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A DIRECT MEANS OF COMPUTING SOUND INTENSITY ALONG A RAYPATH IN --ETC(U)
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(6) A DIRECT MEANS OF COMPUTING SOUND INTENSITY ALONG A RAYPATH IN A CONTINUOUS BROKEN-LINE SOUND VELOCITY FIELD.

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Abstract: Two formulas, equations (20) and (21), are derived for the intensity along a raypath in terms of the horizontal distance traveled by the wave front and the angle which the ray makes with the horizontal. Both formulas are amenable to digital computation in parallel with a regular digital raypath calculation.

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Background

For over a decade, the standard means for computing intensity along a sound raypath in waters with variable sound velocity (versus depth) has been to approximate the sound velocity function by a continuous broken-line function, and, then, compute the vertical distance separating two closely adjacent rays. This vertical distance is compared with the initial angle between the rays in order to determine an approximate value of the divergence. The difficulties with this approach concern the sound velocity approximation and the closeness with which the two neighboring rays must be taken to yield reasonable intensities. Obviously, the closer a pair of rays are taken in initial angle, the more accurate the calculation must be for each ray to preserve a significant value of vertical distance between them.

There are two methods by which the standard calculations of the pairs of rays can be effected: sequentially or simultaneously. In the former, one computes the first ray and stores the z (vertical) ordinates for a certain predetermined x (horizontal) values; then the same is done for the second ray; and finally the stored z-ordinates are differenced to form Δz's. In the latter, one has two computers of matched accuracy for the computation of the raypaths; the computation yielding the respective z ordinates for the same running variable x; then a differencing calculation, either continuous or discrete, between the z's to form Δz as a function of x follows. Both means require high accuracy of raypath calculation (higher than if one just wished to compute the ray itself); both are amenable to digital calculation; only the latter seems suitable for analog implementation. The former requires a very extensive and fast memory and a large computer to achieve any considerable speed at making a complete sound raypath and intensity plot. The latter requires matched accuracies that are difficult and expensive to obtain and maintain from an analog unit.

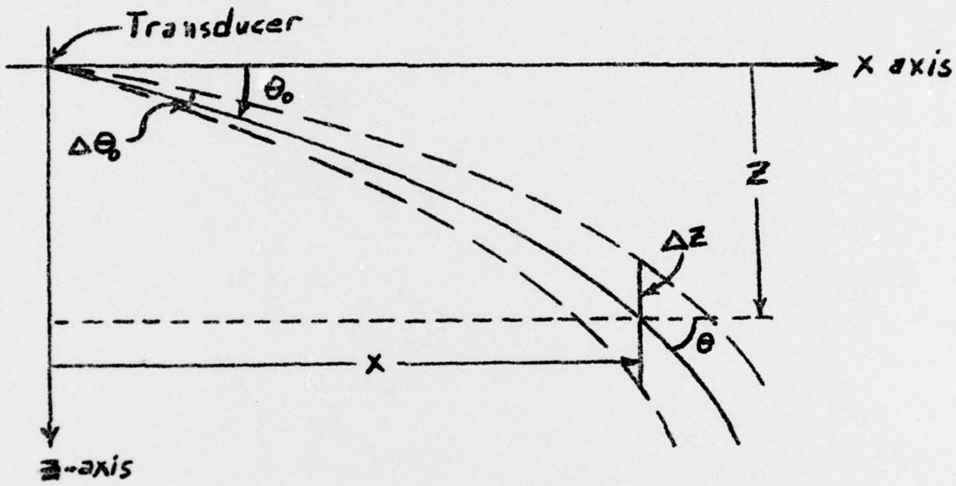
The goal behind the approach initiated here is to find an auxiliary means of computing the sound intensity change along a raypath. The form of the calculation would be to calculate the coordinates of a single ray and while so doing, put these into a second equation which would compute the sound intensity (or some function of it) directly. It is hoped that in this manner, some of the high accuracy and matched unit requirements can be eliminated and reasonable analog means arrived at.

Sound Raypath Intensity Calculation

Because the lateral extent of the oceans is many times greater than the depth dimension, the fall off in intensity along a ray is measured in terms of horizontal distance x. Acoustic intensity is power per unit area and its loss is measured from some standard sphere of unit radius. The intensity as a function of x along the raypath is given approximately by

$$I \cong \frac{P \cos \theta_0 (\Delta\phi)(\Delta\theta_0)}{x(\Delta\phi)(\Delta z) \cos \theta}$$

where the θ, x, and z quantities are as shown in Figure 1 and ϕ is angle in the horizontal plane. As can be seen, the usual assumptions apply: the direction of the sound raypath does not vary in the horizontal plane, thus sound velocity is a z dependent function only; z is the depth dimension and is measured with increasing size downward; θ is the angle the raypath makes with the horizontal axis measured positively from x to z; and θ_o is the initial angle at which the ray leaves the transducer.



Depth Plane showing θ_o, θ, x, z along a Raypath

Figure 1

The intensity along the raypath can be defined as follows:

$$I \equiv \lim_{\substack{\Delta\phi \to 0 \\ \Delta\theta_o \to 0}} \frac{P \cos \theta_o}{x \cos \theta} \left(\frac{\Delta\theta_o}{\Delta z}\right)$$

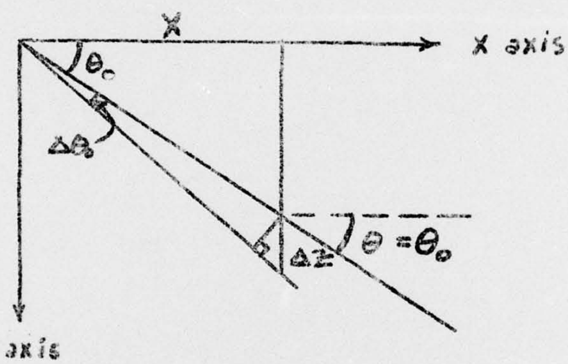
$$I = \frac{P \cos \theta_o}{x \cos \theta} \left(\frac{d\theta_o}{dz}\right) \qquad (1)$$

where it must be remembered to take Δz or dz in the positive direction.

The problem of evaluating I reduces to evaluating $d\Theta_o/dz$ or inversely $dz/d\Theta_o$.

$dz/d\Theta_o$ for the No Gradient Case

This is simply done from reference to the sketch.



$$\Delta z = \sec \Theta_o \, (x \sec \Theta_o) \, \Delta\Theta_o$$

$$\frac{\Delta z}{\Delta\Theta_o} = x \sec^2 \Theta_o$$

$$\frac{dz}{d\Theta_o} = x \sec^2 \Theta_o$$

Substituting into equation (1) one gets

$$I = \frac{P \cos \Theta_o}{x \cos \Theta_o \; x \sec^2 \Theta_o} = \frac{P}{x^2 \sec^2 \Theta_o} \tag{2}$$

Equation (2) reduces to the inverse square law along the path length, R, when one notes that

$$R = x \sec \theta_o .$$

$dz/d\theta_o$ for the Linear Gradient Case

It is well known that for the linear gradient case, the raypaths become arcs of circles. Let the velocity equation be given by

$$V(z) = V_o + V' \, (z - z_o) \tag{3}$$

where V_o is the sound velocity at depth z_o and V' represents the gradient -- it is a negative number for decreasing sound velocity with increasing depth.

Snell's law along a particular sound ray can be written

$$V(z) = \frac{V_o}{\cos \theta_o} \cos \theta . \tag{4}$$

Differentiate and obtain

$$V' \frac{dz}{d\theta} = \frac{-V_o}{\cos \theta_o} \sin \theta$$

or

$$\frac{dz}{d\theta} = \frac{-V_o}{V' \cos \theta_o} \sin \theta .$$

But

$$\frac{dz}{dx} = \tan \theta$$

and, therefore,

$$\frac{dx}{d\theta} = \frac{-V_o}{V' \cos \theta_o} \cos \theta .$$

Let us define A_o by

$$A_o \equiv \frac{-V_o}{V' \cos \Theta_o} \qquad (5)$$

and thus we get as the diffential equation for the sound raypath

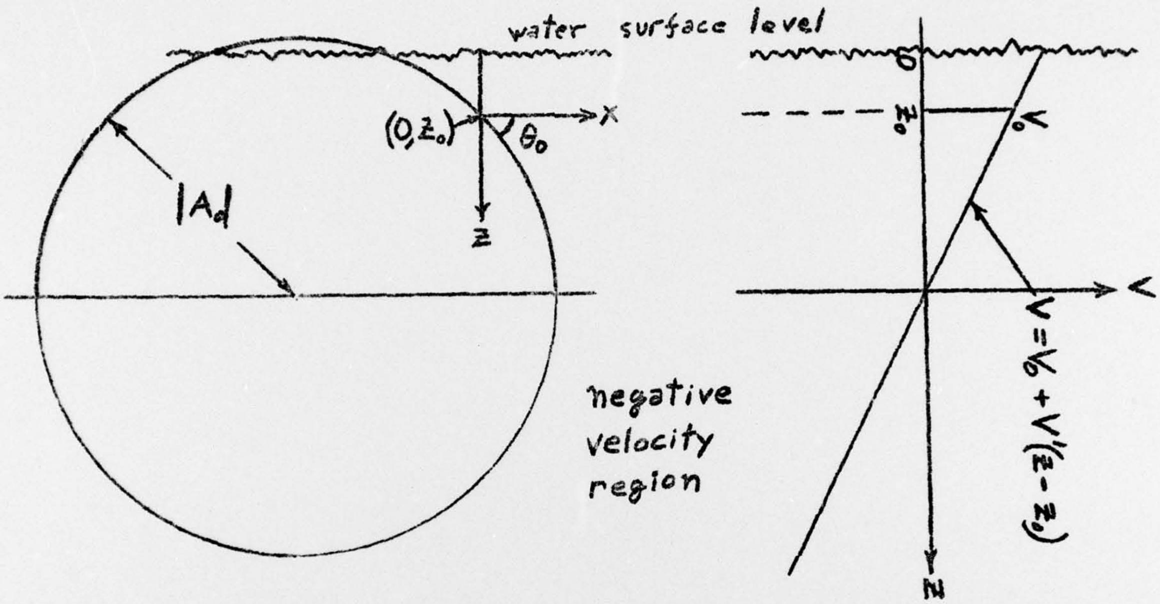
$$\begin{cases} \dfrac{dz}{d\Theta} = A_o \sin \Theta \\[2ex] \dfrac{dx}{d\Theta} = A_o \cos \Theta \end{cases} \qquad (6)$$

which are evidently circles of radius A_o. Integrating and applying the conditions $x = 0$, $z = z_o$ at $\Theta = 0_o$ gives the parametric equations of the raypath circle:

$$\begin{cases} z - x_o = -A_o (\cos \Theta - \cos \Theta_o) \\[2ex] \quad x = A_o (\sin \Theta - \sin \Theta_o) \end{cases} \qquad (7)$$

Elimination of the parameter Θ gives the equation of the circle in the closed form

$$0 = (z - z_o)^2 + x^2 - 2A_o \cos \Theta_o (z - z_o) + 2A_o \sin \Theta_o x. \qquad (8)$$



Sound Raypath Circle

Sound Velocity Curve

Figure 2

It is evident from equations (3) and (4) and Figure 2 that a physical raypath will not travel around a complete circle because to do so requires a negative velocity. In fact, since in the oceans the sound velocity rarely varies by more than 8 o/o total, only small arcs of these circles will be encountered in practice.

The computation of $dz/d\theta_o$ is done as a limit of the ratio $\Delta z/\Delta\theta_o$. Suppose what we take two nearby rays emitted from the same transducer, a θ_o ray and a θ_1 ray. The equation for the circular raypaths are given by

$$\theta_1 \text{ ray: } (\bar{z} - z_o)^2 + \bar{x}^2 - 2A_1 \cos\theta_1 (\bar{z} - z_o) + 2A_1 \sin\theta_1 \bar{x} = 0$$

θ_o ray : same as equation (8)

where

$$A_1 = \frac{-V_o}{V' \cos \theta_1} \quad ; \quad A_o = \frac{-V_o}{V' \cos \theta_o}$$

We wish to compute $\Delta z \equiv \bar{z} - z$ for identical values of x. Solutions of the equations for each ray are:

$$\bar{z} = z_o - \frac{V_o}{V'} + \frac{V_o}{V'} \left[1 + 2 \tan \theta_1 \left(\frac{V'x}{V_o} \right) - \left(\frac{V'x}{V_o} \right)^2 \right]^{1/2}$$

$$z = z_o - \frac{V_o}{V'} + \frac{V_o}{V'} \left[1 + 2 \tan \theta_o \left(\frac{V'x}{V_o} \right) - \left(\frac{V'x}{V_o} \right)^2 \right]^{1/2}$$

From these, it is evident that

$$\frac{dz}{d\theta_o} = \lim \frac{\Delta z}{\Delta \theta_o}$$

$$\frac{dz}{d\theta_o} = \frac{V_o}{V'} \frac{\delta}{\delta \theta_o} \left[1 + 2 \tan \theta_o \left(\frac{V'x}{V_o} \right) - \left(\frac{V'x}{V_o} \right)^2 \right]^{1/2}$$

where δ denotes differentiation with respect to explicit variables only

$$\frac{dz}{d\theta_o} = x \sec^2 \theta_o \left[1 + 2 \tan \theta_o \left(\frac{V'x}{V_o} \right) - \left(\frac{V'x}{V_o} \right)^2 \right]^{-1/2} \tag{9}$$

Therefore, equation (1), becomes

$$I = \frac{P \left[1 + 2 \tan \theta_o \left(\frac{V'x}{V_o} \right) - \left(\frac{V'x}{V_o} \right)^2 \right]^{1/2}}{x^2 \sec^3 \theta_o \cos \theta}$$

But, from equation (7b), one can compute that the radical is nothing more than $\cos\Theta \sec\Theta_o$, therefore,

$$I = \frac{P\cos\Theta\sec\Theta_o}{x^2\sec^2\Theta_o\cos\Theta}$$

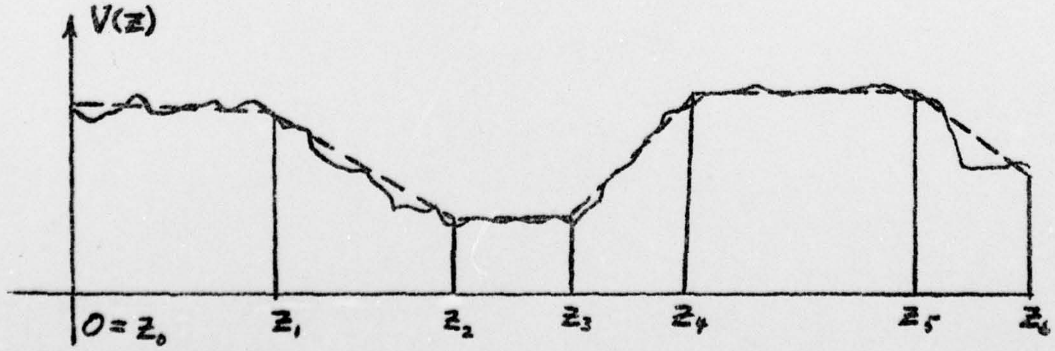
$$I = \frac{P}{x^2\sec^2\Theta_o} \tag{10}$$

Equations (2) and (10) are identical in form. However, in the interpretation of equation (10), the distance along the raypath, R, is no longer equal to $x \sec\Theta_o$. Consequently, the raypath intensity contours differ from the isovelocity case.

What has been derived to this point, equation (10), is not new; it is, for example, derived more elegantly in C. B. Officer's "Introduction to the Theory of Sound Transmission," McGraw-Hill, 1958, pgs. 48-50, 59-61.

Continuous Linear Segment Approximation to the Non-linear Gradient.

It has been customary for some time to approximate non-linear sound-velocity gradients by continuous linear segments for the purposes of approximate calculation of sound raypaths in the ocean. The raypaths which result are arcs of circles, as above, but with different radii in each depth segment. The tangent lines to the resulting raypath are continuous over the boundaries of the depth segments. In general, the linear segment approximation is an "eye" fit with unequal depth segments.



The equations governing the segments are:

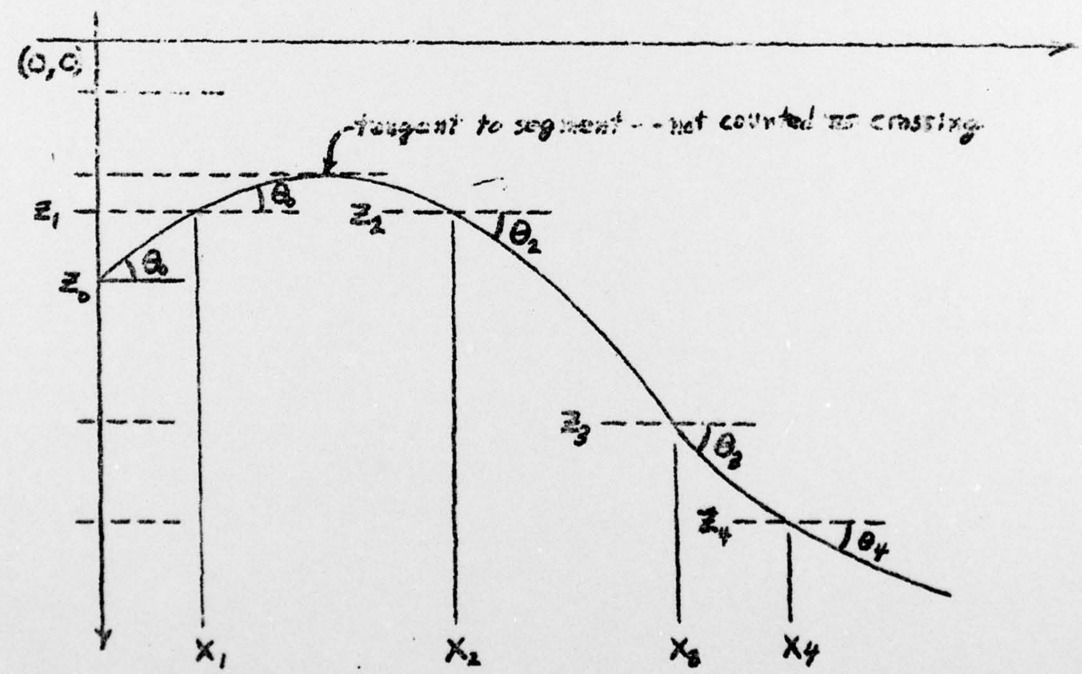
$$V(z_i) = V_i$$

$$V(z) = V_i + (z - z_i)\, V_i' \qquad (11)$$

$$V_{i+1} = V_i + (z_{i+1} - z_i)\, V_i'$$

where all subscripts run over $i = 0, 1, \ldots$ and where the second of equation (11) applies only between z_i and z_{i+1}.

$dz/d\theta_0$ for Continuous Linear Segment Approximation



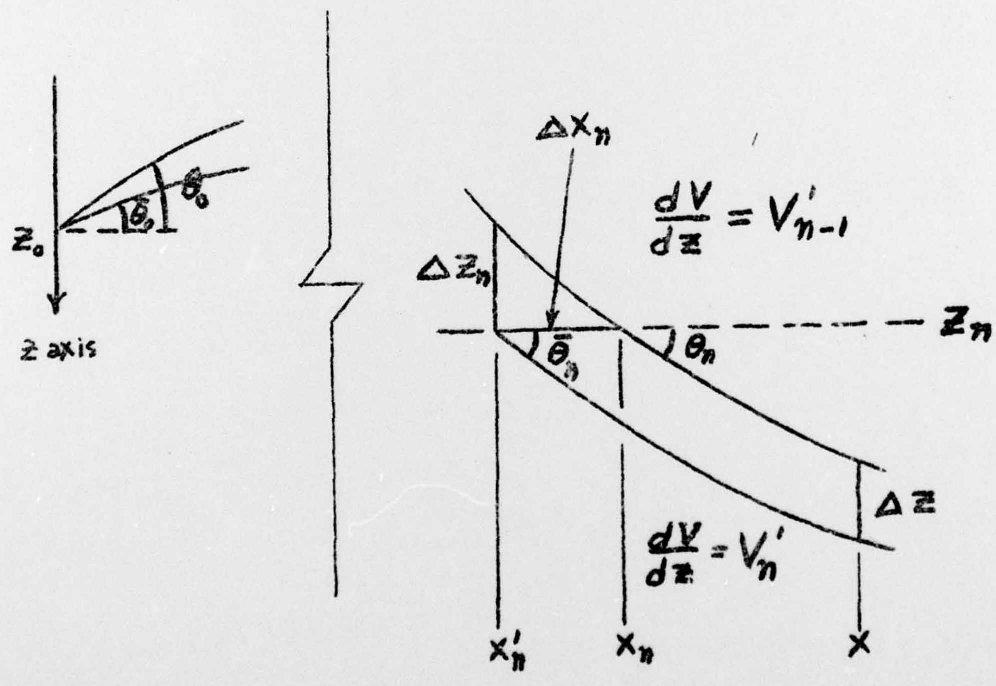
Raypath with x, z, and θ Notation

Figure 3

Here, again, we investigate the limit of $\Delta z/\Delta\theta_o$ but for the situation of the raypath generated by the linear segment approximation. The notation is different from the above; the running subscript n refers to the n-th depth segment crossed by the ray while z_o refers to the initial depth of the ray (transducer depth) which is not necessarily on a segment boundary. The ray picture may look something like Figure 3.

In Figure 3, the boundaries of the depth segment are shown by dashed lines intersecting the z-axis. We observe that the sequence x_n is increasing while the sequences z_n and θ_n are not necessarily so. If, as is drawn in Figure 3, the raypath reaches a point of tangency with a segment boundary, the ray is considered not to have crossed the boundary and will continue on in the same velocity gradient region as before the point of tangency.

For the calculation of the ratio $\Delta z/\Delta\theta_o$, the notation of Figure 4 applies.



Geometry and Notation for $\Delta z/\Delta\theta_o$ Calculation

Figure 4

Care must be taken in using the geometry of Figure 4 because of the different relative magnitudes of x_n and x_n'. The rule is

$$\begin{aligned} &\theta_n \text{ (positive)} \rightarrow x_n' - x_n < 0 \text{ (as shown)} \\ &\theta_n \text{ (negative)} \rightarrow x_n' - x_n > 0 \text{ (not shown)} \end{aligned} \tag{12}$$

From the figure, the equations governing the arcs of the θ_o-ray (and $\bar{\theta}_o$ ray) from x_n (and x_n') to x are given by equation (13), (respectively)

$$\begin{aligned} &(z - z_n)^2 + (x - x_n)^2 + \frac{2 V_n}{V_n'} (z - z_n) - \frac{2 V_n}{V_n'} \tan \theta_n (x - x_n) = 0 \\ &(\bar{z} - z_n)^2 + (x - x_n')^2 + \frac{2 V_n}{V_n'} (\bar{z} - z_n) - \frac{2 V_n}{V_n'} \tan \bar{\theta}_n (x - x_n') = 0 \end{aligned} \tag{13}$$

From Snell's law, along each curve one gets

$$\tan \theta_n = \pm \frac{\sqrt{V_o^2 \sec^2 \theta_o - V_n^2}}{V_n}$$

$$\tan \bar{\theta}_n = \pm \frac{\sqrt{V_o^2 \sec^2 \bar{\theta}_o - V_n^2}}{V_n}$$

where the sign is positive for θ_n positive and negative for θ_n negative. The solution of equation (13) for z and $\bar{z}$ yield:

$$z = z_n - \frac{V_n}{V_n'} + \frac{V_n}{V_n'}\left[1 \pm \frac{2\,V_n'}{V_n}\,\frac{\sqrt{V_o^2 \sec^2 \theta_o - V_n^2}}{V_n}\,(x - x_n) - \left(\frac{V_n'\,(x - x_n)}{V_n}\right)^2\right]^{1/2}$$

$$\bar{z} = z_n - \frac{V_n}{V_n'} + \frac{V_n}{V_n'}\left[1 \pm \frac{2\,V_n'}{V_n}\,\frac{\sqrt{V_o^2 \sec^2 \bar{\theta}_o - V_n^2}}{V_n}\,(x - x_n') - \left(\frac{V_n'\,(x - x_n')}{V_n}\right)^2\right]^{1/2}$$

where the sign conventions again follow θ_n. The expression for Δ_z, $\bar{z} - z$, divided by $\Delta\theta_o$, $\bar{\theta}_o - \theta_o$

$$\frac{\Delta z}{\Delta\theta_o} = \frac{1}{\Delta\theta_o}\left\{\frac{V_n}{V_n'}\left[1 \pm \frac{2\,V_n'}{V_n}\,\frac{\sqrt{V_o^2 \sec^2 \bar{\theta}_o - V_n^2}}{V_n}\,(x - x_n') - \left(\frac{V_n'\,(x - x_n')}{V_n}\right)^2\right]^{1/2}\right.$$

$$- \frac{V_n}{V_n'}\left[1 \pm \frac{2\,V_n'}{V_n}\,\frac{\sqrt{V_o^2 \sec^2 \theta_o - V_n^2}}{V_n}\,(x - x_n)\right.$$

$$- \left(\frac{V_n' (x - x_n)}{V_n} \right)^2 \Bigg] ^{1/2} \Bigg\}$$

By noting that $x - x_n' = x - x_n \mp \Delta x$ with the same convention as is in (12) and then expanding the first of the radicals as a function of Δx, one gets

$$\frac{\Delta z}{\Delta \Theta_o} = \frac{1}{\Delta \Theta_o} \Bigg\{ \frac{V_n}{V_n'} \left[X(\bar{\Theta}_o) - X(\Theta_o) \right] +$$

$$\frac{1}{X(\bar{\Theta}_o)} \left[\frac{\sqrt{V_o^2 \sec^2 \bar{\Theta} - V_n^2}}{V_n} \mp \frac{V_n'}{V_n} (x - x_n) \right] \Delta x$$

$$+ (\Delta x)^2 \, (\text{etc}) \Bigg\}$$

where the definition of $X(\Theta_o)$ is

$$X(\Theta_o) = \left[1 \pm \frac{2 V_n'}{V_n} \frac{\sqrt{V_n^2 \sec^2 \Theta_o - V_n^2}}{V_n} (x - x_n) - \left(\frac{V_n' (x - x_n)}{V_n} \right)^2 \right]^{1/2}$$

If we multiply through by one over $\Delta \Theta_o$ and pass to the limit, we arrive at the preliminary form of the recursion formula

$$\frac{dz}{d\Theta_o} = \frac{V_n}{V_n'} \frac{\partial X(\Theta_o)}{\partial \Theta_o} + \frac{1}{X(\Theta_o)} \left[\frac{\sqrt{V_o^2 \sec^2 \Theta_o - V_n^2}}{V_n} \mp \frac{V_n' (x - x_n)}{V_n} \right] \frac{dx}{d\Theta_o} \qquad (14)$$

The procedure is now straight forward: compute the partial derivative, express the derivative $dz/d\theta_o$ in terms of $dz_n/d\theta_o$, and simplify

$$\frac{\delta X}{\delta \theta_o} = \pm \frac{V'_n}{V_n} \frac{V_o^2 \sec^2 \theta_o \tan \theta_o (x - x_n)}{X(\theta_o) V_n \sqrt{V_o^2 \sec^2 \theta_o - V_n^2}} \tag{15}$$

The expression for $dz_n/d\theta_o$ can be evaluated as follows:

$$\frac{\Delta z_n}{\Delta \theta_o} = \frac{z_n - z}{\Delta \theta_o} \text{ for } \theta_n \text{ positive and}$$

$$\frac{\Delta z_n}{\Delta \theta_o} = \frac{\bar{z} - z_n}{\Delta \theta_o} \text{ for } \theta_n \text{ negative, where the equations for } z \text{ and } \bar{z} \text{ are:}$$

$$(z - z_n)^2 + (\Delta x)^2 + \frac{2 V_n}{V'_n - 1} (z - z_n) + \frac{2 V_n}{V'_n - 1} \tan \theta_n (\Delta x) = 0$$

$$(\bar{z} - z_n)^2 + (\Delta x)^2 + \frac{2 V_n}{V'_n - 1} (\bar{z} - z_n) + \frac{2 V_n}{V'_n - 1} \tan \bar{\theta}_n (\Delta x) = 0$$

It follows then that in the limit as $\Delta\theta_o$ goes to zero, the formal expressions for the derivatives will be the negative of each other. Therefore, the joint calculation can be written as

$$\frac{\Delta z_n}{\Delta \theta_o} = \pm \frac{1}{\Delta \theta_o} \left\{ \frac{V_n}{V'_n - 1} - \frac{V_n}{V'_n - 1} \left[1 - \frac{2 V'_n - 1}{V_n} \tan \theta_n (\Delta x) \right. \right.$$

$$+\left(\frac{V_n' - 1}{V_n}\Delta x\right)^2\Bigg]^{1/2}\Bigg\}$$

Where the sign convention is the upper when Θ_n is positive and the lower when Θ_n is negative. Expanding the radical as a function of Δx yields

$$\frac{\Delta z_n}{\Delta\Theta_o} = \pm \frac{1}{\Delta\Theta_o}\left\{-\frac{V_n}{V_n' - 1}\left[-\frac{V_n' - 1}{V_n}\tan\Theta_n\right]\Delta x + (\Delta x)^2\ (\text{etc.})\right\}$$

which upon passing to the limit becomes, exactly,

$$\frac{dz_n}{d\Theta_o} = \pm \tan\Theta_n \frac{dx}{d\Theta_o} \quad \text{or} \quad = \left|\tan\Theta_n\right| \frac{dx}{d\Theta_o} \qquad (16)$$

in which the sign convention is the usual one. This is certainly the expected result for this calculation. Using equations (14), (15), (16), and the definition of $\tan\Theta_n$, one gets

$$\frac{dz}{d\Theta_o} = \frac{1}{X(\Theta_o)}\left\{\left[1 - \frac{V_n'(x - x_n)}{V_n \tan\Theta_n}\right]\frac{dz_n}{d\Theta_o} + \frac{\tan\Theta_o (x - x_n)}{\cos\Theta_n \sin\Theta_n}\right\} \qquad (17)$$

This is the recursion relationship which carries the derivative over the boundary from one gradient to another. To further simplify the equation, the following two relationships are noted:

$$X(\Theta_o) = \cos\Theta \sec\Theta_n$$

$$1 - \frac{V_n'(x - x_n)}{V_n \tan\Theta_n} = \frac{\sin\Theta}{\sin\Theta_n}$$

and, therefore,

$$\frac{dz}{d\theta_o} = \frac{\cos\theta_n}{\cos\theta}\left\{\frac{\sin\theta}{\sin\theta_n}\;\frac{dz_n}{d\theta_o} + \frac{\tan\theta_o\,(x - x_n)}{\cos\theta_n\,\sin\theta_n}\right\}$$

or

$$\cos\theta\,\frac{dz}{d\theta_o} = \cos\theta_n\,\frac{dz_n}{d\theta_o}\left[\frac{\sin\theta}{\sin\theta_n}\right] + \frac{\tan\theta_o\,(x - x_n)}{\sin\theta_n} \qquad (18)$$

Repeated application of equation (18), decreasing the index n, yields the desired formula:

$$\cos\theta\,\frac{dz}{d\theta_o} = \tan\theta_o\left[\sum_1^n\frac{(x_i - x_{i-1})\sin\theta}{\sin\theta_{i-1}\,\sin\theta_i} + \frac{x - x_n}{\sin\theta_n}\right] \qquad (19)$$

The reciprocal of the intensity function is given by

$$\frac{1}{I} = \frac{x\sin\theta_o}{P\cos^2\theta_o}\left[\sum_1^n\frac{(x_i - x_{i-1})\sin\theta}{\sin\theta_{i-1}\,\sin\theta_i} + \frac{x - x_n}{\sin\theta_n}\right] \qquad (20)$$

Equation (20) has the following properties:

I. It reduces to the inverse of equation (10) for the first term only case:

$$\frac{1}{I} = \frac{x^2}{P\cos^2\theta_o}$$

II. The sum of two adjacent terms of the series are identically equal to a corresponding single term if, and only if, the velocity gradients across the boundary are the same:

$$\frac{x_{n+1} - x_n}{\sin \theta_{n+1} \sin \theta_n} + \frac{x_n - x_{n-1}}{\sin \theta_n \sin \theta_{n-1}} = \frac{x_{n+1} - x_{n-1}}{\sin \theta_{n+1} \sin \theta_{n-1}}$$

if and only if $V'_n = V'_{n-1}$.

III. The inverse intensity can be written as a sum of a term which reduces to the linear gradient loss plus a sum of gradient dependent terms which becomes zero in the absence of a change of gradient:

$$\frac{1}{I} = \frac{x}{P \cos^2 \theta_o} \left[\frac{- V_o (\sin \theta - \sin \theta_o)}{V'_o \cos \theta_o} + \sin \theta_o \sin \theta \sum_{2}^{n+1} \frac{(x_i - x_{i-1})}{\sin \theta_i \sin \theta_{i-1}} \left(1 - \frac{V'_{i-1}}{V'_o} \right) \right] \quad (21)$$

where $x_{n+1} = x$ and $\theta_{n+1} = \theta$. That the first term reduces to x when the gradient is linear can be seen from (7b).

Equation (20) is not really new either; its essence can be found in equation (89) of Chapter 3 of "Physics of Sound in the Sea" and elsewhere. However, I have not seen it written in this form -- usually it is written from crossing to crossing of the velocity break points -- nor have I seen the condition that only actual crossings of break points (and not tangencies to them) need be considered in the summation.