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DISCOVERING HIDDEN TOTALLY LEONTIEF SUBSTITUTION SYSTEMS.(U)
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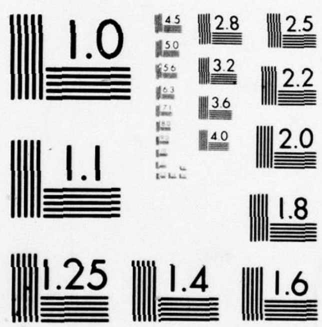
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BY

GEORGE B. DANTZIG and ARTHUR F. VEINOTT, JR.

TECHNICAL REPORT NO. 32
DECEMBER 20, 1977

PREPARED UNDER
OFFICE OF NAVAL RESEARCH CONTRACT
N00014-75-C-0493 (NR-042-264)

DEPARTMENT OF OPERATIONS RESEARCH
STANFORD UNIVERSITY
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*Also partially supported by Office of Naval Research Contracts N00014-75-C-0267, N00014-75-C-0865; and National Science Foundation Grants MCS76-81259, MCS76-22019, and ENG76-12266; and ERDA Contract EY-76-S-03-0326 PA #18.

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Abstract

A constructive procedure is given for determining the existence of and evaluating (when it does exist) a nonsingular matrix that transforms a system of linear equations in nonnegative variables into a totally Leontief substitution system. The computational effort involved is about that required to optimize the given m-row linear system with $m+1$ different linear objective functions.

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The system of m linear equations in n nonnegative variables

$$(1) \qquad Ax = b \; , \quad x \geq 0$$

is called a Leontief substitution system [2] if (i) each column A_j of A has at most one positive element, (ii) $b \gg 0$ and (iii) the set of solutions to (1) is nonempty. If also that set is bounded, (1) is called a totally Leontief substitution system [6]. In either case, it is known that A has rank m. Such systems are discussed in [1] - [8].

Saigal [8], [7] calls (1) a hidden totally Leontief substitution system if there exists a nonsingular matrix Π such that

$$(2) \qquad (\Pi A)x = \Pi b \; , \quad x \geq 0$$

is a totally Leontief substitution system. The purpose of this paper is to give a constructive method for determining whether or not (1) has this property, and if so, to find Π .

Substitution Classes. Associated with any feasible $m \times m$ basis $B = (B_i)$ for (1) is, for $i = 1,\ldots,m$, the set S_i of column indices j such that A_j , if substituted for B_i , forms a feasible basis. Each substitution class S_i is nonempty since it includes a j with

$A_j = B_i$. In general the S_i depend on B and b and can be overlapping when there are degenerate basic feasible solutions. For the totally Leontief substitution case, however, the S_i are independent of the choice of B and $b >> 0$; indeed, S_i consists of all j such that A_j has a positive element in the same row as B_i . Also the S_i partition the column indices $1,\ldots,n$. Thus every submatrix B consisting of m columns of A with a positive element in each row forms a (nondegenerate) feasible basis, and conversely.

The Algorithm.

Step 1. Find a feasible $m \times m$ basis B and determine substitution classes $S_1,\ldots,S_m$ with respect to B , b . Terminate if there is no feasible basis or if the substitution classes do not partition the column indices of A . Otherwise go to Step 2.

Step 2. Solve the linear program of maximizing $z = \Sigma x_j$ subject to (1). Terminate if z is unbounded above. Otherwise go to Step 3.

Step 3. For each $i = 1,\ldots,m$, determine the i^{th} row of Π as any vector π_i such that

$$(3) \qquad \begin{aligned} &\pi_i b > 0 \\ &\pi_i A_j \leq 0 \quad \text{for all } j \notin S_i . \end{aligned}$$

Terminate if for any $i = 1,\ldots,m$ the system (3) is infeasible. Otherwise terminate with Π .

Theorem. If the algorithm terminates with Π , then Π is nonsingular and (2) is a totally Leontief substitution system. Otherwise (1) is not a hidden totally Leontief substitution system.

Proof. If (1) is a hidden totally Leontief substitution system, then there is a feasible basis, the associated substitution classes partition the column indices of A and z in Step 2 is bounded above, because these properties are invariant under nonsingular transformations Π . Also there is a nonsingular matrix Π whose i^{th} row π_i satisfies (3) for each i . Thus if the algorithm terminates without obtaining Π , then (1) is not a hidden totally Leontief substitution system.

If the algorithm does terminate with Π, then ΠA has at most one positive element in each column (from Steps 1 and 3), $\Pi b >> 0$ (from Step 3) and (2) has a solution (from Step 1), so (2) is a Leontief substitution system. Hence ΠA has rank m, implying Π is nonsingular and so (1) and (2) have the same solution set. Thus the boundedness of the solution set of (1) implies that is so of (2), so (2) is a totally Leontief substitution system.

Computational Remarks. The computational effort required to execute the algorithm is about that required to solve the linear program of minimizing cx subject to (1) with $m+1$ different objective-function vectors $c = (c_j)$. To determine the substitution classes in Step 1 requires computing $b' = (b'_k) = B^{-1}b$ and $A'_j = (A'_{jk}) = B^{-1}A_j$ for each j . Then the substitution classes partition the column indices of A if and only if for each j there is a unique $k = k(j)$ that minimizes b'_k/A'_{jk} subject to $A'_{jk} > 0$. In that event $j \in S_{k(j)}$ for each j . Step 2 involves solving the linear program with $c_j = -1$ for all j . Finally, Step 3 necessitates solving m linear programs. The i^{th}, $1 \leq i \leq m$, of these has $c_j = 1$ for all $j \in S_i$ and $c_j = 0$ otherwise. If optimal simplex multipliers π_i exist therefor, they satisfy (3). If no such multipliers exist, (3) is infeasible. Incidentally, Step 3 can be streamlined somewhat by modifying the i^{th} linear program so that all but an (arbitrary) one of the variables x_j with $j \in S_i$ is omitted.

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 32	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) DISCOVERING HIDDEN TOTALLY LEONTIEF SUBSTITUTION SYSTEMS		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) George B. Dantzig and Arthur F. Veinott, Jr.		8. CONTRACT OR GRANT NUMBER(s) N00014-75-C-0493
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Operations Research Stanford University Stanford, California 94305		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS (NR-042-264)
11. CONTROLLING OFFICE NAME AND ADDRESS Logistics and Mathematical Statistics Branch Office of Naval Research Arlington, Virginia		12. REPORT DATE December 20, 1977
		13. NUMBER OF PAGES 5
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

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Also partially supported by ONR Contracts N00014-75-C-0267, N00014-75-C-0865; and National Science Foundation Grants MCS76-81259, MCS76-22019, and ENG76-12266; and ERDA Contract EY-76-S-03-0326 PA #18.

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Totally Leontief substitution systems
Markov decision chains
Equivalent polyhedra

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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