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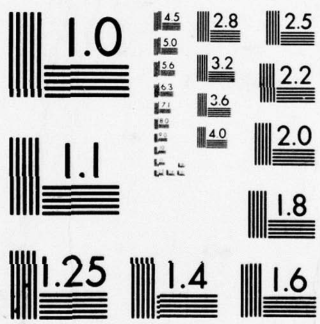
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THE ROLE OF THE INTERACTOR IN DECOUPLING

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Abstract

Given any proper rational transfer matrix, T(s), a special lower left triangular polynomial matrix, $\xi_T(s)$, called the interactor has been defined and shown to be (together with the rank of T(s)) a complete invariant under dynamic compensation. In this paper, the interactor is used to develop results on decoupling and pole placement via feedback. For example, it is shown that triangular decoupling with arbitrary pole assignment is always possible using state feedback and that decoupling with arbitrary pole assignment is always possible using dynamic compensation.

1. Introduction

Given any proper rational transfer matrix $T(s)$, a special lower left triangular matrix, $\xi_T(s)$, called the interactor has been defined and shown to be (together with the rank of $T(s)$) a complete invariant under dynamic compensation ([1]). In this paper, the interactor is used to develop results on decoupling and pole placement. In particular, it is shown that a square system can be decoupled via linear state feedback if and only if the interactor is diagonal, that triangular decoupling with arbitrary pole assignment is always possible using state feedback, that decoupling with arbitrary pole placement is always possible using dynamic compensation (state feedback and input dynamics) ([2]), and that certain of these properties are "generic".

In section 2, the interactor is defined for reference and some basic results relating the interactor to decoupling via state feedback are developed. The question of triangular decoupling with arbitrary pole assignment is examined in section 3. Some comments and extensions are considered in section 4.

2. The Interactor and State Feedback

Let S be the set of all proper rational transfer matrices $T(s)$ of full rank ρ_T such that the first ρ_T rows, $T_{\rho_T}(s)$, are independent. If $T(s)$ is an element of S, then the interactor is defined by means of the following lemmas ([1]):

Lemma 2.1: Let $T(s)$ be an $m \times m$ element of S. Then there is a unique nonsingular matrix $\xi_T(s)$ of the form

$$\xi_T(s) = H_T(s)\mathrm{diag}[s^{f_1},\ldots,s^{f_n}] \quad (2.2)$$

where

$$H_T(s) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ h_{21}(s) & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ h_{m1}(s) & h_{m2}(s) & \cdots & 1 \end{bmatrix} \quad (2.3)$$

and $h_{ij}(s)$ is divisible by s (or is zero) such that

$$\lim_{s\to\infty} \xi_T(s)T(s) = K_T \quad (2.4)$$

with K_T nonsingular.

Lemma 2.5: Let $T(s)$ be a $p \times m$ element of S with $p < m$. Then there is a unique nonsingular lower left triangular $p \times p$ matrix $\xi_T(s)$ of the form (13) such that

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$$\lim_{s\to\infty} \xi_T(s)T(s) = K_T. \qquad (2.6)$$

__Lemma 2.7: Let $T(s)$ be a $p \times m$ element of S with $p > m$ and let $T_m(s)$ be the nonsingular matrix consisting of the first m rows of $T(s)$. Then there is a unique nonsingular lower left triangular $p \times p$ matrix $\xi_T(s)$ of the form__

$$\xi_T(s) = \begin{bmatrix} \xi_{T_m} & 0 \\ -\gamma_1(s) & \gamma_2(s) \end{bmatrix} \qquad (2.8)$$

__where $\gamma_1(s), \gamma_2(s)$ are relatively left prime and $\gamma_2(s)$ is a nonsingular lower left triangular $(p-m) \times (p-m)$ matrix in Hermite normal form with monic diagonal entries such that__

$$\lim_{s\to\infty} \xi_T(s)T(s) = K_T$$

__with K_T a constant matrix of rank m whose final $p - m$ rows are zero.__

The interactor is of critical importance in questions relating to decoupling as will be shown in the sequel.

Let $T(s)$ be an element of S. Then it is well-known that $T(s)$ can be written in the form $R(s)P^{-1}(s)$ where $R(s), P(s)$ are relatively right prime polynomial matrices and $P(s)$ is column proper. Under linear state variable feedback of the form $u = Fx + Gv$, G nonsingular, the open loop transfer matrix $T(s)$ is transformed into the closed loop transfer matrix $T_{F,G}(s)$ given by

$$T_{F,G}(s) = R(s)[P(s)-F(s)]^{-1}G = R(s)P_{F,G}^{-1}(s) \qquad (2.9)$$

where $F(s) = FS(s)$ with

$$S(s) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ s & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ s^{\partial_1 - 1} & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & s^{\partial_2 - 1} & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & \vdots \\ 0 & 0 & & s^{\partial_m - 1} \end{bmatrix} \qquad (2.10)$$

and $\{\partial_1, \ldots, \partial_m\}$ being the column degrees of P ([4]). The feedpack pair (F,G) can be chosen to obtain any arbitrary column proper $P_{F,G}(s)$ having the same column degrees as $P(s)$. It follows that $T_c(s) = P(s)P_{F,G}^{-1}(s)$ and its inverse, $P_{F,G}(s)P^{-1}(s)$, are both proper and hence, that linear state variable feedback can be represented by the dynamic compensator $T_c(s)$ (i.e., by postmultiplication of $T(s)$ by $T_c(s)$). This leads to:

__Proposition 2.11: The interactor $\xi_T(s)$ is an invariant under linear state variable feedback.__

__Proof__: By definition, $\lim_{s\to\infty} \xi_T(s)T(s) = K_T$ with K_T nonsingular. However, $\lim_{s\to\infty} P(s)P_{F,G}^{-1}(s) = G$ and so it follows that $\lim_{s\to\infty} \xi_T(s)R(s)P_{F,G}^{-1}(s)$ $= \lim_{s\to\infty} \xi_T(s)R(s)P^{-1}(s)P(s)P_{F,G}^{-1}(s) = K_T G$ is nonsingular. The proposition follows from the uniqueness of the interactor.

__Theorem 2.12__ (cf. [4]) __A system characterized by a nonsingular, proper, rational $m \times m$ transfer matrix $T(s)$ can be decoupled via linear state variable feedback if and only if the interactor $\xi_T(s)$ is diagonal.__

Proof: If $T(s) = R(s)P^{-1}(s)$ can be decoupled using state feedback, then there is a feedback pair (F,G) such that $R(s)P_{F,G}^{-1}(s) = D(s)$ with $D(s)$ a diagonal transfer matrix. Since $\xi_T(s)$ is invariant under state feedback, $\lim_{s\to\infty} \xi_T(s)D(s) = K$ a diagonal non-singular matrix and so, $\xi_T(s)$ is diagonal.

Conversely, if $\xi_T(s)$ is diagonal, then $\xi_T(s)R(s)$ can be written in the form $P_{F,G}(s)$ for an appropriate feedback pair (F,G) (where $T(s) = R(s)P^{-1}(s)$) since $R(s)$ is nonsingular (i.e. $\det R(s) \neq 0$). It follows that $T_{F,G}(s) = R(s)P_{F,G}^{-1}(s)$ $= \xi_T^{-1}(s)$ is the diagonal matrix with diagonal entries s^{-f_i}. In other words, $T_{F,G}(s)$ is the transfer matrix of an integrator decoupled system.

Suppose that $T(s)$ is a non-singular, proper, rational $m \times m$ transfer matrix add that $\xi_T(s)$ is diagonal so that $T(s)$ can be decoupled using state feedback. How many poles can be arbitrarily assigned while simultaneously decoupling the system? To answer this question, let $T(s) = R(s)P^{-1}(s)$ with $R(s), P(s)$ relatively right prime and $P(s)$ column proper. Then $R(s)$ can be written in the form

$$R(s) = R_d(s)\tilde{R}(s) \tag{2.13}$$

where $R_d(s)$ is a diagonal matrix with diagonal entries $r_i(s)$ such that $r_i(s)$ is the greatest common divisor of the i^{th} row of $R(s)$. Let $\rho_i = \deg r_i(s)$ and let f_i be as in the definition of $\xi_T(s)$. If $D(s)$ is a diagonal matrix with diagonal entries $d_i(s)$ where each $d_i(s)$ is an arbitrary (Hurwitz) polynomial of degree $f_i + \rho_i$, then $D(s)\tilde{R}(s)$ is a column proper polynomial matrix with $\deg d_i = \partial_i$, the i-th colum degree of $P(s)$. It follows that there a feedback pair (F,G) such that $P_{F,G}(s) = D(s)\tilde{R}(s)$ and hence, that the closed loop transfer matrix $T_{F,G}(s)$ $= R(s)R^{-1}(s)D^{-1}(s) = \tilde{R}_d(s)D^{-1}(s)$. Thus, $\sum_1^m (f_i+\rho_i)$ poles can be assigned arbitrarily using this technique. If $\deg(\det R(s)) = q$ and $\deg(\det R_d(s))$ $\sum_1^m \rho_i = \rho$, then $\deg(\det \tilde{R}(s)) = q - \rho$ and $\sum_1^m ((f_i+\rho_i) = n - q + \rho = n - (q-\rho)$. The $q - \rho$ poles which cannot be assigned correspond to system zeros (those of $\det R(s)$) which must be "cancelled" via state feedback.

It should also be noted that if $T(s)$ is a $p \times m$ transfer matrix in S with $p < m$, then the above results can be used by simply adjoining additional rows to $R(s)$ in an appropriate fashion.

Theorem 2.14: Let $T(s)$ be a proper rational $p \times m$ transfer matrix with $p \leq m$ and suppose that $T(s)$ is of full rank p. Let $D(s)$ be any proper rational $p \times p$ diagonal transfer matrix such that $\xi_T(s)\xi_D^{-1}(s)$ is proper. Then there is an element $T_c(s)$ of S such that $T(s)T_c(s) = D(s)$.

Proof: An immediate consequence of Theorem 4.5 of [1].

This theorem (cf. [2]) essentially states that decoupling with "arbitrary" pole placement is always possible using a combination of state feedback and input dynamics (i.e. so-called dynamic compensation).

3. Triangular Decoupling Using State Feedback

In this section, it will be shown that triangular decoupling with arbitrary pole assignment is always possible using state feedback.

Theorem 3.1: A system characterized by a nonsingular, proper, rational $m \times m$ transfer matrix $T(s)$ can always be triangularly decoupled with all closed loop poles arbitrarily assigned using linear state variable feedback.

Proof: Let $T(s) = R(s)P^{-1}(s)$ with $R(s), P(s)$ relatively right prime, $R(s)$ lower left triangular and $P(s)$ column proper. Let $U(s)$ be a unimodular polynomial matrix which reduces $\xi_T(s)R(s)$ to row proper, lower left triangular Hermite normal form i.e.,

$$\xi_T(s)R(s)U(s) = \begin{bmatrix} q_{11}(s) & 0 & 0 \dots & 0 \\ q_{21}(s) & q_{22}(s) & 0 \dots & 0 \\ \vdots & \vdots & & \\ q_{m1}(s) & q_{m2}(s) & \dots & q_{mm}(s) \end{bmatrix} \tag{3.2}$$

where $q_{ii}(s)$ is a nonzero monic polynomial and $\deg q_{ii}(s) = k_i$. Note that

$$\sum_1^m k_i = n = \deg(\det P(s)) = \deg(\det \xi_T(s)R(s)).$$

Now let $D(s)$ be a diagonal matrix with diagonal entries $d_i(s)$ where each $d_i(s)$ is an arbitrary (Hurwitz) polynomial of degree k_i. If $D(s)$ is right divided by $[\xi_T(s)R(s)U(s)]^{-1}$, then

$$D(s)[\xi_T(s)R(s)U(s)]^{-1} = A(s)[\xi_T(s)R(s)U(s)]^{-1} + E(s) \tag{3.3}$$

where

$$\lim_{s\to\infty} A(s)[\xi_T(s)R(s)U(s)]^{-1} = 0 \tag{3.4}$$

and

$$E(s) = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ e_{21}(s) & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & & \\ e_{m1}(s) & e_{m2}(s) & \cdot & \dots & 1 \end{bmatrix}. \tag{3.5}$$

It follows from (3.3) that $D(s) = A(s) + E(s)\xi_T(s)R(s)U(s)$ or, equivalently, that

$$D(s) = A(s) + \xi_T(s)R(s)U(s) + [E(s)-I]\xi_T(s)R(s)U(s) \tag{3.6}$$

Let $\tilde{P}(s) = A(s) + \xi_T(s)R(s)U(s)$. Then $\tilde{P}(s)$ is triangular and $\det \tilde{P}(s) = \det D(s) = \prod_1^m d_i(s)$ in view of (3.6). By virtue of (3.4),

$$\lim_{s\to\infty} \tilde{P}(s)[\xi_T(s)R(s)U(s)]^{-1} = I \tag{3.7}$$

which implies

$$\lim_{s\to\infty} \xi_T(s)R(s)[\tilde{P}(s)U^{-1}(s)]^{-1} = I \tag{3.8}$$

In other words, $\tilde{P}(s)U^{-1}(s)$ is column proper and of the same column degrees as $\xi_T(s)R(s)$ (a fortiori as $P(s)$). Thus, there is a feedback pair (F,G) such that $P_{F,G}(s) = \tilde{P}(s)U^{-1}(s) = A(s)U^{-1}(s) + \xi_T(s)R(s)$ and $\det \tilde{P}_{F,G}(s) = \alpha \prod_1^m d_i(s)$ for some constant α. Thus, the theorem is established.

4. *Comments and Extensions*

The results obtained here are

indicative of the role the interactor can play in decoupling problems using state feedback. Extensions to the case of output feedback can be readily developed for, under linear output feedback of the form $u = -Hy + Gv$, G nonsingular, the open loop transfer matrix $T(s)$ is transformed into the closed loop transfer matrix $T_{H,G}(s)$ given by

$$\begin{aligned} T_{H,G}(s) &= R(s)[P(s) - HR(s)]^{-1}G \\ &= R(s)P_{H,G}^{-1}(s). \end{aligned} \tag{4.1}$$

This leads, for example, to an immediate translation of Theorem 2.12 for output feedback.

Since the pole placement results are constructive, they lead to specific procedures for implementing the requisite compensators. However, the questions of minimality and stability of the compensators remain to be treated.

Finally, there is the question of to what extent the results obtained here are "generic". The answer to this question rests on showing that the interactor $\xi_T(s)$ is, in an appropriate sense, a continuous function of the transfer matrix $T(s)$. This is examined in [6].

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is always possible using dynamic compensation.