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ANALYSIS OF TECHNIQUES FOR IMAGING THROUGH THE ATMOSPHERE

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ANALYSIS OF TECHNIQUES FOR IMAGING THROUGH THE ATMOSPHERE

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This report is the second interim technical report on Contract F30602-77-C-0021. The problem of generating appropriate one-dimensional slices of a target object ~~is treated~~. Methods are developed for generating these image patterns for subsequent use in a simulation of a one-dimensional version of the non-stationary postdetection compensation technique described in the previous interim technical report. The resulting one-dimensional image will correspond to a n^{-2} power spectrum target object with randomly chosen target details.

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SUMMARY

In preparation for a computer simulation demonstration of non-stationary post-detection compensation, we have concerned ourselves in this report with the task of generating a one-dimensional image signal with a κ^{-1} power spectrum, matching a two-dimensional image with a κ^{-2} power spectrum. Using the insight provided by our previous work on nonstationary post-detection compensation, we formulate the image as a set of rectangular pulses of various widths, randomly distributed across the field-of-view. By relating the distribution of pulse height to pulse width, we are able to keep the average pulse height independent of width, but make the second moment of pulse height depend on pulse width. This allows us to adjust the average pulse count as a function of width so that the power spectrum is κ^{-1} , and yet maintain that approximately one-half of the "object region" is empty — a general characteristic of satellite images, and a key item in exploitation of nonstationary post-detection compensation.

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1. Introduction

In a previous report,* we developed the concept of nonstationary post-detection compensation as a way to take account of the information available a posteriori in defining a filter for compensating an image. The nonstationary filter concept as presented there was developed by means of a combination of analytic and heuristic techniques. This left the demonstration of the concept's suitability as a matter for further study. As a practical matter, we believe that the only really sound approach to this demonstration is an empirical one. Accordingly, we have sought to apply our nonstationary post-detection compensation concept to a one-dimensional problem, so as to facilitate computer demonstration.

It is the objective of this report to define, in sufficient detail to support subsequent computer programming, a procedure for generating appropriate one-dimensional images manifesting appropriate target image statistics. As noted earlier, the key feature of a realistic two-dimensional target image in a region where an interesting abundance of detail is present is the fact that its power-spectrum behaves as κ^{-2}, where κ denotes a spatial frequency, and the power-spectrum is the fully two-dimensional power-spectrum. In restricting ourselves to treatment of the one-dimensional image problem, we cause the power-spectrum of interest to take on a κ^{-1}-dependence. In the next section, we shall discuss the other characteristics we seek in the image pattern we shall generate. In the sections after that, we shall present a method of generating such an image, and then will prove that the one-dimensional image so generated does indeed have the desired properties.

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* D. L. Fried, "Analysis of Techniques for Imaging Through the Atmosphere," Rome Air Development Center Report No. RADC-TR-77-196, June 1977; Chapter 2, AD# B019 482L.

2. Desired Image Properties

First and foremost, the one-dimensional image we seek to generate will have a power-spectrum behaving as κ^{-1} . This leads us to hypothesize that the image should be considered to be formed of a randomly dispersed set of rectangular pulses, with the number of pulses of each width and the average intensity for each pulse so chosen that the composite pattern has a κ^{-1} power-spectrum. We shall incorporate this hypothesized formulation of the total image pattern as a sum of rectangular pulses into our presentation of the other desired image properties.

We consider the object pattern to be formed of a randomly selected array of pulses. We expect the pulse sizes to cover a range from some maximum size, comparable to about one-eighth of the total image field-of-view, to a minimum size. The minimum size is chosen simply for lack of interest in smaller details, with the understanding, however, that there actually is no smallest size cut-off.

We assume that the number of pulses of each width is a random variable, as is the positioning of each pulse and the intensity of each pulse. These values are, however, subject to certain constraints. To maintain the compactness of the total image within the field-of-view, i.e., the desired property that the image appear to be that of a contiguous whole rather than of a uniform random distribution over the entire available field-of-view, we assume some constraint on the distribution of positioning of the pulses. To maintain the concept of the target object as made up of different size elements of related characteristics, we shall assume that the average intensity (i.e., the pulse height) associated with each pulse width be independent of the pulse width. (However, we allow the distribution of pulse height to be a function of pulse widths — providing that the function does not affect the average pulse height.)

The average number of pulses of each width is constrained by the need to achieve a κ^{-1} power-spectrum, but to the extent that the relationship between the mean and second moment of the pulse intensity can be made a function of the pulse width, we can correspondingly adjust the mean number of pulses as a function of pulse width. Our concern, in remarking on this fact, is that examination of typical real objects of interest shows that in the compact region which contains most of a two-dimensional object, about three-quarters of the space is entirely empty. For a one-dimensional object pattern, the corresponding factor would be that about one-half of the space will be entirely empty. In our choice of the intensity (i.e., pulse height) distribution for each element size (i.e., for each pulse width), we shall choose the distribution so that the second moment implies an average number of elements (i.e., pulses) of that size, which when "summed" over all element sizes considered results in the desired 50% vacancy.

It should be remarked that this procedure only specifies the average number of elements of each size. The actual number of elements will be a random variable with this average value — except that for the largest element, we shall impose the requirement that there is always one, and only one element. This will not be a random variable.

With the image properties as set forth here, we are now ready to consider definition of a procedure for generating a randomly "chosen" sample image with these properties. This is taken up in the next section.

3. Sample Image Generation

We propose to consider an image defined on a "field-of-view" of 2^n points (where in practice we shall use $n = 10$, $2^n = 1024$). We number these positions as $0 , 1 , 2 , \ldots . 2^n - 1$. To maintain the compactness of the object, we define a standard deviation, σ_s , defining the image pattern spread, and consider the center point of each pulse making up the image to be randomly distributed in accordance with a gaussian probability distribution with this standard deviation and mean value $\bar{\ell} = 2^{(n-1)}$. Thus, if some pulse is centered at ℓ , then ℓ will be chosen as a gaussian random variable with probability density

$$P_c(\ell) = (2\pi\ \sigma_s^2)^{-1/2} \exp\left[-\tfrac{1}{2}(\ell - \bar{\ell})^2/\sigma_s^2\right] \tag{1}$$

We will use $\sigma_s = 2^{(n-2)}$, or for $n = 10$, $\sigma_s = 2^8 = 256$, and $\bar{\ell} = 512$. This means that the nominal width of our object pattern will be spread over about $3\ \sigma_s = 768$ points, or three-quarters of the field-of-view, and that the pattern will be nominally centered in the middle of our field-of-view.

We shall consider our largest element to be a pulse of width

$$a_0 = 2^{(n-3)} = 2^7 = 128 \tag{2}$$

We shall consider a set of pulse widths

$$a_W = a_0 2^{-W} \quad , \qquad \text{for } W = 0 , 1 , 2 , \ldots , n\text{-}3 \ , \tag{3}$$

so that for the $n = 10$ size field-of-view we intend to consider, W takes 8 values (i.e., $W = 0 , 1 , 2 , \ldots , 7$).

To interpret the requirement that approximately one-half of the object area be entirely empty of pulse elements, we need first of all to define what we mean by the object area. Arguing from insight gained from a study of sample targets with κ^{-1} power spectra, we would suggest that the object area be considered to be about six times the size of the largest element. Within this size region, the target elements take up about one-half of the space in one-dimension (and one-quarter of the space in two-dimensions). This means that the object area is $6\ a_0$. Assuming a power law dependence for the average number of elements of size $a_0 2^{-W}$, the average number is

$$\bar{N}_W = b^W , \tag{4}$$

then the average space taken up by all the elements will be

$$\mathscr{S} = \sum_{W=0}^{7} a_W \bar{N}_W = \sum_{W=0}^{7} (a_0 2^{-W})\, b^W$$

$$= \sum_{W=0}^{7} a_0 (b/2)^W \tag{5*}$$

But for half the object space, i.e., half of $6\ a_0$, to correspond to $\mathscr{S}$, it follows that

$$3 = \sum_{W=0}^{7} (b/2)^W = \frac{(b/2)^8 - 1}{(b/2) - 1}$$

$$\approx \frac{1}{1 - (b/2)} \tag{6}$$

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* This formulation is strictly valid only where the area density of elements is so low that there is a negligible probability of element overlap. With our 50% area density objective, this approximation is not strictly valid. However, at this density the overlap correction factor will be close enough to unity that we can ignore it — doing so being no greater an approximation than our "estimation" that the area fill factor should be 50%. The result of this approximation in Eq. (5) will be that the patterns we generate will actually have a slightly less than 50% area fill factor.

Solving Eq. (6) for b , we get

$$b = 1.3333 \, , \tag{7}$$

which implies that $\overline{N}_W = \{1 \, , \, 1.3333 \, , \, 1.7778 \, , \, 2.3704 \, , \, 3.1605 \, , \, 4.2140 \, , \, 5.6187 \, , \, 7.4915\}$.

We shall show in our analysis that for this set of values of $\overline{N}_W$ to be compatible with a κ^{-1} one-dimensional power-spectrum, it is necessary that the ratio of the second moment of pulse height (i.e., of element intensity) divided by the square of the first moment, i.e., $\langle (I_W)^2 \rangle / \langle I_W \rangle^2$, depend on W according to the relationship

$$\frac{\langle (I_W)^2 \rangle}{\langle I_W \rangle^2} = c^W \, , \tag{8}$$

where

$$c = 1.5 \tag{9}$$

A proof of this will be presented in the next section. In order to achieve this ratio of second moment to first moment squared, we shall assume that the pulse heights are random variables, distributed for each pulse width according to a log-normal distribution, with the basic parameters of the distribution dependent on the pulse width.

The log-normal distribution is characterized by two parameters. The first is the logarithmic standard deviation, σ_L , and the second is the logarithmic mean, $\overline{L}$. If I_W obeys a log-normal distribution with these parameters, then the probability density associated with I_W is

$$p_1 (I_W) = (2\pi \, \sigma_L^2)^{-1/2} \, I_W^{-1} \exp \{ -\tfrac{1}{2} [\ln(I_W/I_0) - \overline{L}]^2 / \sigma_L^2 \} \, , \tag{10}$$

where I_0 is a reference intensity value corresponding to the median value of intensity.

Accordingly, we can write for the first moment

$$\langle I_W \rangle = \int_0^{\infty} dI\, p_1(I)\, I$$

$$= \bar{I}_0 \int_{-\infty}^{+\infty} d\ell\, (2\pi\, \sigma_L{}^2)^{-1/2} \exp\left[-\tfrac{1}{2}\, \frac{(\ell - \bar{L})^2}{\sigma_L{}^2}\right] \exp(\ell) \quad , \tag{11}$$

where we have introduced the notation $\ell = \ln(I/\bar{I}_0)$ and replaced the variable of integration I by ℓ. Completing the square in the exponential, this can be rewritten as

$$\langle I_W \rangle = \bar{I}_0 (2\pi\, \sigma_L{}^2)^{-1/2} \int_{-\infty}^{+\infty} d\ell \exp\left[-\tfrac{1}{2}\, \frac{(\ell - \sigma_L{}^2 - \bar{L})^2}{\sigma_L{}^2}\right]$$

$$\times \exp\left[+\tfrac{1}{2}\, \frac{(\sigma_L{}^2 + \bar{L})^2 - \bar{L}^2}{\sigma_L{}^2}\right]$$

$$= \bar{I}_0 \exp\left[+\tfrac{1}{2}\, \frac{(\sigma_L{}^2 + \bar{L})^2 - \bar{L}^2}{\sigma_L{}^2}\right] \tag{12}$$

It follows from Eq. (12) that the first moment, $\langle I_W \rangle$ will be independent of W (and equal to the mean pulse height for the largest element), if

$$\bar{L} = -\tfrac{1}{2}\, \sigma_L{}^2 \tag{13}$$

In the same manner, we can write for the second moment

$$\langle I_W^2 \rangle = \int dI\, p_1(I)\, I^2$$

$$= \bar{I}_0{}^2 \int_{-\infty}^{+\infty} d\ell\, (2\pi\, \sigma_L{}^2)^{-1/2} \exp\left[-\tfrac{1}{2}\, \frac{(\ell - \bar{L})^2}{\sigma_L{}^2}\right] \exp(2\ell) \quad . \tag{14}$$

Completing the square in the exponential, this can be rewritten as

$$\langle I_W^2 \rangle = \bar{I}_0^2 (2\pi\ \sigma_L^2)^{-1/2} \int_{-\infty}^{+\infty} d\ell\ \exp\left[-\tfrac{1}{2}\ \frac{(\ell - 2\,\sigma_L^2 - \bar{L})^2}{\sigma_L^2}\right]$$

$$\times \exp\left[\tfrac{1}{2}\ \frac{(2\ \sigma_L^2 + \bar{L})^2 - \bar{L}^2}{\sigma_L^2}\right]$$

$$= \bar{I}_0^2\ \exp\left[\tfrac{1}{2}\ \frac{(2\ \sigma_L^2 + \bar{L})^2 - \bar{L}^2}{\sigma_L^2}\right] \tag{15}$$

Combining Eq.'s (12) and (15), and making use of Eq. (13), we get

$$\frac{\langle I_W^2 \rangle}{\langle I_W \rangle^2} = \exp\ (\sigma_L^2) \tag{16}$$

It follows from this that Eq.'s (8) and (9) will be satisfied if

$$\sigma_L^2 = W \ln (c)$$

$$= 0.40547\ W \quad , \tag{17}$$

and from Eq. (13) that

$$\bar{L} = -0.20273\ W \tag{18}$$

In practice, we would generate the random pulse height, I_W , by generating the random variable, L_W , where L_W has the gaussian probability density

$$p_L\ (L_W) = (2\pi\ \sigma_L^2)^{-1/2}\ \exp\left[-\tfrac{1}{2}\ \frac{(L_W - \bar{L})^2}{\sigma_L^2}\right] \quad , \tag{19}$$

and generate I_w from our random choice of L_w by the relationship

$$I_w = \overline{I}_0 \exp(L_w) \tag{20}$$

With this method of choosing the random pulse height, I_w, the mean pulse height will be independent of pulse width, while the second moment of pulse height will be so dependent on pulse width that, in combination with the dependence of mean pulse count on pulse width [cf. Eq. (4)], the κ^{-1} nature of the image power-spectrum is insured.

At this point, having completed our definition of a method of generating the random sample image, we need to prove that this method does indeed yield an image with a κ^{-1} power-spectrum. We take this up in the next section.

4. Image Power Spectrum

The image generated by the preceding process is a random selection from an ensemble of possible images, the nature of the ensemble being such that the image power spectrum presumably behaves as κ^{-1} . In this section we shall show that it does indeed behave in this manner.

The randomly selected image can be written as

$$S(p) = \sum_{W=0}^{7} \sum_{M=0}^{N_W} \theta\,(p;W\,,\,\ell_{W,M})\, I_{W,M} \quad , \tag{21}$$

where we have used the dual subscript on $I_{W,M}$ to indicate a separate choice of I for each pulse of width a_W , the choice being random over the set M = 0 to N_W . We let p = 0 , 1 , 2 , . . . , 1023 denote the positions in the field-of-view. The quantity N_W is a Poisson distributed random variable with mean value $\overline{N}_W$, calculated in accordance with Eq.'s (4) and (7). The quantity $I_{W,M}$ is a log-normally distributed random variable, calculated from Eq. (20) with L a gaussian random variable selected in accordance with Eq.'s (17), (18), and (19). The quantity $\ell_{W,M}$ is a gaussian distributed random variable selected in accordance with Eq. (1). The function $\theta(p;W,\ell)$ denotes a pulse of width, W , centered at ℓ . Thus,

$$\theta(p;W,\ell) = \begin{cases} 1 & \text{if} \quad |p-\ell| \le \frac{1}{2}\, a_W \quad , \\ 0 & \text{if} \quad |p-\ell| > \frac{1}{2}\, a_W \quad . \end{cases} \tag{22}$$

To prove that the form of S(p) has a κ^{-1} power spectrum, we start by considering the fourier transform of θ . Thus we write

$$\tilde{\theta}(\kappa;W,\ell) = \int_{-\infty}^{+\infty} dp\, \exp(-i\kappa p)\, \theta(p;W,\ell)$$

$$= \int_{\ell-a_W/2}^{\ell+a_W/2} dp\, \exp(-i\kappa p)$$

$$= \exp(-i\kappa\ell) \int_{-a_W/2}^{a_W/2} dp'\, \exp(-i\kappa p')$$

$$= \exp(-i\kappa\ell)\, a_W \,\frac{\sin(\kappa a_W/2)}{\kappa a_W/2}, \qquad (23)$$

which is a random variable inasmuch as ℓ is a random variable.

Based on this, we can write the fourier transform of the randomly generated image signal, $S(p)$, as

$$\tilde{S}(\kappa) = \int_{-\infty}^{+\infty} dp\, \exp(-i\kappa p)\, S(p)$$

$$= \sum_{W=0}^{7} \sum_{M=0}^{N_W} I_{W,M} \int_{-\infty}^{+\infty} dp\, \exp(-i\kappa p)\, \theta(p;W,\ell_{W,M})$$

$$= \sum_{W=0}^{7} \sum_{M=0}^{N_W} I_{W,M}\, \tilde{\theta}(\kappa;W,\ell_{W,M})$$

$$= \sum_{W=0}^{7} a_W \,\frac{\sin(\kappa a_W/2)}{\kappa a_W/2} \left\{ \sum_{M=0}^{N_W} I_{W,M} \exp(-i\kappa \ell_{W,M}) \right\} \qquad (24)$$

Here the quantities in the curly brackets have been isolated because they contain all of the random aspects of $\tilde{S}(\kappa)$. At this point, to develop a relationship between the random fourier transform and the power spectral density, we make use of the fact that the power spectral density function associated with the random image signal S , namely PSD(κ) can be written as

$$\mathrm{PSD}(\kappa) = 2\pi \int d\kappa' \langle \tilde{S}^*(\kappa)\, \tilde{S}(\kappa') \rangle \tag{25}$$

From this, it follows that

$$\mathrm{PSD}(\kappa) = 2\pi \sum_{W=0}^{7} \sum_{W'=0}^{7} a_W\, a_{W'} \int d\kappa' \; \frac{\sin(\kappa\, a_W/2)}{\kappa\, a_W} \; \frac{\sin(\kappa'\, a_{W'}/2)}{\kappa'\, a_{W'}}$$

$$\times \left\langle \sum_{M=0}^{N_W} \sum_{M'=0}^{N_W} \langle I_{W,M}\, I_{W',M}\rangle \langle \exp[\,i\,(\kappa\, \ell_{W,M} - \kappa'\, \ell_{W',M})] \rangle \right\rangle . \tag{26}$$

Here we have separated the ensemble average over $I_{W,M}$ and over $\ell_{W,M}$ based on the argument that the intensity and position of each element are statistically independent. The outermost ensemble average is over the choice of N_W , the random number of elements. To evaluate the ensemble average over $\ell_{W,M}$ in Eq. (26), we first of all note that we are not interested in spatial frequencies lower than $(2\, a_0)^{-1}$, i.e., in values of κ any smaller than

$$\kappa_0 = 2\pi/(2\, a_0) = \pi/a_0 \tag{27}$$

We shall evaluate the ensemble average over $\ell_{W,M}$ by separately treating the two cases in which $(W,M) = (W', M')$, and when it is not. We argue that the case when $(W,M) \neq (W', M')$ makes a negligible contribution, since in this case $\ell_{W,M}$ and $\ell_{W',M}$ are statistically independent gaussian random variables, so that

$$\langle \exp [i(\kappa \ell_{W,M} - \kappa' \ell_{W',M'})] \rangle$$

$$= \exp [-\tfrac{1}{2}(\kappa^2 + \kappa'^2)\sigma_s^2 + i(\kappa - \kappa')\bar{\ell}] \qquad (28)$$

Considering that $\sigma_s = 2a_0$, it follows that for all values of κ of interest to us, $\frac{1}{2}\kappa^2\sigma_s^2$ is greater than $2\pi^2$, and then, since $\exp(-2\pi^2) = 2.67 \times 10^{-9}$, the value of the ensemble average is negligibly small.

This leaves us with only the case where $(W, M) = (W', M')$ as a possible contribution to the power spectral density. In this case, since $\ell_{W,M}$ and $\ell_{W',M'}$ are identical, we can write

$$\{\langle \exp [i(\kappa \ell_{W,M} - \kappa' \ell_{W',M'})] \rangle\}_{W=W',\, M=M'}$$

$$= \exp [-\tfrac{1}{2}(\kappa - \kappa')^2 \sigma_s^2 + i(\kappa - \kappa')\bar{\ell}] \qquad (29)$$

Because of the relatively large value of σ_s, we can treat the gaussian in Eq. (29) as a delta function, i.e., as $(2\pi\sigma_s^2)^{1/2}\,\delta(\kappa - \kappa')$ in evaluating the κ'-integration in Eq. (26). Thus we get

$$\mathrm{PSD}(\kappa) = (2\pi)^{3/2}\sigma_s \sum_{W=0}^{7} (a_W)^2 \left[\frac{\sin(\kappa a_W/2)}{\kappa a_W/2}\right]^2 \left\langle \sum_{M=0}^{N_W} \langle I_{W,M}^2 \rangle \right\rangle , \qquad (30)$$

where here the ensemble average is only over the choice of N_W for each value of W, and the choice of $I_{W,M}$ for each value of (W, M). These two random features are statistically independent, and factorable in terms of their effect on the power spectral density as defined in Eq. (30). Thus, since

$$\langle I_{W,M}^2 \rangle = \langle I_W^2 \rangle \qquad (31)$$

i.e., the value of $\langle I_{W,M}{}^2 \rangle$ is independent of M, (though as we have set up the statistics of I, it retains a dependence on W), we can rewrite Eq. (30) as

$$PSD(\kappa) = (2\pi)^{3/2} \sigma_s \sum_{W=0}^{7} (a_W)^2 \, \overline{N}_W \, \langle I_W{}^2 \rangle \left[\frac{\sin(\kappa a_W/2)}{\kappa a_W/2} \right]^2 \tag{32}$$

Making use of Eq.'s (3), (4), (7), (8), and (9), and the statement immediately following Eq. (12), we can rewrite Eq. (32) as

$$PSD(\kappa) = (2\pi)^{3/2} \sigma_s \sum_{W=0}^{7} a_0{}^2 \, 2^{-2W} \, b^W \, \overline{I}_0{}^2 \, c^{-W}$$

$$\times \left[\frac{\sin(\kappa a_0 \, 2^{-(W+1)})}{\kappa a_0 \, 2^{-(W+1)}} \right]^2$$

$$= (2\pi)^{3/2} \sigma_s \, a_0{}^2 \, \overline{I}_0 \sum_{W=0}^{7} 2^{-W} \left[\frac{\sin(\kappa a_0 \, 2^{-(W+1)})}{\kappa a_0 \, 2^{-(W+1)}} \right]^2 \tag{33}$$

It is easy to see that the power spectral density is the sum of a set of seven power spectra each of which is constant at low frequencies, but then breaks at some "knee-frequency", and after that falls off at 12 dB per octave. Each of the seven successive spectra has a constant region value that is a factor of two less than the preceding, but a "knee-frequency" that is a factor of two higher. Since 12 dB per octave corresponds to a factor of four fall-off for each factor of two increase in frequency, it is apparent that at each knee frequency there is some one of the sequence of seven subsidiary power spectra that is dominant, with the one just preceding and the one just following reduced by a factor of two. Thus the sum of all the subsidiary power spectra should vary with frequency as the response at the knee-frequency varies with knee-frequency. This is a κ^{-1} dependence. This situation is indicated in Fig. 1, where we show the curves for W = 0 , 1 , 2 , . . . , 6 (W = 7 is off the graph), and the sum of these curves making up the power spectral density. The sum quite clearly follows the knees of the W-curves, and therefore has the κ^{-1}-dependence.

5. Comments

With the technique defined in the preceding section, we have established a method of generating a one-dimensional image with a κ^{-1} power spectrum. Our next objective will be to use this procedure to develop sample images which we will then filter to simulate the effect of the CIS transfer function, and after that corrupt with Poisson distributed noise, i.e., replacing the filtered intensity at each signal point with a Poisson distributed random variable with that mean value. We will then subject such a one-dimensional image to the nonstationary post-detection compensation procedure which we described earlier.

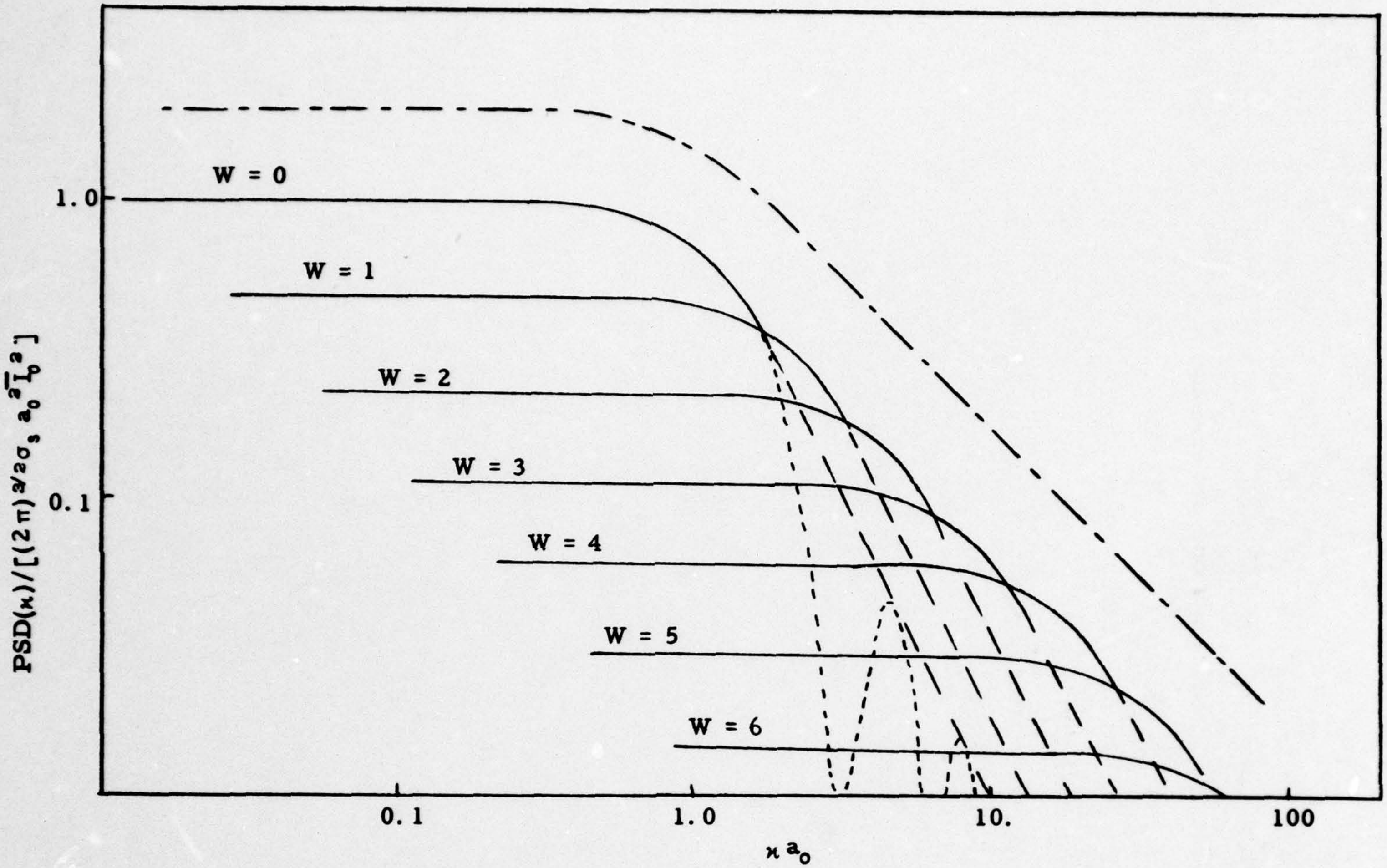

Figure 1. Power Spectral Density As A Function Of Spatial Frequency. The dotted line shows the exact $[\sin^2(x)/x]^2$ dependence for , but in the oscillatory region we have suppressed this oscillation and replaced the curve with the dashed line, which is its envelope. The envelope curves are shown for , 1, . . . , 6. The dot-dash curve is the sum of these envelope curves and represents the power spectral density. Its κ^{-1} dependence is apparent for spatial frequencies above $\kappa\ 1/a_o$.

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RADC plans and conducts research, exploratory and advanced development programs in command, control, and communications (C^3) activities, and in the C^3 areas of information sciences and intelligence. The principal technical mission areas are communications, electromagnetic guidance and control, surveillance of ground and aerospace objects, intelligence data collection and handling, information system technology, ionospheric propagation, solid state sciences, microwave physics and electronic reliability, maintainability and compatibility.

AMERICAN REVOLUTION BICENTENNIAL
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