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A STUDY OF SHIP TRACK PARAMETER ESTIMATION. (U)

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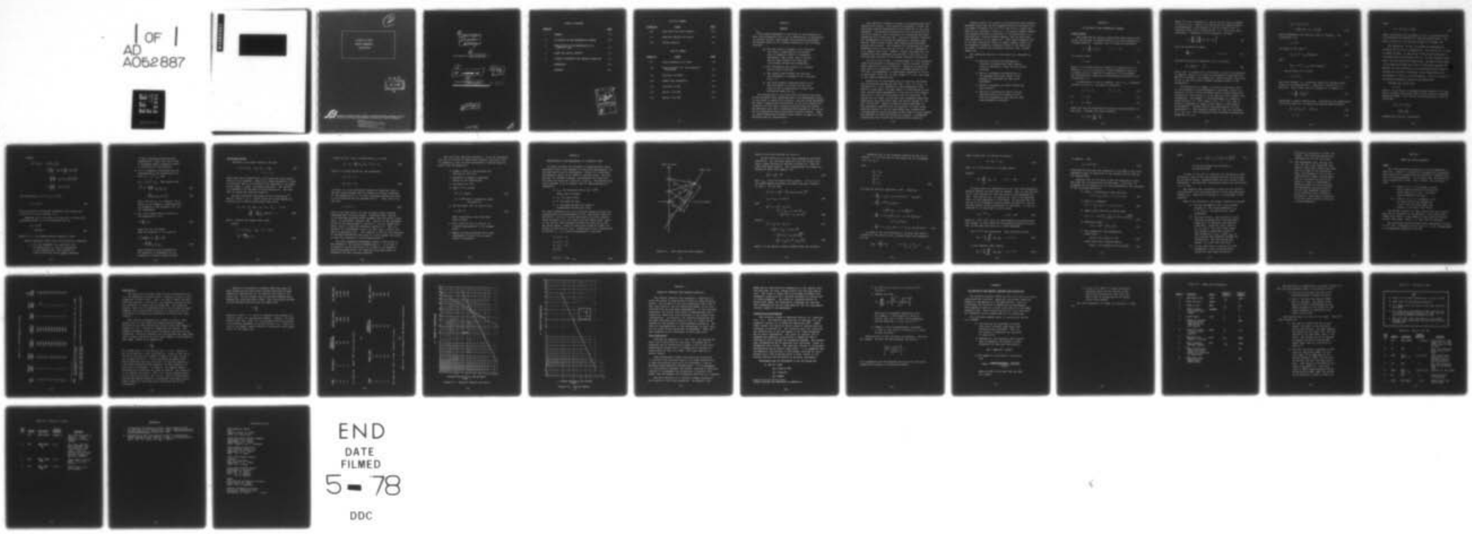

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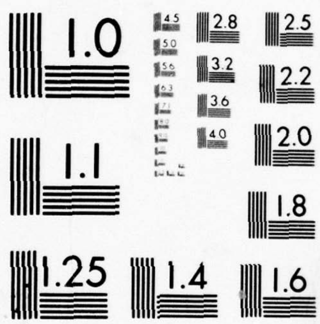

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A STUDY OF SHIP TRACK PARAMETER ESTIMATION

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A STUDY OF SHIP TRACK PARAMETER ESTIMATION

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by

Dr. Joseph Czika

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SECTION 1

SUMMARY

The problem addressed in this study is the estimation of ship track parameters from a set of observations on differences in acoustic arrival time between bottom mounted hydrophones. The studied model of this problem consisted of the following features:

- The ship track is assumed to be a straight line which is described by the closest point of approach (CPA) to a reference hydrophone, time of CPA, ship's velocity, and the angle between the track and a line passing through the hydrophones.
- The hydrophones are arrayed in a straight line at known locations.
- The acoustic path between the ship and each hydrophone is assumed to be a straight line.
- The error between a measured value of the time difference of arrival and the true value is due to limitations in sensor and processing capability.

The primary goal of the study was to determine the error in the estimated track parameters due to the measurement errors. In this error or sensitivity analysis, the true track parameters are used in appropriate calculations, so the error results are indicative of the "best" accuracy available from the system. An additional goal was to determine which subsets of the entire set of observations have sufficiently accurate results. (That is, which subsets gave estimate errors nearly as small as those obtained from all observations.)

This method of analysis is based on the generalized least squares error theory. This theory was extended to treat the non-linear problem represented by the estimation of the track parameter. The extension provided not only the formalism for the error analysis, but a technique for estimating the track parameters for a given data set. The error analysis formalism was implemented by constructing appropriate computer codes. Several cases of ship tracks, data sampling and hydrophone spacing were studied. The technique for estimating the track parameters from a real data set is not, as yet, implemented in a computer code form. However, issues concerning that implementation were addressed and resolved to the point of indicating what features of the present code must be modified.

In this initial study, a limited number of track parameter cases were examined to determine the salient features of parameter variation on error estimation. Numerical calculations were based on a measurement "error" (standard deviation) of 10^{-2} sec, corresponding to a measurement bandwidth of 100 Hz. In most cases "data" out to slant ranges of 5 n.mi. were used in the results presented here.

Results of two structured sets of cases are noteworthy. One set involved varying the CPA from 1 n.mi. to 4 n.mi. in increments of 1 n.mi. keeping all other parameters fixed. Results, using nearly the same number of observations in each case, showed slight degradation of the ratio of CPA standard deviation to true CPA (from nearly 2.5% to nearly 6%). In another set of cases, the CPA and other parameters were held fixed while the track-array angle was varied through 0^o, 10^o, 30^o, 60^o and 90^o. Results indicated severe impairment of CPA estimation capability for the parallel (0^o case) track. Detailed studies of the 10^o case showed that observations corresponding to track locations on both sides of array endfire are required for good CPA estimation accuracy. Furthermore, observations near endfire may be particularly important.

Typical results for cases not involving this array endfire phenomenon show that the CPA standard deviation decreases with the number of samples (N) as $N^{-\frac{1}{2}}$ so long as N is larger than about 10 (at one-minute intervals). For smaller sample numbers, the standard deviation is sensitive to the particulars (i.e., ship position, interval between samples, and the particular case parameters) of the sample set. This result suggests that if the data acquisition system is sample number limited (to few numbers of samples), judicious choice of the sample set can result in better CPA estimation accuracy than a random sample set.

The remaining sections of this report are organized as follows:

- Section 2 outlines the mathematical formalism of generalized least squares theory as applied to linear and non-linear problems.
- Section 3 presents the application of the extended non-linear theory to the problem of estimating the ship track parameters.
- Section 4 presents the cases studied and their results.
- Section 5 concludes the report with an identification of the issues involved with the treatment of real data for ship track parameter estimation.

SECTION 2

AN OUTLINE OF THE MATHEMATICAL METHOD

Linear System

The method may be called a generalized least squares estimation and may best be described with an initial discussion of a linear problem.[1] Consider a set of linear relationships,

$$Y_i = \sum_{j=1}^{M} A_{ij} X_j \qquad i = 1,\ldots N \tag{1}$$

or in matrix form,

$$Y = AX, \tag{2}$$

where Y is a column vector of length N of quantities to be measured, A is a N x M matrix of known parameters, and X is a column vector of length M of quantities to be estimated. The constraint $M<N$ indicates an overdetermined system for the estimations of X.

An observation of the Y_i variable, call it y_i, in general, contains an error e_i, for which we can write,

$$e_i = Y_i - y_i \qquad i = 1,\ldots N \tag{3}$$

or $$e = Y-y, \tag{4}$$

or $$e = AX-y, \tag{5}$$

where the last two expressions are matrix form equivalents of the first. Consider the scalar function

$$S \equiv e^T e = \sum_{k=1}^{N} e_k^2 \tag{6}$$

where e^T is the transpose of e and is the row vector composed of the set $\{e_i\}$. Clearly S is the sum of the squared errors and minimizing it with respect to the X_i's gives a set of equations which can be solved for $\hat{X}_i$, i.e., the vector which minimizes S. Expanding on this discussion, we have

$$S = \sum_{k=1}^{N} \left\{ \sum_{j=1}^{M} A_{kj} X_j - y_k, \right]^2 \tag{7}$$

which we minimize by taking

$$\frac{\partial S}{\partial X_i} = 0 \qquad\qquad i = 1..M \tag{8}$$

Solving the system of equations in (8), we obtain

$$\hat{X} = \left(A^T A\right)^{-1} \quad A^T y \quad . \tag{9}$$

Recall that A is known and y is observed, so with this composite of numbers, equation (9) gives the procedure by which the estimate $\hat{X}$ is obtained. For reasons not indicated here[1], $\hat{X}$ as given in (9) is the best linear unbiased estimator (BLUE) of the parameters X.

We backtrack for a moment to discuss an important point. If the variances of the measurements y are known, either by a series of trials or error estimates based on sensor resolution capabilities, the function S in (6) does not fully exploit this information. For example, if the measurement y_1, carries a large variance, σ_1^2 , with respect to y_2, ${\sigma_1}^2 > {\sigma_2}^2$, then it would be reasonable to weigh y_2 more than y_1 in arriving at an estimate $\hat{X}$. Consider then a covariance matrix C on the measurements y. The diagonal elements of C are the variances of y_1, $y_2 \ldots y_N$. The off-diagonal terms represent correlations among the y's, e.g.,

$$C_{ij} = \mathrm{Cov}\ (y_i\ y_i)$$

$$\equiv \left\langle (y_i - \overline{y}_i)\ (y_j - \overline{y}_j) \right\rangle , \tag{10}$$

where $\langle\ \rangle$ denotes the expected value or "average". For convenience, let

$$W = C^{-1}, \tag{11}$$

for which we note that if

$$C_{ij} = \sigma_i^{\ 2}\ \ \delta_{ij}\ \text{(diagonal)}, \tag{12}$$

then

$$W_{ij} = 1/\sigma_i^{\ 2}\ \ \delta_{ij}\ \text{(also diagonal)}. \tag{13}$$

Now we modify (6) to read

$$S = e^T\ W\ e\quad . \tag{14}$$

This form "weighs" in a consistent manner the variance information on the observations y. Note that C and W are N x N matrices. Note further that for diagonal W, S reduces to

$$S = \sum_{k=1}^{N}\ e_k^2 \Big/ \sigma_k^{\ 2}\quad , \tag{15}$$

indicating a simple weighing case. Carrying out the minimization (8) on the form (14), we obtain the weighted BLUE estimator

$$\widehat{X} = (A^T\ W\ A)^{-1}\quad (A^T\ W\ y) \tag{16}$$

$$= D\ y\quad , \tag{17}$$

where

$$D \equiv (A^T W A)^{-1} (A^T W) \quad . \tag{18}$$

Again, everything in D is known, so (16) is the prescription for obtaining the estimator $\hat{X}$ from the observations y. We shall continue to work with the forms (16) through (18).

The discussion, so far, is focused on obtaining the estimator $\hat{X}$ of a set of parameters X. Note that $\hat{X}$ is, in fact, a random variable. In particular, $\hat{X}$ has a mean value and a covariance matrix associated with it. By the structure given above, but not explicitly shown there, the mean of $\hat{X}$ is equal to the population mean and is not of particular concern here. However, we are interested in the covariance matrix since it relates how "errors" (variances) in the observations are propagated to "errors" in the estimator. This is the Law of Covariance Propagation which we now demonstrate.

Consider the linear form

$$V = AZ$$

where Z is the vector of observed random variables, A is the matrix of known coefficients and V is the vector of dependent random variables. Let P be the covariance matrix of the observations,

$$P_{ij} = \mathrm{Cov}\,(z_i z_j)$$

$$= \langle z_i \; z_j \rangle \ ,$$

assuming zero mean for convenience.

Forming

$$\begin{aligned} \text{Cov}\ (v_i v_j) &= \left\langle v_i v_j \right\rangle \\ &= \left\langle \sum_k A_{ik} z_k \sum_l A_{jl} z_l \right\rangle \\ &= \sum_k \sum_l A_{ik} A_{jl} \left\langle z_k z \right\rangle \\ &= \sum_k \sum_l A_{ik} A_{jl} P_{kl} \quad , \end{aligned}$$

hen defining $Q_{ij} = \text{Cov}\ (v_i v_j)$, we have

$$Q = A\ P\ A^T \tag{19}$$

This is the Law of Covariance Propagation and relates the "errors" in Z to the "errors" in V.

Applying (19) to the form (17) with (18), we obtain the covariance matrix, Q, of the estimator $\hat{X}$ to be

$$\begin{aligned} Q &= D\ C\ D^T \\ &= (A^T\ WA)^{-1} \end{aligned} \tag{20}$$

where W C = 1, and standard matrix algebra is used.

Several important points about (20) should be recognized:

- Q is not dependent on a particular set of observations, y, only on covariances of the observations. The elements of C may be selected from a priori estimates of basic sensing system resolution capabilities. Thus, (20) can be used to propagate these irreducible errors to the errors in the estimator.

- If C is diagonal, indicating that the measurements y_i are uncorrelated, so that $C_{ij} = \sigma_i^2 \delta_{ij}$, then

  $W_{ij} = (1/\sigma_i^2)\, \delta_{ij}$. This implies that

$$(Q^{-1})_{ij} = \sum_k \sum_l A_{ki} W_{kl} A_{lj} \tag{21}$$

$$= \sum_k A_{ki} A_{kj} (1/\sigma_k^2). \tag{22}$$

  Thus, even if $\sigma_k = \sigma =$ constant, the Q matrix is not diagonal, indicating that the elements of the estimator, $\hat{X}_i$, are not independent.

- Let f be a linear scalar function of the parameters X, viz.,

$$f = \sum_i b_i X_i \tag{23}$$

  where the $\{b_i\}$ are known.
  Then the variance of f is given by

$$\sigma_f^2 = \left\langle \sum b_i X_i \sum_j b_j X_j \right\rangle$$

$$= \sum_i \sum_j b_i b_j Q_{ij}, \tag{24}$$

  which relates how the covariance of the estimator is propagated to the variance of a scalar random variable.

Non-Linear System

Consider a non-linear system of the form

$$Y_i = g\ (a_{1i} \ldots . a_{ni};\ X_1, \ldots . X_M) \tag{25}$$

$$i = 1, \ldots N,$$

where there are N quantities Y_i to be observed, and M quantities X_i to be estimated. There are n known parameters a, each of which may take on a different value depending on the observation index, i, hence, the double subscript. The function g is assumed to be the same for each observation. The form (25) is called the condition equation.

We shall linearize the system in the following way.[2] Select a point X' (i.e., known values for X) and expand the function g in a Taylor series about that point,

$$g\ (\{a\}\ ;\ X_1 .. X_M) = g\ (\{a\}\ ;\ X_1', \ldots\ X'\ M)$$

$$+ \sum_{k=1}^{M} \left. \frac{\partial g}{\partial X_k} \right|_{X'} (X_k - X'_k) + \varepsilon, \tag{26}$$

where ε contains the higher order terms.

Define

$$Y_i = g\ (a_{1i} \ldots . a_{ni},\ \ X'_1 \ldots X'_M)$$

$$A_{ij} = \left. \frac{\partial g}{\partial X_j} \right|_{X = X'} \tag{27}$$

so when we have a set of observations y_i, we form

$$y_i - Y_i = \sum_j A_{ij} (X_j - X'_j) + e_i , \tag{28}$$

which is a linear system for the differences

$$V_i = y_i - Y_i$$

$$Z_j = X_j - X'_j \tag{29}$$

In words, V_i is the difference between the observed value y_i and the value obtained from the condition equation evaluated at the assumed point X'; Z is the difference between the point X to be estimated and the assumed point X'. Thus, (28) is in the form

$$V = A Z + e. \tag{30}$$

This is the same form as (5) and the whole linear analysis discussed above follows through. Interpretations, however, of the derived equations differ somewhat from the purely linear case. In particular, consider (20). Since C is the covariance matrix of the observations y_i and the Y_i are not random variables, then C is also the covariance matrix on V. The A in equation (5) corresponds to the A in (30), but of course, the later is evaluated at X'. The Q matrix in (20) is the covariance on the BLUE estimator $\hat{Z}$, but since X' is not random, the matrix is also the covariance of the estimated point X.

Now (20) propagates measurement errors to the errors in the estimate X, for a given assumed point X'. If in fact X' is chosen to be the true point, as is done in our specific problem discussed below, the resulting covariance matrix Q represents the best accuracy possible.

One last point requires discussion. If we are interested not only in an error or sensitivity analysis, but also a way to estimate X, then the above mathematical structure must be used in the following way:

- Assume a point X' and evaluate the partials of (27) at X'.
- Construct or assume a covariance matrix on the observations.
- Calculate Q in (20).
- Apply (17) to obtain

$$\hat{Z} = D\,V, \text{ where} \tag{31}$$

$$D = Q\,A^T W \text{ (and is completely known for given } X'\text{).}$$

- Use the result (31) in (29) to find

$$X = \hat{Z} + X' \tag{32}$$

  where everything on the right-hand side is known.
- The X given in (32) is used as X", a second approximation to the assumed point.
- Repeat the above procedure until some convergence criterion (as yet undefined) is satisfied.

SECTION 3

APPLICATION OF THE METHODOLOGY TO A SPECIFIC CASE

We shall consider the problems of estimating ship track parameters from a time sequence of time delay measurements for an ocean bottom line of hydrophones. Figure 3-1 illustrates the geometry, which is not drawn to relative scale. For simplicity, we consider 3 phones, one defining the origin and the other two along the y-axis at y_1 and y_2. The surface ship track (assumed to be a straight line) is characterized by 4 parameters,

- r_{co}, the horizontal CPA to the "0"$\underline{th}$ phone (one at origin)
- t_o, the time of CPA
- V, the speed of ship
- b, the angle between the array of hydrophones and the ship track.

The track parameters are to be estimated (phone positions are assumed known) from measurements of the time delays, which we now define. Let $T_1(t)$ be the difference in arrival time between the phone at y_1 (1st phone) and the zeroth phone. $T_2(2)$ is the arrival time difference between the phone at y_2 and the zeroth phone. Since the ship moves along the track, the delays are functions of time. Taking time to be a discrete set of points $t_1, \ldots t_n$, we have measurements of the form

$$
\begin{aligned}
T_1\,(t_1) &\equiv Y_1 \\
T_2\,(t_1) &\equiv Y_2 \\
T_1\,(t_2) &\equiv Y_3 \\
T_2\,(t_2) & \\
&\;\;\vdots \\
T_2\,(t_N) &\equiv Y_{2N}
\end{aligned}
\qquad (33)
$$

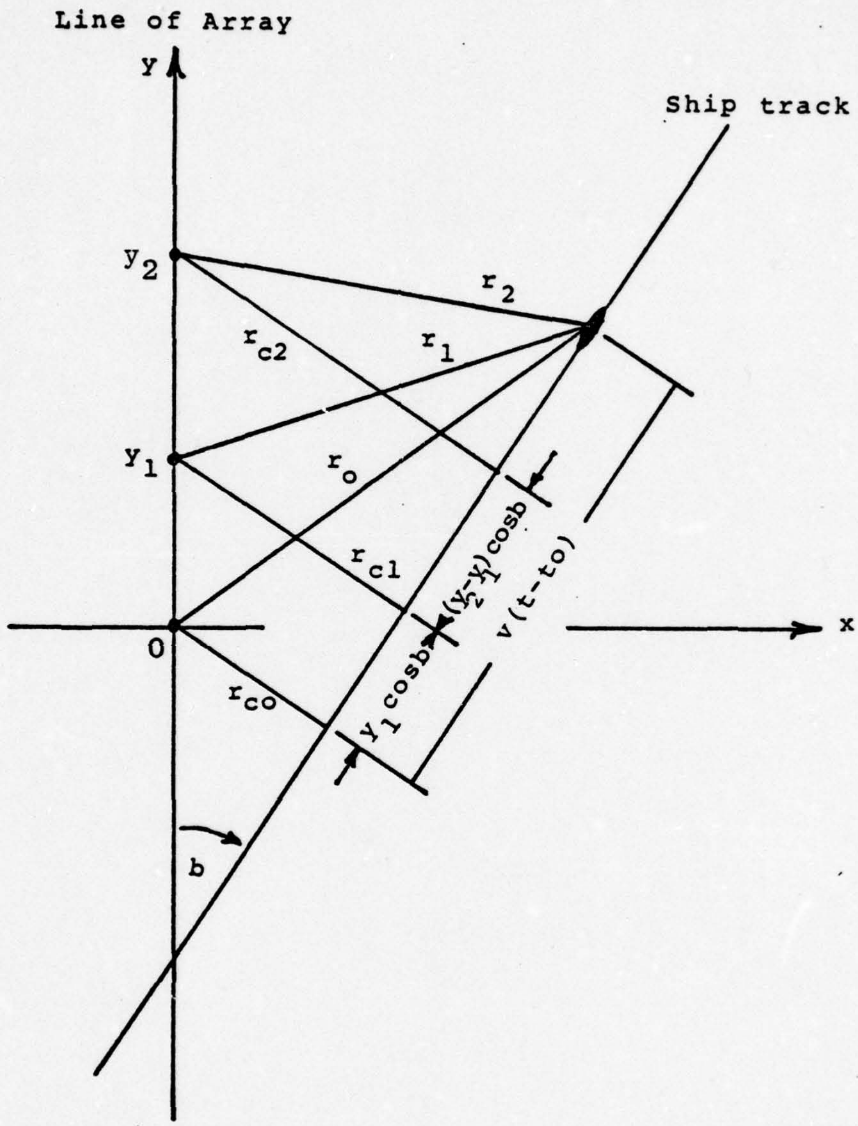

Figure 3-1. Ship Track and Array Geometry

where we have also defined the vector Y.

We now relate $T_i(t)$ to the track parameters and known quantities. Let the ocean depth be d and the surface horizontal range variables be defined on Figure 3-1; they are lower case Roman symbols. The corresponding 3-D ranges are defined as upper case symbols, e.g.,

$$R_{ci}^2 = r_{ci}^2 + d^2. \tag{34}$$

Here, r_{ci} is the horizontal CPA to phone i. Let $r_i(t)$ be the instantaneous horizontal distance from ship to phone i at time t. We see that

$$r_i^2(t) = r_{ci}^2 + \left\{V(t-t_o)-y_i \cos b\right\}^2$$

$$r_{ci} = r_{co} + y_i \sin b \tag{35}$$

then

$$\begin{aligned} R_i^2(t) &= d^2 + r_i^2(t) \\ &= d^2 + (r_{co} + y_i \sin b)^2 \\ &\quad + \left[V(t-to) - y_i \cos b\right]^2 . \end{aligned} \tag{36}$$

Finally,

$$cT_i \equiv R_i(t) - R_o(t) \tag{37}$$

$$\begin{aligned} &= \Big\{d^2 + (r_{co} + y_i \sin b)^2 \\ &\quad + \left[V(t-t_o) - y_i \cos b\right]^2\Big\}^{\frac{1}{2}} \\ &\quad - \left\{d^2 + r_{co}^2 + V^2 (t-t_o)^2\right\}^{\frac{1}{2}} \end{aligned} \tag{38}$$

where c is the speed of sound (assumed known and constant).

Equation (38) is the condition equation we set out to obtain. It is in the form of (25) where the $\{a\}$ correspond to d, y_i, y_2, t

and

$$X_1 = r_{co}$$
$$X_2 = b$$
$$X_3 = V$$
$$X_4 = t_o \qquad (39)$$

We need the partials indicated in (27). These are:

$$c \frac{\partial T_i}{\partial r_{co}} = (r_{co} + y_i \sin b)/R_i(t) - r_{co}/R_o(t)$$

$$c \frac{\partial T_i}{\partial t_o} = V^2(t-t_o)/R_o(t) - V\left[V(t-t_o) - y_i \cos t\right]/R_i(t)$$

$$c \frac{\partial T_i}{\partial V} = (t-t_o)\left[V(t-t_o) - y_i \cos b\right]/R_i(t) - V(t-t_o)^2/R_o(t)$$

$$c \frac{\partial T_i}{\partial b} = y_i \left(r_{co} \cos b + V(t-t_o) \sin b\right)/R_i(t) \qquad (40)$$

In order to set up the problem in the form (20), where Y is given by (33) and X by (39), we must define the components of A as

$$A_{kj} = \frac{\partial T_i}{\partial X_j}(t_n) \qquad i = 1,2, \quad j = 1,\ldots 4, \quad n = 1,\ldots N \qquad (41)$$

where k goes from 1 to 2N and is given by

$$k = 2n - i\,\delta_{i1} \tag{42}$$

Thus, A is a 2N (rows) by 4 (columns) matrix.

Finally,

$$Y_k = \sum_{j=1}^{4} A_{kj}\, X_j \qquad k = 1,\ldots 2N\ . \tag{43}$$

We shall base our computations on the true track parameters, which we specify for each case we examine. That is, the partials in (41) are evaluated at the true values of the parameters in (39). The time set $\{t_i\}$ is selected in a way to be described later, so the components of A are known. We can then use (20) for the error propagation analysis where we specify C, the co-variance matrix on the observations of time delay. In the calculations to be described, we have chosen

$$C_{ij} = \sigma^2\, \delta_{ij} \qquad i, j=1,\ldots 2N \tag{44}$$

where $\sigma^2 = 10^{-4}$ $\sec^2$, thus, all measurements are uncorrelated with respect to each other and the standard deviation is given, say, by the time resolution in a 100 Hz bandwidth.

Let $P = Q^{-1}$ for convenience. Then from (22) we have

$$P_{ij} = \frac{1}{\sigma^2} \sum_{k=1}^{2N} A_{ki}\, A_{kj}\ ,\ i,j = 1,4 \tag{45}$$

In the computer code a matrix

$$B_{ij} = \frac{1}{c^2} \sum_{k=1}^{2N} A_{ki}\, A_{kj}\ ,\ i,j = 1,4 \tag{46}$$

is computed. Thus

$$Q = \sigma^2 c^2 B^{-1}, \qquad (47)$$

from which we note that the covariance on the BLUE of the track parameters scale in proportion to the variance on the time delay observations.

Equation (20) is the focus of what has been done to date. One other set of calculations is also made. Using (24), we propagate the covariances in the track parameters to the variances of the following quantities:

- Ship's x coordinate at each time step,

$$x(t) = r_{co} \cos b + v(t-t_o) \sin b \qquad (48)$$

- Ship's y coordinate,

$$y(t) = -r_{co} \sin b + v(t-t_o) \cos b \qquad (49)$$

- Aspect angle from bow to zeroth phone

$$\theta(t) = \Pi - \sin^{-1}\left\{ r_{co} \Big/ \left[r_{co}^{\,2} + \left(V(t-t_o) \right)^2 \right]^{\frac{1}{2}} \right\} \qquad (50)$$

- Depression angle

$$\phi(t) = \tan^{-1} \left\{ d/r_o(t) \right\} \qquad (51)$$

- Two transmission loss expressions,
  single path:

$$TL_1(t) = 20 \log R(t) + 66 \qquad (52)$$

  single path from a surface dipole:

$$TL_2(t) = 20 \log \left(R(t)/\sin \phi(t) \right) + 66, \qquad (53)$$

where

$$R(t) = \left\{ d^2 + r_{co}{}^2 + \left[V(t-t_o) \right]^2 \right\}^{\frac{1}{2}} \quad (54)$$

is the slant range from the ship to the zeroth phone.

In order to use (24) to calculate the variances of these parameters, each of the above expressions must be linearized by the Taylor series' expansion about the true (input) track parameters. The resultant linearization gives the coefficients $\{b_i\}$ in (24).

The computer code, developed to carry out the required calculations of the covariance matrix in (48) and other parameters (e.g., Equations 48 through 54), is discussed in the Appendix.

Some of the features of the present formulation include:

(1) We have implemented (46), which assumes diagonal C. The track parameters, r_{co}, b, v, and t_o, are specified as input parameters.

(2) The time points t_i are selected by first choosing a maximum ship-to-zeroth-sensor range which is feasible under physical conditions. Using the input value of r_{co}, b, and V, the total track length of interest is determined, which is then divided by $V \cdot T_{inc}$, where T_{inc} is the selected time increment between the observations. The time points with respect to t_o are then calculated.

(3) On the first run of a new set of track parameters, a file is created which stores the time steps and partial

derivatives (evaluated at those time steps). The covariance matrix Q is evaluated for all the time steps.

(4) Upon subsequent runs, the user can call the file, just described, and specify, upon input, which time steps are to be used in the calculation. Thus, the procedure in (2) is not a limitation but simply a convenience. That is, the user can look at any subset of times (to approximate a physical condition) for evaluating the appropriate covariance matrix on the track parameter estimator. It is noted that with 2 sensors a minimum of 2 time steps is required for a well posed solution (there are 4 track parameters).

SECTION 4

CASES AND PARTIAL RESULTS

Cases

The track parameters examined so far are summarized in Table 4-1. Figure 3-1 identifies the geometrical parameters. This limited set of cases does not represent a complete variation of track parameters. In the first look, we wanted only some representative cases. Note, however, two structured sets:

(i) Cases 4, 10, 11, 9 represent a cross-array track for which the surface CPA varies from 1 to 4 n.mi. All other track parameters are equal.

(ii) Cases 5, 8, 6, 7, 4 represent a set for which the CPA is constant at 1 n.mi., but the track angle with respect to the array line varies.

One primary interest in these initial data was to determine how the variances of the estimator, for a given case, change with respect to the number of time steps and the particular time steps. Cases 1, 4, and 8 were so analyzed.

All calculated quantities have used a measurement variance of 10^{-4} sec^2, consistent with an error of .01 sec corresponding to a measurement bandwidth of 100 Hz.

Table 4-1. Track Parameters for Cases

Case No.	Surface CPA (NMI)	Track Angle (DEG)	Location (FEET)	Location (FEET)	Range (NMI)	Increment (MIN)	No. of Time Steps
1	2	30	1000	2000	8	1	60
2	2	30	1000	2000	5	.25	138
3*	0	30	-1000	1000	8	1	62
4	1	90	1000	2000	8	1	62
5	1	0	1000	2000	8	1	62
6	1	30	1000	2000	8	1	62
7	1	60	1000	2000	8	1	62
8	1	10	1000	2000	8	1	62
9	4	90	1000	2000	8	1	54
10	2	90	1000	2000	8	1	60
11	3	90	1000	2000	8	1	58

NOTE: All cases use ship velocity, V = 15 knots and time of CPA, T_O = 0 HRS, Ocean Depth, D = 1.5 N.MI.

* In this case, the zeroth phone is between the other two and the ship passes directly over the zeroth phone.

Some Results

The results of the cross track set with varying CPA item (i), are summarized in Table 4-2(a). Results of item (ii) are summarized in Table 4-2(b). Note, for case 5, where the ship traverses parallel to the array, the standard deviation on the CPA estimator is larger than the known CPA. The indicated dramatic reduction in standard deviation with increasing angle suggests that a significant reduction in track error can be obtained by including some measurements with ship near array "endfire."

This feature is supported by a study of Case 8. Here, several sets of time steps were chosen and the standard deviation of the CPA estimate was plotted versus the number of samples in a set. Figure 4-1 summarizes the results. The crosses correspond to sets where the points near end-fire are eliminated. The dots represent sets which extend the same range (which crosses end-fire) but with fewer number of time steps in that range. Based on an assumed law

$$\sigma = \frac{X}{N^{x}} ,$$

the crosses have $\alpha = 1.69$ and the dots $\alpha = .50$. Figure 4-1 may be interpreted in the following way: Start at the top of the solid line where the number of samples is high, giving a low standard deviation. If one decreases the sample size, moving downward along the solid line, by removing samples in a way which preserves those samples at or near endfire, the solid line is traversed. If, however, the sample removal systematically eliminates the steps at endfire, the transition to the dashed line occurs. Thus, for a given standard deviation on the CPA estimate, say .5 N.mi., sets of points which discriminate against endfire or crossarray time steps require larger numbers of time steps than those sets that do not discriminate.

Figure 4-2 illustrates a property which may exist for most tracks and data sets which include sufficiently large numbers of observations for the ship on both sides of the array line. On this figure, data points were removed while keeping the track end points fixed. The relationship between the CPA standard deviation and the sample size

$$\sigma = \frac{x}{N^{\alpha}}$$

appears to have $\alpha \simeq .5$, which is expected from standard statistical theory for uncorrelated samples. However, for small sample sizes (say, under 10), the standard deviation appears to be strongly dependent on the particulars (i.e., ship location of the sample, sample interval, and specific case parameters) of the sample set.

CASE	CPA (N.Mi.)	NUMBER OF OBSERVATIONS	ST.DEV. (N.Mi.)	ST. DEV./CPA
4	1	62	.0246	2.46%
10	2	60	.0569	2.84%
11	3	58	.1202	4.01%
9	4	54	.2268	5.67%

All cases use: Angle = 90^{o} (cross-array track)

(a)

CASE	ANGLE (DEG)	NUMBER OF OBSERVATIONS	ST. DEV. (N.Mi.)
5	0	62	1.369
8	10	62	.281
6	30	62	.056
7	60	62	.034
4	90	62	.025

All cases use: CPA = 1 N.Mi., Velocity = 15 knots

(b)

Table 4-2. Partial Results For Two Parametric Variations

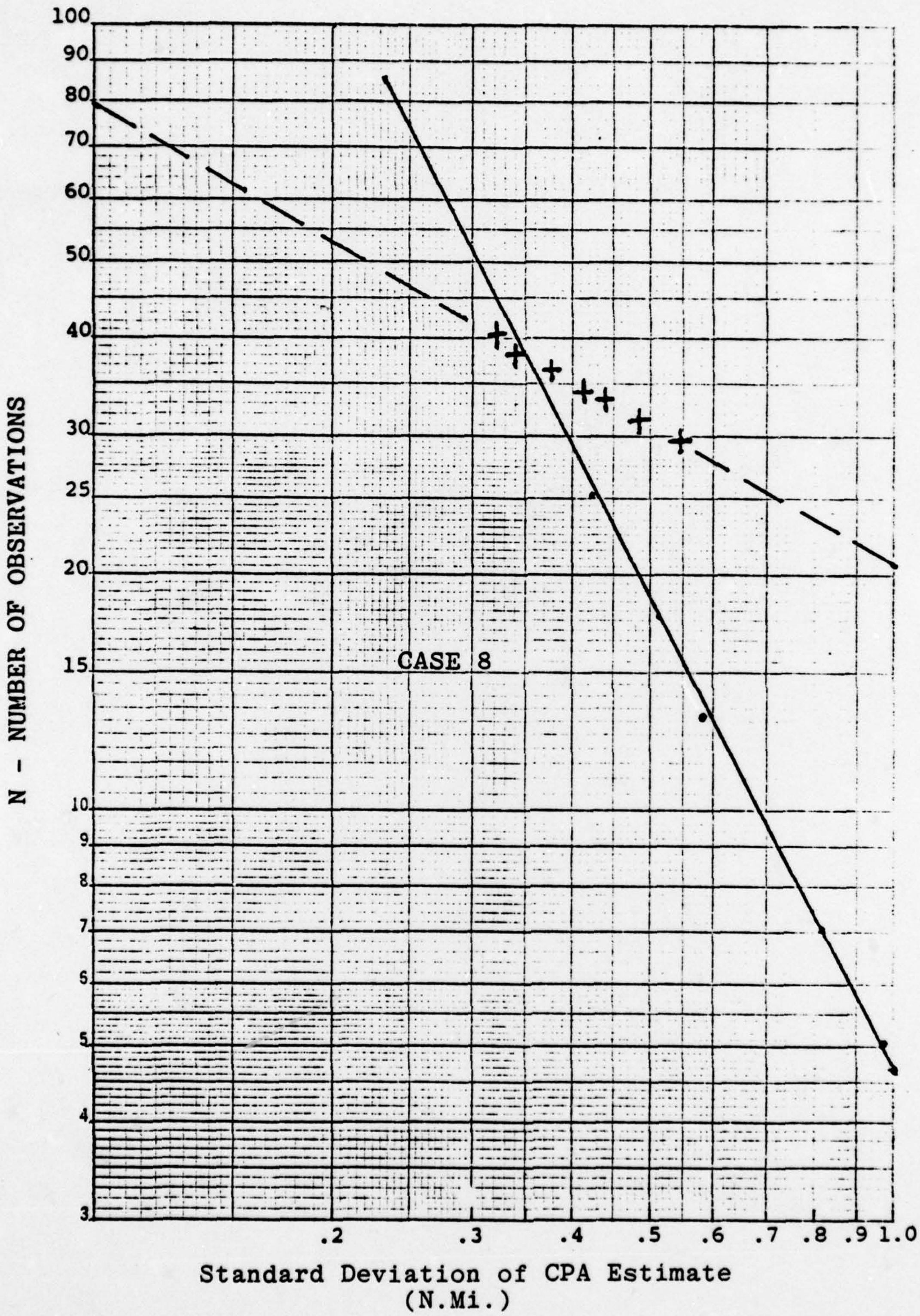

Figure 4-1. Numerical Results for Case 8

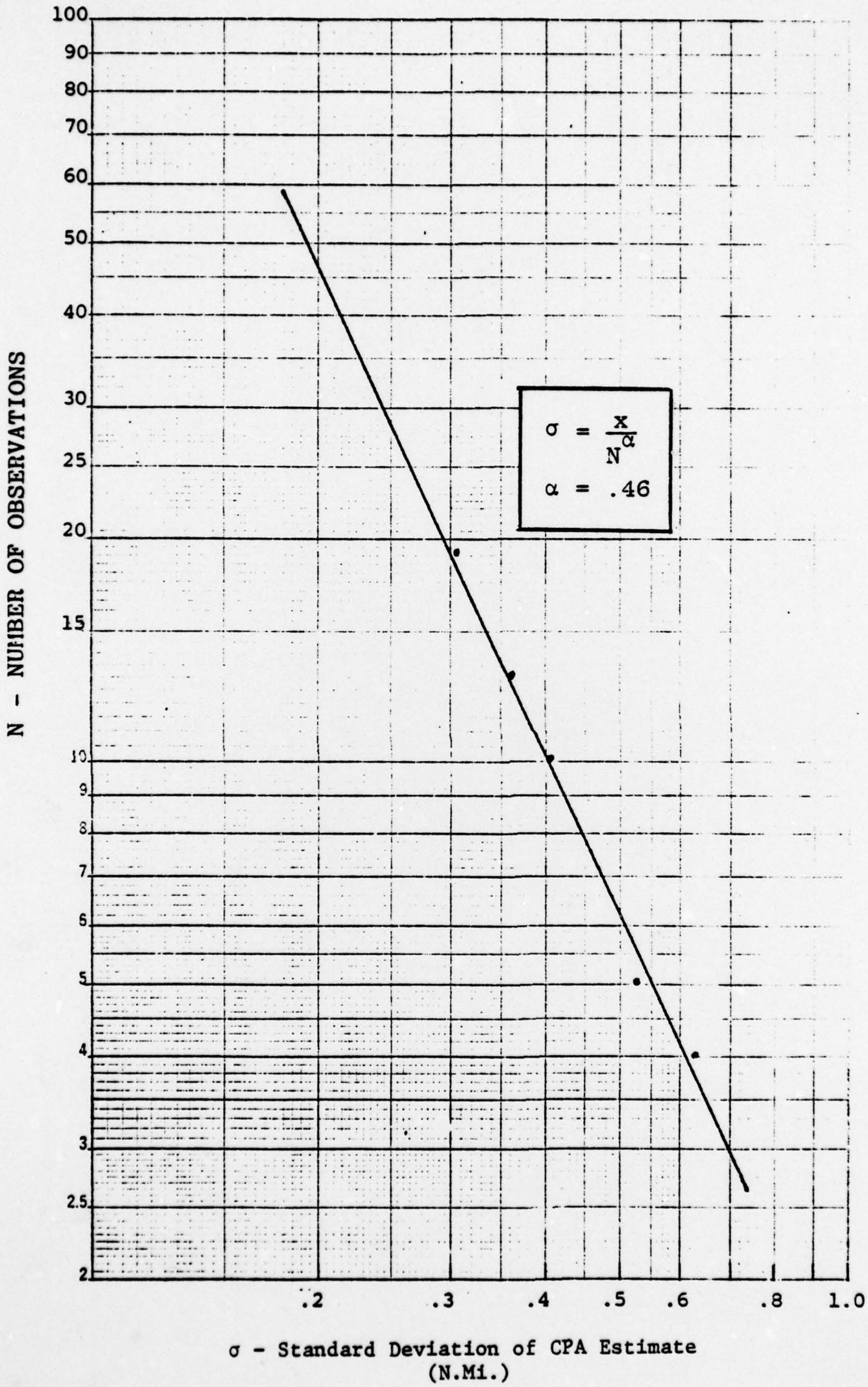

Figure 4-2. Typical Results

SECTION 5

ISSUES ON EXTENDING THE PROGRAM CAPABILITY

The present computer code represents a capability to study the effect of observational errors on track estimator errors. The estimate of the track parameters (CPA, time of CPA, ship velocity, and the array-track angle) is not constructed from the time delay measurements. In the following discussion, we outline some issues in implementing such a capability, especially as an extension of the present code configuration. In Section 2, an iterative methodology was outlined (discussion near Equation (31) ff), which can be adapted for the purpose of track parameter estimation. Besides the implementation of this methodology in code, there are two other factors which must be addressed. These are data preparation and convergence criteria selection.

Data Preparation

Data may be supplied in a "raw" form. Not knowing the exact form of much data at this point, we assume that a processor can be constructed to supply a set of time delay observations between each of M phones and a reference on zeroth phone at each of N times. The times need not be equally spaced.

Some processing of these time delays may be desired to calculate the covariance matrix C of these observations. If such calculations turn out to be infeasible, C may be estimated from bandwidth considerations. If a non-diagonal C is calculated or assumed, the present code must be modified (easily) to accommodate the calculation of equation (20) rather than the present use of Equation (22) with σ_k = constant.

The data processor is a iterative scheme requiring a first guess for the track parameters. An adequate first guess may be constructed by examination of the display from one beam when the ship crosses endfire and the beam centerline. The times of these events plus range estimates from broadband correlation will uniquely determine an initial set of track parameters. For ships not crossing endfire in direct-path range, two beam-center crossings are required. This technique has not yet been applied, and its convergence criteria remain to be determined.

<u>Iteration and Convergence</u>

The iteration scheme is described earlier (cf. equations (31), ff.). The required calculations in equation (31) are presently carried out in TRACK and subroutine COM*. These codes contain I/O functions which should be omitted when the iteration is carried out. The iterative step (32) involving the construction of the next guess to the track parameters may be easily implemented. The issue is, however, how to condense the essential calculations in TRACK and COM to efficiently cycle through the successive guesses. The present code configuration may be modified with sufficient "flags" to accomplish this cycling. The resultant add-ons to the present configuration may be unduly complicated. It is suggested that the essential calculations of the present code may be incorporated into a new structure which is more "tuned" to the cycling function and the method of deciding convergence.

Convergence may be determined in the following way:

- Let X_1 = CPA

  X_2 = Time of CPA

  X_3 = Velocity

  X_4 = Angle

---

*These routines are described in Appendix A.

- Let $X_j^{(n)}$ be the value of X_j after the n^{th} iteration.

- Compose the index

$$I_n = \sum_{k=1}^{4} W_k \left[\frac{X_k^{(n)} - X_k^{(n-1)}}{X_k^{(n-1)}} \right]^2$$

  where W_k is an assigned weighting factor. For example, if CPA accuracy is more important than that of other track parameters, one may choose $W_4 = 4$, $W_3 = W_2 = W_1 = 1$.

- Compare I with a predetermined "tolerance" ε, that is, the first n for which $I_n < \varepsilon$ turns off the iteration.

Modifications to this criterion are possible. One can, for example, use only the CPA parameter and check if

$$\left| \frac{X_1^{(n)} \; 1 \; X_1^{(n-1)}}{X_1^{(n-1)}} \right| < \varepsilon .$$

It is suggested that the required accuracies for the track parameters be based on the physical problem.

APPENDIX

AN OUTLINE OF THE PRESENT COMPUTER CODE CAPABILITY

Two separate programs (TRACK and MIN) with their associated subroutines and functions, constitute the present computing capability implemented on a CDC 6600 machine. Table A-1 illustrates the structure of TRACK and its subprograms. This program is used for a first calculation of a new TRACK parameter case. Basic results are put on a file and saved for subsequent use by program MIN and its subprograms.

The following comments apply to calculations carried out in TRACK:

- Calculations are performed in Hrs-NMi units until printout stage where some conversions are performed. Thus, input values of parameters are converted to these units.

- Immediately, the horizontal CPA variable name is changed to RT and RCO is used, thereafter for the slant CPA given by

  RCO = SQRT(D*D + RT*RT)

- The number of time steps is calculated from

$$\text{NTMS} = \frac{\text{2*SQRT(RMAX*RMAX - RCO*RCO)}}{\text{V*TINC}}$$

  where now RCO is the slant CPA and TINC is in hours.

- A data file "TAPE 7" is made containing pertinent input and the results of the derivatives evaluated at each time step. TAPE 7 must be categorized by the appropriate Scope demand.

The input parameters for TRACK are indicated in Table A-2.

Table A-2. TRACK Input Parameters

Card #	Variable	Units	Algebraic Symbol	Computer Symbol
1	Horizontal CPA	N.MI	r_{co}	RCO
1	Ship Velocity	KNOTS	V	V
1	Time of CPA	HRS.	t_o	TO
1	Angle between array and track lines	DEGREES	b	B
1	Ocean depth	N.MI	d	D
2	Number of sensors (excluding the implied one at the origin)	-	-	NSS
3	Locations along Y axis of the sensors	FEET	y_i	Y(I)
4	Maximum slant range of interest	N.MI	R_m	RMAX
4	Time increment between samples	MIN	t_{inc}	TINC
5	Flag for printing time step information (=1 printing is suppressed)	-	-	IFL
5	Flag (No longer functional - leave blank)	-	-	JFL

The structure of program MIN is outlined in Table A-3. The following comments apply to the running of MIN:

- Only one of the above options can be performed during a single execution.
- Tape 7, the data file made by TRACK, must be attached. The header information on TAPE 7 identifying the case under consideration is printed so that handy reference is contained on the output of MIN runs.

MIN requires input in either of two forms. These two input options are:

- The user can select the set of time steps to be used in the covariance matrix calculations. The user supplies the number of time steps, JFL, then the indices of the time steps, NS(1),---NS (JFL). The program can cycle through any number of sets by setting NJFL. A sample input deck is illustrated in Table A-4.
- The user can select a sequence of time steps by identifying IBEG, the time step index of the beginning of the desired sequence, IEND, the time step of the end, and INC, the index increment. This option is activated by setting a flag INTNL $\neq 0$. The actual value of INTNL causes program cycling using multiple IBEG, IEND, INC values. An illustration is shown in Table A-5.

Table A-3. Structure of MIN

MIN
• Read file of stored information on case at hand. • Read cards for processing parameters. • If a flag is set for special time step selection, call SORT. • Call COM, which performed the same functions as in the TRACK program package, but uses only the time step information provided to MIN. • COM calls VAR, which performs as in the TRACK package, again using only the provided time step information.

Table A-4. Option 1 for MIN

CARD NO.	FORMAT	VARIABLES	SAMPLE CONTENTS	COMMENTS
1	2I5	NJFL,INTNL	3 (Blank)	Three sets of time steps will be used. INTNL must be blank or zero.
2	I5	JFL	4	Four time steps are used in the first pass.
3	16I5	NS(I); I=1,...4	1,14,16,20	Time steps whose indices are 1,14, 16,20 are used in the calculation.
4	I5	JFL	6	Six time steps are used in the second pass.
5	16I5	NS(I) I=1,...6	1,2,3,4,5,6	Indices of time steps
6	I5	JFL	2	Third pass has 2 time steps.
7	16I5	NS(1)NS(L)	1, 40	Time steps 1 and 40 are used.

Table A-5. Option 2 for MIN

CARD NO.	FORMAT	VARIABLES	SAMPLE CONTENTS	COMMENTS
1	2I5	NJFL,INTNL	(BLANK),3	Any value for NJFL is ignored. Three passes will be performed.
2	3I5	IBEG,IEND, INC	1,6,1	All time steps between indices 1 and 6 (inclusive) are used. (This is another way of doing the second pass in Option 1 example.)
3	3I5	IBEG, IEND, INC	1,6,2	Time steps 1,3,5 are used. Note truncation.
4	3I5	IBEG,IEND, INC	1,18,5	Time steps 1,6,11, 16 are used.

REFERENCES

1. An excellent discussion of linear least squares theory is contained in, W. Jenkins and D. Watts, Spectral Analysis and Its Applications, Holden-Day, 1968.

2. Extensions of the least squares theory to treating non-linear systems may be found in, G. Box, "Fitting Empirical Data," Ann. N.Y. Acad. Sci. 86, 3 (1960.)

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