (s,S) policies respectively, have performances comparable to H1 but are outperformed by H2. The heuristics H8, however, compares well with heuristic H2. Heuristic H2 performs best in three of the test problems while H8 is slightly better in test problem b.

	heuristic				
test problem	H1	H2	H6	H7	H8
a	22.50	17.41*	20.72	20.56	18.26
b	28.91	26.31	28.92	30.18	26.14*
c	22.93	18.58*	20.64	21.51	19.32
d	29.54	28.51*	29.54	31.59	29.16

Figure 3: Simulation Results for Identical-Cost Problem

* Indicates best performing heuristic for test problem.

To verify these observations, a set of statistical tests has been constructed from the simulation runs. For each of the simulation runs, costs are accumulated over 20,000 periods. By breaking the simulation run into 100 blocks of 200 periods each, a sample of 100 cost observations is obtained; each cost observation corresponds to the average cost for a heuristic on a test problem over a block of 200 periods. Note that the sample points for a heuristic are not totally independent since the costs incurred in one period are dependent on the costs incurred in the previous period. This dependency between sample points will grow smaller as the size of the blocks increases. From observation of the behavior of the sample points, a block size of 200 periods is sufficient to reduce the dependency to a point where it can be safely ignored. In this manner, 100 cost sample points are obtained for each heuristic for each problem.

To test the null hypothesis that two heuristics are equivalent for a specific test problem, a student's t-statistic can be formed from the differences between the two matched cost samples. Figure 4 reports these t-statistics when the four heuristics H1, H6, H7, and H8 are

	heuristic			
test problem	H1	H6	H7	H8
a	6.49	6.44	5.82	4.14
b	4.03	6.08	7.08	-0.35
c	6.27	6.50	5.42	1.18
d	2.18	2.65	5.04	1.47

Figure 4: t-tests* for Heuristics Compared Against H2 for Identical-Cost Problem

* All t-values are with 99 degrees of freedom.

compared individually against H2, the composite product heuristic. The difference between H2 and H1, H6, and H7 is highly significant for each of the test problems; the significance level is less than .001 for each instance, except for problem d where the significance level is .025 for H1 and .005 for H6. Hence the null hypothesis that H2 is equivalent to any of these three heuristics can be rejected. In comparing H2 with H8, H2 clearly outperforms H8 only for problem a. For the other three test problems, the null hypothesis cannot be rejected with 95% confidence. Note, however, that despite the lack of statistical significance, H2 seems to perform better than H8. The differences between H2 and H8 on the three test problems for which H2 is best are greater and more significant than the difference between the two heuristics for problem b. Hence, based on this limited evidence the composite product heuristic seems to be the most effective procedure for the identical-cost problem.

5. The Correlated-Demand Problem

For the identical-product problem and the identical-cost problem, it is assumed that demand across products is independent. In this section the correlated-demand problem assuming identical products is examined. Five heuristics are proposed for the correlated-demand problem, and are tested by simulation on six test problems.

The five heuristics to be tested are H1, H2, H6, H7, and H8. In [3] it is shown how the composite-product heuristic H2 can be used for the correlated-demand problem. Heuristic H1 which only uses the analysis of single products is not altered when applied to the correlated-demand problem. Assuming identical products, H6, H7, and H8 are not altered in form at all for the correlated-demand problem. It is expected that the effect of the demand correlation on the heuristics H6, H7, and H8 will be reflected in parameter choices yielding higher safety stock levels for increasing positive correlation.

These five heuristics are tested on a set of six problems. Each of the problems assumes two identical products with machine utilization of 90%. Demand is assumed to be a bivariate Poisson process with a correlation coefficient of ρ where $|\rho| \leq 1$ (see [1]). A test problem is characterized by specifying a value for ρ and the setup cost. The setup cost is set so that each individual product has an EOQ equal to 2 or 5 units. The correlation coefficient ρ is set to 0, .33, or .66. This results in six test problems. A test problem is denoted by the pair ρ/EOQ. Note that ρ=0/EOQ=2 and ρ=0/EOQ=5 are identical to problems 2/90/2 and 2/90/5, respectively, in Figure 1.

The simulation results for the test problems are presented in Figure 5. Heuristic H2, utilizing the composite product, is a dramatic improvement over H1. Indeed, the magnitude of the benefit from the composite product increases as the demand correlation increases. Alternative heuristics H6 and H7, again, do not perform well with respect to H2. Heuristic H8 does better than H2 in two of the six test problems; however, for one of these problems (ρ=0/EOQ=5), the composite-product heuristic will perform better than H8 by modifying H2 to include a lead-time adjustment (see Figure 1). Thus, over the six test problems, the composite-product heuristic H2 seems best.

test problem	heuristic H1	H2	H6	H7	H8
ρ=0/EOQ=2**	13.52	11.21	15.69	15.54	13.49
ρ=.33/EOQ=2	17.37	12.77*	14.37	16.07	13.74
ρ=.66/EOQ=2	20.91	14.18	15.50	17.25	14.07*
ρ=0/EOQ=5**	16.37	14.17	21.57	17.44	13.77
ρ=.33/EOQ=5	20.17	15.10*	19.75	19.25	15.77
ρ=.66/EOQ=5	23.64	16.21*	19.15	20.29	16.58

Figure 5: Simulation Results for Correlated-Demand Problem

To test for the dominance of H2, a statistical analysis similar to that for the identical-cost problem has been done. For each test problem the heuristic H2 is compared with each of the alternative heuristics by means of a student's t-test. Figure 6 gives the t-values for this analysis. In comparing H2 with either H1, H6, or H7, the null hypothesis that the pair of heuristics are identical is easily rejected for all test problems. However, when comparing H2 with H8, there is no simple conclusion on the relative performance of the two heuristics. In testing the null hypothesis that the two heuristics are identical against the alternative hypothesis that H2 outperforms H8, the null

* Best performance for test problem.

** Best performance for test problem was with heuristics using lead-time adjustments; see Figure 1.

hypothesis can be rejected with 95% confidence for three of the test problems and with 90% confidence for four of the test problems. In addition, for the one problem (ρ=0/EOQ=5) for which H8 is significantly better than H2, we know that by using the lead-time adjustment, the composite-product heuristic will perform as well as H8. Hence, even though the evidence presented is not conclusive, the heuristic H2 appears to be the best heuristic over a reasonable range of problems. Note that the relative performance of H2 does not seem to be sensitive to the level of demand correlation as measured by ρ.

	heuristic			
test problem	H1	H6	H7	H8
ρ=0/EOQ=2	4.67	6.13	6.30	4.88
ρ=.33/EOQ=2	4.58	3.20	4.79	2.18
ρ=.66/EOQ=2	5.06	2.84	5.86	-0.75
ρ=0/EOQ=5	4.46	7.32	5.26	-2.24
ρ=.33/EOQ=5	4.97	5.71	5.10	1.58
ρ=.66/EOQ=5	5.34	6.62	6.31	1.69

Figure 6: t-tests* for Heuristics Compared Against H2 for Correlated-Demand Problem

* All t-values are with 99 degrees of freedom.

6. Summary

In this paper the multi-product production cycling problem has been studied by means of simulation. Three classes of problems have been considered, and for each class, heuristic decision procedures, that were proposed in [3], were tested by simulation.

The simplest multi-product problem considered was the identical-product problem. For this problem it was shown that a heuristic which incorporated both the notion of composite products and the lead-time adjustment, was the most effective heuristic that was considered over a wide range of test problems.

The identical-cost problem represented an attempt to model the general multi-product problem. It was argued in [3] that if a multi-product scheduling problem exists, it can often be formulated as an identical-cost problem. For this problem, the composite-product heuristic performed best. It is believed that improvements in this composite-product heuristic could be made by incorporating a lead-time adjustment. Unfortunately, though, theory for such a lead-time adjustment for the identical-cost problem has not been developed.

The correlated-demand problem was a natural extension to the original identical-product problem. Here a simple two-product version of the problem with bivariate Poisson demand was considered. The conclusions for this problem are similar to those for the identical-cost problem. The composite product is very useful in scheduling the problem; the heuristic which utilizes composite products was shown to be the best heuristic. Again, this composite-product heuristic would likely be improved if a suitable lead-time adjustment could be found for the correlated-demand problem.

References

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