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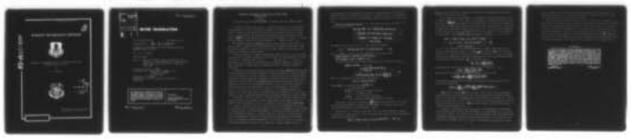

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FOREIGN TECHNOLOGY DIVISION

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G. F. Sigalov

EDITED TRANSLATION

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SUPERSONIC FLOW=AROUND OF THIN BODIES WITH MACH NUMBER
CLOSE TO UNITY

by

G.F. Sigalov

(Presented by Yu.A. Mitrpolskiy, Member of Academy of Science , Ukrainian SSR)

In studying of flows in the region of tran-sonic speeds, one has to study problems described by non-linear differential equations. In connection with this, basic results in solving such problems were obtained by a time curve method and a method of series approximations. The difficulties which appear using these methods are well known. A ~~surch~~ search for general methods of solution led to development of series of new approximation methods of study of own country problems for transonic flows (1-3).

The asymptotic method for studying problems of around-sonic non-accented gas dynamics, based on a problem of full approximation (6) was developed in the works (4,5). Applying his method to a series of non-linear problems showed its effectiveness, community and reliability of results(4,5,7).

Further development of a complete approximation method was given in the said work, and on its basis was shown a possibility to investigate non-linear problems of supersonic range of supersonic flows at $M_\infty > 1$, such that $M_\infty^2 - 1 = O(\varepsilon)$, $\varepsilon \ll 1$ - small parameter of destruction. The method has a sufficient community of aim, which makes it possible to study space, axis symmetrical, conical and other problems of sonic flows.

In applying the asymptotic methods to mechanical problems, important is the question of real possibility to construct algorithms of higher approximations. The basic complexities of calculation of non-linear effects are connected with a great labour-consuming realization of higher algorithms approximations, so that a problem to construct approximation above the second is often hopeless. On the other hand, corresponding final information of exactness of the mathematical model in a series of problems determines the contentment of second approximation. So, the entrophy change in a transonic flow is in the order of O (ε^3) and the assumption of flow isotrophy determines the contentment of second approximation. Simultaneously, the construction of a third approximation requires entrophy calculation, which significantly complicates problem solving.

In connection with this , there is a current problem to construct effective algorithm and methods of construction of second approximation. With regard to this, the problem of full approximation was set as a problem to select a physical space reflection in some new space, in which the solution of the first approxsimation would give, at a transition into a physical space, solution of two first approximations. The reflexion is realized by means of a special deformation of coordinates, which transforms a non-linear differential equation in a physical space into a linear in auxiliary space, a space of approximation. Therefore, the method of full approximation is by ideology close to the method od deformated coordinates (8). The central question in a problem of full approximation is a proof of its solution for a type of equation which is being studied. In this work, problem solving of full approximation of non-linear differential equation of supersonic gas dynamics in a physical space is investigated by means of linear equation of space approximation which is proved by the following theorem:

Theorem on asymptotic relation. Let a large number of elements $E(\Omega) \in C^2(\Omega)$ satisfy the equation (9):

$$\varphi_{yy} + \varphi_{zz} - (M_\infty^2 - 1)\varphi_{xx} = M_\infty^2 \Big[\frac{\gamma - 1}{2}(2\varphi_x + \varphi_y^2 + \varphi_z^2 + \varphi_x^2) \times \times (\varphi_{xx} + \varphi_{yy} + \varphi_{zz}) + (2\varphi_x + \varphi_x^2)\varphi_{xx} + \varphi_y^2\varphi_{yy} + \varphi_z^2\varphi_{zz} + + 2\varphi_y\varphi_z\varphi_{yz} + 2(1 + \varphi_x)\varphi_z\varphi_{zx} + 2(1 + \varphi_x)\varphi_y\varphi_{yx}\Big]; \quad q \in \Omega;\ \varphi \in E(\Omega); \tag{1}$$

and number of elements $E_1(\Omega_1) \in C^2(\Omega_1)$ satisfy the equation

$$\varphi_{\zeta\zeta} + \varphi_{\eta\eta} - \varphi_{\xi\xi} + 0(\varepsilon^{2+\alpha}) = 0;\ q_1 \in \Omega_1;\ \varphi \in E_1(\Omega_1). \tag{2}$$

If: 1) a number of elements $E(\Omega)$ and $E_1(\Omega_1)$ are of equal power;

2) $M_\infty^2 - 1 = 0(\varepsilon)$; $\lambda = 0(\varepsilon^{-\alpha/2})$; $\nu = 0(\varepsilon^{\alpha/2})$; $0 < \alpha < 1$;

3) $\varphi = \varepsilon\bar{\varphi}$; $\|\bar{\varphi}\| \sim 1$; $\varepsilon \ll 1$; $\varepsilon = 0(\delta^n)$; $0 < n < 1$;

4) operator P, which realizes the reflection, gives the relation of elements $q(x, y, z)$ i $q_1(\xi, \eta, \zeta)$ in the form

$$P = \begin{vmatrix} q\{\xi + B\varphi(\xi, \eta, \zeta);\ \eta\beta^{-1};\ \zeta\beta^{-1}\} \\ q_1\{\xi, \eta, \zeta\} \end{vmatrix}; \quad \eta = \frac{\bar{\eta}}{\nu};\ \zeta = \lambda\bar{\zeta};\ \beta = (M_\infty^2 - 1)^{1/2};\ B = \frac{M_\infty^4(\gamma + 1)}{3(M_\infty^2 - 1)}. \tag{3}$$

This has an asymptotic connectio between the solutions

$$\varphi(x, y, z) = \varphi(\xi, \eta, \zeta) + 0(\varepsilon^{2+\alpha}).$$

where: $0(\varepsilon^{2+\alpha})$ has characteristic $\lim\limits_{\varepsilon \to 0} \frac{0(\varepsilon^m)}{\varepsilon^m} \to N$,

N is certain constant

Ω i Ω_1 are regions of the potential flow

q qnd q_1 are regional points

$\varepsilon, \nu, \lambda, \delta$ are dimentionless parameters

a, n are exponents, certain from condition of limits.

Result No.1. For small distructions $\frac{\varepsilon M_\infty^2}{M_\infty^2 - 1} \ll 1$ and the transition (3) transfers into transition of Prandtl - Glauert, and the equation (1) transfers into a linear differential equation of small destructions theory.

Result No.2. In selecting of transition (3) in the form of Prandtl - Glauert, the equation (1) transfers into a non-linear differential equation of small destruction theory for transonic flow (2).

$$[M_\infty^2 - 1 + M_\infty^2(\gamma + 1)\varphi_x]\varphi_{xx} - \varphi_{yy} - \varphi_{zz} + 0(\varepsilon^{2+\alpha}) = 0. \tag{4}$$

The proof of the theorem is analogous to the proof of the theorem for non-accetuated flows (4, 5) and is not shown here.

As an example, let us investigate a flow problem of a bodily profile, at an angle of atack and Mach value a little larger than unity, by an isotropic transonic gas flow. The angle of ~~atack~~ attack, the Mach number and the profile edge are such that on the front and rear edges are formed joined jumps of condensation with a supersonic flow at both sides of jumps and the deformation function $B_\varphi(\pm 1, 0, 0) = 0$. In this case, the conditions in the theory of the second approximation are not used on jumps in a visible form. If the form equation is $y^* = \delta \bar{F}(x);\ \bar{F} = 0(1)$, the problem in a physical flow area will be

$$[M_\infty^2 - 1 + M_\infty^2(\gamma+1)\varphi_x]\varphi_{xx} - \varphi_{yy} + 0(\varepsilon^{2+\alpha}) = 0;\ q\in\Omega;$$

$$\varphi_{y\pm} = (1+\varphi_{x\pm})F_{x\pm};\ x\in[-1 \div +1];\ q\in y'; \tag{5}$$

$$\varphi = \varphi_x = 0;\ |x| > 1.$$

Using the evaluations 2, 3 from the theorem of a limiting condition, we shall obtain an expression for exponent $n = \frac{2}{2+\alpha}$, at which $\alpha = 1$ shall have known value of n=2/3 (10). Since the flows in an upper (+) and lower (-) areas are indepemdent, let us investigate the upper area. According to the characteristic equation theorem in the area of approximation $\frac{d\eta}{d\xi} = 1$ in physical area is

$$\frac{dy}{dx} = \frac{1}{\beta}\left(1 - \frac{\varepsilon M_\infty^2(\gamma+1)}{3(M_\infty^2-1)}\bar{\varphi}_\xi\right).$$

Integrating this expression, we shall obtain equation of curves φ = const

$$x - \beta y\left(1 + \frac{\varepsilon M_\infty^2(\gamma+1)}{3(M_\infty^2-1)}\bar{\varphi}_\xi\right) = \text{const.}$$

We shall write the solution of the problem equation (5) in the form

$$\varphi = f_1\left(x - \beta y\left(1 + \frac{\varepsilon M_\infty^2(\gamma+1)}{3(M_\infty^2-1)}\bar{\varphi}_\xi\right)\right) + 0(\varepsilon^{2+\alpha}).$$

Using limiting condition, we shall obtain and expression for φ_x, that means also for the pressure coefficient:

$$C_{p+} = \frac{2F_x}{\sqrt{M_\infty^2-1}} + \frac{2}{3}\,\frac{M_\infty^2(\gamma+1) - 3(M_\infty^2-1)}{(M_\infty^2-1)^2}F_x^2 + 0(\varepsilon^{2+\alpha}).$$

These results entirely ~~egree~~ agree with the known results of the second approximation theory of Buseman and Van-Dyke, which were obtained by method of consecutive approximations (9) and method of deformed coordinates (8). However, our solution has a considerable overweight; it is evenly useful in all of the flow area, providing that it does not contain parts which lead to a cumulative effect (8), and at the same time gives an equation of corrected Mach lines.

It should be pointed out that all qualitative results, obtained in a non-linear flow of profiles, come from the obtained formulas (11). The obtaining of the non-linear result already in the solution of the first approximation makes the method of full approximation approach the method of accelerated convergence (12). The proof of the theorems (4, 5) together with the one given in this paper is constructive; it gives also a selection of presentation. To the discussed solution , one may apply also the method of the (4, 5, 7) works. The procedure of solving in this case is somewhat longer, however, it leads to the same results.

The overweight of the method that is developed is apparent if one compares the method of full approximation with the method of accelerated convergence. In the method of accelerated convergence, beginning with the second step one has to find integrals of Pausson equations or nonhomogenous wave equations, which is connected with great calculating difficulties (9, 13) and , in addition, it leads to non-homogenous solutions, connected with a cumulative effect. Formalization of the full approximation method by itself already determines the problem of liquidation of these integrals. This significant overweight of the method, at last, determines its great possibilities.

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