



FORECASTING OFFICER LOSSES-AN EXAMINATION OF METHODS

TECHNICAL REPORT

BY

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MODELING BRANCH SYSTEMS DEVELOPMENT AND SUPPORT DIVISION DIRECTORATE OF PERSONNEL DATA SYSTEMS AIR FORCE MILITARY PERSONNEL CENTER RANDOLPH AFB, TEXAS 78148



### ABSTRACT

Air Force Personnel Managers must be able to accurately forecast the force size. This need is explicit in meeting statutory budget limitations. Further officer losses drive accession, training, and promotion, thus the need for accuracy in forecasting losses cannot be over emphasized. To accomplish this objective loss rates have been generated using Ordinary Least Squares (OLS) stepwise regression. The objective of this paper is to expose the relative efficacies of alternative methods which could be used viz. Maximum Likelihood Estimation (MLE) and OLS standardized coefficient (Beta) predictor models.

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### PART I. INTRODUCTION

### Background

The Directorate of Personnel Data Systems of the Air Force Military Personnel Center is the prime agency for providing officer loss rates for all personnel management actions within HQ USAF. Currently this objective is satisfied by using OLS stepwise regression and an OLS derivative technique called Odds for Effectiveness (OFE). It is presumed that readers of this paper are thoroughly familiar with; (1) The need for and various modeling/simulation applications of loss rates, and (2) The fundamentals and applications of OLS. Therefore the objective of this paper will be to compare candidate methods to produce viable loss rates, and not a tutorial on theories as to why or how loss rates may be generated.

A project was initiated late in 1976 to evaluate candidate methods for producing reliable officer loss rates. Since much has been written of late concerning MLE (See Nerlove and Press #1, Dempsey and Fast #2, Lockman & Warner #3), and these authors as well as others contend that MLE is more reliable for predicting the dichotomous dependent variable, the MLE method was the top contender to replace OLS ... MLE is theoretically more stable over time (i.e. the concomitant data shift). In search of alternative methods which may also achieve time stability, the standardized coefficient (Beta) model of OLS was also investigated. Four cases were examined:

1. Contrived data with controlled data shifts.

- 2. 74-→75 Colonels' Retirements, Line officers.
- 3. 75->76 Colonels' Retirements, Line officers.

4. Fourth year group Non-Rated Line Separations 75-76.

CASE 1: CONTRIVED DATA WITH CONTROLLED SHIFTS

Three primary objectives were sought in this case; (1) Known and controlled data, (2) A small case, and (3) Controlled data shifts. To meet these objectives the following test was built:

N=50 Dependent Variable: Off (Ø) 70% On (1) 30% Variable #1 was distributed as below in the sample test file and the two x-validation files:

ON/OBS*	TEST FILE VALUE	X-VAL FILE #1 VALUE	X-VAL FILE #2 VALUE
1/10	1	0	0
2/10	8	9	5
3/10	27	22	22
4/10	64 ·	72	. 60
5/10	125	150	110

Variables 2 and 3 were held constant in all three files as below:

		¢ +	VAR 2 0	1
17A D	2	0	0/12	7/13
VAR 3	1 (	7/13	1/12	

Variable four was distributed as follows in the three files:

ON/OBS	TEST FILE VALUE	X-VAL FILE #1 VALUE	X-VAL FILE #2 VALUE
1/10	1	3	• 0
3/10	2	5	1
7/10	3	6	2
3/10	4	7	2
1/10	5	9	3

CASE 2: COLONELS' RETIREMENT 74->75, LINE OFFICERS

This case used historical data from the Air Force Military Personnel System with an objective of building a "weak" predictor model. The following six attributes were chosen from the '74 data to meet this objective:

\*Ten observations have a value of 1 in the sample test file. One of these observations is "ON". 1. Total Active Federal Commissioned Service (TAFCS)

- 2. Age (years)
- 3. Officer Effectiveness Report, Weighted Mean
- 4. Officer Effectiveness Report, Current
- 5. Number Permanent Passovers
- 6. Below the Zone Selection to Colonel (1/0)

CASE 3: COLONELS' RETIREMENT 75->76, LINE OFFICERS

A more comprehensive analysis was done in this case to produce a "strong" predictor model. Historical data were analyzed for colonels' retirements in 1973, 1974 and 1975 to select the attributes and attribute values which would be used. In this analysis three conditions were imposed: (1) Consistency, (2) Discrimination, and (3) Representative incumbency. Use of these constraints provided 13 attributes which were then transformed to provide yet a "stronger" model, or at least one with a higher r-squared. Stepwise regression results subsequently reduced the number of attributes used to six:

- 1. Total Active Federal Commissioned Service (TAFCS)
- 2. Number of Dependents
- 3. Source of Commission
- 4. Permanent Grade
- 5. Duty Air Force Specialty Code
- 6. Officer Effectiveness Report, Weighted Mean

CASE 4: FOURTH YEAR GROUP NON-RATED LINE SEPARATIONS 75-76

Statistical phenomena which occur at or near .50 should be more difficult to model. Intuitively one expects the potential discriminators also to be distributed at or near a fifty-fifty split just as the dependent variable. Fourth year separations approach fifty percent for non-rated line officers as this point is the end of obligated service for all except Air Force Academy accessions. Just as in the colonels' cases the constraints of discrimination and representative incumbency were applied...consistency over time was not. Transforming the data and applying stepwise regression reduced the predictor attributes to the following nine:

- 1. Permanent Grade
- 2. Source of Commission
- 3. Officer Effectiveness Report, Current
- 4. Service Component
- 5. DOB (year)
- 6. Major Air Command Assigned
- 7. Officer Effectiveness Report, Weighted Mean
- 8. Race
- 9. Academic Specialty

### METHOD

In each of the four casesprediction equations were developed using the three methods to be evaluated; MLE, OLS Simple B, and OLS standardized (Beta). Then these equations were applied to the sample test file to compute the probability of attrition for each observation in the following manner: P(A) = $\sum_{coefficient*F(observed value)$ . In this application the function of the observed value concept was used because of the diversity of the three models, in which the B model uses raw values, the Beta model uses standardized variates and the MLE model uses deviation from the mean.

A cut score was determined which correctly identified the observed number of attriters in the sample test file. For example in the Beta Model the observations which have a computed probability of greater than .64 might account for the known number of observations which were attriters. This cut score would be used to predict the 1/0 (attrit/non-attrit) status of the observations in the cross validation file.

The predictor equations were then applied to the cross validation files. In each of the three live data cases, these files were the next years population which is analogous to the real-world problem of predicting into the future year. However, historical data were employed for cross validation purposes and actual attrition results were known and used for analysis of prediction efficiency. Likewise in the contrived data case, the results of the "future" were known and used to measure predictive strength of the various models.

In all cases the data is displayed not only as the number of predicted attriters but also in the classical hits/falsepositive/false-negative format commonly used in screening applications. Additionally the expected value concept was examined. Expected value might be employed in a stochastic model wherein the probability of attrition is computed and compared to a uniform random number to make the 1/0 determination. For this reason, the expected value of a cross validation run (the sum of all the computed probabilities) is a pertinent index of predictive strength in the arena of officer loss rates.

### PART II. RESULTS

### CASE 1 - CONTRIVED DATA

Correlation matrices are revealing tools in the analysis of data and are, in fact, the "guts" of OLS. As can be seen in the table below the correlation matrix for the sample test file does not protend for a strong predictor model and the resulting R-Squared of 0.19 supported this conclusion.

Intercorre.	lation	Matrix	for	Case	1

	Y	x,	×2.	x,	X
Y	1.00	.29	.04	.04	0
x,	.29	1.00	57	20	.43
x2	.04	57	1.00	04	37
x,	.04	20	04	1.00	85
XH	0	.43	37	85	1.00

Stepwise regression produced the following results:

Variable	B Coefficient	Beta Coefficient	F
X,	.00517	.51460	9.14191
X,	.45993	.50182	5.00405
X	.46047	.50241	1.92134
X.+	.12701	.39195	1.01117
Constant	77401		

Multiple R = 0.44001R-Squared = 0.19360 Standard Error = 0.43377

Albeit that the last two variables are not significant at .05 by application of the F statistics, the four variable model was used because the Beta Coefficients indicated significant influence. Application of the B model to the sample test file produced a cutting score of 0.33, i.e. classifying all observations with a computed probability of 0.33 as "1" or "on" resulted in the correct (observed)number of "ons". As indicated above the prediction equation and cutting score were then applied to the cross-validation files. Crossvalidation file #1 was designed to test the models under extreme positive data shifts. To meet this objective the mean of  $X_1$  was increased from 45 to 51, and the mean of  $X_4$  was increased from 3 to 6, while  $X_2$  and  $X_3$  were left unchanged. Using the cut score produced 49 observations identified as "on" and one observation labeled "off". The expected number of "ons" was 36. Displayed below is the hits/false-positive/ false-negative results of this prediction:



Since the number of observed records which were "on" was 15, the tentative conclusion can be drawn that the OLS B model is not reliable with strong positive data shift. Next the negative data shift in cross-validation #2 was processed with the B model.

In this case the mean of X1 was shifted from 45 to 39, X4 from 3 to 1.6, and X2 and X3 remained unchanged. Use of the cut score predicted 5 observations to be "on". The expected number of "ons" was 6.7. Below are displayed the hits/falsepositive/false-negative data:



Again the conclusion can be drawn that the B model is not viable under extreme data shifts.

Beta model results are shown below:

### Cross-validation #1 - Positive Data Shift

		Predi	cted
		on	off
	on	10	5
Actual	off	7	28

Cut score prediction = 17 Expected value = 15

Cross-validation #2 - Negative Data Shift



Cuts score prediction = 16 Expected value = 15

MLE results follow:

## Cross-validation #1 - Positive Data Shift

	Predicted			
	on	off		
n	10	5		
ff	7	28		

on Actual off

Cut score prediction = 17 Expected value = 16

Cross-validation #2 - Negative Data Shift

		Predicted	
		on	off
3 - + 1	on	. 9	6
ACTUAL	off	2	33

Cut score prediction = 11 Expected value = 14

Case 1 summary is shown below for the three models evaluated:

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	·B	BETA	MLE
% Hits	32	76	76
% "Ons" = on	30	59	59
% "Offs" = off	100	85	85
<pre>% False-positive</pre>	68	14	14
<pre>% False-negative</pre>	0	10	10
Cut score prediction	49	17	17
Expected value	36	15	16
Observed "on"	15		

Cross-validation #1 - Positive Data Shift

# Cross-validation #2 - Negative Data Shift

	В	BETA	MLE
% Hits	80	74	84
% "Ons" = on	100	2	85
% "Offs" = off	78	56	82
<pre>% False-positive</pre>	0	14	4
<pre>% False-Negative</pre>	20	12	12
Cut score prediction	5	16	11
Expected value	6.7	15	14

Observed "on" 15

CASE 2 - LINE COLONELS' RETIREMENTS, '75 FROM '74

In this case a "weak" predictor model was designed as is indicated by the validity vector below and the resultant R-Squared of .16:

# Validity Vector for Case 2

Variable	
Tenure (TAFCS)	0.37613
Age:	0.29107
Old OER Mean:	-0.17088
No. permanent passovers:	0.11924
Last OER:	-0.07066
Below Zone Selection:	-0.11534

Stepwise regression produced the following results:

Variable B	Coefficient	Beta Coefficient	F
Tenure (TAFCS)	.04861	.32330	215.559
# Permanent Pass	.05433	.07597	20.004
Age	.00728	.05772	6.892
Old OER Mean	00453	02754	1.954
Last OER	00955	01005	.317
Below Zone Selection	.00429	.00330	.037
Constant	94884		

Multiple R = .39049 R-Squared = .15248 Standard Error = .39870

Using the B model and Beta model on the cross-validation file, the 1975 Line Colonels, produced prediction results as follows:

# OLS Predicted 1975 Line Colonels' Retirements

	B	Model		
	Pr	edicted	1	
	1	377	409	7
Actual	0	211	2422	]
Cut sco Expecte	re p d va	redicti lue	lon = 5 = 6	88 597
Observe	d		= 7	186



	Pre	dicted	
		1	0
Natura I	1	434	352
ACTUAL	0	340	2293
		774	
		857	•

\* 1 = Retire, 0 = Stay

The MLE results are shown below:

MLE Predicted 1975 Line Colonels' Retirements

1

Actual

	1	0
1	446	340
0	361	2272

Cut score prediction = 807 Expected value = 859 Observed = 786

Dredicted

Case 2 summary results are as follows:

### Cross-validation - 1975 Line Colonels' Retirements

	В	BETA	MLE
% Hits	82	79	79
% "Retirees" that retired	64	56	55
% "Stayers" that stayed	. 86	87	87
<pre>% False-positive</pre>	6	10	11
<pre>% False-negative</pre>	12	10	10
Cut score prediction	588	774	807
Expected value	697	857	859
Observed Retirees	786		

CASE 3 - LINE COLONELS' RETIREMENT, '76 FROM '75

As outlined earlier, extensive analysis and data transformation were undertaken to build a "strong" predictor model. The resultant validity vector is shown below. An R-Squared of 0.26 was achieved.

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# Validity Vector for Case 3

Tenure (TAFCS):	.49036
No. of Dependents:	.20485
Source of Commission:	.22527
Permanent Grade:	:27404
DAFSC (first two):	.08344
Old OER Mean:	.19063

Stepwise regression results are displayed below:

Variable	<b>B</b> Coefficient	Beta Coefficient	F
Tenure (TAFCS)	.00952	.46560	566.528
No. Of Dependents	.00349	.07148	21.129
Source of Commission	.00290	.06529	16.294
Permanent Grade	00206	05677	8.950
DAFSC (first two)	.00518	.04323	8.468
Old OER Mean	.00226	.04306	7.701

Application of the B model and the Beta model to the 1976 Colonels file produced the following prediction results:

OLS Predicted 1976 Line Colonels' Retirements

	B Mo	del			Bet	ta Model		
	Pred	icted	0		Pro	edicted	0	
Actual	0	276	2528	Actual	0	341	332 2422	
Cut sco Expeqte Observe	re pr d val d	edicti ue	on = 501 = 623 = 673			678 690		

The MLE prediction, consistent with case one and two, varies only slightly from the Beta prediction.

MLE Predicted 1976 Line Colonels' Retirements

	Pre	dicted 1	0
	1	343	342
Actual	0	333	2414

Cut score prediction = 676 Expected value = 691 Observed = 673

Case 3 summary results are as follows:

	В	BETA	MLE
% Hits	82	81	80
<pre>% "Retirees" that retired</pre>	55	50	51
<pre>% "Stayers" That stayed</pre>	86	88	88
<pre>% False-positive</pre>	7	10	10
<pre>% False-negative</pre>	12	10	10
Cut score prediction	501	678	676
Expected value	623	690	691
Observed Retirees	673		

Cross-validation - 1976 Line Colonels' Retirements

In the interest of brevity, only the detail and summary results will be displayed for Case 4. The methodology followed has been sufficiently detailed in Cases 1 thru 3.

OLS Predicted 1976 4th Year Group Non-rated Line Separations

	B M Pre	dicted			Beta	Model Micted	
	1	1193	0		1	1 1230	0
Actual	0	212	2020	Actual	0	317	1915
Cut sco Expecte Observe	re pr d val d	ediction ue	$n = 1405 \\ = 1437 \\ = 1557$			1547 1674	

MLE Predicted 1964 4th Year Group Non-rated Line Separations

	Pre	dicted	0
	1	1207	350
Actual	0	301	1931

Cut score prediction = 1508 Expected value = 1730 Observed = 1557

% Hits	B	BETA 83	MLE 83
% "Separatees" that separated	85	80	80
% "Stayers" that stayed	85	85	85
<pre>% False-positive</pre>	• 6	8	8
<pre>% False-negative</pre>	10	9	9
Cut score prediction	1405	1547	1508
Expected value	1437	1,674	1730
Observed Separations	1557		•

Cross-validation - 1976 4th Year Group Non-rated Line Separations

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### PART III. SUMMARY

Based on the empirical data presented, the conclusion must be drawn that the usual B model application of OLS is not a viable solution to the loss rate prediction problem. (And by this is meant the prediction of the correct number without regard to the correct classification). However, either the OLS Beta model or MLE model provide highly reliable predictions of the correct count. To reinforce this point the predicted count for the three models is shown below for the five crossvalidation cases.

### Cut Score Predictions

				Observed	в	Beta	MLE
Case	1	-	Positive data shift	15	49	17	17
Case	1	-	Negative data shift	15	5	16	11
Case	2	-	1975 Colonels' Retirement	786	588	774	807
Case	3	-	1976 Colonels' Retirement	673	501 .	678	676
Case	4	-	1976 4th Yr Gp Separation	1557	1405	1547	1508

### Expected Value Predictions

		Observeu	D	Dela	PILE
Case 1 -	Positive data shift	15	35	15	16
Case 2 -	Negative data shift	15	7	15	14
Case 2 -	1975 Colonels' Retirement	786	697	857	859
Case 3 -	1976 Colonels' Retirement	673	623	690	691
Case 4 -	1976 4th Yr Gp Separation	1557	1437	1674	1730

Further examination of the data shown above leads to the conclusion that the cut score predictions of the Beta and MLE models are consistently more reliable than the expected value predictions. This suggests that it is more difficult to correctly map the individual probabilities into the future than it is to map a 1/0 criterion such as is done with a cut score applied to the estimates of the probabilities.

Significance tests at .05 were applied to Cases 2 and 3, the Line Colonel Retirement data. The conclusion was drawn that the B model predictions were not equivalent to the observed values while both the Beta and MLE predictions were equivalent to the observed values. Further the predictions of the Beta and MLE models were found to be equivalent at .05. Thus the selection of a model reduces to one of economics.

Lockman and Warner (# 3) noted that MLE will require more resources than OLS. In this study, the MLE program employed required from 2.3 to 5 times the computer processor time required by OLS. It should be further noted that the cases analyzed were small, compared to typical real-world problems, and that MLE tends toward exponentially increasing resource consumption with increase in the number of independent variables. Thus the analyst is left on the horns of a dilemma....MLE is theoretically more accurate but at the same time may be significantly more costly to use. The OLS Beta model will always cost less than MLE but may not be reliably accurate over the spectrum of problems which must be solved.

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### PART IV. CONCLUSION

Loss rate generation can be dichotomized into two classes... first, the problem of predicting the correct number for applications such as force structure modeling, and second the problem of classification which finds application in screening for selective admission. The empirical evidence presented for a diversity of data in this study supports the use of the Beta model for the prediction of the number of losses. Cut score predictions for the Beta model in all cases were equally as good as the MLE predictions. In addition the Beta model can be used with only marginal increase in costs over the B model (Beta requires the Mean and Standard Deviation of the independent data in the observations to be predicted) while the MLE model incurs greater resource costs. There is, however, one serious shortcoming of the Beta model, also shared by the MLE model. Neither of the techniques consistently provided reasonable Expected Value predictions. Thus, until further research unravels the Expected Value anomalies, the recommendation is made that the Beta model using cut score for the 1/0 criterion be used for forecasting loss rates.

### PART V. ET CETERA

Several loose ends, questions, and conjectures are left which did not appropriately fit in the above discussion. This section will present those remaining elements for deliberation and possible resolution.

First and foremost is the contention in the literature that the OLS B Model and Beta Model are mathematically equivalent. They are, indeed, equivalent when applied to the sample test file and when applied in the typical manner of cross-validation (see discussion below). However, the cases examined in this study clearly indicate that the two models are not equivalent when applied to real-world, or controlled data, problems. As is suggested by Fast and Dempsey (#4 ), the OLS B model perishes under the influence of data shifts. The Beta model does not. Therefore the two models are not equivalent in application, or when cross-validated in the loss forecasting arena.

Cross-validation as documented, taught, and practiced has no relevance to loss forecasting. Typically the researcher will randomnly divide a data file into two statistically equivalent halves, build a predictor model with one half, and crossvalidated into the other half. In practice, this technique will always cross-validate if the researcher is successful in producing statistically equivalent files. The additional step of constructing independent predictor models in each file and cross-validating into the other file contributes nothing to loss forecasting. It is also the incorrect approach. In loss forecasting, the objective is to build a predictor model with an historical population which predicts the losses for a population in the future which usually will not be statistically equivalent to the historical population due to data shifts. To accomplish this, the researcher should employ an analog of the process to be modeled by using two previous years of data. One to build the predictor model and the subsequent year to cross-validate the predictor model. Only in this way can any conclusions be drawn regarding the efficiency of models for predicting losses.

Predicting losses over time leads to the concomitant shift of data over time. Any model nominated for this task <u>must</u> be able to accommodate shifts in data values. Consideration of this requirement resulted in the conjecture that the three models evaluated are equal if there is no shift of data in the crossvalidation file, as is the case with statistically equivalent files. Is the Beta model equivalent to the MLE model? Or even more accurate? Isolating the question to the five cases presented would result in a strong affirmative to the first question and a "perhaps" to the second. However, such is not the case. Without a doubt, cases could be contrived, particularly bivariate cases, in which the Beta model would be inferior to the MLE model. This is an important consideration because loss rates will usually present a multivariate problem. With this type of problem, and more particularly with data which provide low R-Squares, it is conjectured that the sigmoid curve produced by MLE will be essentially flat and thus the equivalent of the Beta model.

Finally, the classification question. In officer loss rate generation, correct classification is not germane; however, the serendipity which is evident should not be ignored, for the Air Force personnel managers are responsible for accession actions in which classification is essential. Reference to the summary data shown on pages 7-12, above, reveals that the Beta and MLE model classify with approximately the same accuracy. One other form of the Beta model was investigated as a result of informal correspondence with Dr. Joe Ward at the Air Force Human Resources Laboratory. Ward suggested that a more accurate Beta model could be derived by using the validity vector from the sample test file and the correlation matrix from the cross-validation file. This is in the form of  $[\beta] = [R_2]^{-1} [V_1]$ . Such a prediction equation was built and tested in Case 4, the 4th Year Group Non-rated Line Separations. For comparison purposes the summary table is presented.

Cross-val	lidation	#4	-	4th	Year	Group	1976	Non-	-rated	Line	Se
and an internet when the second se	and the second se					and a second of the second of the second	a financial statement in the second statement of the second statement	the second se	a final state of the second state of the secon	the second se	the second se

	B	Beta	MLE	WARD
* Hits	, 85	83	.83	85
% "Separatees" that separa	ated 85	. 80	. 80	86
% "Stayers" that stayed	85 .	85	85	85
<pre>% False-positive</pre>	6	8	8	5
<pre>% False-negative</pre>	10	. 9	9	9
Cut score prediction	1405	1547	1508	1396
Expected value	,1437 ,	1674	1673	1673
Observed Senarations	1557			

Although the Ward method resulted in the worst cut score prediction (90% of actual separations), the accuracy on classification is the best of the four methods. This suggests an area for further research for those who have an interest in screening tools.

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