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A DYNAMIC THEORY OF CONTRACTUAL INCENTIVES.(U)

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A DYNAMIC THEORY OF CONTRACTUAL INCENTIVES

by

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I. Introduction

A topic of increasing importance in public-sector management is the design and implementation of financial incentive systems that will encourage lower-level government units and profit-making organizations under contract to these units to make efficient use of government funds and to do so in a way that will satisfy non-financial government objectives as well. One such incentive system is the design-to-cost system implemented for many major weapons acquisition projects in the Department of Defense, in which a design-to-cost goal is established for each project and any deviations from the goal are corrected by making changes in the performance of the weapons system and/or the number of systems produced. The purpose of this paper is to construct a simple model of the information, incentive, and decision aspects of ~~such~~ a ~~process~~ financial incentive system and to offer insights into the problem of a high-level government unit that wishes to encourage lower-level units and private contractors to behave in consonance with its financial and non-financial objectives.

This problem is a special case of a more general incentives problem that has been examined in some length by economists concerned with the central control of decentralized activities and by persons concerned with the proper structuring of intra-bureaucratic incentive systems and incentive contracts. Three general problems arise in this context. The first is the identification of a feasible class of incentives, which is usually determined by institutional and legal constraints. The second is the design and implementation of information processing and monitoring systems to support the incentives. Such systems must (1) provide information to those in the lower levels of the hierarchy so that they may adjust their behavior in accordance with their performance and the incentives established by the higher-level units and (2) provide information to higher level units so that they may influence behavior through incentives. The third problem is legitimacy - that is, determination of the willingness of outside parties (i.e., contractors) to participate in the contractual arrangement of which the incentive system is a part.

As stated above, in this paper we are concerned with the incentival properties of the design-to-cost (DTC) system used by the Department of Defense (DoD).[1] Although this system has been viewed as a life cycle costing system and a production cost control system,[2] we will view it as an incentive system in which lower levels of DoD are rewarded for effecting cost savings and penalized for incurring cost overruns. Our work emphasizes two characteristics of the DTC system. The first is its dynamic properties. The weapons acquisition process is viewed as a multistage process whose characteristics change substantially over time. The process consists of (at least) three steps: (1) a development stage in which two or more contractors receive funds to design and test a prototype system, at the end of which a single contractor is awarded a production contract, (2) a production stage in which the winning contractor produces one or more copies of the system, and (3) an implementation and maintenance stage in which the system is maintained and modified in the field, often with some contractor support. Since the principal interactions occur between the first two stages (i.e., contractors behave differently than they otherwise would during the development stage in the hope of being awarded the production contract), we will confine our analysis to these two stages. We assume that the development contract is a fixed-price contract and the production contract includes (1) full cost recovery, (2) a reqard or penalty depending on the cost of production relative to a preselected cost target, and (3) a reward or penalty depending on system performance relative to a preselected performance target.

The second characteristic of the DTC system emphasized in our research is the hierarchical nature of the system. Important informational and decision processes occur at four levels of the government/contractor hierarchy. At the highest level, representing the Congress and the administration, DTC goals and allowable probabilisties of exceeding these goals are established. (System cost and performance are assumed to be random variables whose mean values are controllable). At a second level, representing DoD and the appropriate military service, the DTC goal is partitioned into two subgoals, one for the development stage and one for the production stage, and the contractors participating in the development stage are selected. At a third level, representing the military service and its project managers, most of the parameters in the incentive system are established and the production contract is awarded. (In our analysis, we assume that the decisions at this level are established by decision rules known in advance to both the government and the contractors.) At the fourth level, representing the contractors, one of the parameters in the production stage is established as are the levels of contractor effort (hiring personnel, purchasing raw material, etc.) for the two stages.

A growing literature on incentives is available for addressing problems of this type, although it will be shown that the problem posed above has features not adequately treated in previous literature. The literature may be partitioned into two principal categories: (1) the theory of contracts and (2) incentives in organizational design. The first category assumes two purposeful entities, a Principal and an Agent. This concept was developed in a paper by Ross,[3] who assumes that the Agent may select an action leading to an uncertain output, after which he is rewarded by the Principal according to a fee schedule. The Principal's utility function depends on the random output and the fee he must pay the Agent, and Ross examines and compares the several categories of fee schedules. This work has been extended by Harris and Raviv,[4] who assume that the fee schedule does not depend directly on the random output, but on a random observable quantity (the result of a monitoring system) that in turn depends both on the output and on the action taken by the Agent. The authors define a "contract" as the fee schedule and the observable quantity, and they examine the characteristics of optional monitoring systems.

The second category of incentives literature, incentives in organizational design, assumes that one or more Agents are engaged in a productive enterprise and are rewarded by a Central Planner. For example, Bonin[5] considers the case where the reward is a simple function of a random outcome relative to a target set by the Agent and estimates the impact of the parameters in the reward function on the Agent's selection of the target. Kleindorfer and Sertel[6] consider an enterprise in which a group of Agents produce a joint output which is shared among themselves according to a rule established by the Central Planner, and they compare the effects on the Agents of the reward system and of imperfect knowledge of the activities of the other Agents. In addition, the work of Hurwicz[7] on resource allocation methods and of Groves[8] on team theory is also of interest, although it is less directly relevant than the work cited above.

An examination of this literature discloses two gaps. The first is the lack of consideration of a dynamic incentive process - that is, a multistage process in which contractor behavior during any one stage is affected by the incentives operative during that stage and also by an expectation of rewards or punishments in the subsequent stages. Yet such a condition prevails whenever a development effort must precede a production effort. The second gap is a lack of consideration of any hierarchy other than that represented by Principal and Agent. Yet in a government/contractor effort of any size the government is represented by at least three distinct organizations (the Congress, the administration, and the bureaucracy), and the contracting agent may be represented by several organizations as well (e.g., contractors and subcontractors). Thus, the DTC problem posed above is of interest not only because of its practical significance, but also because it leads to an examination of two important topics not adequately addressed in the literature on incentives.

In the following section a basic model of the DTC process is developed. As an example of the nature of the results derived, we show that under certain conditions the contractors will compete during the development stage but that the winner of the production contract will put forth no effort during the production stage. The characteristics of an incentive system that gives rise to this behavior will be identified and a revised model will be solved to determine the impact of the contractors' anticipation of rewards during and after the production stage on their performance during the development stage.

II. The Basic Model

We consider a given project and assume that Congress has established a Design-to-Cost (DTC) goal, G, for the project. G is understood to be a constraint on total project cost and it is assumed that G may be exceeded only with ex ante probability γ. One might anticipate that DoD would set γ strategically to trade off the transactions costs of exceeding budgets and exposing itself to (re-) appropriations hearings against the internal transactions costs which may be assumed to occur if γ is small.

We assume that n firms have been preselected as candidates for carrying out the project, which occurs in two stages. In the first stage, development, the n firms compete against one another in producing the best design. In the second stage the firm with the best first-stage design is awarded (the opportunity to bid on) a production contract. To state the problem precisely we need the following notation.

e_{si} = Effort expended by firm i in stage s. In the development stage, s = d, and in the production stage, s = p;

$Q_{si}(e_{si})$ = Quality (or performance level) achieved by firm i in stage s;

$C_{si}(e_{si})$ = Costs incurred by firm i in stage s as a function effort expended.

DoD is assumed to let the following contract types. All development contracts are firm fixed fee with each of the n firms involved receiving G_d/n dollars. $G_d \leq G$ is therefore the total development cost to the government. The production contract, if awarded to firm i, is assumed to be a general incentive contract with payments above costs to firm i specified as:

$$\text{(1)} \quad \tilde{\Pi}_{pi}(T_{pi}, e_{pi}, \tilde{Q}_d) = a\, T_{pi} + b\,[T_{pi} - \tilde{C}_{pi}(e_{pi})] + R_i\,(\tilde{Q}_d + \tilde{Q}_{pi}(e_{pi})),$$

where random quantities have a ~ over them, and where

T_{pi} = Target cost rate, negotiated by firm i at the beginning of the production stage;

Q_d = Cumulative progress on the project during the development stage, the assumed starting point for the production stage;

a,b = Contract incentive parameters, where $a \geq 0$, $0 < b \leq 1$;

$R(q)$ = Performance incentive payment, a function of total performance level achieved over both stages.

At the end of the development stage, DoD will have expended exactly G_d dollars, leaving $G_p = G - G_d$ dollars in the overall project budget. Suppose firm i achieves the best performance in the development stage, i.e. suppose

$$(2) \quad \tilde{Q}_{di}(e_{di}) = \tilde{Q}_d = \underset{1 \leq j < n}{\text{Max}} \tilde{Q}_{dj}(e_{dj}).$$

We assume that if (2) obtains, then firm i is given the exclusive right to bid on a production contract. In a realistic setting, one might assume that more than one of the leading firms at the end of the development stage is given the opportunity to bid on a production contract. This possibility is excluded here. Thus, it is assumed that the leading development firm, say i, is interested at the beginning of the production stage in setting T_{pi}, e_{pi}, a and b so as to maximize

$$(3) \quad \bar{U}_{pi}(T_{pi}, e_{pi}, Q_d) = E\{\tilde{\Pi}_{pi}(T_{pi}, e_{pi}, Q_d) + F_i(\tilde{Q}_{di}(e_{di}), \tilde{Q}_{pi}(e_{pi})) | \tilde{Q}_{di}(e_{di}) = Q_d\},$$

where $F_i(q_d, q_p)$ represents expected follow-on benefits to firm i (e.g., in terms of maintenance contracts, future benefits from the technology developed, etc.) $\tilde{\pi}_{pi}$ is given in (1), and $\tilde{\Pi}_{pi} + \tilde{C}_{pi}$ represents total (incentive plus cost) payments made by the government in the production stage.

Of course, firm i will be subject to some constraints in indulging its preferences as represented by (3). Indeed we assume that a is fixed in advance by the Government and that whatever (T_{pi}, e_{pi}, b) are set the following holds:

$$(4) \quad \Pr\{\tilde{P}_{pi}(T_{pi}, e_{pi}, Q_d) + \tilde{C}_{pi}(e_{pi}) \geq G_p\} \leq \gamma,$$

where γ is specified by the Congress and the Administration. The fact that firms accept (4) as a constraint, of course, presumes that acceptable auditing practices can expose and penalize firms which cannot make a credible ex post case in the event of cost overruns that (4) was observed in their planning - the dependence just outlined of contractual incentive on (legitimate) enforcement and monitoring procedures cannot be overemphasized.

Beyond fixing a and imposing (4), we will assume that production contracts are negotiated through one of two methods[9] (firm i is the leading development firm):

M1: b is fixed ex ante and any T_{pi}, e_{pi} satisfying (4) will be accepted by DoD.

M2: Firm i and DoD negotiate (T_{pi}, e_{pi}, b) at the beginning of the production stage such that (4) is satisfied and such that a Pareto efficient point is reached between firm i (with preferences represented by (3)) and DoD, which is assumed to have preferences represented by a utility function $U_d(C, CO, Q)$, where $Q = \tilde{Q}_d + \tilde{Q}_{pi}(e_{pi})$ is final project quality, $C = G_d + \tilde{\Pi}_{pi} + \tilde{C}_{pi}$ is total project cost, and $CO = C - G$ is the cost overrun.

Summarizing, the production stage decision processes are assumed described by:

(5) M1: Maximize (3) with respect to (T_{pi}, e_{pi}), subject to (4).

(5') M2: $$\max_{(T_{pi}, e_{pi}, b)} \alpha \, \overline{U}_{pi}(T_{pi}, e_{pi}, Q_d) + (1-\alpha)\, E\{U_D(G_d + \tilde{\Pi}_{pi} + \tilde{C}_{pi}, G_d + \tilde{\Pi}_{pi} + \tilde{C}_{pi} - G, Q_d + \tilde{Q}_{pi}(e_{pi}))\}$$

s.t. (4) and $0 \leq b < 1$.

where α is between 0 and 1 and reflects the relative bargaining power of the contractor versus that of DoD, $\overline{U}_{pi}$ is defined in (3), $\tilde{\Pi}_{pi}$ is given in (1), $\tilde{C}_{pi} = \tilde{C}_{pi}(e_{pi})$ is the cost for the production stage, and Q_d is the observed realization of (2). We will define the optimal solution value to (5) (or (5')) as $V_{pi}(Q_d)$, the optimal expected return for firm i if the ending quality level in (2) is Q_d and firm i is awarded the production contract.

Now consider the development stage. Each of the n firms involved may be assumed to maximize the sum[10] of present benefits and expected follow-on benefits ($V_p(Q_d)$ if firm i is allowed to bid on the production contract). Expected follow-on benefits may then be written:

$$\text{(6)}\quad \text{Benefits} = \begin{cases} 0 & \text{if } \tilde{Q}_{di}(e_{di}) < \tilde{Q}_d. \\ V_{pi}(\tilde{Q}_d) & \text{if } \tilde{Q}_{di}(e_{di}) = \tilde{Q}_d. \end{cases}$$

From (6) we see that an expected profit maximizing contractor would solve the following problem in determining his level of effort e_{di} in the development stage:

$$\text{(7)}\quad \underset{e_{di}}{\text{Max}}\; E\,\{(G_d/n) - \tilde{C}_{di}(e_{di}) + V_{pi}(Q_{di}(e_{di}))\tilde{A}_i(e_{di},\ldots,e_{dn})\},$$

where $\tilde{C}_{di}(e_{1i})$ is realized in stage d for firm i and where $\tilde{A}_i(e_{11},\ldots,e_{1n})$ is equal to 1 if $\tilde{Q}_{di}(e_{di}) = \tilde{Q}_d = \underset{j}{\text{Max}}\,\{\tilde{Q}_{dj}(e_{dj})\}$ and 0 otherwise. Note that the probability that firm i is allowed to bid on the production contract (i.e., $\Pr\{\tilde{A}_i = 1\}$) depends on the level of effort of all the n firms involved. Denote the optimal solution value in (7) by $V_{di}(\underline{e}_d,G_d,n)$, where $\underline{e}_d = (e_{d1},\ldots,e_{dn})$.

The final step is the determination of $\underline{e}_d$. This problem may be formulated as a non-cooperative game, with utility functions $V_{di}(\underline{e}_d,G_d,n)$. We are interested in a Nash solution $\hat{\underline{e}}(G_d,n)$ to this game which is characterized by

$$\text{(8)}\quad V_{di}(\hat{\underline{e}}_{dj},\hat{\underline{e}}^{i}_{d},G_d) = \underset{e_{di}\geq 0}{\text{Max}}\; V_{di}\,(\,\cdot\,,\,\hat{\underline{e}}^{i}_{d},\,G_1),\quad i = 1,\ldots,n;$$ where $\underline{e}^{i}_{d}$ represents the vector $(e_{d1},\ldots,e_{di},e_{di+1},\ldots,e_{dn})$.

Suppose for the moment that $\hat{\underline{e}}_d(G_d,n)$ is unique for each G_d. DoD is then interested in determining G_d (and possibly also n) so that its expected utility $U(C,CO,Q)$ is maximized. If firm i is awarded the production contract, then

$$\text{(9)}\quad C = \text{COST} = G_d + (\tilde{\Pi}_{pi} + \tilde{C}_{pi})$$

$$\text{(10)}\quad CO = \text{COST OVERRUN} = C - G,$$

$$\text{(11)}\quad Q = \text{Quality} = \tilde{Q}_{di} + \tilde{Q}_{pi}.$$

Thus, DoD wishes to set G_d (and possibly n) so as to

$$\text{(12)}\quad \underset{0\leq G_d<G}{\text{Max}}\; \sum_{i=1}^{n} E\,\{U(G_d+\tilde{\Pi}_{pi}+\tilde{C}_{pi},(G_d+\tilde{\Pi}_{pi}+\tilde{C}_{pi}-G),\,\tilde{Q}_{di}+\tilde{Q}_{pi})\}\cdot \Pr\{\tilde{A}_i = 1\},$$

where all quantities are evaluated at $\hat{\underline{e}}_d(G_d)$, e.g.,

$$\tilde{\Pi}_{pi} = \tilde{\Pi}_{pi}(\hat{T}_{pi}(\tilde{Q}_{di}(\hat{e}_{di})),\ \hat{e}_{pi}\ (\tilde{Q}_{di}(\hat{e}_{di})),\ \tilde{Q}_{di}(\hat{e}_{di})),$$

where $\hat{T}_{pi}(Q_d), \hat{e}_{pi}(Q_d)$ are the optimal solution to (5)-(5') for given Q_d. The major problems in solving matters are in solving (5)-(5') and in obtaining $\hat{e}_d(G_d, n)$, to which we now turn.

III. Solution - Method 1

In order to obtain analytical results it is necessary to make assumptions about the forms of the probability distributions and reward functions. Specifically we assume for each $i = 1,\dots,n$ that:

1. $\hat{C}_{di}(e_{di})$ is random quantity with expected value e_{di}^2.
2. $\tilde{Q}_{di}(e_{di})$ is exponentially distributed, independently of $\{\tilde{Q}_{dj}(e_{dj}) | j \neq i\}$ with expected value $q_{di}e_{di}$, where $q_{di} > 0$.
3. $\tilde{C}_{pi}(e_{pi})$ and $\tilde{Q}_{pi}(e_{pi})$ are jointly normal with respective means, e_{pi}^2 and $q_{pi}e_{pi}(q_{pi} > 0)$, respective variances, σ_{pi}^2 and η_{pi}^2, and with positive correlation coefficient δ_{pi}.
4. $R_i(Q) = c(Q-\underline{Q})$ where $\underline{Q}$ is some desired minimal level of quality and $c > 0$.
5. $F_i(Q_d,Q_p) = H_i + h_{di}Q_d + h_{pi}Q_p$, where $H_i, h_{di} \geq 0, h_{pi} \geq 0$ are constants.

For this data we may write (5) as

$$(13) \max_{T_{pi} e_{pi}} [(a+b)T_{pi} - b\, e_{pi}^2 + c(Q_d + q_{pi}e_{pi} - \underline{Q}) + H_i + h_{di}Q_d + h_{pi}q_{pi}e_{pi}]$$

Subject to:

$$(14) \Pr\{(a+b)T_{pi} - b\,\tilde{C}_{pi}(e_{pi}) + c(Q_d + \tilde{Q}_{pi}(e_{pi}) - \underline{Q}) + \tilde{C}_{pi}(e_{pi}) \geq G_p\} \leq \gamma.$$

Collecting terms, (14) may be rewritten as:

$$(15) \Pr\{[(1-b)\,\tilde{C}_{pi}(e_{pi}) + c\tilde{Q}_{pi}(e_{pi})] \geq [G_p - (a+b)\,T_{pi} - c(Q_d - \underline{Q})]\} \leq \gamma$$

Since $(\tilde{C}_{pi},\tilde{Q}_{pi})$ is jointly normal, we see that $(1-b)\,\tilde{C}_{pi} + c\tilde{Q}_{pi}$ is normal with mean $[(1-b)\,e_{pi}^2 + c\,q_{pi}e_{pi}]$ and variance $[(1-b)^2\,\sigma_{pi}^2 + c^2\,\eta_{pi}^2 + 2(1-b)\,c\sigma_{pi}\eta_{pi}\delta_{pi}]$ so (15) may be expressed as

$$(16) [(1-b)\,e_{pi}^2 + cq_{pi}e_{pi} + (a+b)T_{pi} + c(Q_d - \underline{Q}) - G_p]$$
$$+ K(\gamma)\,[(1-b)^2\,\sigma_{pi}^2 + c^2\,\eta_{pi}^2 + 2(1-b)\,c\sigma_{pi}\eta_{pi}\delta_{pi}]^{1/2} \leq 0$$

where $K(\gamma)$ is the $(1-\gamma)^{th}$ fractile of the unit normal, i.e.

$$\Pr\{\tilde{N}(0,1) \geq K(\gamma)\} = \gamma.$$

Define $k_{pi}(\gamma,b,c)$ through

(17) $k_{pi}(\gamma,b,c) = K(\gamma)\,[(1-b)^2\,\sigma_{pi}^2 + c^2\tau_{pi}^2 + 2(1-b)c\,\sigma_{pi}\tau_{pi}\delta_{pi}]^{1/2}$.

Then (16) becomes

(18) $[(1-b)e_{pi}^2 + cq_{pi}e_{pi} + (a+b)\,T_{pi} + c(Q_d-\underline{Q}) - G_p] \leq k_{pi}(\gamma,b,c)$.

Since $b \leq 1$, we see that (18) defines a convex region for every value of Q_d. Note also that $\partial k_{pi}/\partial\gamma < 0$, and if $\delta_{pi} \geq 0$, $\partial k_{pi}/\partial b < 0$ and $\partial k_{pi}/\partial c > 0$. Thus, as γ or b decrease or c increases the constraint region becomes larger. Similarly, as Q_d decreases the constraint region becomes larger.[10]

To find the optimal T_{pi}, e_{pi} in (13) note that whatever e_{pi} is, the optimal T_{pi} will be set so that (18) holds as an equality since otherwise firm i could simply make T_{pi} higher with consequent higher profits. Solving for $(a+b)T_{pi}$ in (18) we see that

(19) $(a+b)T_{pi} = k_{pi}(\gamma,b,c) + G_p - c(Q_d-\underline{Q}) - (1-b)e_{pi}^2 - cq_{pi}e_{pi}$.

Thus, substituting in (13) for $(a+b)T_{pi}$ we have the following problem for e_{pi}:

(20) Max $[-e_{pi}^2 + k_{pi}(\gamma,b,c) + G_p + H_i + h_{di}\,Q_d + h_{pi}\,q_{pi}\,e_{pi}]$,

subject to $e_{pi} \geq 0$.

This leads to the solution

(21) $\hat{e}_{pi} = \dfrac{h_{pi}\,q_{pi}}{2}$,

(22) $\hat{T}_{pi} = \dfrac{k_{pi}(\gamma,b,c) + G_p - c(Q_d - \underline{Q}) - [\frac{(1-b)h_{pi}^2}{4} + \frac{ch_{pi}}{2}]\,q_{pi}^2}{(a+b)}$,

and finally,

(23) $V_{pi}(Q_d) = K_{pi} + h_{di}\,Q_d$,

where

(24) $K_{pi} = k_{pi}(\gamma,b,c) + G_p + H_i + \dfrac{h_{pi}^2\,q_{pi}^2}{4}$.

Notice from (21) that firm i will expend only the minimum effort (here $\hat{e}_{pi} = 0$) in stage P under Method 1 contracting unless there is some promise of follow-on rewards from such effort (i.e., unless $h_{pi} > 0$).

From (7) and (23) we see that firm i solves the following problem in determining its level of development effort e_{di}:

(25) $\max_{e_{di}\geq 0} \; E\{(G_d/n) - e_{di}^2 + [K_{pi} + h_{di}\,\tilde{Q}_{di}(e_{di})]\,\tilde{A}_i(e_{di}, e_d^i, n)\}$,

where we have used the assumption $E\{\tilde{C}_{di}(e_{di})\} = e_{di}^2$ and we recall that $\tilde{A}_i(\underline{e}_d, n) = 1$ precisely when firm i achieves the maximum in (2); otherwise $\tilde{A}_i(\underline{e}_d, n) = 0$.

We first evaluate the following expression in (25):

(26) $EP = E\{[K_{pi} + h_{di}\,\tilde{Q}_{di}(e_{di})]\,\tilde{A}_i(e_{di}, e_d^i, n)\}$.

EP represents the expected returns from the production stage as seen by firm i at the beginning of stage d.

We first note from (2) that

(27) $Pr\,\{\tilde{A}_i(\underline{e}_d, n) = 1\} = Pr\,\{\tilde{Q}_{dj}(e_{dj}) \leq \tilde{Q}_{di}(e_{di}) / \text{ for all } j = 1,\ldots,n\}$,

or using the assumed independence of $\{\tilde{Q}_{dj} / j = 1,\ldots,n\}$

(28) $Pr\,\{\tilde{A}_i(\underline{e}_d, n) = 1\} = \prod_{j=i} Pr\,\{\tilde{Q}_{dj}(e_{dj}) \leq \tilde{Q}_{di}(e_{di})\}$.

Thus, if $F_{dj}(q, e_{dj}) = Pr\,\{\tilde{Q}_{dj}(e_{dj}) \leq q\}$ is the cumulative distribution function of $\tilde{Q}_{dj}(e_{dj})$, we may write (28) as

(29) $Pr\,\{\tilde{A}_i(\underline{e}_d, n) = 1\} = \prod_{j\neq i} F_{dj}(\tilde{Q}_{di}(e_{di}), e_{dj})$.

Finally, using (29), (26) becomes

(30) $EP = \int_{-\infty}^{\infty} ([K_{pi} + h_{di}\,x] \prod_{j\neq i} F_{dj}(x, e_{dj}))\, f_{di}(x, e_{di})\, dx$,

where $f_{di}(x, e_{di})$ is the probability density function of $\tilde{Q}_{di}(e_{di})$.

In the exponential case[11] considered here, (30) becomes

(31) $EP = \dfrac{1}{q_{di}\, e_{di}} \displaystyle\int_{-\infty}^{\infty} ([K_{pi} + h_{di}x] \prod_{j\neq i} [1-\exp(-\frac{x}{q_{dj}\, e_{dj}})])\, \exp(-\frac{x}{q_{di}\, e_{di}})\, dx$

Restricting attention to n = 1 or 2, we obtain

(32) $EP\,(n=1) = \dfrac{1}{q_{di}\, e_{di}} \displaystyle\int_0^{\infty} [K_{pi} + h_{di}\, x]\, \exp(-\frac{x}{q_{di}\, e_{di}})\, dx$

$= K_{pi} + h_{di}\, q_{di}\, e_{pi}$,

and setting $j \neq i$

$$(33)\quad EP(n=2) = \frac{1}{q_{di}\ e_{di}} \int_0^\infty ([K_{pi} + h_{di}\ x]\ [1 - \exp\ (1- \frac{x}{q_{dj}\ e_{dj}})])\ \exp\ (- \frac{x}{q_{di}\ e_{di}})\ dx$$

$$= [K_{pi}+h_{di}\ q_{di}\ e_{di}](\frac{q_{di}\ e_{di}}{q_{dj}\ e_{dj} + q_{di}\ e_{di}})\ .$$

Comparing (32) and (33), it is interesting to note that for any given level of effort during the development stage the *ex* ante expected returns from the production stage, which we denoted EP above, are less for firm i if 2 firms compete for the production contract than if firm i alone is to bid on the production contract.

Now, given (32) - (33), we may easily solve (25) for the optimal development effort $\hat{e}_{di}$, assuming the other firm's effort fixed at e_{dj}.

When n = 1, of course, there is no other competing firm and substituting (32) in (25) yields the following as the appropriate problem for firm i (if firm i is the only development firm):

$$(34)\quad \text{Max}\ [G_d - e_{di}^2 + K_{pi} + h_{di}\ q_{di}\ e_{di}],$$

which has the unique solution

$$(35)\quad \hat{e}_{di} = \frac{h_{di}\ q_{di}}{2}$$

yielding overall profits for firm i of

$$(36)\quad V_{di}(e_{di},G_d) = G_d + K_{pi} + \frac{h_{di}^2\ q_{di}^2}{4}\ .$$

When n = 2, matters are more complicated. Substitution of (33) in (25) yields

$$(37)\quad \underset{e_{di}\geq 0}{\text{Max}}\ [(G_d/2) - e_{di}^2 + [K_{pi} + h_{di}\ q_{di}\ e_{di}]\ (\frac{q_{di}\ e_{di}}{q_{dj}\ e_{dj} + q_{di}\ e_{di}})\ .$$

Taking first-order conditions in (37), while assuming e_{dj} fixed, we obtain

$$(38)\quad e_{di} = \frac{K_{pi}\ q_{di}\ q_{dj}\ e_{dj}}{[2\Delta^2 - h_{di}\ q_{di}^2\ (\Delta + q_{dj}\ e_{dj})]}$$

where

$$(39)\quad \Delta = q_{d1}\ e_{d1} + q_{d2}\ e_{d2}\ .$$

We seek a Nash solution, defined by (8), which would be a simultaneous solution to (38) and the corresponding equation for firm j, i.e., to (38) and

$$(40)\quad e_{dj} = \frac{K_{pj}\, q_{dj}\, q_{di}\, e_{di}}{[2\Delta^2 - h_{dj}\, q_{dj}^2\, (\Delta + q_{di}\, e_{di})]} .$$

Assuming Δ fixed, and $h_{di}, h_{dj} > 0$, the simultaneous solution to (38) and (40) is

$$(41)\quad \hat{e}_{di}(\Delta) = \frac{\Delta 1}{[A(2\Delta - h_{di}q_{di}^2)\, h_{dj}\, q_{di}\, q_{dj}^2 - h_{di}\, K_{pj}\, q_{di}^3\, q_{dj}^2]}$$

$$(42)\quad \hat{e}_{dj}(\Delta) = \frac{\Delta 1}{[\Delta(2\Delta - h_{dj}q_{dj}^2)\, h_{di}\, q_{di}^2\, q_{dj} - h_{dj}\, K_{pi}\, q_{di}^2\, q_{dj}^3]}$$

where

$$(43)\quad \Delta 1 = [K_{pi}\, K_{pj}\, q_{di}^2\, q_{dj}^2 - (2\Delta - h_{di}\, q_{di}^2)(2\Delta - h_{dj} q_{dj}^2)\, \Delta^2] .$$

Now, from (39) the Nash solution $\hat{\underline{e}}_d = \hat{\underline{e}}_d(\hat{\Delta})$ we seek must clearly satisfy (41)-(42) and

$$(44)\quad q_{d1}\, \hat{e}_{d1}(\hat{\Delta}) + q_{d2}\, \hat{e}_{d2}(\hat{\Delta}) = \hat{\Delta} .$$

Thus, multiplying (41) (resp., (42)) by q_{di} (resp., q_{dj}) and adding the results leads to (44), which in general is a polynomial of degree 6 for the sought for $\hat{\Delta}$. Numerical solution procedures easily yield $\hat{\Delta}$ in general, and once $\hat{\Delta}$ is obtained so also is the desired Nash point $\hat{\underline{e}}_d$ from (40)-(41), from which all other desired information may be obtained. In this paper we will not proceed further with the general case. We do note, however, two cases which may be solved analytically.

(1) The Case $h_{di} = 0$ for all i: In this case (38)-(40) can be solved directly to yield

$$(45)\quad \hat{e}_{di} = T\sqrt{K_{pi}}, \qquad \hat{e}_{dj} = T\sqrt{K_{pj}},$$

where

$$(46)\quad T = \frac{(\sqrt{q_{d1}\, q_{d2}})\, (K_{p1}\, K_{p2})^{1/4}}{\sqrt{2}\, (q_{d1}\sqrt{K_{p1}} + q_{d2}\sqrt{K_{p2}})} .$$

In this case it can be shown that $\partial\hat{e}_{di}/ q_{di}$ has the same sign and $\partial\hat{e}_{di}/ K_{pj}$ has the opposite sign of $(q_{dj}\sqrt{K_{pj}} - q_{di}\sqrt{K_{pi}})$. As expected $\partial\hat{e}_{di}/\partial K_{pi} > 0$

always holds.

(2) <u>The case of two identical firms</u>: When $q_{di} = q_{dj} = q_d$, $h_{di} = h_{dj} = h_d$, and $K_{pi} = K_{pj} = K_p$ we can again solve (38)-(40) explicitly, obtaining

$$(47)\quad \hat{e}_{d1} = \hat{e}_{d2} = \frac{3h_d q_d + \sqrt{9h_d^2 q_d^2 + 32K_p}}{16} .$$

Here all the relative change effects are obvious and in the expected (positive) direction. An interesting point to note from (47) (or (45)-(46)) is that when $h_d = 0$, the amount of effort expended in development is independent of quality or performance returns to effort.

This concludes our discussion of Method 1 contracting (see (5)). Before considering further the government's problem in this regard, let us turn our attention briefly to Method 2 contracting (see (5')).

IV. Solution - Method 2

We continue to make the cost and distributional assumptions 1 - 5 of the previous section. In Method 2 contrasting the stage p behavior of the production contracting firm, say i, is determined as a solution to (5'), except that we further restrict b so that $b \geq \underline{b} \geq 0$, with $\underline{b}$ some minimal sharing rate set by Congress.[12] We assume the DoD utility function is specified linearly as

(48) $U_D(C,CO,Q) = -g_1 C - g_2 CO + g_3 Q,$

where $g_i > 0$, $i = 1,2,3$. Then, for given $\alpha \in (0,1)$, we may write the problem (5') as follows:

$$(49)\ \underset{b,T_{pi},e_{pi}}{\text{Maximize}}\ EV = \alpha E\{\tilde{\Pi}_{pi} + \tilde{F}_i\} + (1-\alpha) E\{U_D(\tilde{C},\tilde{CO},\tilde{Q})|\tilde{Q}_{di} = Q_d\}$$
$$= \alpha[(a+b)T_{pi} - be_{pi}^2 + c(Q_d + q_{pi}e_{pi} - \underline{Q})$$
$$+ (H_i + h_{di}Q_d + h_{pi}q_{pi}e_{pi})]$$
$$+ (1-\alpha)[-g_1(G_d + E\{\tilde{\Pi}_{pi} + \tilde{C}_{pi}(e_{pi})\})$$
$$- g_2(G_d + E\{\tilde{\Pi}_{pi} + \tilde{C}_{pi}(e_{pi})\} - G)$$
$$+ g_3(Q_d + q_{pi}e_{pi})].$$

Subject to: (4) and $\underline{b} \leq b \leq 1$.

Note that the expected total project cost (to the government) and quality given Q_d) are, respectively, $G_d + E\{\tilde{\Pi}_{pi} + \tilde{c}_{pi}(e_{pi})\}$ and $Q_d + E\{\tilde{Q}_{pi}(e_{pi})\} = Q_d + q_{pi}e_{pi}$. Now we note that

$$(50)\ E\{\tilde{\Pi}_{pi} + \tilde{C}_{pi}(e_{pi})\} = (a+b)T_{pi} + (1-b)e_{pi}^2 + c(Q_d + q_{pi}e_{pi} - \underline{Q}).$$

Now, under our assumptions, (4) may be rewritten in the form (18). Moreover, as in section III, it may be shown here that for any fixed $b \in [\underline{b},1]$ the solution to (49) is on the boundary of the constraint set (18) provided only that[13]

$$(51)\qquad \alpha > \frac{g_1 + g_2}{1 + g_1 + g_2}.$$

Condition (51) may be viewed as a lower bound on the bargaining power of firm i. We henceforth assume (51) so that (4) (i.e., (18)) holds as an equality. Just as in section III, we can now substitute (19) in (49) to obtain the final problem of interest:

(52) $$\underset{b,T_{pi},e_{pi}}{\text{Maximize}} \left(-\alpha e_{pi}^2 + (\alpha h_{pi} + (1-\alpha)\, g_3)\, g_{pi} e_{pi} + Q_d(\alpha h_{di} + c(1-\alpha)(g_1+g_2)) + TV(b),\right.$$

Subject to: $T_{pi} \geq 0$, $e_{pi} \geq 0$, $\underline{b} \leq b < 1$.

where the term TV is independent of e_{pi} and Q_d and is given by

(53) $$\begin{aligned} TV(b) = (&\underline{Q}(1-\alpha)c + \alpha H_i + (1-\alpha)\,[g_3 - (g_1+g_2)]c \\ &+ G_p\,[\alpha - (1-\alpha)(g_1+g_2)] - G_d(g_1+g_2)(1-\alpha) \\ &+ k_{pi}(\gamma,b,c)\,[\alpha - (1-\alpha)\,(g_1+g_2)]), \end{aligned}$$

We may first note that (51) implies $[\alpha - (1-\alpha)(g_1+g_2)] > 0$, and this coupled with (see (17)) $\delta k_{pi}(\gamma,b,c)/\delta b < 0$ implies that the optimal solution for b in (52) is $\hat{b} = \underline{b}$ (note that the only term containing b is $[\alpha - (1-\alpha)(g_1+g_2)]\, k_{pi}(\gamma,b,c)$). To obtain $\hat{e}_{pi}$ we take first-order conditions in (52) and find

(54) $$\hat{e}_{pi} = \frac{q_{pi}[\,h_{pi} + (1-\alpha)\,g_3]}{2\alpha} > 0.$$

$\hat{T}_{pi}$ is found by substituting $\hat{e}_{pi}$, $\hat{b}$ into (19) and solving.

Substituting $\hat{b} = \underline{b}$ and $\hat{e}_{pi}$ in (54) into (52), we see that Method 2 leads to exactly the same form of solution value (see (23)) as Method 1 (where we use a' to distinguish Method 2 values):

(55) $$V'_{pi}(Q_d) = K'_{pi} + h'_{di}Q_d,$$

where for Method 2

(56) $$K'_{pi} = \frac{q_{pi}^2\,[\alpha h_{pi} + (1-\alpha)\,g_3]^2}{4\alpha} + TV(\underline{b})$$

and

(57) $$h'_{di} = [\alpha h_{di} + (1-\alpha)\,c\,(g_1+g_2)].$$

From this we see that the solution procedure and results for Method 1 in stage d are completely transferable to Method 2, with K'_{pi} and h'_{di} are substituted everywhere for K_{pi} and h_{di}.

Before closing our analysis of Method 2 it is of interest to note, comparing (21) and (54), that effort expended in the production stage is always greater under Method 2 than under Method 1 contracting. More detailed comparative analysis of the other parameters and decisions awaits further research.

V. Summary and Conclusions

Our framework and results thus far may be summarized as follows.

1. Congress sets certain institutional and financial limits on specific projects. These took the form here of specifications of G and b (or $\underline{b}$).

2. DoD then splits up the budget into development and production funds and negotiates with contractors. Thus, DoD sets G_d, G_p, γ and enforces constraint (4) through its auditing activities.

3. Preselected firms then play a multistage game against one another to determine their own effort contributions in the development stage and, subject to renegotiation, in the production stage.

Our main results at this point are those of Sections III and IV which provide solutions for firm behavior under certain cost and distributional assumptions. These will then allow a detailed examination of the following typical questions in follow-on research:

. What percentage of total budget should be allocated to development?

. When should parallel development efforts be undertaken and when not?

. What forms of contractual agreement (e.g. in terms of present values of a and b (or $\underline{b}$) are cost efficient? Performance efficient?

. What constraints or auditing procedures should Congress undertake to better control cost? What would the performance and private sector profit consequences of such procedures be? In particular, under what project and contractor conditions is Design-to-Cost a viable arrangement?

. What are ball-park estimates for parameters of the above model for a few selected projects? Is the model predictive?

The above questions may be a bit heroic in scope given the fairly detailed assumptions which we found necessary to make to derive our results. Nonetheless, to the best of our knowledge, our theoretical framework, even with its limitations, seems to offer the first sufficiently general structure within which to ask these and similar hierarchy and time-related questions.

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FOOTNOTES

1. See "Design to Cost," a special issue of the Defense Management Journal, September 1974.

2. See "Design to Cost - Problem Definition, Survey of Potential Actors and Observations on Limitations" by James D. McCullough, Paper P-928, Institute for Defense Analysis, January 1973, and "Application of Design-to-Cost Concept to Major Weapon Systems Acquisitions," Report to the Congress by the Comptroller General of the United States, U.S. General Accounting Office, Washington, D.C., June 23, 1975.

3. "On the Economic Theory of Agency and the Principle of Similarity" by S.A. Ross, in Essays on Economic Behavior Under Uncertainty, ed. by M. Balch, D. McFadden, and S. Wu, North Holland, 1974.

4. "Optimal Incentive Contracts with Imperfect Information," by Milton Harris and Artur Raviv, Working Paper No. 70-75-76, GSIA, Carnegie-Mellon University.

5. "On the Design of Managerial Incentive Structures on a Decentralized Planning Environment," by John P. Bonin, American Economic Review, Vol. 66, No. 4, September 1976, pp. 682-687.

6. "An Exploration in Optimal Enterprise Design via Incentives," by Paul R. Kleindorfer and Sertel, International Institute of Management, West Berlin, January 1975.

7. "The Design of Mechanisms for Resource Allocation" by Leon Hurwicz, The American Economic Review, Vol. 63, 1973, pp. 1-30.

8. "Incentives in Terms," by T. Groves, Econometrics, Vol. 41 (1973), pp. 701-710.

9. Method 1 was analyzed by McCall for a static problem and neglecting (4). He showed the possibility of a bias in favor of inefficient firms arising from opportunity cost considerations. Such effects are ignored here. Method 2 is in the spirit of Canes and Cummins, who also did not consider any constraint similar to (4).

10. We ignore discounting for the moment.

11. The authors would be grateful for suggestions as to other cases which might be analytically tractable.

12. See also Canes [1975] for a similar assumption and a discussion of some rationale for establishing such a lower bounding sharing rate.

13. When (51) does not hold, the solution to (49) appears to be somewhat complicated as the solution need no longer be on the boundary of (18). Details for this more general case have not yet been worked out.