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RECENT RESULTS OBTAINED
IN THE MODELING OF TURBULENT
FLOWS BY SECOND-ORDER CLOSURE

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1. INTRODUCTION

Two separate and distinct problems in the modeling of turbulent flows are under investigation at A.R.A.P. under AFOSR support under Contract No. F44620-76-C-0048.

1. A method for extending the calculation of turbulent dynamics by second-order closure to include the effect of "structure" as well as scale.

2. The development of a unique closure for the equations governing molecular mixedness and turbulent chemistry in turbulent flows.

Significant progress has been made toward the solution of problem 1 above and will be reported in Section 2 of this report. Although the results are very new, we believe that a new principle has been discovered which permits rational closures for turbulent mixing and turbulent chemically reacting flows. The closures are unique and satisfy all the statistical constraints and all the limiting known conditions that we have been able to check the principle against.

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2. MODELING TURBULENT STRUCTURE

In current second-order closure models for turbulent flows, use is made of the complete equations for the Reynolds tensor $\overline{u_i u_j}$. These models contain two scales associated with the turbulent motion: an energy containing scale Λ , and a dissipative scale λ . Since the large scale Λ is related to the integral scale of the turbulence, which is not the same for all velocities and all directions, our equations which use Λ in the sense of an average scale, do not allow one to take account of turbulent structure. As an example we expect, in general, that the scale associated with streamwise velocity fluctuations (at two points) can differ significantly from the scale associated with crosswise fluctuations. Further, the scale depends on the orientation of the two points.

This information is fully contained in the tensor

$$R_{ij}(\vec{x}, \vec{\zeta}) = \langle u_i'(\vec{x} + \frac{\vec{\zeta}}{2}) u_j'(\vec{x} - \frac{\vec{\zeta}}{2}) \rangle$$

In particular:

(i) as $\zeta \rightarrow 0$, $R_{ij} \rightarrow \overline{u_i u_j}$

(ii) The integral of R_{ij} along a direction of $\vec{\zeta}$ yields the integral scale corresponding to that direction.

We propose to develop and analyze a model equation for the tensor R_{ij} . It follows from (i) that the model equation for R_{ij} must be chosen so as to be compatible with the equation chosen for $\overline{u_i u_j}$.

Our first work on the problem of structure was directed at exhibiting the difference in the integral scales associated with streamwise and crosswise components of velocity fluctuations. We thus introduced the scale tensor Λ_{ij} which measures the scale for the velocity components i and j averaged over the orientation of the two points:

$$\frac{q^2}{3} \Lambda_{ij}(\vec{x}) = \int \frac{R_{ij}(\vec{x}, \vec{\zeta})}{4\pi\zeta^2} d\vec{\zeta}$$

We further define the mean scale by

$$\Lambda = \frac{1}{3} \Lambda_{kk}$$

Note that this is consistent with the meaning of Λ as it is now used in second-order-closure calculations. For isotropic turbulence we can express Λ in terms of G.I. Taylor's integral scales as

$$\Lambda = \frac{L_f + 2L_\varepsilon}{3} = .666L_f$$

and it is surprising to find that a great many real flows are adequately calculated using this simple result.

To obtain the governing equation for the scale tensor, we took the following steps:

1. R_{ij} is expanded in terms of its moments with respect to $\vec{\zeta}$.
2. The expanded equation is integrated over relative separation.

The resulting equation for Λ_{ij} can be decomposed without approximation into two equations: a scalar equation for the mean scale Λ and a (tracelen tensor) equation for the deviator

$$\Delta_{ij} = \Lambda_{ij} - \frac{1}{3} \Lambda_{kk} \delta_{ij}$$

Since the equation for Δ_{ij} contains the source $q^2 \Lambda \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$,

we expect that the deviator will reach an asymptotic (super-equilibrium) value on scales of space and time short compared to those on which the mean scale Λ will evolve. To demonstrate this new result, by a special example,* we choose the simple form

*This new result and the results which follow were presented at a lecture given at the ONR Martime Meteorology Workshop, Boston, September 8-9, 1977, by Dr. Coleman duP. Donaldson.

$$\Delta_{ij} = \sigma \Lambda \left(\frac{\overline{u_i' u_j'}}{q^2} - \frac{1}{3} \delta_{ij} \right)$$

and obtain a pair of coupled equations for the mean scale Λ and for the anisotropy parameter σ . These equations are:

$$\begin{aligned} \frac{D\Lambda}{Dt} = & -2\left(\frac{\sigma}{3} - 1\right) \Lambda \frac{\overline{u_i' u_j'}}{q^2} \frac{\partial U_i}{\partial x_j} + \\ & + \frac{v_{e1}}{q^2} \left\{ \frac{\partial}{\partial x_k} \left[q \Lambda \frac{\partial}{\partial x_k} (q^2 \Lambda) \right] + \right. \\ & \quad \left. - \frac{1}{\Lambda} \frac{\partial}{\partial x_k} \left[q \Lambda \frac{\partial q^2}{\partial x_k} \right] \right\} + \\ & + p'_{c2} 3q \frac{\partial \Lambda}{\partial x_k} \frac{\partial \Lambda}{\partial x_k} + \left(\frac{4}{3} b + 3v'_{c2} \right) q \end{aligned}$$

and

$$\begin{aligned} \frac{D\sigma}{Dt} = & 2\left(\frac{\sigma}{3} - 1\right) \left[\sigma + \frac{q^4}{\overline{u_i' u_j'} \overline{u_i' u_j'} - \frac{1}{3}} \right] \overline{u_e' u_n'} \frac{\partial U_e}{\partial x_k} \\ & + \frac{v_{e1}}{\Lambda} \left\{ \frac{\partial}{\partial x_k} \left[q \Lambda \frac{\partial}{\partial x_k} (\sigma \Lambda) \right] - \sigma \frac{\partial}{\partial x_k} \left[q \Lambda \frac{\partial \Lambda}{\partial x_k} \right] \right\} \\ & - \sigma \left(\frac{2}{3} + \frac{v'_{c2}}{b} \right) \frac{2bq}{\Lambda} \end{aligned}$$

The parameters v_{e1} and b are known from analysis of the Reynolds' tensor, $v'_{e2} \approx -.03$ from observations of Λ in grid turbulence, p'_{c2} still remains to be determined (we have given it the name "eddy stripping coefficient").

We note that the production coefficient in the Λ equation is $2\left(\frac{\sigma}{3} - 1\right)$. Thus, it is not a constant or a flow independent quantity as has been assumed in most closure calculations made to date. For parallel flows a popular choice for this nonconstant has been

$$-2\left(\frac{\sigma}{3} - 1\right) \approx .35$$

which gives $\sigma \approx 2.35$.

Since parallel flow can generally be estimated by super-equilibrium methods, solving the σ equation under the assumption of superequilibrium yields

$$\sigma \approx 2.7$$

which is in surprisingly good agreement with the value currently popular. We note, however, that σ can take on values from 0 to 3, depending on the type of mean under consideration. The superequilibrium values for the streamwise and crosswise scales, using the σ just obtained are

$$\frac{\Lambda_{11}}{\Lambda} \approx 1.44 \quad , \quad \frac{\Lambda_{22}}{\Lambda} = \frac{\Lambda_{33}}{\Lambda} \approx .78$$

in rough agreement with observations.

When it was found that this simple example showed that the "tensor" character of the scale evolved very rapidly compared to the mean scale $\Lambda_{kk}/3$, it suggested that the same result would hold for the two-point correlation tensor R_{ij} , thus permitting a far more detailed analysis of the turbulence structure than afforded by the angular averaged Λ_{ij} .

Following this line of reasoning, we separate from R_{ij} the scalar quantity $\frac{1}{3} R_{\alpha\alpha}$ (which determines Λ) and obtain the deviator D_{ij}

$$D_{ij} = R_{ij} - \frac{1}{3} R_{\alpha\alpha} \delta_{ij}$$

To extend current computations to include "structure" to a first approximation, we intend to determine D_{ij} from superequilibrium considerations while $R_{\alpha\alpha}$ will be calculated dynamically. To implement the latter part of the program, we only need a model for two terms in the equation for $R_{\alpha\alpha}$ that vanish when $\vec{\zeta} \rightarrow 0$ and thus do not appear in the equation for the Reynolds tensor. An attractively simple hypothesis for one of these terms (the cascading term) is being tested against grid turbulence data. This work is continuing and is, we believe, the logical way to proceed to introduce "structure" as well as scale into the calculation of turbulent flows.

3. MODELING OF MIXING

During the reporting period we have further developed our models for scalar variables. We have obtained a consistent second-order closure for a binary mixture with disparate masses by establishing a box model which is completely determined by the moments calculated by the second-order closure equations ($\bar{\alpha}$, $\overline{\alpha^2}$, $\overline{\rho\alpha}$, $\overline{\rho^2}$), with the added criterion that the box model should correspond to the most unmixed distribution consistent with those moments. The criterion is motivated by three observations: (i) the energy containing eddies, which are of large scale are now thought to be responsible for the creation of new contact surfaces and, if this is so, the mixedness will be less the larger the eddy; thus, the choice of minimum mixing consistent with the means and second-order correlation of scalar qualities that are being computed seems appropriate; (ii) in the presence of a fast chemical reaction, it is essential that this particular choice be made if a consistent calculation for these fast reaction rates is to be achieved; (iii) this particular choice must also be made if the limiting solutions for zero molecular transport are to be recovered from the closure equations.

Presently, this model is being programmed to calculate the experimental results being obtained at CalTech by Roshko and his coworkers. In addition, the extension of these ideas to more than two species is being studied theoretically.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) We have obtained an approximate solution for the tensor scale equation reported previously. The solution agrees with both numerical and laboratory experiments for parallel flows. Furthermore, we believe that substantial progress has been made towards rational closure of the equations for turbulence mixing and turbulence chemistry.		

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