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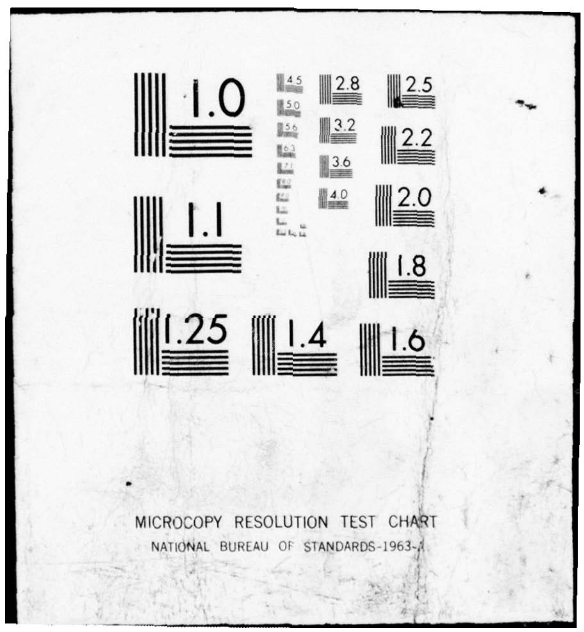
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Studies in Support of the Application of Statistical Theory to Design and Evaluation of Operational Tests. Annex A.

A METHODOLOGY FOR DETERMINING THE POWER OF MANOVA WHEN THE OBSERVATIONS ARE SERIALLY CORRELATED.

Final rept.,

Master's A THESIS,

Presented to

The Faculty of the Division

of Graduate Studies

By

Norviel Robert Eyrich

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In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Operations Research

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Bayesian Theory	Multivariale statistics	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report is a summary report of four studies in support of the application of statistical theory to design and evaluation of operational tests. The four topics are:

a. "A Methodology for Determining the Power of MANOVA when the Observations are Serially Correlated" by Norviel R. Eyrich, CPT, Artillery.

b. "An Application of Multiple Response Surface Optimization to the Analysis of Training Effects in Operational Test and Evaluation" by Vernon M. Bettencourt, Jr., CPT, Artillery.

c. "A Cost Optimal Approach to Selection of Experimental Designs for Operational Testing under Conditions of Constrained Sample Size" by Sam W. Russ, MAJ, Signal Corps.

d. "An Application of Bayesian Statistical Methods in the Determination of Sample Size for Operational Testing in the US Army" by Robert M. Baker, CPT, Infantry.

A METHODOLOGY FOR DETERMINING THE POWER OF MANOVA WHEN THE OBSERVATIONS ARE SERIALLY CORRELATED

A THESIS

Presented to

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A METHODOLOGY FOR DETERMINING THE POWER OF MANOVA
WHEN THE OBSERVATIONS ARE SERIALLY CORRELATED

Approved:

Douglas C. Montgomery, Chairman

Leslie G. Callahan, Jr.

Russell G. Heikes

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I would like to express my appreciation to all those who assisted me in preparing this thesis. I am particularly grateful to the members of my reading committee for their guidance and patience which greatly aided me in accomplishing this research.

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SUMMARY

This research addresses two related problems of multivariate statistical analysis. First, the effects of a multivariate time series on the MANOVA power function are investigated through the use of an experimental design. Second, a generalized procedure is developed for incorporating the multivariate time series into the MANOVA power function so that the effectiveness of ANOVA and MANOVA models in evaluating command and control systems, on the basis of powers of the tests, may be made.

In order to make the analysis possible, a procedure to determine the power of MANOVA test was required. The MANOVA power function is not known in a closed or usable form; consequently, a Monte Carlo procedure was used to determine the power of the MANOVA test. The maximum likelihood form of the MANOVA test statistic was utilized due to its ease of computation.

Previous research has found the following general results to hold for the MANOVA power function:

1. Power is a decreasing function of the dimension of the multiresponse.
2. Power is an increasing function of the size departure from the null hypothesis.
3. Power is an increasing function of sample size.

4. Power is an increasing function of the probability of Type I error.

5. Power is an increasing function of $-\log |\underset{\sim}{P}|$, where $\underset{\sim}{P}$ is the correlation matrix of the multi-response.

An investigation of the effects of a multivariate time series on the MANOVA power function would have little meaning without simultaneously considering the other factors which influence the power function. Two full 2^5 factorial experiments, using the factors in 2-5 above and exponentially decaying autocorrelated response vectors as factors, were run and analyzed using ANOVA. The results verify statements 2-5 and indicate the MANOVA power is an increasing function of the autocorrelation coefficient. In addition, many two factor interactions were found to be significant indicating an extremely complex interrelationship between the various factors.

The power of the MANOVA appears to be a decreasing function of the dimension of the response, as in 1 above. It was found that the dimension of the response could not be separated from the other factors and thus the two experiments were run with the dimension of the response, p, set at 2 and 3. There is a decrease in the power from $p = 2$ to $p = 3$, with other factors held constant, lending support to the hypothesis; however, there is no statistical evidence to support the statement.

To accomplish the second objective a procedure is proposed which uses the MANOVA Monte Carlo procedure, for comparing the effectiveness of the ANOVA with MANOVA for a multivariate time series. An example of the use of this procedure is given. A FORTRAN IV listing of the MANOVA Monte Carlo power program is also included.

CHAPTER I

INTRODUCTION

Background

Department of the Army Major Systems Acquisition Procedure

The acquisition of major defense systems by the Department of Defense is accomplished through the use of a well defined decision procedure with safeguards to prevent the acquisition of unsatisfactory or unnecessary systems. The procedure used by the Department of the Army closely parallels that of the Department of Defense and is an essential element of the Department of Defense acquisition procedure. Measures are taken to insure that only those systems for which a valid need exists are acquired by the Department of Defense. The measures are discussed at some length in various Department of Defense directives [7,22,23].

After the Army staff has determined a valid requirement exists for a proposed system, the system must pass through three phases prior to full production. The first phase is the conceptional development phase during which the system hardware is in an experimental prototype configuration. The second phase is the full scale development phase during which the systems hardware is in an engineering development prototype configuration. The third phase is the full scale development phase during which the systems hardware is in a production prototype configuration [7].

At each phase transition point the Secretary of Defense may terminate the system, permit the system to proceed to the next phase, or retain the system in its present phase for remedial action [23]. To assist the Secretary of Defense in these decisions a permanent advisory body, the Defense Systems Acquisition Review Council (DSARC), is in being. The DSARC provides information and recommendations to the Secretary of Defense whenever program decisions become necessary. A scheduled meeting of the DSARC precedes the Secretary of Defense's decision concerning the disposition of a system at each phase transition point.

Within the Department of the Army there exists a permanent advisory body, the Army Systems Acquisition Review Council (ASARC), which provides the DSARC with the Army's recommendations at each phase in the acquisition process. The ASARC is chaired by the Vice Chief of Staff of the Army and has as its principal members the Commander U. S. Army Material Command, the Commander U. S. Army Training and Doctrine Command, the Chief of Research, Development, and Acquisition, and various assistant secretaries of the Army. Scheduled meetings of the ASARC precede those of the DSARC.

<u>Requirement for Testing</u>

Normally three distinct Developmental Tests (DT) and

Operational Tests (OT) are conducted for each major system. One DT and OT is conducted prior to the three meetings of the ASARC and DSARC. Results of the DT and OT at each phase are reported directly to the ASARC for inclusion in its recommendations to the DSARC. The DT and OT are required to be evaluated independently of each other [7].

DT is conducted to determine if the engineering design and development is complete, to determine if design risks have been minimized, and to determine if the system will meet its specifications. OT is conducted to estimate the system's military worth in comparison with competitor systems, to determine its operational effectiveness and suitability in its environment, and to determine if the system requires modification [7]. This research will be concerned with OT only.

Operational Testing

The U. S. Army Test and Evaluation Agency is designated as the agency responsible for OT on major defense systems [5,6]. OT will emphasize the comparative evaluation of the new system with existing systems and competitor developmental systems. The OT agency is independent of the developing/procuring and using organization. OT is accomplished using typical users/operators, crews, or units in as realistic an operational environment as possible. OT is conducted to provide the necessary data to estimate:

1. The military utility, operational effectiveness,

and operational suitability of the system.

2. The system's desirability, operational benefits, and burdens from the user's viewpoint.

3. The need for modification of the system.

4. The adequacy of doctrine, organization, operating techniques, tactics, and training for the system.

5. The adequacy of maintenance support for the system.

6. The systems performance in a countermeasures environment.

Command and Control Systems

In recent years the U. S. Army has expended a great deal of money and time to develop and deploy sophisticated tactical command and control systems. Tactical command and control systems currently under development include the Tactical Operations System (TOS), a division level command and control system; TSQ-73, an air defense command and control system; and TACFIRE, air artillery fire control and fire support command and control system.

Measures of effectiveness employed in the evaluation of command and control systems vary; however, the measures of effectiveness are rarely independent [58]. For instance, the fraction of available time passed to subordinate echelons and time required to prepare staff actions, two possible measures of effectiveness, are highly correlated [58].

Both analysis of variance (ANOVA) and multivariate analysis of variance (MANOVA) appear to be appropriate statistical methods to be used for analysis of command and control experimental data. Recent research has developed a methodology for determining which statistical method, or combination of methods, is most appropriate for a particular system [16]. This research has not, however, considered that in addition to the various measures being correlated, that in the case of computer assisted systems they may also constitute a multivariate time series. A promising area of research appears to exist in developing a methodology for identifying, analyzing, and incorporating this additional information into the methodology developed by Burnette for determining the appropriateness and effectiveness of ANOVA and MANOVA in the analysis of command and control systems [16].

Objective, Procedure and Scope

The primary objective of this research is to investigate the effects of a multivariate time series on the MANOVA power function and develop a methodology for incorporating time series information into the MANOVA power generator previously developed by Burnette [16]. Using the methodology developed by Burnette for comparing the effectiveness of the ANOVA and MANOVA the methodology will be demonstrated.

The scope of this research will be limited by four assumptions. First, due to the standard scenarios used in OT, only the fixed effects model of the ANOVA and MANOVA will be considered appropriate. Second, equal cell sample sizes only will be considered appropriate. Third, due to the high cost and time factor in training more than one command and staff group to operate each alternative command and control system, operators of the alternative system will not be considered a factor. Fourth, for practical reasons only stationary multivariate time series will be considered. To limit the computer programming involved only two factor completely crossed designs will be considered. In addition, only those elements of ANOVA necessary to demonstrate the methodology will be reviewed. Burnette has an excellent discussion of the ANOVA model if additional information is required.

CHAPTER II

REVIEW OF APPLICABLE STATISTICAL RESULTS AND TECHNIQUES

Introduction

This chapter is a brief review of statistical results and techniques necessary to develop a methodology for use in comparing the applicability of ANOVA with MANOVA. The essential elements of time series analysis necessary to incorporate this information into the MANOVA power function will also be reviewed.

Univariate Analysis of Variance

The appropriate univariate statistical model for comparing several systems is analysis of variance (ANOVA). The model and assumptions for the two-factor case will be reviewed. These results may be easily extended to the general case. Only completely crossed designs and fixed-effects models will be considered.

Model and Required Assumptions

The two-factor fixed-effects ANOVA model is

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + e_{ijk} \tag{2.1}$$

$$i = 1, \ldots, a$$

$$j = 1, \ldots, b$$

$$k = 1, \ldots, n$$

μ is the mean effect common to all observations, α_i is the effect due to level i of factor A, β_j is the effect due to level j of factor B. γ_{ij} is the effect due to the interaction of level i of factor A and level $_j$ of factor B. e_{ijk} is the effect due to random error in the k th observation with factor A at level $_i$ and factor B at level $_j$ [34].

The following assumptions are necessary for estimation, inference and hypothesis testing.

$$\sum_{i=1}^{a} \alpha_i = 0 = \sum_{j=1}^{b} \beta_j \tag{2.2}$$

$$\sum_{i=1}^{a} \alpha_{ij} = 0 \quad j = 1, \ldots, b \tag{2.3}$$

$$\sum_{j=1}^{b} \alpha_{ij} = 0 \quad i = 1, \ldots, a \tag{2.4}$$

e_{ijk} are distributed independently $N(0, \sigma^2)$ (2.5)

Hypothesis Testing

Appropriate hypotheses we may want to test include:

H_{10}: No effect due to factor A or $\alpha_i = 0$, $i = 1, \ldots, a$

against

H_{11}: Not H_{10}

H_{20}: No effect due to factor B or $\beta_j = 0$, $j = 1, \ldots, b$

against

H_{21}: Not H_{20}

H_{30}: No effect due to interaction or $\gamma_{ij} = 0$,

$i = 1, \ldots, a$

$j = 1, \ldots, b$

against

H_{31} = Not H_{30}

The ANOVA test procedure consists of partitioning the total variation in the observations into the contributions due to main effects, the interaction, and the error component. For the two factor model the partitioning is:

$$SS_T = SS_A + SS_B + SS_{AB} + SS_E. \qquad (2.6)$$

Methods for determining the sums of squares are well known and will not be elaborated on here [34].

The test statistic for use in the ANOVA model is based upon the F distribution. The null hypothesis, say H_{10}: no effect due to factor A, would be rejected if:

$$F_0 = \frac{SS_A/(a-1)}{SS_E/ab(n-1)} > F_{\alpha,a-1,ab(n-1)} \tag{2.7}$$

where $F_{\alpha,a-1,ab(n-1)}$ is the upper $(1-\alpha)$ percentage point of the F distribution with (a-1) numerator degrees of freedom and ab(n-1) denominator degrees of freedom [34]. Similar test statistics can be constructed for the other hypotheses.

When the ANOVA model is used the test for interaction effect should be made first. If the interactions are not found to be significant then we may test the hypothesis on the main effects. However, if we reject the hypothesis of no interaction effect then tests on main effects may have little meaning. For a further discussion, see Press [43].

Power of the Analysis of Variance

When constructing hypotheses there are two probability measures we are concerned with. First, the probability of rejecting the hypothesis given it is true; or α. Second, the probability of rejecting the hypothesis given it is false, or the power of the test, $(1-\beta)$. It has been shown that the alternative hypothesis is distributed as a non-central F distribution. Pearson and Hartley [34] have constructed charts which plot the probability of type II error (1-power) for various V1, V2, α, and parameter Φ, where for the case of H_{10}

$$V1 = a-1$$

$$V2 = ab(n-1)$$

$$\Phi^2 = \frac{2}{a}\lambda \qquad (2.8)$$

$$\lambda = \frac{n \sum_{i=1}^{a} \alpha_1^{\,2}}{2\sigma^2}$$

Since σ^2 is seldom known the ratio of $\frac{\sum_{i=1}^{a} \alpha_i^{\,2}}{\sigma^2}$ which you desire to detect is normally used.

Multivariate Analysis of Variance

Model and Required Assumptions

The appropriate multivariate model for comparing several multiresponse systems is the multivariate analysis of variance (MANOVA). The model and assumptions for the two-factor case will be reviewed and may easily be extended to the general case. Only the fixed-effects model will be considered.

The two-factor fixed-effects model is

$$\underset{(PX1)}{\underset{\sim}{y}_{ijk}} = \underset{(PX1)}{\underset{\sim}{\mu}} + \underset{(PX1)}{\underset{\sim}{\alpha}_i} + \underset{(PX1)}{\underset{\sim}{\beta}_j} + \underset{(PX1)}{\underset{\sim}{\gamma}_{ij}} + \underset{(PX1)}{\underset{\sim}{e}_{ijk}} \qquad (2.9)$$

$$i = 1, \ldots, a, \; j = 1, \ldots, b, \; k = 1, \ldots, n$$

The vector $\underset{\sim}{\mu}$ is the effect common to all observations. The vector $\underset{\sim}{\alpha}_i$ is the effect due to level i of factor A, the vector $\underset{\sim}{\beta}_j$ is the effect due to level j of factor B, and the vector $\underset{\sim}{\gamma}_{ij}$ is the interaction effect due to level i of factor A and level j of factor B. The vector e_{ijk} is the effect due to the random error with factor A at level i and factor B at level j on the k th observation [46,52].

Several assumptions are necessary for estimation, inference, and hypothesis testing. The following assumptions are made concerning the effects due to levels of factors and interactions:

$$\sum_{i=1}^{a} \underset{\sim}{\alpha}_i = \phi = \sum_{j=1}^{b} \underset{\sim}{\beta}_j \qquad (2.10)$$

$$\sum_{i=1}^{a} \underset{\sim}{\gamma}_{ij} = \phi \quad j = 1, \ldots, b \qquad (2.11)$$

$$\sum_{j=1}^{b} \underset{\sim}{\gamma}_{ij} = \phi \quad i = 1, \ldots, a \qquad (2.12)$$

e_{ijk} are independently distributed $N(\underset{\sim}{\phi},\underset{\sim}{\Sigma})$, $\underset{\sim}{\Sigma} > \phi$.

Hypothesis Testing

The hypotheses we might want to test include:

H_{10}: No effect due to factor A or $\underset{\sim}{\alpha}_i = \phi$ $i = 1, \ldots, a$ against

H_{11}: Not H_{10}

H_{20}: No effect due to factor B or $\underset{\sim}{\beta}_j = \phi$ $j = 1, \ldots, b$

against

H_{21}: Not H_{20}

and

H_{30}: No effect due to the interaction or $\underset{\sim}{\gamma}_{ij} = \phi$

$i = 1, \ldots, a$

$j = 1, \ldots, b$

against

H_{31}: Not H_{30}

There are three widely used MANOVA hypothesis testing criteria. They are the likelihood ratio criterion, the trace criterion, and the largest characteristic root [44]. The likelihood ratio criterion will be used due to its ease of computation and attendant power considerations [54]. The likelihood ratio criterion requires, for the two-factor case, that the dimension of the response, $p \leq ab(n-1)$ [46].

The MANOVA hypothesis testing procedure consists of partitioning the total variation of the observations in a manner similar to the ANOVA partitioning. Specific computational formulae will not be given; however, relevant matrices will be defined.

$\underset{\sim}{E}$ --matrix of error sums of squares and cross products.

$\underset{\sim}{H1}$--matrix of factor A sums of squares and cross products.

$\underset{\sim}{H2}$--matrix of factor B sums of squares and cross products.

$\underset{\sim}{H}3$--matrix of interaction sums of squares and cross products.

The likelihood ratio test for H_{10}: $\underset{\sim}{\alpha}_i = \phi$ is to reject H_{10} if

$$\frac{|\underset{\sim}{E}|}{|\underset{\sim}{E} + \underset{\sim}{H}1|} < \text{Constant} \qquad (2.13)$$

under H_{10}

$$\mathscr{L}\left\{\frac{|\underset{\sim}{E}|}{|\underset{\sim}{E}| + |H_1|}\right\} = \mathscr{L}\{U_p, q_1, n\}$$

where p is the dimension of the response, $q_1 = a-1$, and $n = ab(n-1)$. Thus, we reject H_{10} if the test statistic is less than $U_{p,\, q_1,\, n}$. The values of U are determined using a second order χ^2 approximation developed by Box [10,46]. The test statistics for H_{20} and H_{30} are found similarly.

The hypothesis of no interaction effect is conducted first. If we fail to reject this hypothesis, we would then test the hypotheses on the main effects. If we reject the hypothesis of no interaction effects, we must use other techniques to determine if the main effects are significant [46].

Power of the Multivariate Analysis of Variance

The power function of the MANOVA test criteria is not available in closed form. Recently, the noncentral distributions of the largest characteristic root and likelihood ratio statistics have been studied; however, to date research has not yielded a useable power function for the MANOVA tests. Roy, Mikhail [53], and others have shown that the MANOVA power is a monotonically increasing function of the noncentrality parameters of the criteria distribution [53]. Gnanadisikan [54], using Monte Carlo methods, showed that the MANOVA test power is monotonically decreasing with increasing dimension of the response, p, and is monotonically increasing with increasing probability of type I error. The lack of a usable power function has resulted in most research being accomplished via Monte Carlo simulations.

Correlation Analysis

Simple Correlation

If multivariate statistical analysis is to be appropriate it is necessary to have at least two measures which are significantly correlated. The most elementary expression of correlative structure involves the simple correlation coefficient, ρ. Let $y_1, y_2, \ldots, y_n$ be n independent observations of a p-dimensional random vector Y. The covariance between the i th and j th component of Y, Y^i and Y^j is

$$\sigma_{ij} = COV(Y^i,Y^j) = E[(Y^i-EY^i)(Y^j-EY^j)] \qquad (2.14)$$

where σ_{ii} is the variance of Y^i. The pxp matrix of population covariances is defined as

$$\underset{\sim}{\Sigma} = (\sigma_{ij}) \qquad (2.15)$$

The correlation coefficient between Y^i and Y^j is defined as

$$\rho_{ij} = \frac{\sigma_{ij}}{(\sigma_{ii}\ \sigma_{jj})^{1/2}} \qquad -1 \leq \rho_{ij} \leq 1 \qquad (2.16)$$

The pxp matrix of population correlation coefficients is defined as

$$\underset{\sim}{P} = (\rho_{ij}) \qquad (2.17)$$

The sample covariance matrix, $\underset{\sim}{S}$, and the sample correlation matrix, $\underset{\sim}{R}$, are found by replacing the population covariances and correlations with their maximum likelihood estimators. Thus, the sample correlation coefficient between Y^i and Y^j is

$$r_{ij} = \frac{S_{ij}}{(S_{ii}\ S_{jj})^{1/2}} \qquad -1 \leq r_{ij} \leq 1 \qquad (2.18)$$

where S_{ij} is the maximum likelihood estimator of σ_{ij}.

Fisher has shown that under the assumption of joint normality the transformation

$$Z = \tanh^{-1} r_{ij} \tag{2.19}$$

produces an asymptotic normal variate with mean

$$q \approx 1/2 \log \left(\frac{1+\rho_{ij}}{1-\rho_{ij}}\right) \tag{2.20}$$

and variance

$$\mathrm{Var}\,(Z) \approx \frac{1}{N-3} \tag{2.21}$$

when N, the number of observations, becomes large.

Using the Z-transform it is possible to test

$$H_0 : \rho_{ij} = \rho_o$$

against

$$H_1 : \rho_{ij} \neq \rho_o .$$

The hypothesis is rejected if

$$|Z-q_o| \sqrt{N-3} > Z_{\alpha/2} \tag{2.22}$$

where q_o is the z-transform of $r = \rho_o$ and $Z_{\alpha/2}$ is the upper 100 $(1-\alpha/2)$ percentage point of the standard normal distribution [44].

Multiple Correlation

For a p-dimensional response vector the multiple correlation coefficient, P_i, of one response component, P_i, with a linear combination of the other P-1 response components is defined as

$$P_i = \max \text{corr}\ (y^i, \underset{\sim}{\alpha}'\underset{\sim}{X}) \tag{2.23}$$

where $\underset{\sim}{\alpha}$ is a P-1 dimensional contrast vector and $\underset{\sim}{X}$ is the vector of the other P-1 response variables. P_i is the largest possible correlation between Y^i and any linear combination of the remaining P-1 response variables. The sample multiple correlation coefficient may be determined from either the sample correlation matrix or the sample covariance matrix. To find the multiple correlation of Y^i, R^i, rearrange the appropriate matrix by replacing the 1^{st} response with the i^{th} response and partition the matrix. When using the sample covariance matrix the partitioning is as follows

$$\begin{matrix} S_{11} & \underset{\sim}{S}_{12}' \\ \underset{\sim}{S}_{12} & \underset{\sim}{S}_{22} \end{matrix} \tag{2.24}$$

where S_{11} is now S_{ii}, $\underset{\sim}{S}_{22}$ is the P-1 covariance matrix of the remaining response components, and S_{12} is the P-1 vector of sample correlations between response i and the other P-1 response components. With the matrix so partitioned the multiple correlation coefficient, R_i, is defined as

$$R_i^2 = R_1^2 = \frac{\underset{\sim}{S}'_{12}\ \underset{\sim}{S}_{22}^{-1}\ \underset{\sim}{S}_{12}}{S_{11}} \tag{2.25}$$

The appropriate hypothesis to determine if Y^i is independent of the remaining response components is to test

$$H_0 : P_i = 0$$

against

$$H_1 : P_i > 0$$

The hypothesis would be rejected if

$$Q = \frac{R_i^2\ (n-p)}{1-R_i^2\ (p-1)} > F_{\alpha,p-1,n-p} \tag{2.26}$$

where n is the number of observations, p is the dimension of the response, and F is the upper 100 (1-α) percentage point of the F distribution [46].

Independence of K Variates

To determine if a set of k multivariate normal response variates are independent can be accomplished by testing

$$H_0 : \underset{\sim}{P} = \underset{\sim}{I}$$

against

$$H_1 : \underset{\sim}{P} \neq I$$

where $\underset{\sim}{P}$ is the k x k population correlation matrix and $\underset{\sim}{I}$ is the k x k identity matrix. The null hypothesis is rejected if

$$\chi_o^2 = -\left(N-1 - \frac{2k+5}{6}\right) \log |\underset{\sim}{R}| > \chi^2_{\alpha,1/2\ k\ (k-1)} \qquad (2.27)$$

where N is the number of independent observations, R is the k x k sample correlation matrix, and χ^2 is the upper-tail χ^2 distribution [44]. This test is appropriate prior to any multivariate analysis.

<u>Independence of k sets of Variates</u>

In addition to determining if a set of k responses are independent, it will also be of interest to determine if k sets of multivariate normal variates are mutually independent. If the j^{th} of the k^{th} sets contains P_j variates, then the gross covariance matrix may be partitioned into submatrices $\underset{\sim}{\Sigma}_{ij}$ of dimension $P_i \times P_j$. The appropriate hypothesis to test is

$$H_0 : \underset{\sim}{\Sigma}_{ij} = \phi \quad \text{for} \quad i \neq j$$

against

$$H_1 : \underset{\sim}{\Sigma}_{ij} \neq \phi \qquad i \neq j .$$

For N independent observations from a multivariate normal population, compute R, the sample correlation matrix, and partition as above. To test H_0 use the test statistic

$$V = \frac{|\underset{\sim}{R}|}{|\underset{\sim}{R}_{11}|\,|\underset{\sim}{R}_{22}| \cdots |\underset{\sim}{R}_{kk}|} . \tag{2.28}$$

It has been shown the statistic

$$\chi_o^2 = -\frac{(N-1)}{C} \log V \sim \chi^2_{\alpha,f} \tag{2.29}$$

where

$$C^{-1} = 1 - \frac{(2S_1 + 3S_2)}{12f(N-1)} \tag{2.30}$$

$$f = S_2/2$$

$$S_j = \left(\sum_{i=1}^{k} P_i\right)^j - \sum_{k=1}^{k} P_i^j \qquad j = 1,2 . \tag{2.31}$$

H_0 would be rejected if $\chi_o^2 > \chi_{\alpha,f}^2$ [10,44].

Stationary Multivariate Time Series

Multivariate Time Series

When more than one measure of a time-varying process is required to properly describe its behavior, then the process is called a multivariate (vector, multidimensional) time series. Thus, the position or state of the process at each instant of time can be represented by a vector of time dependent measurements

$$X_t = \begin{pmatrix} X_1(t) \\ X_2(t) \\ \vdots \\ X_p(t) \end{pmatrix}$$

only those multivariate time series for which the components are univariate time series will be considered. This restriction appears to have little effect on the current investigation since it is reasonable to expect each component, or subset of the components, to exhibit this characteristic.

Univariate Stationary Time Series

A stochastic process is said to be strictly stationary if

$$P(X(t_1+\tau)\varepsilon S_1, \ldots, X(t_n+\tau)\varepsilon S_n) =$$

$$P(X(t_1)\varepsilon S_1, \ldots, X(t_n)\varepsilon S_n) \tag{2.32}$$

for all $t_1 <\ldots<t_n$, real events $S_1, \ldots, S_n$, and $\tau, -\infty<\tau<\infty$. Note that the distributions depend on the relative time separations of the random variables and not their absolute time locations.

When the mean and variance of the random variables exist, it is easily established that stationarity implies

$$E_x(t) = E_x(0) = m \qquad -\infty<t<\infty \tag{2.33}$$

and

$$EX(t+\tau)x(t) = EX(\tau)x(0) = C(t) \qquad -\infty<t<\infty \tag{2.34}$$

Thus, the mean values are constant in time and the covariances depend on the time displacement τ, but not on t. The function $C(\tau)$ is called the autocovariance function.

If condition (2.32) is discarded and we assume only that the random variables of the process have the property

$$\text{Var } x(t) = C(0) < \infty \tag{2.35}$$

and satisfy properties (2.33) and (2.34), then the process is said to be weakly stationary. Since the joint multivariate normal distributions of a Gaussian process, sometimes called a normal process, depend only on the mean vector and covariance matrix of the random variables and these functions have properties (2.33) and (2.34), the joint distributions will have property (2.32). Thus, stationarity and weak stationarity are equivalent for normal processes [41].

Identification of Time Series

There are a number of statistical tests available to determine if a set of time indexed observations constitute a significant autoregressive process or are pure white noise [30]. Perhaps the simplest and most commonly used procedure is periodogram analysis. Let

$$X_t = \sum_{v=1}^{p} (a_v \cos\lambda t + b_t \sin \lambda_{vt}) + e_t \qquad (2.36)$$

where a_v, b_v, and λ_v are real constants with $0 < \lambda_v < \pi$ and e_t is pure white noise. We desire to detect the periods $2\pi/\lambda v$ that have been masked by the random disturbances e_t. For this purpose the following statistic has been proposed.

$$I_n(\lambda) = \frac{1}{2\pi n} \left| \sum_{t=1}^{n} X_t \, e^{-i\lambda t} \right|^2 = \frac{1}{4\pi} A^2(\lambda) + \frac{1}{4\pi} B^2(\lambda) \qquad (2.37)$$

where

$$A(\lambda) = \sqrt{\frac{2}{n}} \sum_{t=1}^{n} \chi_t \cos t\lambda \tag{2.38}$$

$$B(\lambda) = \sqrt{\frac{2}{n}} \sum_{t=1}^{n} \chi_t \sin t\lambda \tag{2.39}$$

$I_n(\lambda)$ is called the periodogram and is suggested by Fourier analysis treating the time series as if it were just the undisturbed trigometric sum.

R. A. Fisher developed a test procedure to determine the significance of the periods of the periodogram. Fisher's null hypothesis is that the process has no period, that is $\chi_t = e_t$, and the e's are distributed normally with unknown mean m and variance σ^2.

Let the number of observed values be odd, say $n = 2m+1$, and consider the m values of the periodogram at points $L_r = 2\pi r/(2m+1)$, $r=1, \ldots, m$. Due to the orthogonality of the trigometric coefficients (2.38) and (2.39), the stochastic variables

$$A(L_r),\ r = 1, \ldots, m$$

$$B(L_r),\ r = 1, \ldots, m$$

are 2m independent normal variables with mean zero and variance σ^2. Hence

$$\frac{S_r}{\sigma^2} = \frac{A^2(L_r)+B^2(L_r)}{\sigma^2} \qquad r = 1, \ldots, m \tag{2.40}$$

are independent X^2-variables. Define

$$g = \frac{\max S_i,\ i = 1,2,\ldots,m}{(S_1+S_2+\ldots+S_m)} \tag{2.41}$$

where the values of S_i are computed using (2.40). The distribution of g under the null hypotheses is

$$P(g>\chi) = m(1-\chi)^{m-1} - \frac{m(m-1)}{2}(1-2\chi)^{m-1} + \\ + \frac{m(m-1)(m-2)}{3.2}(1-3\chi)^{m-1} - \ldots \tag{2.42}$$

where the summation should be extended as long as the terms in the brackets are positive. The null hypothesis, no period present is rejected if $g > g_p$, where g_p is some appropriate percentile of the distribution given by (2.42) [30].

Parameterization and Estimation of Multivariate Time Series

Once it has been determined that a time series is not only noise, it is important to determine the parameters which fully describe the system. For a discrete time series the system is adequately described by the matrix

$$C(\tau) = \begin{pmatrix} C_{1,1}(\tau) & C_{1,2}(\tau) & \ldots & C_{1,p}(\tau) \\ C_{2,1}(\tau) & C_{2,2}(\tau) & \ldots & C_{2,p}(\tau) \\ \vdots & \vdots & & \vdots \\ C_{p,1}(\tau) & C_{p,2}(\tau) & \ldots & C_{p,p}(\tau) \end{pmatrix} \tag{2.43}$$

where $C_{jj}(\tau)$ is the auto-covariance of the j^{th} component and $C_{jk}(\tau)$ is the cross-covariance of the j^{th} and k^{th} components. $C(\tau) = [C_{j,k}(\tau)]$ is a positive definite matrix [51].

For the purpose of estimation usually at least 50 observations are necessary. In addition, for useful results only the first $K \leq N/4$ autocovariance and crosscovariance coefficients are useful [38]. The theoretical autocorrelation for a p-dimensional multivariate time series is

$$C_{ii}(\tau) = E[(X_i(t)-\mu)(X_i(t+\tau)-\mu)] \qquad \begin{matrix} \tau = 0,1,2,\ldots \\ i = 1, \ldots, p \end{matrix}$$

The theoretical autocorrelation function is never known with certainty and must be estimated. A satisfactory estimate of $C_{ii}(\tau)$ is the sample autocorrelation function.

$$\hat{C}_{ii}(\tau) = \frac{1}{N-\tau} \sum_{t=1}^{N-\tau} (\chi_i(t))(\chi_i(t+\tau))^\tau \qquad \begin{matrix} = 0, 1, \ldots \\ i = 1, \ldots, p \end{matrix} \tag{2.44}$$

where N is the number of observations, i is the component of time time series, and τ is the lag. The crosscorrelation function may be estimated as follows

$$\hat{C}_{ij}(\tau) = \frac{1}{N-\tau} \sum_{t=1}^{N-\tau} (\chi_i(t)(X_j(t+\tau)) \qquad \begin{matrix} \tau = 0, 1, \ldots \\ i \neq j \\ i,j \leq p \end{matrix} \tag{2.45}$$

where N is the number of observations, i and j are the components, and τ is the lag.

Within the literature there are a number of statistical tests available for the analysis of multivariate time series. For example, test statistics similar to (2.19), simple correlation, and (2.26), multiple correlation, may be constructed. However, these tests are based on spectral distributions that are specified in the frequency domain and are not particularly relevant to the current development. If the reader is interested, an excellent discussion is contained in [41].

Generation of Multivariate Time Series

Generation of Univariate Normal Random Variates

To investigate the MANOVA power function in the presence of a multivariate time series it will be necessary to generate a multivariate time series. In order to generate these time series we require a procedure to generate independent univariate normal deviates. A number of procedures are available, however, the method proposed by Box and Muller appears to be the most efficient [45]. Let U_j and U_{j+1} be independent deviates from a uniform (0,1) distribution; these deviates can be obtained from any valid uniform deviate generator. To generate the $N(\mu,\sigma^2)$ variates the uniform deviates are transformed as follows:

$$X_j = \mu + (-2\sigma^2 \log U_j)^{1/2} \cos(2\pi U_{j+1}) \qquad (2.46)$$

$$X_{j+1} = \mu + (-2\sigma^2 \log U_j)^{1/2} \sin(2\pi U_{j+1}) \qquad (2.47)$$

X_j and X_{j+1} will be independent variates from $N(\mu,\sigma^2)$ [13].

Generation of Multivariate Time Series

There are two specific cases under which we may desire to generate multivariate time series. First, it may be desired to generate a multivariate time series based on a subjective estimate of the autoregression of X_t on X_{t-1}. This may occur when insufficient observations are available to accurately determine the autocorrelation and cross-correlation structure of the response but it is felt the structure does exist. Second, sufficient observations are available and all parameters have been determined. Each procedure will be developed below.

When only a subjective estimate of the autoregressive structure of the time series is available a rather simple procedure may be developed for generating the time series. In order to generate p-dimensional random vectors from the multivariate normal population $N(\mu,\Sigma)$ we use a fundamental theorem of multivariate analysis. If $(Z_1, Z_2, \ldots, Z_p)$ are p independent observations from $N(0,1)$, then the p-dimensional vector, $\underset{\sim}{X}$ from $N(\underset{\sim}{\mu},\underset{\sim}{\Sigma})$ may be represented as

$$\underset{\sim}{X} = \underset{\sim}{C}\,\underset{\sim}{Z} + \underset{\sim}{\mu} \tag{2.48}$$

where $\underset{\sim}{C}$ is a unique lower triangular matrix satisfying (2.49).

$$\underset{\sim}{\Sigma} = \underset{\sim}{C}\,\underset{\sim}{C}' \tag{2.49}$$

The matrix $\underset{\sim}{C}$ may be computed by the routine reported by Scheuer and Stoller [50].

We may generate autocorrelated vectors, each with the same autoregressive structure and exponential decay, by a simple change to the above procedure. For the univariate case it is known that exponential smoothing is based on the recursive relationship

$$Z_t' = \lambda Z_{t-1}' + (1-\lambda)Z_t, \quad 0<\lambda<1 \tag{2.50}$$

where Z_t are mutually independent variables with mean zero and variance σ^2. We may apply (2.50) to each component of $\underset{\sim}{Z}_t'$ to obtain an autocorrelated vector time series. Thus, the procedure is as follows:

1. Compute the $\underset{\sim}{C}$ matrix.
2. Generate p independent variates from N(0,1) and designate Z_o.
3. Apply (2.48) to the above to get the 1^{st} vector.

4. Generate P independent variates from N(0,1) and designate Z_t, t = 1, 2, ...
5. Apply (2.50) to each component of Z_t and Z_{t-1} to get the t^{th} observation, Z_t'.
6. Repeat steps 2-5 until the desired number of observations have been generated.

When sufficient information is available to estimate all necessary information to fully describe the time series a different approach may be used. We have shown that a multivariate time series is adequately described by $C(\tau)$ (2.43). It is possible to construct a correlation matrix to fully describe the first k observations of a discrete multivariate time series as follows:

$$
\begin{array}{cc|ccccccccccc|}
 & p & 1 & 2 & \cdots & \rho & \cdots & 1 & 2 & \cdots & p \\
p & t & 0 & 0 & \cdots & 0 & \cdots & k & k & \cdots & k \\
\hline
1 & 0 & 1 & \rho_{12}(0) & \cdots & \rho_{1p}(0) & \cdots & \rho_{11}(k) & \rho_{12}(k) & \cdots & \rho_{1p}(k) \\
2 & 0 & & 1 & \cdots & \rho_{2p}(0) & \cdots & \rho_{21}(k) & \rho_{22}(k) & \cdots & \rho_{2p}(k) \\
\vdots & \vdots & & & \ddots & 1 & \cdots & \rho_{p1}(k) & \rho_{p2}(k) & \cdots & \rho_{pp}(k) \\
p & 0 & & & & & \ddots & & & & \\
\vdots & \vdots & & & & & & \ddots & & & \\
1 & k & & & & & & 1 & \rho_{12}(0) & & \rho_{1p}(0) \\
2 & k & & & & & & & 1 & & \rho_{2p}(0) \\
\vdots & \vdots & & & & & & & & \ddots & \vdots \\
p & k & & & & & & & & & 1
\end{array}
\tag{2.51}
$$

where t is the time index, p is the component, and $\rho_{ij}(\tau)$ is the correlation of the i^{th} and j^{th} component at lag τ. The k observations from the multivariate time series may then be generated as follows:

1. Compute the matrix $\underset{\sim}{C}$ such that $\underset{\sim}{C}\,\underset{\sim}{C}' = \underset{\sim}{\Sigma}$ where $\underset{\sim}{\Sigma}$ is given by (2.51).
2. Generate kp independent variates from N(0,1).
3. Apply (2.48) and separate the components to form the multivariate time series.

These two procedures will be of great utility in studying the effects of a multivariate time series on the MANOVA power function.

CHAPTER III

MANOVA POWER GENERATION

Introduction

To perform a meaningful analysis, we require a procedure which will enable us to obtain the power of the MANOVA test in a form useful to us in operational testing. Previous research has addressed this problem and only those points necessary for an adequate development will be reviewed. If further information is desired an excellent development is presented in [16].

MANOVA Power Criteria

Under the usual MANOVA assumptions we would be interested in determining the power of the test,

$$P\,\{\text{Reject Ho} \mid \text{Ho is false}\}\ , \qquad (3.1)$$

in terms of the hypothesis we are testing. As in the case of the ANOVA (2.8), the MANOVA power function appears to be directly related to the departures you desire to detect. Three useful forms of the departures have been proposed [16].

The departures may be specified in either euclidean norm, supremum norm, or individual component departures. Thus, the euclidean norm is

$$D_2 = \left|\left| \sum_{i=1}^{a} \frac{(\alpha_i^1)^2}{\sigma_{11}}, \sum_{i=1}^{a} \frac{(\alpha_i^2)^2}{\sigma_{22}}, \ldots, \sum_{i=1}^{a} \frac{(\alpha_i^p)^2}{\sigma_{pp}} \right|\right| \quad (3.2)$$

where α_i^j is the departure of the i^{th} level of factor A on component j. The Supremum norm is

$$D_s = \max_j \sum_{i=1}^{a} \frac{(\alpha_i^j)^2}{\sigma_{jj}} \qquad j = 1, 2, \ldots, p \quad (3.3)$$

If individual component departures are to be specified we would desire to detect

$$D_j = \sum_{i=1}^{a} \frac{(\alpha_i^j)^2}{\sigma_{jj}} \quad (3.4)$$

where the other p-1 component departures are set at levels from the distribution uniform $(0, D_j/R)$ where R = 1, 2, ..., to be selected.

Monte Carlo Power Generation

A Monte Carlo approach to determining the power of the MANOVA appears appropriate and necessary since the MANOVA power function is not available in a usable form. Our general approach will be to generate random observations which satisfy the MANOVA model, the multivariate time series, and the size and type component departures we desire to detect. Once we generate the observations, we compute the MANOVA test to determine whether to reject the null hypothesis and record the results. We repeat this procedure a large number of times and the power of the test is the ratio of the number of times we rejected the null hypothesis to the total number of tests conducted.

In addition to the usual MANOVA calculations, with sample size n, and component departures we desire to detect, we must be able to accomplish the following:

1. Randomly assign the p component departures in such a manner that they satisfy the MANOVA power criteria we desire to use.
2. For each j = 1, ..., p randomly assign the a components, a^j, for each D_j such that
$$\sum_{i=1}^{a} \frac{(\alpha_i^j)^2}{\sigma_{jj}} = D_j \text{ and } \sum_{i=1}^{a} \alpha_i^j = 0.$$
3. Obtain an estimate of the response correlation structure in the form of a pxp correlation matrix.
4. Generate a p-dimensional multivariate time series of error vectors.

Procedures to accomplish items 1 through 3 are covered in detail by Burnette [16]. Item 4 has been previously discussed in Chapter II.

MANOVA Power Generation Procedure

In order to simplify our computations, we will use a standardization transformation on all responses. This transformation is:

$$y^{j\prime} = \frac{y^j - \mu^j}{(\sigma_{jj})^{1/2}} \qquad (3.5)$$

For original $\underset{\sim}{y}$ distributed $N(\underset{\sim}{\mu},\underset{\sim}{\Sigma})$, the transformed $\underset{\sim}{y}'$ will be distributed $N(\underset{\sim}{\phi},\underset{\sim}{P})$, where $\underset{\sim}{P}$ is the population correlation matrix. It should be noted at this point that the MANOVA test procedure requires the population correlation matrix and must be estimated from the transformed observations. The observations will be generated such that they compose a multivariate time series. This transformation will greatly simplify the MANOVA power calculations and permit us to express the component departures in standardized units of component variances of 1.

The procedure we will use to determine the power of the MANOVA test for a given probability of type I error, α, sample size, n, is as follows:

1. Select the MANOVA model, for example, a completely crossed, two factor, p-dimensional MANOVA model.
2. Estimate the multivariate time series parameters.
3. Select the hypothesis to be tested, for example, no effect due to factor A.
4. Select the size and type component departures we desire to detect.
5. Select the number of Monte Carlo iterations, NR, we desire to run.
6. For each Monte Carlo iteration, randomly assign

the component departures and component departure levels, as appropriate.

7. For each model index combination, for example, the two-factor MANOVA model above, generate an error vector $\underset{\sim}{e}_{ijk}$ from the multivariate time series and apply the model with all effects levels zero except the effect being tested.
8. Compute the MANOVA test statistic, compare it with the critical value of the test, and record the results.
9. Repeat steps 5-8 NR-1 times.
10. Compute the power of the MANOVA test:

$$\text{power} = \frac{\text{number of hypothesis rejected}}{\text{NR}}$$

Previous experience has shown that NR 500 is adequate and will be used unless otherwise specified.

A complete FORTRAN IV program with necessary subroutines for use on the CDC CYBER 74 appears in Appendix A. The program is a conversion of the program developed and validated by Burnette for use on the UNIVAC 1108 [16]. The program has been modified to generate autocorrelated error vectors based on a subjective estimate of the autocorrelation structure. The program may be easily modified to generate error vectors when there is sufficient information to totally describe the multivariate time series.

CHAPTER IV

INVESTIGATION OF THE EFFECTS OF A MULTIVARIATE TIME SERIES ON THE MANOVA POWER FUNCTION

Introduction

We turn our attention now to a primary objective of this research; that is, investigating the effects of a multivariate time series on the MANOVA power function. In Chapters II and III we developed the procedures necessary to determine the power of the MANOVA test criteria for a given set of parameters. We have previously noted that the MANOVA power function is also dependent upon a number of other factors and any investigation would not be complete without simultaneously considering all parameters which affect the MANOVA power function.

Those factors which have been found are listed below for easy reference. They are:

1. Power is a decreasing function of the dimension of the multiresponse.

2. Power is an increasing function of the size departure from the null hypothesis.

3. Power is an increasing function of sample size.

4. Power is an increasing function of the probability of Type I error.

5. Power is an increasing function of $-\log |\underset{\sim}{P}|$, where $\underset{\sim}{P}$ is the correlation matrix of the multiresponse. We desire to construct an experiment which will enable us to simultaneously consider all factors which affect the MANOVA power function.

Analysis of Effects

It was decided that an appropriate method to simultaneously investigate the effects would be to use a factorial design and analyze the results by ANOVA. Prior to selecting the design, either a 2^k or a 3^k, it was necessary to determine if the main effects were linear or of some higher order. Thus, six individual experiments were conducted to determine the nature of the main effects. In each experiment the effect under investigation was varied over the range of interest while the other effects were held constant. In each case there appears to be a linear trend in the main effect, with the exception of the response dimension, and thus, it was felt that a 2^k experimental design would be appropriate.

The effect of the dimension of the response was investigated by the procedure described above. We found that the dimension of the response could not be separated from the other factors and thus could not be included as a factor. It was then decided to run two full 2^5 factorial experiments with the dimension of the response, p, set at 2 in the first and 3 in the second. By this procedure we hoped to be able to determine visually if the power of the MANOVA did decrease with the dimension of the response.

The experimental design is shown in Tables 1 and 2 with the high and low levels of each factor in each experiment. The eculidean norm was specified in the MANOVA power generator and was adjusted so that the norm for P = 2 and P = 3 were of the same relative magnitude.

The experiments were run on the CDC Cyber 74 and the complete ANOVA for each are in Tables 3 and 4. The experiment was not replicated since the number of replications of the MANOVA power generator, NR = 500, results in little or no variation in the responses. The effects in each experiment were plotted on normal probability paper, Figures 1 and 2, in accordance with the procedure outlined by Montgomery in [43]. If the fourth and fifth order interactions fall along that portion of the plot where the effects may be represented by a straight line then ANOVA is appropriate. In both experiments this requirement is met and the error sums of squares is estimated using the fourth and fifth order interactions. The results of the ANOVA are given in Tables 3 and 4.

The analysis of both experimental designs verify that all main effects are highly significant. We also note that the results also indicate a number of second-order interactions are significant while no third-order interactions are significant. However, if we examine the percentage of total

Table 1. Experimental Design #1

Factor Level	Response Dimension p	Sample Size n	Departure to detect D_2	Probability of Type I Error α	Auto-Correlation Coefficient λ	Value of $\lvert \underset{\sim}{P} \rvert$ $\lvert \underset{\sim}{P} \rvert$
Low	2	4	.5	.05	.2	.4
High	2	6	1.0	.10	.5	.8

Table 2. Experimental Design #2

Factor Level	Response Dimension p	Sample Size n	Departure to Detect D_2	Probability of Type I Error α	Auto-Correlation Coefficient λ	Value of $\lvert \underset{\sim}{P} \rvert$ $\lvert \underset{\sim}{P} \rvert$
Low	3	4	.61	.05	.2	.4
High	3	6	1.225	.10	.5	.8

Table 3. Complete ANOVA for Experiment 1

Source	df	SS	MS	F_o
Probability of Type I error, α	1	202.5	202.5	14.25*
Autocorrelation Coefficient, λ	1	1,430.4	1,430.4	100.63*
$\alpha \times \lambda$	1	4.1	4.1	0.29
Norm to Detect, D_2	1	1,926.5	1,926.5	135.54*
$\alpha \times D_2$	1	45.8	45.8	3.22
$\lambda \times D_2$	1	234.3	234.3	16.48*
$\alpha \times \lambda \times D_2$	1	3.1	3.1	0.22
Sample Size, n	1	746.5	746.5	52.52*
$\alpha \times n$	1	6.1	6.1	0.43
$\lambda \times n$	1	153.7	153.7	10.81**
$\alpha \times \lambda \times n$	1	5.8	5.8	0.41
$D_2 \times n$	1	118.3	118.3	8.33**
$\alpha \times D_2 \times n$	1	.2	.2	0.01
$\lambda \times D_2 \times n$	1	31.0	31.0	2.18
The value of the Determinant, $\lvert \tilde{P} \rvert$	1	2,268.0	2,268.0	159.56*
$\alpha \times \lvert \tilde{P} \rvert$	1	52.9	52.9	3.72
$\lambda \times \lvert \tilde{P} \rvert$	1	287.2	287.2	20.21*
$\alpha \times \lambda \times \lvert \tilde{P} \rvert$	1	20.7	20.7	1.46

Table 3 (concluded)

Source	df	SS	MS	F_o
$D_2 \times \lvert \underset{\sim}{P} \rvert$	1	246.0	246.0	17.31*
$\alpha \times D_2 \times \lvert \underset{\sim}{P} \rvert$	1	5.5	5.5	0.39
$\lambda \times D_2 \times \lvert \underset{\sim}{P} \rvert$	1	18.0	18.0	1.26
$n \times \lvert \underset{\sim}{P} \rvert$	1	92.4	92.4	6.50**
$\alpha \times N \times \lvert \underset{\sim}{P} \rvert$	1	3.8	3.8	0.27
$\lambda \times n \times \lvert \underset{\sim}{P} \rvert$	1	52.0	52.0	3.66
$D_2 \times n \times \lvert \underset{\sim}{P} \rvert$	1	3.6	3.6	0.25
Error	6	85.3	14.2	

*Indicates significance at the 1-percent level.

**Indicates significance at the 5-percent level.

Table 4. Complete ANOVA for Experiment 2

Source	df	SS	MS	F_o
α	1	948.7	948.7	209.20*
λ	1	4,554.0	4,554.0	1,004.25*
α x λ	1	147.5	147.5	32.52*
D_2	1	2,842.6	2,842.6	626.85*
α x D_2	1	114.2	114.2	25.19*
λ x D_2	1	544.6	544.6	120.11*
α x λ x D_2	1	23.1	23.1	5.10
n	1	1,331.7	1,331.7	293.67*
α x n	1	72.9	72.9	16.08*
λ x n	1	427.7	427.7	94.30*
α x λ x n	1	10.0	10.0	2.21
D_2 x n	1	91.2	91.2	20.11*
α x D_2 x n	1	16.9	16.9	3.73
λ x D_2 x n	1	14.9	14.9	3.28
$\lvert\tilde{P}\rvert$	1	1,811.7	1,811.7	399.52*
α x $\lvert\tilde{P}\rvert$	1	13.9	13.9	3.07
λ x $\lvert\tilde{P}\rvert$	1	348.1	348.1	78.76*
α x λ x $\lvert\tilde{P}\rvert$	1	2.7	2.7	0.60
D_2 x $\lvert\tilde{P}\rvert$	1	128.2	128.2	28.26*

Table 4 (concluded)

Source	df	SS	MS	F_o
α x D_2 x $\|\underset{\sim}{P}\|$	1	6.7	6.7	1.48
λ x D_2 x $\|\underset{\sim}{P}\|$	1	2.5	2.5	0.55
n x $\|\underset{\sim}{P}\|$	1	40.8	40.8	9.00**
α x n x $\|\underset{\sim}{P}\|$	1	0.7	0.7	0.15
λ x n x $\|\underset{\sim}{P}\|$	1	18.0	18.0	3.96
D_2 x n x $\|\underset{\sim}{P}\|$	1	4.4	4.4	0.96
Error	6	27.2	4.5	

*Indicates significance at the 1-percent level.

**Indicates significance at the 5-percent level.

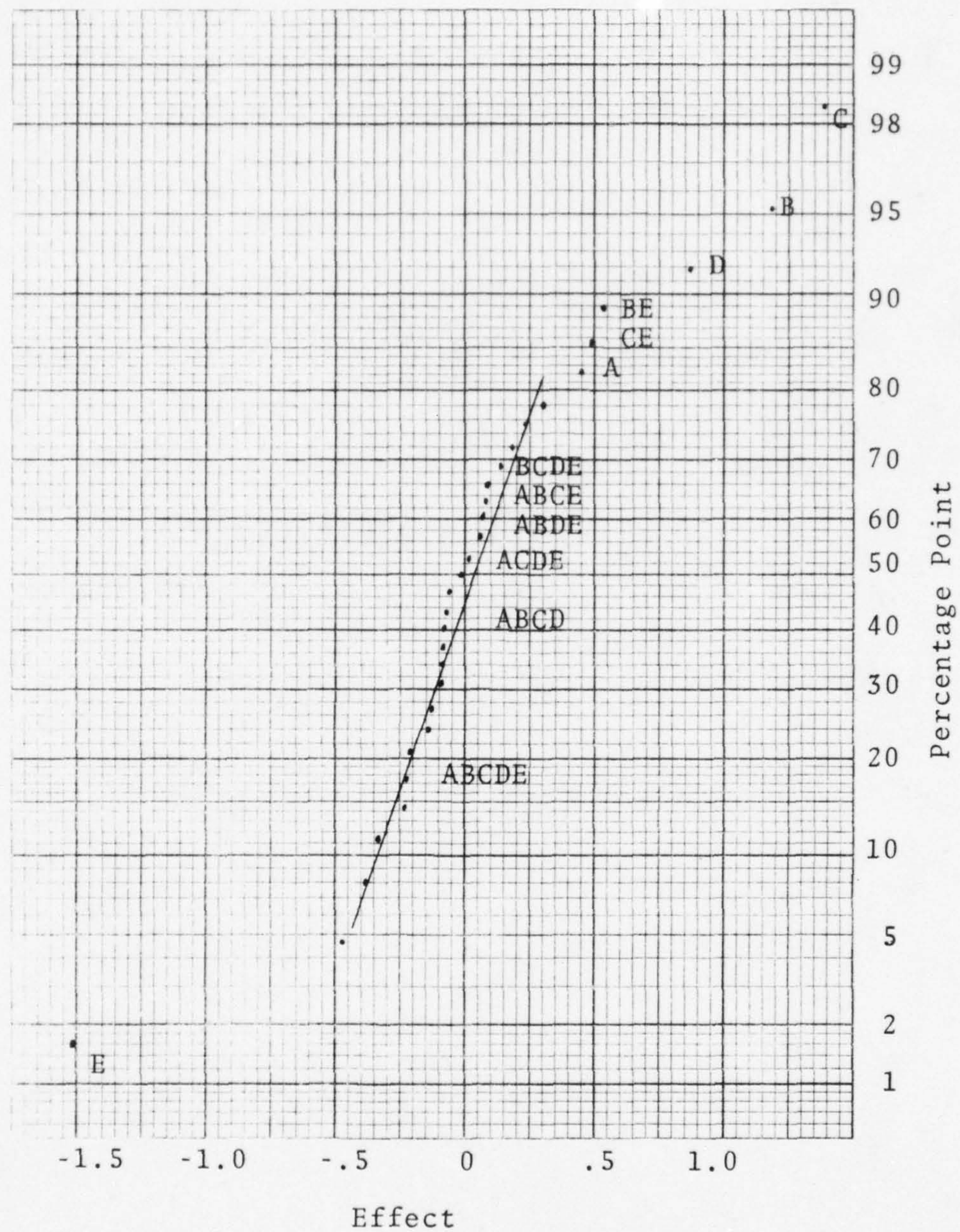

Figure 1. Plot of the Effects for the First Experimental Design on Normal Probability Paper

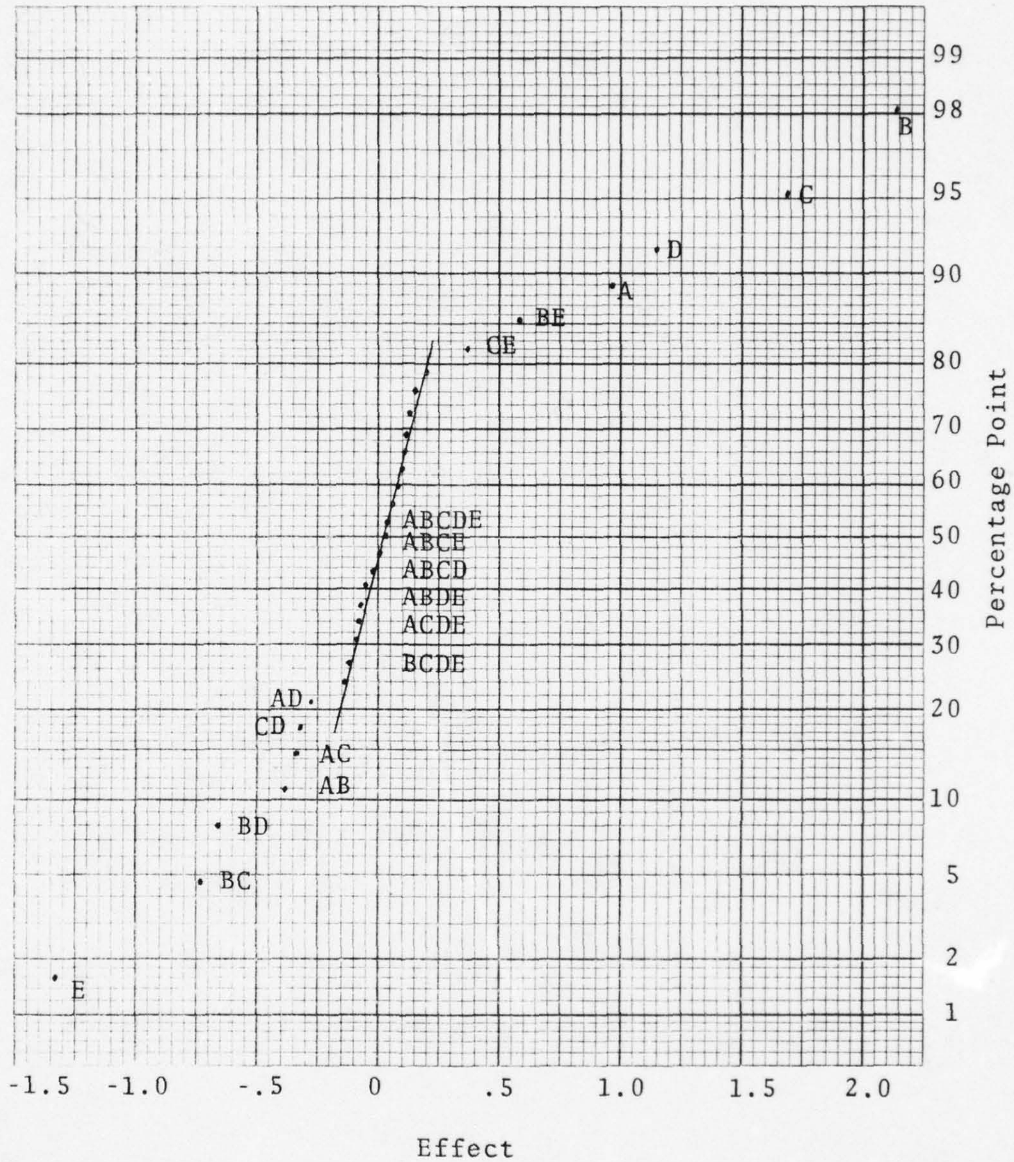

Figure 2. Plot of the Effects for the Second Experimental Design on Normal Probability Paper

variation explained by the main effects, their mean square, and the amount of total variation explained by the second order interactions we may infer that some of the second order interactions are not significant. The $\lambda \times |\underset{\sim}{P}|$, $D_2 \times |\underset{\sim}{P}|$, $\lambda \times D_2$ and the $\lambda \times n$ interactions appear significant in this perspective.

Additional information on the second-order interactions can be acquired through their graphical representation. The interaction of the autocorrelation coefficient with the other factors is graphically displayed in Figures 3 and 4 for Experiments 1 and 2, respectively. The graphical results again confirm the interaction of the autocorrelation coefficient with the other factors and also indicates that the autocorrelation coefficient has its greatest effect on the other factors when they are at their low levels. This result is not surprising since we would expect the greatest increase in the MANOVA power to occur when the MANOVA power is low; that is, when the other factors are at their low levels.

We may now make several general statements concerning the factors which influence the MANOVA power function. They are:

1. All five factors considered in the experimental design significantly affect the MANOVA power function.

2. The numerous second-order interactions make an interpretation of the effects of the factors on the MANOVA

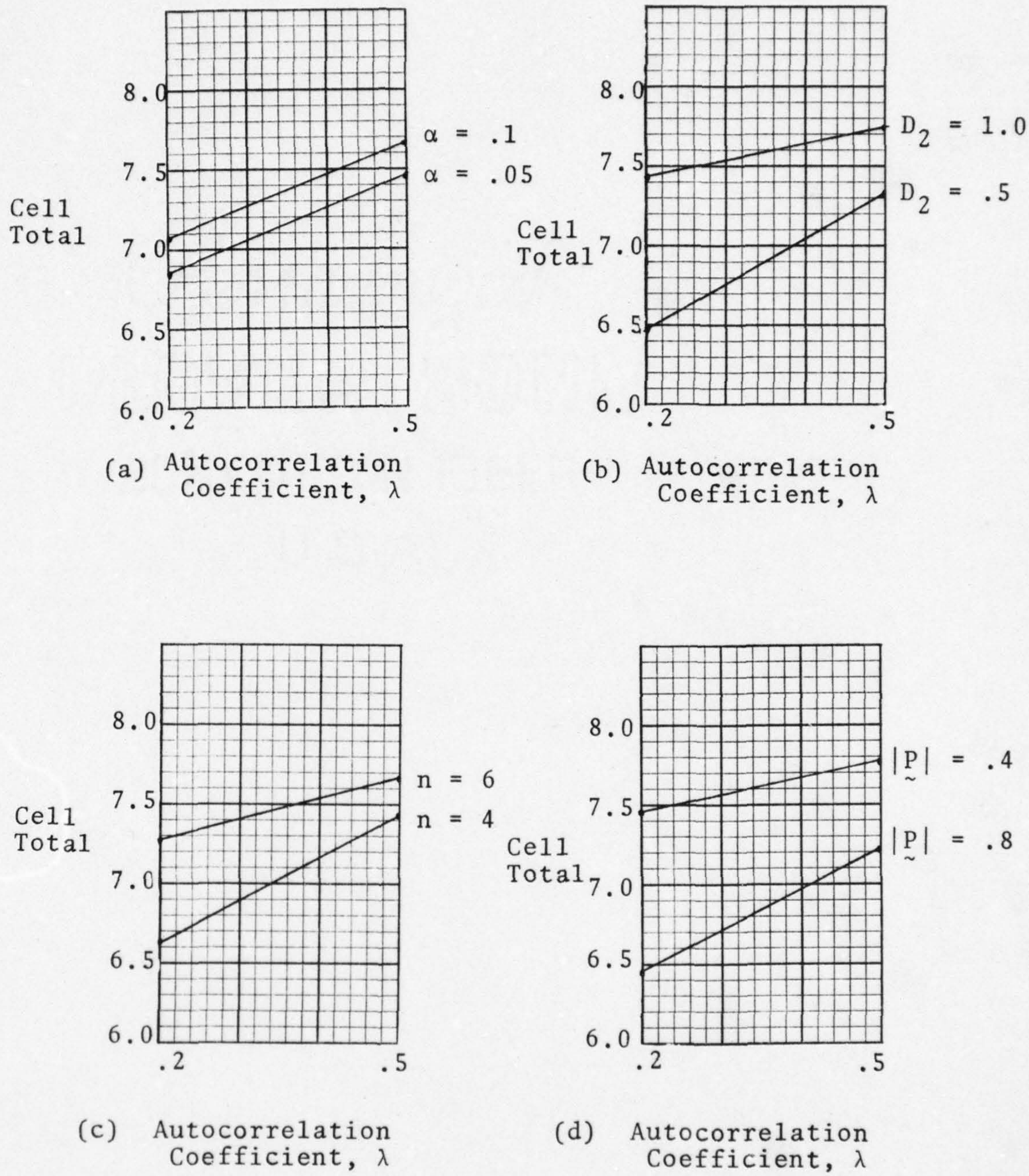

Figure 3. Graphical Results of Experiment 1

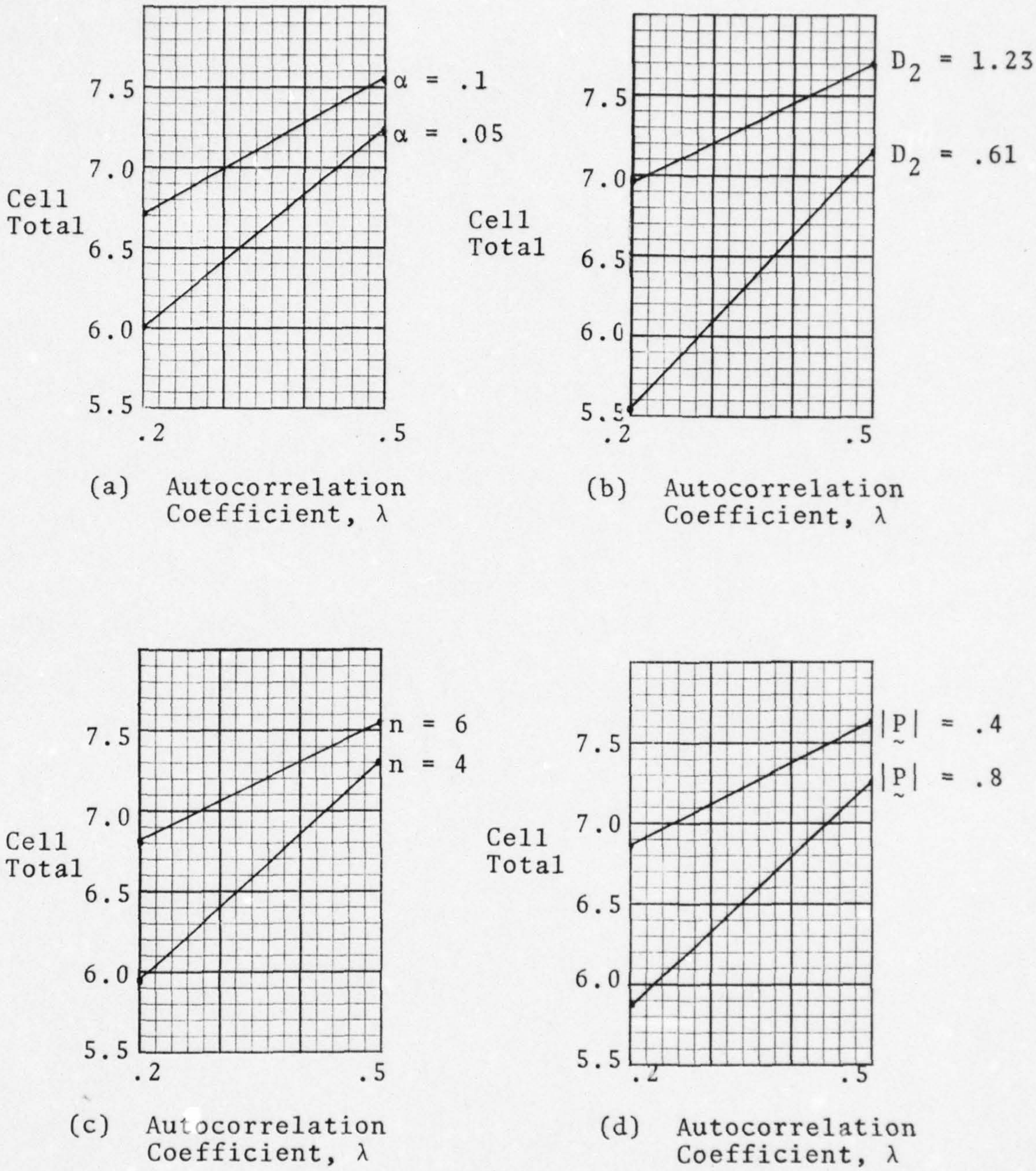

Figure 4. Graphical Results of Experiment 2

power function extremely difficult.

3. The autocorrelation coefficient, λ, the determinant of the correlation matrix, $|\underset{\sim}{P}|$, and the departure, D_2, appear to have a very significant effect on the MANOVA power function through second-order interactions.

4. The power of the MANOVA test statistic decreases with the dimension of the response.

5. The autocorrelation coefficient, λ, has a greater effect on the MANOVA power function when the other factors are at their low levels.

Conclusions

The above analysis of the experimental data leads us to the conclusion that all five factors do in fact influence the MANOVA power function. That is:

1. Power is a decreasing function of the dimension of the response.

2. Power is an increasing function of the size departure from the null hypothesis.

3. Power is an increasing function of sample size.

4. Power is an increasing function of the probability of Type I error.

5. Power is an increasing function of -Log $|\underset{\sim}{P}|$.

We also note that power is an increasing function of the autocorrelation structure of the response vector. That is, power increases as the significance of the multivariate time series increases.

It is also noted that the large number of significant second-order interactions make an interpretation of the response difficult; however, we may note that 1, 2, 4, and the autocorrelation account for a significant portion of the interaction sum of squares. Thus, if subjective estimates are to be made for either λ or $\underset{\sim}{P}$ great care must be exercised due to their impact on the MANOVA power function.

CHAPTER V

A METHODOLOGY FOR COMPARING ANOVA WITH MANOVA

Introduction

We now return to a primary objective of this research: to develop a methodology for comparing the effectiveness of ANOVA with MANOVA for use in the operational test and evaluation of command and control systems. Clearly, MANOVA is the preferred procedure for evaluating systems with correlated measures of effectiveness since it provides a joint comparison of the measures.

Burnette has developed a methodology for comparing the ANOVA with MANOVA on a basis of power of the tests. It was noted by Burnette that the powers of the tests appears to be the only method for comparing the ANOVA and the MANOVA. Our research has not indicated a more appropriate approach; therefore, the essential elements of Burnette's research will be reviewed.

Segregating the Measures of Effectiveness

Separation of Independent Measures

A comparison of the effectiveness of ANOVA with MANOVA is not applicable for independent measures of effectiveness. Our first task should be to separate all independent measures from the rest. We may separate the measures of effectiveness by an application of (2.25) and (2.26). For a system with p measures of effectiveness we would compute the sample multiple correlation coefficients, R_i, i = 1, ..., p, and test the p hypothesis of the form

$$H_0 : P_i = 0$$

against

$$H_1 : P_i > 0.$$

For those hypothesis which we fail to reject we assign the measure to the set of mutually independent measures, I.

<u>Grouping of Independent Sets of Measures</u>

After separating the independent measures we would like to group the remaining measures into k sets which are correlated within sets, but independent between sets. Let us designate the sets C_i, i = 1, ..., k. This grouping may be accomplished using the procedure of (2.28) and (2.29). In addition, we may test to insure that each set is correlated using the procedure of (2.27).

For those k independent sets of correlated measures, C_i, i = 1, ..., k, MANOVA is the appropriate procedure to utilize. For those measures which have been assigned to the set of independent measures, I, only ANOVA is appropriate.

Determining the Powers of the Tests

ANOVA Power

To determine the ANOVA power the following must be specified:

1. α, the probability of Type I error.
2. n_{max}, the maximum sample size permitted.
3. $\sum_{i=1}^{a} \alpha_i^2/\sigma^2 = D$ the component departure to detect.
4. $(1-\beta)$, the power of the test desired.

Based on the above information the sample size required to achieve the desired ANOVA power, n_{anova} is determined. If the desired power can not be achieved by a sample size n_{max} then either the maximum sample size or the departure, or both, must be reconciled. The above procedure is performed for each measure of effectiveness.

MANOVA Power

In addition to the parameters provided for each individual measure of effectiveness, for each independent set of correlated measures, C_i, $i = 1, \ldots, k$, the following must be specified.

1. α, the joint probability of Type I error.
2. $(1-\beta)$, the joint power desired.
3. R, the ratio of the primary component departure to the maximum departure of the other components.

The maximum sample sizes as well as the departures to detect would have previously been specified.

We will use the third form of the norm proposed in Chapter III since it enables us to determine the power of the MANOVA test for each component in the correlated set, C_i, for a specified departure, D_j, probability of Type I error, α, norm ratio, R, and sample size, n_{manova}. Here, n_{manova}, is the minimum sample size required by MANOVA to achieve the desired power.

After completing the above procedure we would have for each measure in the correlated set:

1. α, the probability of Type I error.
2. $(1-\beta)$, the power desired.
3. D_j, the departure to detect.
4. n_{max}, the maximum sample size permitted.
5. n_{anova}, the ANOVA sample size required to achieve the desired ANOVA Power.
6. n_{manova}, the MANOVA sample size to achieve the desired MANOVA power.

Trading Joint Inference for Power

For a correlated set of measures, C_i, i = 1, ..., k, we are constrained by the minimum sample size in the set, $n_{min} = \min(n_{anova\ j})$, so far as MANOVA sample size is concerned with the system as a whole. If we are unable to achieve the desired MANOVA power for each of the p_i measures in the set using n_{min}, then to increase the power, measures may be deleted from the set p_i to increase the power. These measures will be deleted as follows:

1. The measure corresponding to n_{min} will be deleted first.
2. If two measures correspond to n_{min} then the measure with the smallest power will be deleted.
3. If there are only two measures in the set both will be deleted.

Those measures deleted will be assigned to the set I for which ANOVA is more effective than MANOVA.

Summary of the Methodology

A summary of the methodology for comparing the effectiveness of the ANOVA with MANOVA is as follows:

1. Determine the correlation matrix for the measures of effectiveness.
2. Separate the measures into mutually independent measures, I, and correlated measures, C_i, $i = 1, \ldots, k$.
3. Determine the probability of Type I error, α, and the power of the test, $(1-\beta)$, to be utilized.
4. For each measure determine the maximum sample size permitted, n_{max}, and the univariate departure to detect.
5. For each measure of effectiveness determine the sample size, n_{anova}, to achieve the required power.
6. For each set of correlated measures of effectiveness, C_i, $i = 1, \ldots, k$, perform the following.

(a) For each measure of effectiveness, Y^j, $j = 1, \ldots, p_i$, determine the sample size, n_{manova}, required to achieve the desired MANOVA power.

(b) If the n_{manova} are less than or equal to $n_{min} = \min_j (n_{anova\ j})$ for the desired power, stop; MANOVA is more effective than ANOVA for the measures in the set.

(c) If the n_{manova} are greater than n_{min} for one or more measures in the set, remove from the set the measure corresponding to n_{min}. If more than one measure corresponds to n_{min}, remove from the set the measure with the lowest power which corresponds to n_{min}. Renumber the measures in the set which remain; set $p_i = p_{i-1}$. If $p_i = 1$, stop; ANOVA is more effective than MANOVA for all original measures in the set C_i. If $p_i > 1$, repeat steps a through c.

The methodology will be demonstrated in Chapter VI.

CHAPTER VI

AN APPLICATION TO OPERATIONAL TESTING

Introduction

In this chapter we shall apply the methodology developed in Chapter V to an operational testing problem. We will use the hypothetical command and control system used by Burnette so that the results may be compared. The hypothetical command and control system will be known as the Brigade Antiarmor Command and Control System (BACCS). Two competing forms of BACCS are under consideration for acquisition and are designated BACCS-I and BACCS-II.

For OT-II, the commander, U. S. Army Operational Test and Evaluation Agency (OTEA), has approved a comparative operational test of the two systems consisting of three scenarios. The commander has also approved seven measures of effectiveness designated MOE-1 through MOE-7. In addition, the commander has approved a completely crossed two-factor experiment with equal numbers of observations per cell. He desires to determine for which MOE MANOVA will be most effective, powerwise, than ANOVA.

Correlation Structure of the MOE

An objective estimate of the correlation structure of the MOE correlation matrix is:

	1	2	3	4	5	6	7
1	1.0	.00	-.06	-.12	.00	-.17	.16
2	.00	1.0	.01	-.11	.01	-.04	.76
3	-.06	.01	1.0	.68	-.49	.56	.07
4	-.12	-.11	.68	1.0	-.21	.72	-.04
5	.00	.01	-.49	-.21	1.0	-.26	-.11
6	-.17	-.04	.56	.72	-.26	1.0	-.08
7	.16	.76	.07	-.04	-.11	-.08	1.0

OT-1 test results indicated that each response vector was related to the previous response vector. However, insufficient information was available to obtain an objective estimate; therefore, a subjective estimate of the autocorrelation coefficient, $\lambda = 0.3$, was made by the BACCS project manager and the U. S. Army Training and Doctrine Command.

Based upon a knowledge of BACCS, we feel that MOE-1 is independent of all other MOE. We test the hypothesis

$$H_0 : P_1 = 0$$

against

$$H_1 : P_1 \neq 0$$

Using a computer program (Appendix B) we compute the sample multiple correlation coefficient

$$R_1 = 0.293452$$

and

$$R_1{}^2 = 0.86114$$

applying (2.26) we obtain the test statistic

$$Q \; \frac{R_1{}^2 \; (n-p)}{1-R_1{}^2 (p-1)} \quad \frac{(.086114)(42-7)}{(1-.086114)(7-1)} \; 0.5515$$

We desire to test the hypothesis at $\alpha = 0.05$ and determine the critical value of the test is $F_{.05,6,35} = 2.36$. The test statistic is less than the critical value of the test; hence, we fail to reject the hypothesis that MOE 1 is independent of the other MOE. MOE-1 is assigned to the set of mutually independent measures, I.

Our knowledge of BACCS indicates that MOE-2 and MOE-7 are correlated, but independent of the other MOE. We also feel that MOE-3, MOE-4, MOE-5, and MOE-6 are correlated but independent of the other MOE. We assign MOE-2 and MOE-7 to correlated set C_1. We assign MOE-3, MOE-4, MOE-5, and MOE-6 to correlated set C_2. The correlation matrix for the set C_1 is now the 2 x 2 matrix

2	1.0	.76
7	.76	1.0

and the correlation matrix for set C_2 is now the 4 x 4 matrix

3	1.0	.68	-.49	.56
4	.68	1.0	-.21	.72
5	-.49	-.21	1.0	-.26
6	.56	.72	-.26	1.0

We desire to test the hypothesis that set C_1 and set C_2 are mutually independent using the procedures of (2.28) and (2.29) with $\alpha = 0.05$. Using a computer program (Appendix C), we determine the test statistic

$$\chi_0^2 = 4.1630$$

and the critical value of the test

$$\chi^2_{.05,8} = 15.5072$$

The test statistic is less than the critical value of the test; hence, we fail to reject the hypothesis of independence and conclude that C_1 and C_2 are independent. We must now check to determine if the MOE within the mutually independent sets C_1 and C_2 are independent.

We observe that set C_1 has only two MOE and thus has a bivariate normal distribution. We may then make use of the

Fisher Z-transformation and test the hypothesis

$$H_{10} : \rho_{27} = 0$$

against

$$H_{11} : \rho_{27} \neq 0.$$

Using (2.19) through (2.22) we find

$$z = \tanh^{-1} (.76) = 0.638$$

and the test statistic is

$$|Z| \sqrt{N-3} = 0.638 \sqrt{42-3} = 3.984.$$

The critical value of the test with $\alpha = .05$ is $Z_{.05} = 1.96$. The test statistic exceeds the critical value of the test; hence, we reject H_{10} and conclude MOE-2 and MOE-7 are correlated.

We test the following hypothesis

$$H_{20} : \underset{\sim}{P}_{c2} = \underset{\sim}{I}$$

against

$$H_{21} : \underset{\sim}{P}_{c2} \neq I$$

to determine if MOE-3, MOE-4, MOE-5, and MOE-6 are correlated. Using the results of (2.27) and a computer program (Appendix D) we determine the test statistic

$$X_0^2 = -\left(N-1 - \frac{2k+5}{6}\right) \text{Log } |\underset{\sim}{R}| = -\left(42-1 - \frac{2\cdot 4+5}{6}\right) \text{Log } |\underset{\sim}{R}|$$

$$X_0^2 = 65.81137.$$

With $\alpha = .05$ the critical value of the test is

$$X^2_{.05,6} = 12.59120.$$

The test statistic exceeds the critical value of the test; hence, we conclude the members of C_2 are correlated.

The above tests have enabled us to separate the MOE into three mutually independent sets:

I = MOE-1

C_1 = MOE-2, MOE-7

C_2 = MOE-3, MOE-4, MOE-5, MOE-6.

ANOVA is appropriate for MOE-1, the sole member offset I; therefore, MOE-1 will not be used for a comparison of the effectiveness of MANOVA with ANOVA.

ANOVA Power/Sample Size for the MOE

The Commander of OTEA has specified the following probability levels be used for BACCS OT-II:

Probability of Type I error, -.05

Power of the test (1-β) -.75.

These parameters will apply to both ANOVA and MANOVA. In addition, the maximum sample size, n_{max}, and the departure to be detected, D, have been specified for each MOE. These parameters are shown in Table 5.

Table 5. MOE Maximum Sample Sizes and Departures

MOE	Maximum Sample Size n_{max}	Departure to Detect D
1	6	1.5
2	6	1.5
3	4	2.0
4	6	1.5
5	6	1.5
6	7	1.0
7	6	1.5

Using the information in Table 5 we compute for each MOE the minimum sample size, n_{anova}, required to achieve the desired power. We accomplish this by using the results reviewed in Chapter II. The results are shown in Table 6.

Table 6. MOE Sample Sizes for Required Power

MOE	Maximum Sample Size n_{max}	Departure to Detect D	Minimum Sample Size n_{anova}
1	6	1.5	5
2	6	1.5	5
3	4	2.0	4
4	6	1.5	5
5	6	1.5	5
6	7	1.0	7
7	6	1.5	5

Comparing the Effectiveness of MANOVA with ANOVA

For the two sets of correlated measures, C_1 and C_2, we are now interested in determining for which members of these sets MANOVA is more effective than ANOVA from the standpoint of power. The Commander of OTEA has approved a ratio R = 2 for use in setting the random levels of the MOE in the sets other than those under consideration.

For set C_1 = {MOE-2, MOE-7} we find that n_{min} =

min $\{n_{anova\ 2}, n_{anova\ 7}\} = 5$ (Table 5). Using the two-factor MANOVA program (Appendix A), we set levels of factor A = 2, levels of factor B = 3, D = 1.5, sample size = $n_{min} = 5$, $\lambda = .3$, R = 2, Monte Carlo iterations = 500, and correlation matrix $\underset{\sim}{P}_{c_1}$. The results are tabulated in Table 7 with the results of Burnette's research for ease of comparison.

Table 7. MOE Power 1

MOE	MANOVA Sample Size n_{manova}	Departure to Detect D	Power Achieved by Burnette	Power Achieved by this Research
2	5	1.5	.762	.866
7	5	1.5	.824	1.000

The MANOVA power is greater than the ANOVA power with sample size n_{min}; thus, MANOVA is more effective than ANOVA for members of set C_1.

For set C_2 = {MOE-3, MOE-4, MOE-5, MOE-6} we use the same two factor MANOVA power program. We set $\alpha = .05$, levels of factor A = 2, levels of factor B = 3, Monte Carlo iterations = 500, $\lambda = .3$, and R = 2. For the four MOE $n_{min} = 4 = n_{anova\ 3}$. We run the power program for each MOE with sample size $n_{min} = 4$ and departures to detect, $D = D_j$, j = 3, 4, 5, 6. The results are shown in Table 8 for this research and Burnette's for ease of comparison of results.

Table 8. MOE MANOVA Power 2

MOE	MANOVA Sample Size n_{manova}	Departure to Detect D	Power Achieved by Burnette	Power Achieved by this Research
3	4	2.0	.614	.850
4	4	1.5	.482	.824
5	4	1.5	.496	.776
6	4	1.0	.452	.994

We note that again the MANOVA power exceeds the power of the ANOVA for all components, therefore, we conclude that MANOVA is more effective than ANOVA for all members of the set C_2. In summary we have found that MANOVA is superior to ANOVA for both sets C_1 = {MOE-2, MOE-7} and set C_2 = {MOE-3, MOE-4, MOE-5, MOE-6}. This information would be used in to aid in the design of BRACCS OT-II.

Although the example presented in this chapter is hypothetical the methodology as demonstrated here may be applied to any system so long as an estimate of the structure of the response is available. We also note that the introduction of autocorrelated vectors greatly influence the MANOVA power function. Burnette was able to achieve joint inference on only two MOE in set C_2 [16] at the specified power. Our analysis, using the systems information, has enabled us to achieve joint inference on all four MOE of set C_2 at the specified power level greatly enhancing the analysis of the test results.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

Limitations of the Research

This research has been limited by the initial assumptions of two-factor, fixed-effects, crossed models, equal sample sizes per cell, and no effects due to operators. In addition, it was assumed that an estimate of the correlation structure of the measure of effectiveness and the autocorrelation coefficient or all the parameters of a multivariate time series are available.

Conclusions

This research has accomplished two objectives: first, through the use of two experimental designs analyzed by ANOVA it has been shown that:

1. The MANOVA power is a decreasing function of the dimension of the response.

2. The MANOVA power is an increasing function of the size of departure from the null hypothesis.

3. The MANOVA power is an increasing function of sample size.

4. The MANOVA power is an increasing function of the probability of Type I error.

5. The MANOVA power is an increasing function of

-Log $|\underset{\sim}{P}|$, where $\underset{\sim}{P}$ is the correlation matrix of the multiresponse.

6. The MANOVA power is an increasing function of the significance of the time dependence of the response vectors.

7. An extremely complex relationship exists between statements 2-5 since most second order interactions were found to be significant.

Second, it was found that the incorporation of the time series into the MANOVA power function significantly increased the MANOVA power for a given sample size. It was also noted that a reduction in sample size, for a given power, can be achieved when the time series information is incorporated in the MANOVA power function.

Recommendations

Several recommendations for further research arose during the course of this research. One recommendation is to develop an exact statistical test for a multiresponse system when the responses are time dependent. An experiment could then be designed using the exact test and the current procedure to determine if MANOVA is robust to independence of observations. Another recommendation is to extend the MANOVA power program so that it may handle nested, multifactor designs.

APPENDICES

APPENDIX A

This appendix contains a complete FORTRAN IV listing of the two-factor MANOVA power program along with its use. The program is entirely interactive and all input is made in free-field format. This listing is a modification and conversion of previous work [16].

```
** MANØVA PØWER PRØGRAM **

ENTER THE NR ØF STARTUP RUNS FØR UNIF
789

ENTER THE NR ØF LEVELS ØF FACTØR A
2

ENTER THE NR ØF LEVELS ØF FACTØR B
3

ENTER THE DIMENSIØN ØF THE RESPØNSE
4

ENTER THE SAMPLE SIZE
4

ENTER ALPHA
.05

DØ YØU DESIRE TØ SPECIFY ALL NØRM CØMPØNENTS?
YES

ENTER THE NØRM INDEX TØ BE SPECIFIED
1

WHAT NØRM RATIØ DØ YØU WANT TØ USE?
2.

ENTER THE SIZE NØRM YØU DESIRE TØ DETECT
2.

ENTER THE ITERATIØNS SAMPLE SIZE
500

ENTER THE SIGMA MATRIX
1.,.68,-.49,.56
.68,1.,-.21,.72
-.49,-.21,1.,-.26
.56,.72,-.26,1.

ENTER THE MEAN VECTØR
0.,0.,0.,0.

ENTER LAMBDA,THE AUTØCØRRELATIØN CØEFFICIENT
.3
** STARTUP RUNS FØR UNIF=   789
```

** LEVELS ØF FACTØR A = 2

** LEVELS ØF FACTØR B = 3

** SAMPLE SIZE = 4

** VECTØR DIMENSIØN IS 4

** ITERATIØNS SAMPLE SIZE = 500

** ALPHA = .05

**THE VALUE ØF LAMBDA IS .30

** SIZE NØRM TØ DETECT IS 2.00

** NØRM 1 IS SPECIFIED

** NØRM RATIØ IS 2.00

** SIGMA MATRIX **

1.00000	.68000	-.49000	.56000
.68000	1.00000	-.21000	.72000
-.49000	-.21000	1.00000	-.26000
.56000	.72000	-.26000	1.00000

** C MATRIX **

1.00000	0.00000	0.00000	0.00000
.68000	.73321	0.00000	0.00000
-.49000	.16803	.85538	0.00000
.56000	.46262	-.07404	.68330

```
             **  MEAN  VECTØR  **

          0.00000      0.00000      0.00000      0.00000

   IS YØUR INPUT CØRRECT ?
 ? YES

             **  PØWERS ØF THE TESTS

             SAMPLE SIZE          PØWER ØF TEST

                   4                       .81400

   ## DØ YØU DESIRE TØ MAKE ANØTHER RUN?
 ? NØ
     40.234 CP SECØNDS EXECUTIØN TIME
```

```
      PRØGRAM MANØVA(INPUT,ØUTPUT,TAPE5=INPUT,TAPE6=ØUTPUT)
      REAL LAMBDA
      INTEGER ERRØR
      CØMMØN /ØNE/ E(20,20),H1(20,20),HD(20,20)
      CØMMØN /TWØ/ NL
      CØMMØN /THREE/ KØRD(20)
      CØMMØN /FØUR/ KUSED(20)
      CØMMØN /FIVE/ SIGMA(20,20)
      CØMMØN /SIX/ CMAT(20,20)
      CØMMØN /SEVEN/ ZVEC(20),U(20),XVEC(20),BUF(20)
      CØMMØN /NINE/ FAC(20)
      CØMMØN /ELEVEN/ NI
      CØMMØN /TWEL/ LAMBDA
      DIMENSIØN DCØM(20)
      DIMENSIØN A(3,5),C(3,5,20),T(5,20),R(3,20)
      DIMENSIØN Y1A(5,100,20),Y2A(5,100,20),Y3A(5,100,20)
      DIMENSIØN G(20),JD(20),H2(20,20),H3(20,20),Z(20,20),IPR(20)
      DIMENSIØN IEUC(2),ISUP(2)
      EXTERNAL UNIF,RNØRM1,CMAT1,PICHI,VIPDA,CHIPRB
      DATA KNØRM/6HEUC   /
      DATA ISUP /6HSUPREM,6HØM    /
      DATA IEUC /6HEUCLID,6HEAN   /
0101  FØRMAT(1H1,2X,"ERRØR**READ PAST END ØF FILE**")
0103  FØRMAT(1H1,2X,"ERRØR**PRØBLEM IN CHI SQUARED RØUTINE**")
0105  FØRMAT(1H1,2X,"ERRØR**PRØBLEM IN GJR RØUTINE**")
0112  FØRMAT(1H1,10X,"**  MANØVA PØWER PRØGRAM  **")
0114  FØRMAT(//,10X,"**  ALPHA  =",F5.2)
0116  FØRMAT(//,10X,"**  VECTØR DIMENSIØN IS ",I2)
0118  FØRMAT(//,10X,"**  PØWERS ØF THE TESTS")
0120  FØRMAT(//,10X,"**  SIZE NØRM TØ DETECT IS ",F5.2)
0121  FØRMAT(//,10X,"**  LEVELS ØF FACTØR A = ",I3)
0122  FØRMAT(//,10X,"SAMPLE SIZE",9X,"PØWER ØF TEST")
0123  FØRMAT(//,10X,"**  LEVELS ØF FACTØR B = ",I3)
0124  FØRMAT(/,14X,I3,16X,F10.5)
0125  FØRMAT(//,10X,"**  SAMPLE SIZE = ",I3)
0127  FØRMAT(//,10X,"**  ITERATIØNS SAMPLE SIZE = ",I3)
0128  FØRMAT(//,10X,"**  MEAN  VECTØR  **")
0131  FØRMAT(//,10X,"**  SIGMA MATRIX  **")
0133  FØRMAT(//,10X,"**  C  MATRIX  **")
0137  FØRMAT(//,2X,"ENTER THE SIGMA MATRIX")
0139  FØRMAT(//,2X,"ENTER THE MEAN VECTØR")
0141  FØRMAT(//,2X,5(2X,F10.5))
0143  FØRMAT(//,2X,"IS YØUR INPUT CØRRECT ?")
0144  FØRMAT( /,2X,"ENTER THE TYPE NØRM YØU DESIRE TØ USE:EITHER")
0145  FØRMAT(,2X,"EUC FØR EUCLIDEAN ØR SUP FØR SUPREMUM")
0146  FØRMAT( /,10X,"** NØRM USED IS ",2A6)
0147  FØRMAT( //,2X,"## DØ YØU DESIRE TØ MAKE ANØTHER RUN?")
0148  FØRMAT( /,2X,"DØ YØU DESIRE TØ CHANGE ØNLY SAMPLE SIZE,ALPHA,
     1 NØRM?")
0149  FØRMAT( /,2X,"ENTER THE SAMPLE SIZE")
0150  FØRMAT( /,2X,"ENTER ALPHA")
0151  FØRMAT(A6)
0152  FØRMAT( /,2X,"ENTER THE SIZE NØRM YØU DESIRE TØ DETECT")
0153  FØRMAT( /,2X,"ENTER THE NR ØF LEVELS ØF FACTØR A")
0154  FØRMAT( /,2X,"ENTER THE NR ØF LEVELS ØF FACTØR B")
0155  FØRMAT( /,2X,"ENTER THE DIMENSIØN ØF THE RESPØNSE")
0157  FØRMAT( /,2X,"ENTER THE ITERATIØNS SAMPLE SIZE")
0158  FØRMAT( /,2X,"ENTER THE NR ØF STARTUP RUNS FØR UNIF")
0159  FØRMAT( /,10X,"** STARTUP RUNS FØR UNIF= ",I5)
0160  FØRMAT(/,2X,"DØ YØU DESIRE TØ SPECIFY ALL NØRM CØMPØNENTS?")
0161  FØRMAT(/,2X,"ENTER THE NØRM INDEX TØ BE SPECIFIED")
```

```
0162  FØRMAT(/,10X,"** DCØM(",I2,") =",F5.2)
0163  FØRMAT(/,10X,"** NØRM ",I2," IS SPECIFIED")
0164  FØRMAT(/,2X,"WHAT NØRM RATIØ DØ YØU WANT TØ USE?")
0165  FØRMAT(/,10X,"** NØRM RATIØ IS "F5.2)
0166  FØRMAT(/,2X,"ENTER LAMBDA,THE AUTØCØRRELATIØN CØEFFICIENT")
0167  FØRMAT(/,10X,"**THE VALUE ØF LAMBDA IS" F5.2)
      DATA IRES/6HYES   /
C
C  **  INPUT SECTIØN  **
C
      WRITE(6,0112)
      WRITE(6,0158)
      READ(5,*) KSU
      IF(EØF(5)) 9791,20
20    DØ 0800 I=1,KSU
      ZSU=UNIF(A)
0800  CØNTINUE
0900  WRITE(6,0153)
      READ(5,*) NI
      IF(EØF(5)) 9791,21
21    WRITE(6,0154)
      READ(5,*) NJ
      IF(EØF(5)) 9791,22
22    WRITE(6,0155)
      READ(5,*) NL
      IF(EØF(5)) 9791,23
23    WRITE(6,0149)
      READ(5,*) NI4
      IF(EØF(5)) 9791,24
24    WRITE(6,0150)
      READ(5,*) ALPHA
      IF(EØF(5)) 9791,25
25    WRITE(6,0160)
      READ(5,0151) LNØR
      IF(LNØR.NE.IRES) GØ TØ 0902
      WRITE(6,0161)
      READ(5,*) IDX
      WRITE(6,0164)
      READ(5,*) RATIØ
0902  WRITE(6,0152)
      READ(5,*) DC
      IF(EØF(5)) 9791,0904
0904  WRITE(6,0157)
      READ(5,*) NN
      IF(EØF(5)) 9791,26
26    IF(LNØR.EQ.IRES) GØ TØ 0908
      WRITE(6,0144)
      WRITE(6,0145)
      READ(5,0151) NØRM
0908  WRITE(6,0137)
      READ(5,*)((SIGMA(I,J),J=1,NL),I=1,NL)
```

```
      IF(EØF(5)) 9791,27
27    WRITE(6,0139)
      READ(5,*)(U(I),I=1,NL)
      IF(EØF(5)) 9791,28
28    WRITE(6,0166)
      READ(5,*) LAMBDA
      IF(EØF(5)) 9791,41
41    GØ TØ 0915
0910  WRITE(6,0149)
      READ(5,*) NI4
      IF(EØF(5)) 9791,29
29    WRITE(6,0166)
      READ(5,*) LAMBDA
      IF(EØF(5)) 9791,40
40    WRITE(6,0150)
      READ(5,*) ALPHA
      IF(EØF(5)) 9791,30
30    WRITE(6,0152)
      READ(5,*) DC
      IF(EØF(5)) 9791,31
31    WRITE(6,0161)
      READ(5,*) IDX
      IF(EØF(5)) 9791,0915
0915  CALL CMAT1
      WRITE(6,0159) KSU
      WRITE(6,0121) NI
      WRITE(6,0123) NJ
      WRITE(6,0125) NI4
      WRITE(6,0116) NL
      WRITE(6,0127) NN
      WRITE(6,0114) ALPHA
      WRITE(6,0167) LAMBDA
      WRITE(6,120) DC
      IF(LNØR.EQ.IRES) GØ TØ 916
      GØ TØ 917
0916  WRITE(6,0163) IDX
      WRITE(6,0165) RATIØ
      GØ TØ 0930
0917  IF(NØRM.NE.KNØRM) GØ TØ 0920
      WRITE(6,0146) IEUC
      GØ TØ 0930
0920  WRITE(6,0146) ISUP
0930  CØNTINUE
      WRITE(6,0131)
      DØ 940 I=1,NL
      WRITE(6,0141)(SIGMA(I,J),J=1,NL)
940   CØNTINUE
      WRITE(6,0133)
      DØ 950 I=1,NL
      WRITE(6,0141)(CMAT(I,J),J=1,NL)
950   CØNTINUE
```

```
      WRITE(6,0128)
      WRITE(6,0141)(U(I),I=1,NL)
      WRITE(6,0143)
      READ(5,0151) IZ
      IF(IZ.NE.IRES) GØ TØ 0900
      PØWER=0.0
C
C  ** CØMPUTE THE CRITICAL VALUE ØF THE TEST STATISTIC  **
C
      CALL CRIT(NI,NJ,NI4,NL,ALPHA,PICHI,CHIPRB,CRITV,ERRØR)
      IF(ERRØR.EQ.1) GØ TØ 9795
C
C  **  LØØP ØN REPLICATIØN FØR THIS NØRM  **
C
      DØ 8500 I2=1,NN
      IF(LNØR.EQ.IRES) GØ TØ 0990
      CALL ØRDER(NL,UNIF)
      IF(NØRM.NE.KNØRM) GØ TØ 0970
      CALL ASGNØR(DC,DCØM,UNIF)
      GØ TØ 0990
0970  CALL ASGMAX(DC,DCØM,UNIF)
C
C  **  LØØP ØN ITERATIØNS  **
C
0990  IF(LNØR.NE.IRES) GØ TØ 1020
      DØ 1000 LL=1,NL
      IF(LL.EQ.IDX) GØ TØ 0995
      DCØM(LL)=UNIF(A)*DC/RATIØ
      GØ TØ 1000
0995  DCØM(LL)=DC
1000  CØNTINUE
1020  DØ 1050 I3=1,NL
      CALL ØRDER(NI,UNIF)
      CALL FACØM(DCØM(I3))
      DØ 1030 III=1,NL
      DØ 1029 JJJ=1,NI
      A(JJJ,III)=0.0
1029  CØNTINUE
1030  CØNTINUE
      DØ 1040 KC=1,NI
      JR=KØRD(KC)
      A(JR,I3)=FAC(KC)
1040  CØNTINUE
1050  CØNTINUE
C
C  **  GENERATE THE ØBSERVATIØNS  **
C
      DØ 1500  II=1,NI
      DØ 1490  JJ=1,NJ
      DØ 1480 KK=1,NI4
      IREPS=KK
```

```
      CALL XVEC1(RNØRM1,UNIF,IREPS)
      DØ 1470 LL=1,NL
      IF(II.NE.1) GØ TØ 1501
      Y1A(JJ,KK,LL)=A(II,LL)+XVEC(LL)
1501  IF(II.NE.2) GØ TØ 1502
      Y2A(JJ,KK,LL)=A(II,LL)+XVEC(LL)
1502  IF(II.NE.3) GØ TØ 1470
      Y3A(JJ,KK,LL)=A(II,LL)+XVEC(LL)
1470  CØNTINUE
1480  CØNTINUE
1490  CØNTINUE
1500  CØNTINUE
C
C  **  CØMPUTE THE MANØVA  **
C
C  **  CØMPUTE THE CELL MEANS  **
C
      DØ 1600 IC=1,NI
      DØ 1590 JC=1,NJ
      DØ 1580 LC=1,NL
      SUM=0.0
      DØ 1570 KC=1,NI4
      IF(IC.NE.1) GØ TØ 1571
      SUM=SUM+Y1A(JC,KC,LC)
1571  IF(IC.NE.2) GØ TØ 1572
      SUM=SUM+Y2A(JC,KC,LC)
1572  IF(IC.NE.3) GØ TØ 1570
      SUM=SUM+Y3A(JC,KC,LC)
1570  CØNTINUE
      C(IC,JC,LC)=SUM
1580  CØNTINUE
1590  CØNTINUE
1600  CØNTINUE
C
C  **  CØMPUTE THE CØLUMN TREATMENTS  **
C
      DØ 1700 JC=1,NJ
      DØ 1690 LC=1,NL
      SUM=0.0
      DØ 1680 IC=1,NI
      DØ 1670 KC=1,NI4
      IF(IC.NE.1) GØ TØ 1671
      SUM=SUM+Y1A(JC,KC,LC)
1671  IF(IC.NE.2) GØ TØ 1672
      SUM=SUM+Y2A(JC,KC,LC)
1672  IF(IC.NE.3) GØ TØ 1670
      SUM=SUM+Y3A(JC,KC,LC)
1670  CØNTINUE
1680  CØNTINUE
      T(JC,LC)=SUM
1690  CØNTINUE
```

```
C
C  **  CØMPUTE THE RØW TREATMENTS  **
C
      DØ 1800 IC=1,NI
      DØ 1790 LC=1,NL
      SUM=0.0
      DØ 1780 JC=1,NJ
      DØ 1770 KC=1,NI4
      IF(IC.NE.1) GØ TØ 1771
      SUM=SUM+Y1A(JC,KC,LC)
1771  IF(IC.NE.2) GØ TØ 1772
      SUM=SUM+Y2A(JC,KC,LC)
1772  IF(IC.NE.3) GØ TØ 1770
      SUM=SUM+Y3A(JC,KC,LC)
1770  CØNTINUE
1780  CØNTINUE
      R(IC,LC)=SUM
1790  CØNTINUE
1800  CØNTINUE
C
C  **  CØMPUTE THE GRAND TØTALS  **
C
      DØ 1900 LC=1,NL
      SUM=0.0
      DØ 1890 KC=1,NI4
      DØ 1880 JC=1,NJ
      DØ 1870 IC=1,NI
      IF(IC.NE.1) GØ TØ 1871
      SUM=SUM+Y1A(JC,KC,LC)
1871  IF(IC.NE.2) GØ TØ 1872
      SUM=SUM+Y2A(JC,KC,LC)
1872  IF(IC.NE.3) GØ TØ 1870
      SUM=SUM+Y3A(JC,KC,LC)
1870  CØNTINUE
1880  CØNTINUE
1890  CØNTINUE
      G(LC)=SUM
1900  CØNTINUE
C
C  **  CØMPUTE THE H1 MATRIX  **
C
      DØ 2000 IL=1,NL
      DØ 1990 JL=1,NL
      SUM=0.0
      DØ 1980 IC=1,NI
      SUM=SUM+R(IC,IL)*R(IC,JL)
1980  CØNTINUE
      NJK=NJ*NI4
      Y1=NJK
      SUM=SUM/Y1
```

```
      NIJK=NI+NJ+NI4
      Y1=NIJK
      H1(IL,JL)=SUM-G(IL)*G(JL)/Y1
1990  CØNTINUE
2000  CØNTINUE
C
C  **  CØMPUTE THE E MATRIX  **
C
      DØ 2100 IL=1,NL
      DØ 2090 JL=1,NL
      SUM1=0.0
      DØ 2060 IC=1,NI
      DØ 2050 JC=1,NJ
      DØ 2040 KC =1,NI4
      IF(IC.NE.1) GØ TØ 2041
      SUM1=SUM1+Y1A(JC,KC,IL)*Y1A(JC,KC,JL)
2041  IF(IC.NE.2) GØ TØ 2042
      SUM1=SUM1+Y2A(JC,KC,IL)*Y2A(JC,KC,JL)
2042  IF(IC.NE.3) GØ TØ 2040
      SUM1=SUM1+Y3A(JC,KC,IL)*Y3A(JC,KC,JL)
2040  CØNTINUE
2050  CØNTINUE
2060  CØNTINUE
      SUM2=0.0
      DØ 2080 IC=1,NI
      DØ 2070 JC=1,NJ
      SUM2=SUM2+C(IC,JC,IL)*C(IC,JC,JL)
2070  CØNTINUE
2080  CØNTINUE
      Y1=NI4
      E(IL,JL)=SUM1-SUM2/Y1
2090  CØNTINUE
2100  CØNTINUE
      IF(NI4.NE.1) GØ TØ 2600
C
C  **  CØMPUTE THE H2 MATRIX  **
C
      DØ 2200  IL=1,NL
      DØ 2190  JL=1,NL
      SUM=0.0
      DØ 2180 JC=1,NJ
      SUM=SUM + T(JC,IL)*T(JC,JL)
2180  CØNTINUE
      Y1=(NI*NI4)
      SUM=SUM/Y1
      Y1=(NI*NJ*NI4)
      H2(IL,JL)=SUM - G(IL)*G(JL)/Y1
2190  CØNTINUE
2200  CØNTINUE
C
C  **  CØMPUTE THE TØTALS MATRIX  **
```

AD-A052 074 GEORGIA INST OF TECH ATLANTA SCHOOL OF INDUSTRIAL AN--ETC F/G 12/1
STUDIES IN SUPPORT OF THE APPLICATION OF STATISTICAL THEORY TO --ETC(U)
DEC 75 N R EYRICH DAAG39-76-C-0085

UNCLASSIFIED NL

2 OF 2

AD A052 074

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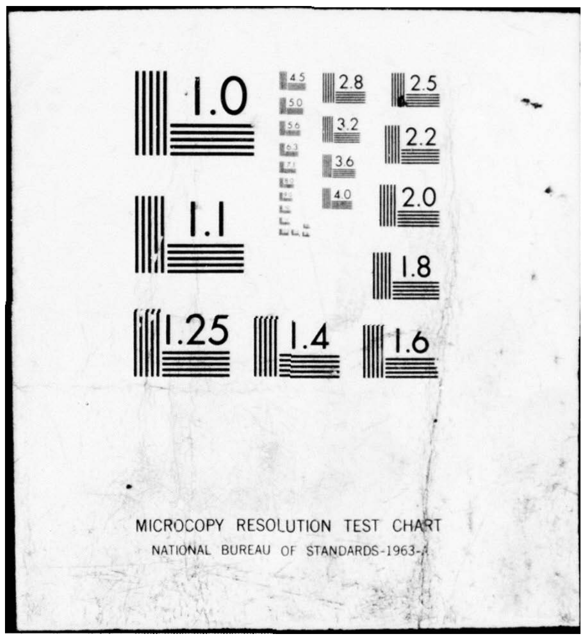
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5.6
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

```
C
      DØ 2300  IL=1,NL
      DØ 2290  JL=1,NL
      SUM1=0.0
      DØ 2260  IC=1,NI
      DØ 2250  JC=1,NJ
      DØ 2240  KC=1,NI4
      IF(IC.NE.1) GØ TØ 2241
      SUM1=SUM1+Y1A(JC,KC,IL)*Y1A(JC,KC,JL)
2241  IF(IC.NE.2)  GØ TØ 2242
      SUM1=SUM1+Y2A(JC,KC,IL)*Y2A(JC,KC,JL)
2242  IF(IC.NE.3) GØ TØ 2240
      SUM1=SUM1+Y3A(JC,KC,IL)*Y3A(JC,KC,JL)
2240  CØNTINUE
2250  CØNTINUE
2260  CØNTINUE
      Y1=(NI*NJ*NI4)
      Z(IL,JL) = SUM1 - G(IL)*G(JL)/Y1
2290  CØNTINUE
2300  CØNTINUE
C
C  **  CØMPUTE THE H3 MATRIX  **
C
      DØ 2400 IL=1,NL
      DØ 2390  JL=1,NL
      H3(IL,JL)=Z(IL,JL)-H1(IL,JL)-H2(IL,JL)-E(IL,JL)
2390  CØNTINUE
2400  CØNTINUE
C
C  **  REPLACE E MATRIX WITH H3 MATRIX  **
C
      DØ 2500 IL=1,NL
      DØ 2490 JL=1,NL
      E(IL,JL)=H3(IL,JL)
2490  CØNTINUE
2500  CØNTINUE
C
C  **  CØMPUTE THE TEST STATISTIC ØF THE MANØVA  **
C
2600  CALL MATADD
      CALL DECØM(E,20,NL,JD,IPR,D1,VIPDA)
      DET=D1
      DØ 10 I=1,NL
10    DET=DET*E(I,I)
      ED=DET
      CALL DECØM(HD,20,NL,JD,IPR,D1,VIPDA)
      DET=D1
      DØ 11 I=1,NL
11    DET=DET*HD(I,I)
```

```
      HT=DET
      CV=ED/HT
C
C  **  TEST THE CRITICAL VALUE ØF THE TEST STATISTIC  **
C
      IF(CV.GT.CRITV) GØ TØ 3000
      PØWER=PØWER + 1.0
3000  CØNTINUE
8500  CØNTINUE
      GØ TØ 9801
C
C  **  ERRØR MESSAGES  **
C
9791  WRITE(6,0101)
      GØ TØ 9801
9793  WRITE(6,0105)
      GØ TØ 9801
9795  WRITE(6,0103)
      GØ TØ 9801
C
C  **  CØMPUTE THE PØWERS BY SAMPLE SIZE  **
C
9801  W=NN
      PØWER=PØWER/W
C
C  **  ØUTPUT SECTIØN  **
C
      WRITE(6,0118)
      WRITE(6,0122)
      WRITE(6,0124) NI4,PØWER
      WRITE(6,0147)
      READ(5,0151) IZ
      IF(EØF(5)) 9791,35
35    IF(IZ.NE.IRES) GØ TØ 9990
      WRITE(6,0148)
      READ(5,0151) IZ
      IF(EØF(5)) 9791,36
36    IF(IZ.NE.IRES) GØ TØ 0900
      GØ TØ 0910
9990  CØNTINUE
      STØP
      END
```

```
      SUBRØUTINE CRIT(NI,NJ,I4,NL,ALPHA,PICHI,CHIPRB,CRITV,ERRØR)
C  **  THIS SUBRØUTINE CØMPUTES THE SECØND-ØRDER APPRØXIMATIØN
C  **  ØF THE CRITICAL VALUE ØF THE MANØVA TEST USING THE BØX
C  **  METHØD BY MEANS ØF A NØNLINEAR SEARCH ØPTIMIZATIØN
C  **  RØUTINE FØR A GIVEN PRØBABILITY ØF TYPE I ERRØR, ALPHA.
      INTEGER ERRØR
      S=1. - ALPHA
      KØUNT=1
      DEL=.01
      P=NL
      Q1=(NI-1)
      SN=(NI*NJ*(I4-1))
      IF(I4.EQ.1) SN=(NI-1)*(NJ-1)
      ZR=NI
      BN=SN+ZR
      Q2=ZR-Q1
      CM=BN-Q2-.5*(P+Q1+1.)
      G=(P*Q1*(P**2+Q1**2-5.))/48.
      KDF1=NL*(NI-1)
      KDF2=KDF1+4
      AKDF1=KDF1
      AKDF2=KDF2
      X=PICHI(S,AKDF1,IR)
      IF(IR.EQ.1.ØR.IR.EQ.2) GØ TØ 900
100   APT=CHIPRB(X,AKDF1,IR)
      BPT=CHIPRB(X,AKDF2,IR)
      Z=APT+(BPT-APT)*G/(CM**2)
      IF(IR.EQ.1.ØR.IR.EQ.2) GØ TØ 900
      XEWF=S-Z
      IF(XEWF.LT.0.0) XEWF=-XEWF
      IF(KØUNT.NE.1) GØ TØ 200
      KØUNT=KØUNT+1
150   Y=X
      X=Y+DEL
      ØLDF=XEWF
      GØ TØ 100
200   IF(XEWF-ØLDF)201,201,202
201   IF(XEWF.LT..00001) GØ TØ 800
      DEL=DEL*3.0
      ØLDF=XEWF
      GØ TØ 150
202   DEL=DEL*(-.5)
      X=Y
      XEWF=ØLDF
      GØ TØ 150
800   CRITV=EXP(X/(-CM))
      GØ TØ 950
900   ERRØR=1
      GØ TØ 990
950   ERRØR=2
990   CØNTINUE
      RETURN
      END
```

```
      SUBRØUTINE ØRDER(N,UNIF)
C  **  THIS SUBRØUTINE RANDØMLY ASSIGNS ØRDER TØ A P-DIMENSIØNAL
C  **  VECTØR'S CØMPØNENTS.
      CØMMØN /THREE/ KØRD(20)
      CØMMØN /FØUR/ KUSED(20)
      DØ 100 I=1,N
      KØRD(I)=0
      KUSED(I)=0
100   CØNTINUE
      LEFT=N
      DØ 500 J=1,N
      X=UNIF(X)
      DØ 400 K=1,LEFT
      Y=FLØAT(K)
      Y=Y/FLØAT(LEFT)
      IF(X.GT.Y) GØ TØ 400
      LU=0
      DØ 300 M=1,N
      IF(KUSED(M).NE.0) GØ TØ 300
      LU=LU+1
      IF(LU.NE.K) GØ TØ 300
      KUSED(M)=1
      KØRD(J)=M
      GØ TØ 450
300   CØNTINUE
400   CØNTINUE
450   LEFT=LEFT-1
500   CØNTINUE
      RETURN
      END


      SUBRØUTINE MATADD
      CØMMØN /ØNE/ E(20,20),H1(20,20),HD(20,20)
      CØMMØN /TWØ/ NL
      DØ 100 I=1,NL
      DØ 90 J=1,NL
      HD(I,J)=E(I,J)+H1(I,J)
90    CØNTINUE
100   CØNTINUE
      RETURN
      END


      FUNCTIØN RNØRM1(UNIF,RNØRM2,U,SIG2)
C  **  THIS FUNCTIØN PRØDUCES INDEPENDENT NØRMAL VARIATES WITH MEAN
C  **  U AND VARIANCE SIG2 BY MEANS ØF THE BØX AND MULLER
C  **  TRANSFØRMATIØN ØF UNIFØRM(0,1) DEVIATES.
      TPI=6.2831852
      A=UNIF(X)
      B=UNIF(X)
      RNØRM1=U+SQRT(-2.0*SIG2*ALØG(A))*CØS(TPI*B)
      RNØRM2=U+SQRT(-2.0*SIG2*ALØG(A))*SIN(TPI*B)
      RETURN
      END
```

```
      SUBRØUTINE ASGMAX(D,DCØM,UNIF)
C  **  THIS SUBRØUTINE RANDØMLY ASSIGNS THE CØMPØNENTS ØF A SUPREMUM
C  **  NØRM SUCH THAT THE CØMPØNENTS ARE IN NØRM EQUAL TØ THE ØRIGINAL
C  **  NØRM.
      CØMMØN /TWØ/ N
      CØMMØN /THREE/ KØRD(20)
      DIMENSIØN DCØM(20)
      M=N-1
      DØ 100 I=1,M
      J=KØRD(I)
      DCØM(J)=D*UNIF(A)
100   CØNTINUE
      J=KØRD(N)
        DCØM(J)=D
      RETURN
      END



      SUBRØUTINE XVEC1(RNØRM1,UNIF,IREPS)
C  **  THIS SUBRØUTINE GENERATES MULTIVARIATE NØRMAL AUTØCØRRELATED
C  **  VECTØRS USING THE TRANSFØRMATIØN Y=CX+U, WHERE C IS THE
C  **  MATRIX FRØM SUBRØUTINE CMAT, LAMBDA IS THE AUTØCØRRELATIØN
C  **  CØEFFICIENT, AND X IS A P-DIMENSIØNAL VECTØR FRØM N(0,1).
      CØMMØN /TWØ/ N
      CØMMØN /SIX/ CMAT(20,20)
      CØMMØN /SEVEN/ ZVEC(20),U(20),XVEC(20),BUF(20)
      CØMMØN /TWEL/ LAMBDA
      DIMENSIØN ØLDVEC(20),VECNEW(20)
      IF(IREPS.NE.1) GØ TØ 90
      DØ 27 I=1,N,2
      ZVEC(I)=RNØRM1(UNIF,RNØRM2,0.0,1.0)
      II=I+1
      ZVEC(II)=RNØRM2
27    CØNTINUE
      GØ TØ 1000
90    DØ 1 I=1,N
      ØLDVEC(I)=ZVEC(I)
1     CØNTINUE
      DØ 28 I=1,N,2
      VECNEW(I)=RNØRM1(UNIF,RNØRM2,0.0,1.0)
      II=I+1
      VECNEW(II)=RNØRM2
28    CØNTINUE
      DØ 29 I=1,N
      ZVEC(I)=LAMBDA*ØLDVEC(I)+(1.-LAMBDA)*VECNEW(I)
29    CØNTINUE
1000  DØ 121 I=1,N
      SUM=0.0
      DØ 111 J=1,N
      SUM=SUM+CMAT(I,J)*ZVEC(J)
111   CØNTINUE
      BUF(I)=SUM
121   CØNTINUE
      DØ 131 K=1,N
      XVEC(K)=BUF(K)+U(K)
131   CØNTINUE
      RETURN
      END
```

```
      SUBRØUTINE ASGNØR(D,DCØM,UNIF)
C  **  THIS SUBRØUTINE RANDØMLY ASSIGNS THE CØMPØNENTS ØF A EUCLIDEAN
C  **  NØRM SUCH THAT THE CØMPØNENTS ARE IN NØRM EQUAL TØ THE ØRIGINAL
C  **  NØRM.
      CØMMØN /TWØ/ N
      CØMMØN /THREE/ KØRD(20)
      DIMENSIØN DCØM(20)
      R=D**2
      M=N-1
      DØ 100 I=1,M
      J=KØRD(I)
      DCØM(J)=R*UNIF(A)
      R=R-DCØM(J)
100   CØNTINUE
      J=KØRD(N)
      DCØM(J)=R
      DØ 200 K=1,N
      DCØM(K)=SQRT(DCØM(K))
200   CØNTINUE
      RETURN
      END
```

```
      SUBRØUTINE CMAT1
C  **  THIS SUBRØUTINE CØMPUTES THE C-MATRIX REQUIRED TØ GENERATE
C  **  MULTIVARIATE NØRMAL RANDØM VECTØRS, SUCH THAT CC'=SIGMA,
C  **  WHERE SIGMA IS THE PØPULATIØN CØVARIANCE MATRIX.
      CØMMØN /TWØ/ N
      CØMMØN /FIVE/ SIGMA(20,20)
      CØMMØN /SIX/ CMAT(20,20)
      DØ 110 J=1,N
      IF(J.GE.2) GØ TØ 91
      DØ 81 I=1,N
      CMAT(I,1)=SIGMA(I,1)/SQRT(SIGMA(1,1))
81    CØNTINUE
      GØ TØ 110
91    DØ 105 I=1,N
      IF(J.GE.I+1) GØ TØ 104
      IF(J.NE.I) GØ TØ 95
      SUB1=0.0
      L=I-1
      DØ 93 K=1,L
      SUB1=SUB1+CMAT(I,K)**2
93    CØNTINUE
      CMAT(I,J)=SQRT(SIGMA(I,J)-SUB1)
      GØ TØ 105
95    SUB2=0.0
      L=J-1
      DØ 97 K=1,L
      SUB2=SUB2+CMAT(I,K)*CMAT(J,K)
97    CØNTINUE
      CMAT(I,J)=(SIGMA(I,J)-SUB2)/CMAT(J,J)
      GØ TØ 105
104   CMAT(I,J)=0.0
105   CØNTINUE
110   CØNTINUE
      RETURN
      END
```

```
      SUBRØUTINE FACØM(D)
      CØMMØN /NINE/ X(20)
      CØMMØN /ELEVEN/ NI
      DØ 50 I=1,NI
      X(I)=0.0
50    CØNTINUE
      IMØD=0
      Y=NI
      NN=NI
      IF(AMØD(Y,2.).GT..1) IMØD=1
      IF(IMØD.EQ.1) NN=NN-1
      Y=NN
      R=D/Y
      R=SQRT(R)
      DØ 100 I=1,NN,2
      X(I)=R
      II=I+1
      X(II)=-R
100   CØNTINUE
      IF(IMØD.EQ.1) X(NI)=0.
      RETURN
      END


      FUNCTIØN UNIF(A)
      DATA IU/31415926531/
      UMX=20007777777777777777B
      IX=16777213
      IU=IU*IX
      U=IU
      U=(U/UMX)
      IF(U.LT.0) U=U+1.
      UNIF=U
      RETURN
      END
```

APPENDIX B

This appendix contains a complete FORTRAN IV listing of the program which computes the multiple correlation coefficients of a set of responses, given the sample correlation or covariance matrix. The program is interactive and input is free-field format. An example of its use is also given.

```
*** MULTIPLE CØRRELATIØN CØEFFICIENT PRØGRAM ***

ENTER THE DIMENSIØN ØF THE RESPØNSE
7

ENTER THE SAMPLE CØVARIANCE MATRIX
1.,.00,-.06,-.12,.00,-.17,.16
.00,1.,.01,-.11,.01,-.04,.76
-.06,.01,1.,.68,-.49,.56,.07
-.12,-.11,.68,1.,-.21,.72,-.04
.00,.01,-.49,-.21,1.,-.26,-.11
-.17,-.04,.56,.72,-.26,1.,-.08
.16,.76,.07,-.04,-.11,-.08,1.

R(1)**2 =   .086114

R(1)    =   .293452

R(2)**2 =   .623385

R(2)    =   .789547

R(3)**2 =   .597057

R(3)    =   .772694

R(4)**2 =   .665777

R(4)    =   .815951

R(5)**2 =   .303452

R(5)    =   .550865

R(6)**2 =   .558023

R(6)    =   .747010

R(7)**2 =   .632303

R(7)    =   .795175

DETERMINANT IS      .06341
    .487 CP SECØNDS EXECUTIØN TIME
```

```
      PRØGRAM MLTCØR(INPUT,ØUTPUT,TAPE5=INPUT,TAPE6=ØUTPUT)
      INTEGER PØS(20)
      DIMENSIØN S(20,20),C(20,20),S12(20,20),S12T(20,20),S22(20,20)
      DIMENSIØN JD(20),A(20,20),B(20,20),IPR(20)
      DIMENSIØN UL(20,20),RR(20,20),X(20,20)
      EXTERNAL VIPDA
      WRITE(6,101)
101   FØRMAT(/,5X,"*** MULTIPLE CØRRELATIØN CØEFFICIENT PRØGRAM ***")
      WRITE(6,103)
103   FØRMAT(/,2X,"ENTER THE DIMENSIØN ØF THE RESPØNSE")
      READ(5,*) NL
      WRITE(6,105)
105   FØRMAT(/,2X,"ENTER THE SAMPLE CØVARIANCE MATRIX")
      READ(5,*)((C(I,J),J=1,NL),I=1,NL)
      N2=NL-1
      N1=1
      DØ 900 IP=1,NL
      IF(IP.NE.1) GØ TØ 175
      DØ 150 IC=1,NL
      PØS(IC)=IC
150   CØNTINUE
      GØ TØ 200
175   PØS(1)=IP
      PØS(2)=1
      IK=2
      DØ 190 IC=3,NL
      IF(IP.EQ.IK) IK=IK+1
      PØS(IC)=IK
      IK=IK+1
190   CØNTINUE
200   CØNTINUE
      DØ 250 IC=1,NL
      IA=PØS(IC)
      DØ 240 JC=1,NL
      JA=PØS(JC)
      S(IC,JC)=C(IA,JA)
240   CØNTINUE
250   CØNTINUE
      DØ 300 IC=1,N2
      IA=IC+1
      DØ 290 JC=1,N2
      JA=JC+1
      S22(IC,JC)=S(IA,JA)
290   CØNTINUE
300   CØNTINUE
      DØ 320 JC=1,N2
      JA=JC+1
      S12T(1,JC)=S(1,JA)
320   CØNTINUE
      DØ 340 IC=1,N2
      IA=IC+1
      S12(IC,1)=S(IA,1)
340   CØNTINUE
      L=0
      CALL INVITR(S22,UL,20,N2,IPR,RR,X,D1,L,DX,KD)
      IF(D1.EQ.0) GØ TØ 950
      CALL FMMX(S12T,X,A,N1,N2,20,20,20,N2)
      CALL FMMX(A,S12,B,N1,N2,20,20,20,N1)
      R=B(1,1)/S(1,1)
      WRITE(6,109) IP,R
109   FØRMAT(//,2X,"R(",I1,")**2 =",F10.6)
```

```
      R=SQRT(R)
      WRITE(6,111) IP,R
111   FØRMAT(/,2X,"R(",I1,")      =",F10.6)
900   CØNTINUE
      CALL DECØM(C,20,NL,JD,IPR,D1,VIPDA)
      DET=D1
      DØ 10 I=1,NL
10    DET=DET*C(I,I)
      D=DET
      WRITE(6,113) D
113   FØRMAT(//,2X,"DETERMINANT IS ",F10.5)
950   CØNTINUE
      END
```

APPENDIX C

This appendix contains a complete FORTRAN IV listing of a program which computes the test statistic used to test if two sets of responses are independent. The program is interactive and the input is in free-field format. An example of its use is also given.

```
**TEST FØR INDEPENDENCE ØF 2 SETS ØF VARIATES**

ENTER THE DIMENSIØN ØF THE RESPØNSE
6

ENTER THE NUMBER ØF VARIATES IN 1ST SET
2

ENTER THE NUMBER ØF VARIATES IN 2ND SET
4

ENTER THE SAMPLE  CØVARIANCE MATRIX
1.,.00,-.11,.01,-.04,.76
.01,1.,.68,-.49,.56,.07
-.11,.68,1.,-.21,.72,-.04
.01,-.49,-.21,1.,-.26,-.11
-.04,.56,.72,-.26,1.,-.08
.76,.07,-.04,-.11,-.08,1.

ENTER THE INDEX NRS ØF 1ST SET ØF VARIATES
1,6

ENTER THE INDEX NRS ØF 2ND SET ØF VARIATES
2,3,4,5

ENTER THE SAMPLE SIZE
42

ENTER ALPHA
.05

   **  DIMENSIØN ØF THE RESPØNSE = 6

   **  NR ØF VARIATES IN 1ST SET = 2

   **  NR ØF VARIATES IN 2ND SET = 4

   **  SAMPLE SIZE = 42

   **  ALPHA =.05

   **  REARRANGED CØVARIANCE MATRIX  **

   1.0000     .7600   0.0000    -.1100     .0100    -.0400

    .7600    1.0000    .0700    -.0400    -.1100    -.0800

    .0100     .0700   1.0000     .6800    -.4900     .5600

   -.1100    -.0400    .6800    1.0000    -.2100     .7200

    .0100    -.1100   -.4900    -.2100    1.0000    -.2600

   -.0400    -.0800    .5600     .7200    -.2600    1.0000

   **  TEST STATISTIC =     4.1630

   **  CRITICAL VALUE =    15.5072

   ** HENCE FAIL TØ REJECT INDEPENDENCE **
    .232 CP SECØNDS EXECUTIØN TIME
```

```
      PRØGRAM INDSET(INPUT,ØUTPUT,TAPE5=INPUT,TAPE6=ØUTPUT)
      INTEGER PØS(20)
      DIMENSIØN R11(20,20),R22(20,20)
      DIMENSIØN R(20,20),C(20,20),JD(20),IPR(20)
      EXTERNAL VIPDA
      WRITE(6,101)
101   FØRMAT(1H1,2X,"**TEST FØR INDEPENDENCE ØF 2 SETS ØF VARIATES**")
      WRITE(6,103)
103   FØRMAT(/,2X,"ENTER THE DIMENSIØN ØF THE RESPØNSE")
      READ(5,*) NL
      IF(EØF(5)) 995,20
20    WRITE(6,105)
105   FØRMAT(/,2X,"ENTER THE NUMBER ØF VARIATES IN 1ST SET")
      READ(5,*) N1
      IF(EØF(5)) 995,21
21    WRITE(6,107)
107   FØRMAT(/,2X,"ENTER THE NUMBER ØF VARIATES IN 2ND SET")
      READ(5,*) N2
      IF(EØF(5)) 995,22
22    WRITE(6,109)
109   FØRMAT(/,2X,"ENTER THE SAMPLE  CØVARIANCE MATRIX")
      READ(5,*)((R(I,J),J=1,NL),I=1,NL)
      IF(EØF(5)) 995,23
23    WRITE(6,111)
111   FØRMAT(/,2X,"ENTER THE INDEX NRS ØF 1ST SET ØF VARIATES")
      READ(5,*)(PØS(I),I=1,N1)
      IF(EØF(5)) 995,24
24     WRITE(6,113)
113   FØRMAT(/,2X,"ENTER THE INDEX NRS ØF 2ND SET ØF VARIATES")
      N3=N1+1
      READ(5,*)(PØS(I),I=N3,NL)
      IF(EØF(5)) 995,25
25    WRITE(6,115)
115   FØRMAT(/,2X,"ENTER THE SAMPLE SIZE")
      READ(5,*) NS
      IF(EØF(5)) 995,26
26    WRITE(6,117)
117   FØRMAT(/,2X,"ENTER ALPHA")
      READ(5,*) ALPHA
      IF(EØF(5)) 995,27
27    DØ 300 IC=1,NL
      IA=PØS(IC)
      DØ 290 JC=1,NL
      JA=PØS(JC)
      C(IC,JC)=R(IA,JA)
290   CØNTINUE
300   CØNTINUE
      WRITE(6,121) NL
121   FØRMAT(/,5X,"**  DIMENSIØN ØF THE RESPØNSE =",I2)
      WRITE(6,122) N1
122   FØRMAT(/,5X,"**  NR ØF VARIATES IN 1ST SET =",I2)
      WRITE(6,123) N2
123   FØRMAT(/,5X,"**  NR ØF VARIATES IN 2ND SET =",I2)
      WRITE(6,124) NS
124   FØRMAT(/,5X,"**  SAMPLE SIZE =",I3)
      WRITE(6,125) ALPHA
125   FØRMAT(/,5X,"**  ALPHA =",F3.2)
      WRITE(6,126)
126   FØRMAT(/,5X,"**  REARRANGED CØVARIANCE MATRIX  **")
127   FØRMAT(/,2X,8(1X,F8.4))
      DØ 200 I=1,NL
```

```
      WRITE(6,127)(C(I,J),J=1,NL)
200   CØNTINUE
      DØ 400 IC=1,N1
      DØ 390 JC=1,N1
      R11(IC,JC)=C(IC,JC)
390   CØNTINUE
400   CØNTINUE
      DØ 500 IC=1,N2
      IA=N1+IC
      DØ 490 JC=1,N2
      JA=N1+JC
      R22(IC,JC)=C(IA,JA)
490   CØNTINUE
500   CØNTINUE
      YN1=N1
      YNS=NS
      YN2=N2
      V1=YNS-(YN1+YN2+1.)/2.
      CALL CHSDEC(C,20,NL,JD,D1,VIPDA)
      DET=D1
      DØ 10 I=1,NL
10    DET=DET*JD(I)
      T=1./(DET*DET)
      CALL CHSDEC(R11,20,N1,IPR,D1,VIPDA)
      DET=D1
      DØ 11 I=1,N1
11    DET=DET*IPR(I)
      B1=1./(DET*DET)
      CALL CHSDEC(R22,20,N2,IPR,D1,VIPDA)
      DET=D1
      DØ 12 I=1,N2
12    DET=DET*IPR(I)
      B2=1./(DET*DET)
      CVT=-ALØG(T/(B1*B2 ))*V1
      AIDF=N1*N2
      ALPHA=1.-ALPHA
      CRIT=PICHI(ALPHA,AIDF,IR)
      IF(IR.EQ.1.ØR.IR.EQ.2) GØ TØ 995
      WRITE(6,131) CVT
131   FØRMAT(/,5X,"**  TEST STATISTIC =",F10.4)
      WRITE(6,133) CRIT
133   FØRMAT(/,5X,"**  CRITICAL VALUE =",F10.4)
      IF(CVT.GT.CRIT) GØ TØ 800
      WRITE(6,135)
135   FØRMAT(/,5X,"** HENCE FAIL TØ REJECT INDEPENDENCE **")
      GØ TØ 900
800   WRITE(6,137)
137   FØRMAT(/,5X,"** HENCE REJECT INDEPENDENCE **")
900   CØNTINUE
995   CØNTINUE
      END
```

APPENDIX D

This appendix contains a complete FORTRAN IV listing of a program used to test whether a set of responses is independent using the results of (2.27). The program is interactive and the input is in free-field format. An example of its use is also given.

```
      **  TEST FØR CØMPLETE INDEPENDENÇE  **

ENTER DIMENSIØN ØF THE RESPØNSE
4

ENTER THE SAMPLE CØRRELATIØN MATRIX
1.,.68,-.49,.56
.68,1.,-.21,.72
-.49,-.21,1.,-.26
.56,.72,-.26,1.

ENTER THE SAMPLE SIZE
42

ENTER ALPHA
.05

   **  DIMENSIØN ØF THE RESPØNSE = 4

   **  SAMPLE SIZE =  42

   **  ALPHA =.050


   **  CØRRELATIØN MATRIX  **

 1.0000   .6800 -.4900   .5600

  .6800 1.0000 -.2100   .7200

 -.4900 -.2100 1.0000 -.2600

  .5600   .7200 -.2600 1.0000

   THE VALUE ØF THE TEST STATISTIC =  65.81137

   THE CRITICAL VALUE =  12.59120

   **  HENCE REJECT INDEPENDENCE  **
    .075 CP SECØNDS EXECUTIØN TIME
```

```
      PRØGRAM INDEP(INPUT,ØUTPUT,TAPE5=INPUT,TAPE6=ØUTPUT)
      DIMENSIØN R(20,20),JD(20),IPR(20)
      EXTERNAL VIPDA
      WRITE(6,101)
101   FØRMAT(1H1,5X,"**  TEST FØR CØMPLETE INDEPENDENCE  **")
      WRITE(6,103)
103   FØRMAT(/,2X,"ENTER DIMENSIØN ØF THE RESPØNSE")
      READ(5,*) NL
      IF(EØF(5)) 999,90
90    WRITE(6,105)
105   FØRMAT(/,2X,"ENTER THE SAMPLE CØRRELATIØN MATRIX")
      READ(5,*)((R(I,J),J=1,NL),I=1,NL)
      IF(EØF(5)) 999,93
93    WRITE(6,107)
107   FØRMAT(/,2X,"ENTER THE SAMPLE SIZE")
      READ(5,*) NK
      IF(EØF(5)) 999,91
91    WRITE(6,109)
109   FØRMAT(/,2X,"ENTER ALPHA")
      READ(5,*) ALPHA
      IF(EØF(5)) 999,92
92    WRITE(6,121) NL
121   FØRMAT(/,5X,"**  DIMENSIØN ØF THE RESPØNSE =",I2)
      WRITE(6,125) NK
125    FØRMAT(/,5X,"**  SAMPLE SIZE =",I4)
      WRITE(6,127) ALPHA
127   FØRMAT(/,5X,"**  ALPHA =",F4.3)
      WRITE(6,122)
122   FØRMAT(//,5X,"**  CØRRELATIØN MATRIX  **")
      DØ 200 I=1,NL
      WRITE(6,123)(R(I,J),J=1,NL)
123   FØRMAT(/,2X,10(1X,F6.4))
200   CØNTINUE
      ALPHA=1.-ALPHA
      CALL DECØM(R,20,NL,JD,IPR,D1,VIPDA)
      DET=D1
      DØ 10 I=1,NL
10    DET=DET*R(I,I)
      ED=ALØG(DET)
      YN=NK-1
      YM=(2*NL+5)
      YM=YM/6.
      YN=-(YN-YM)
      CHISQ=YN*ED
      AIDF=(NL*(NL-1))/2.
      CV=PICHI(ALPHA,AIDF,IR)
      IF(IR.EQ.1.ØR.IR.EQ.2) GØ TØ 900
      WRITE(6,111) CHISQ
111   FØRMAT(/,5X,"THE VALUE ØF THE TEST STATISTIC =",F10.5)
      WRITE(6,113) CV
113   FØRMAT(/,5X,"THE CRITICAL VALUE =",F10.5)
      IF(CHISQ.GE.CV) GØ TØ 800
```

```
      WRITE(6,115)
115   FØRMAT(/,5X,"**  HENCE FAIL TØ REJECT INDEPENDENCE  **")
      GØ TØ 900
800   WRITE(6,117)
117   FØRMAT(/,5X,"**  HENCE REJECT INDEPENDENCE  **")
900   CØNTINUE
999   CØNTINUE
      END
```

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