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DEPARTMENT OF STATISTICS

DIVISION OF MATHEMATICAL SCIENCES

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ON THE PERFORMANCE OF SOME SUBSET SELECTION PROCEDURES*

by

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On the Performance of Some Subset Selection Procedures*

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1. Introduction and statement of the problem

Research in the area of subset selection has progressed steadily since the 1950's. For many problems, there are heuristically proposed procedures. When there are competing procedures for a given problem, performance comparisons are often available. However, these performance comparisons generally do not establish directly any optimality property of the procedures studied. In this paper, we restrict ourselves to the problem of selecting a subset of normal populations. The approaches and results of some previous studies are discussed briefly and then the result of a new Monte Carlo study is presented. We now make the problem precise.

Suppose n independent observations are obtained from each of k independent normal populations having unknown (unequal) means and a common known variance. By sufficiency we can restrict our attention to the sample means. For $i = 1,\dots,k$, let X_i be the sample mean of the n observations from the ith population and let $\underset{\sim}{X} = (X_1,\dots\dots,X_k)$. Without loss of generality, we assume that the common known variance of the X_i is 1, so that the joint distribution of the X_i is $\prod_{i=1}^{k} \Phi(x_i-\theta_i)$ where Φ is the standard normal distribution function and θ_i is the unknown mean of the ith population. Let $\underset{\sim}{\theta} = (\theta_1,\dots,\theta_k)$

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and let $\theta_{[1]} \leq \ldots \leq \theta_{[k]}$ be the ordered means. Populations having larger means are considered better than those having smaller means. The population associated with $\theta_{[k]}$ is considered the 'best' population. If more than one θ_i are tied for $\theta_{[k]}$ then arbitrarily one of them is chosen to be $\theta_{[k]}$. We are interested in selecting the 'best' population. However, if the observed values so indicate, we want to select more than one population, i.e. a subset of populations, to guard against the possibility of making an error. Thus, the action space $\mathcal{G}$ of the subset selection problem can be taken as the set of all non-empty subsets of $\{1,\ldots,k\}$, where taking the action $a \in \mathcal{G}$ means the selection of those populations whose indices are in a.

2. Some proposed procedures and known results

For any $a \in \mathcal{G}$, let

$$CS(\underset{\sim}{\theta},a) = \begin{cases} 1 & \text{if } \theta_{[k]} \in \{\theta_i : i \in a\} \\ 0 & \text{otherwise} \end{cases}.$$

Let $ICS(\underset{\sim}{\theta},a) = 1 - CS(\underset{\sim}{\theta},a)$ and $|a|$ = no. of elements in a. CS and ICS stand for 'correct selection' and 'incorrect selection', respectively. For any subset selection procedure R, if $\delta_R(\underset{\sim}{x},a)$ denotes the probability assigned to a by R having observed $\underset{\sim}{x}$, then let $P_{\underset{\sim}{\theta}}[CS|R] = E_{\underset{\sim}{\theta}}[\sum_{a \in \mathcal{G}} CS(\underset{\sim}{\theta},a)\delta_R(\underset{\sim}{X},a)]$ (the probability of a correct selection) and $E_{\underset{\sim}{\theta}}[S|R] = E_{\underset{\sim}{\theta}}[\sum_{a \in \mathcal{G}} |a|\delta_R(\underset{\sim}{X},a)]$ (the expected subset size). For fixed P^*, $0 \leq P^* \leq 1$, a procedure R is said to satisfy the P^* condition if $\inf_{\underset{\sim}{\theta} \in \mathbb{R}^k} P_{\underset{\sim}{\theta}}[CS|R] \geq P^*$.

For the normal populations problem, Seal (1955) proposed the following class $\mathcal{C}$ of procedures. For $\underset{\sim}{c} = (c_1,\ldots,c_{k-1})$, $c_j \geq 0$ $(j = 1,\ldots,k-1)$, $\sum_{j=1}^{k-1} c_j = 1$, the procedure $R_{\underset{\sim}{c}}(P^*)$ is as follows:

$R_{\underset{\sim}{c}}(P^*)$: Select the ith population iff $X_i \geq \sum_{j=1}^{k-1} c_j X_{[j]} - d_{\underset{\sim}{c}}(P^*)$

where $X_{[1]} \leq \ldots \leq X_{[k-1]}$ are the ordered sample means excluding X_i and $d_{\underset{\sim}{c}}(P^*)$ is the smallest number such that the P* condition is satisfied. When $\underset{\sim}{c} = (1/k-1,\ldots,1/k-1)$, the procedure $R_{\underset{\sim}{c}}(P^*)$ and its associated constant $d_{\underset{\sim}{c}}(P^*)$ will be denoted by $R_{avg}(P^*)$ and $d_{avg}(P^*)$ respectively. We will be interested in the class of procedures $\{R_{avg}(P^*): \ 0 \leq P^* \leq 1\}$. Note however, when $P^* < 1/2$, $R_{avg}(P^*)$ may select an empty set. Hence we modify $R_{avg}(P^*)$ as follows:

$R_{avg}(P^*)$: Select the ith population iff $X_i \geq X_{[k-1]}$ and/or $X_i > \sum_{j=1}^{k-1} X_{[j]}/(k-1) - d_{avg}(P^*)$.

The class of procedures $\{R_{avg}(P^*): \ 0 \leq P^* \leq 1\}$ will henceforth be referred to as 'average type procedures'.

For the more general problem of selecting a subset to contain the population having the largest location parameter, Gupta (1965) proposed the following class of procedures. Let $X_1,\ldots,X_k$ be independent random variables having the joint distribution $\prod_{i=1}^{k} F(x_i-\theta_i)$. To select a subset to contain the population associated with $\theta_{[k]}$, where $\theta_{[k]}$ is defined as before, the procedure $R_{max}(P^*)$ is as follows:

$R_{max}(P^*)$: Select the ith population iff $X_i \geq X_{[k-1]} - d_{max}(P^*)$

where $X_{[k-1]}$ is defined as before and $d_{max}(P^*)$ is the smallest number such that the P* condition is satisfied. We will be interested in the class of procedures $\{R_{max}(P^*): 1/k \leq P^* \leq 1\}$ which will henceforth be referred to as 'maximum type procedures'. Note that when applied to the normal populations problem, $R_{max}(P^*)$ is $R_{(0,\ldots,0,1)}(P^*)$ in $\mathcal{C}$.

For the normal populations problem a number of performance comparisons have been made. Usually attention is restricted to some subset of the parameter space (e.g. parameter points having the slippage configuration, or sequences of parameter points having certain limiting behavior), and the operating characteristics of some competing procedures (e.g. $R_{avg}(P^*)$ and $R_{max}(P^*)$) are compared. A representative but not exhaustive list of studies of this type is Seal (1957), Deely and Gupta (1968), Deverman (1969) and Deverman and Gupta (1969). Generally the results indicate that, in terms of the expected subset size $E_{\underset{\sim}{\theta}}[S|R]$ and related criteria, $R_{max}(P^*)$ is superior to $R_{avg}(P^*)$ over much of the parameter space. This, however, does not establish directly any optimality property of the procedure $R_{max}(P^*)$.

More recently, Berger (1977) proved that $R_{max}(P^*)$ is minimax with respect to $E_{\underset{\sim}{\theta}}[S|R]$ among all procedures satisfying the P^* condition.

In Berger and Gupta (1977), it is proved that $R_{max}(P^*)$ is minimax and admissible with respect to the maximum of the probability of selecting each of the non-best populations among all non-randomized 'just' translation invariant and permutationally invariant procedures satisfying the P^* condition. Hence $R_{max}(P^*)$ is optimal according to the above criteria.

3. The decision-theoretic approach and the loss functions

The approach taken in the present study is to compare the average performance of subset selection procedures, where the average is taken over the parameter space with respect to some prior. Thus the quantities to be compared are the integrated risks. For a given prior, the optimal procedure is the corresponding Bayes procedure by definition. However, Bayes procedures are often difficult to use. Thus, it is reasonable to look for procedures that are easy to use and which are approximately Bayes.

In classical performance studies of subset selection procedures, the measures of loss most often used have been $ICS(\underset{\sim}{\theta},a)$ and $|a|$ and quantities related to $|a|$. More recently, Goel and Rubin (1977) studied the subset selection problem from a Bayesian point of view using loss functions that are linear combinations of $\theta_{[k]} - \max_{j\in a} \theta_j$ and $|a|$. Bickel and Yahav (1977) studied the behavior of Bayes procedures as $k \to \infty$ using loss functions that are linear combinations of $ICS(\underset{\sim}{\theta},a)$ and $\theta_{[k]} - \sum_{j\in a} \theta_j/|a|$. Chernoff and Yahav (1977), employing Monte Carlo techniques, compared the integrated risks with respect to exchangeable normal priors of Bayes, maximum type and fixed-size procedures of Bechhofer (1954) using loss functions that are linear combinations of $\theta_{[k]} - \max_{j\in a} \theta_j$ and $\theta_{[k]} - \sum_{j\in a} \theta_j/|a|$. The present Monte Carlo study parallels Chernoff and Yahav's in that exchangeable normal priors are used but differs in that the loss functions considered are linear combinations of $ICS(\underset{\sim}{\theta},a)$ and $|a|$, and Bayes, maximum type and average type procedures are compared. The four loss combinations that have been used are presented in Figure 1.

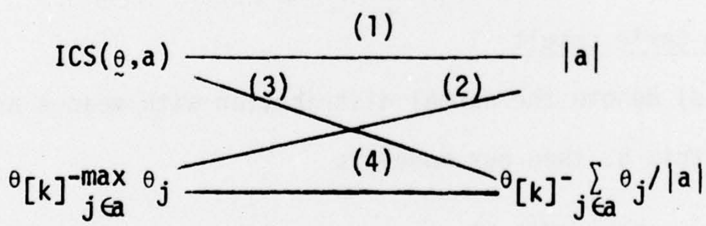

Figure 1

Note that the different combinations have different interpretations. The combinations (1) and (2) correspond to situations where the subset selection procedure is used as a screening procedure. For example, in developing a new drug, a pharmaceutical company may start with a number of ingredients known to have beneficial effects (and side effects) from previous experience, and then obtain a collection of potentially good formulations by combining these ingredients in different proportions. After the first stage of testing, one wants to reject those formulations that are evidently non-best and retain those formulations that still seem potentially best for further study. Eventually, if the development is successful, only one formulation will be marketed. Corresponding to this situation then, loss functions that depend only on the best selected and the size selected are reasonable. On the other hand, the component $\theta_{[k]} - \sum_{j\in a} \theta_j/|a|$ in the combinations (3) and (4) correspond to situations where all those selected will be used. This is the case, for example, when one purchases stocks for long term investment. One purchases stocks of more than one company to guard against the possibility of gross errors, and all the stocks purchased contribute to the gain or loss. We believe the distinction between screening-type situations and non-screening type situations needs to be pointed out. It is true that the loss combinations (3) and (4) each contains a component that corresponds to screening type situations.

4. The Monte Carlo result

Let $N(\underset{\sim}{a},B)$ denote the normal distribution with mean $\underset{\sim}{a}$ and variance-covariance matrix B, then our model is

$$\underset{\sim}{X}|\underset{\sim}{\theta} \sim N(\underset{\sim}{\theta},I)$$

where $\underset{\sim}{X} = (X_1,\ldots,X_k)$, $\underset{\sim}{\theta} = (\theta_1,\ldots,\theta_k)$ and I is the identity matrix. Consider the exchangeable normal prior

$$\underset{\sim}{\theta} \sim N(m\underset{\sim}{1}, rI + sU)$$

where m,r,s are constants, $\underset{\sim}{1} = (1,\ldots,1)$, $U = \underset{\sim}{1}'\underset{\sim}{1}$, $r > 0$ and $-r/k < s < r$.

Then jointly

$$(\underset{\sim}{X},\underset{\sim}{\theta}) \sim N\left((m\underset{\sim}{1},m\underset{\sim}{1}),\begin{pmatrix}(1+r)I+sU & rI+sU\\ rI+sU & rI+sU\end{pmatrix}\right).$$

Hence a posteriori

$$\underset{\sim}{\theta}|\underset{\sim}{X} \sim N(\hat{\underset{\sim}{\theta}},\Sigma)$$

where

$$\hat{\underset{\sim}{\theta}} = m\underset{\sim}{1}+(\underset{\sim}{x}-m\underset{\sim}{1})[(1+r)I+sU]^{-1}(rI+sU)$$

$$= (r/1+r)\underset{\sim}{x} + \text{a multiple of } \underset{\sim}{1}$$

and

$$\Sigma = (rI+sU)-(rI+sU)[(1+r)I+sU]^{-1}(rI+sU)$$

$$= rI-(r^2/1+r)I + \text{a multiple of } U$$

$$= (r/1+r)I + \text{a multiple of } U.$$

Consider the loss function $L(\underset{\sim}{\theta},a) = c_1 ICS(\underset{\sim}{\theta},a)+c_2|a|$ where $c_1,c_2 \geq 0$, $c_1+c_2 = 1$. It is easy to see that for this loss function the Bayes procedure, denoted by R_B, is as follows:

R_B: Select the ith population iff $X_i \geq X_{[k-1]}$ and/or

$$P[\theta_i=\theta_{[k]}|\underset{\sim}{X}] \geq c_2/c_1.$$

If we denote by $\Phi_{\underset{\sim}{a},B}$ the normal distribution function with mean $\underset{\sim}{a}$ and variance-covariance matrix B, then

$$P[\theta_i=\theta_{[k]}|\underset{\sim}{x}]$$

$$= \int I_{\{\theta_i=\theta_{[k]}\}} d\Phi_{(r/1+r)\underset{\sim}{x}+\text{a multiple of } \underset{\sim}{1},\ (r/1+r)I+\text{a multiple of } U}(\underset{\sim}{\theta})$$

$$= \int I_{\{\theta_i=\theta_{[k]}\}} d\Phi_{(r/1+r)\underset{\sim}{x},(r/1+r)I}(\underset{\sim}{\theta}). \tag{4.1}$$

Hence the Bayes procedure is translation invariant and can be obtained by numerical integration. The following computation shows that the integrated risk of any translation invariant procedure is independent of m and s so long as the loss function is translation invariant.

$$\text{Integrated Risk} = \iint \sum_{a\in G} L(\underset{\sim}{\theta},\delta(\underset{\sim}{x},a))d\Phi_{(r/1+r)\underset{\sim}{x}+b\underline{1},(r/1+r)I+gU}(\underset{\sim}{\theta})d\Phi_{m\underset{\sim}{1},(1+r)i+sU}(x)$$

$$= \iint \sum_{a\in G} L(\underset{\sim}{\theta},\delta(\underset{\sim}{x},a))d\Phi_{(r/1+r)\underset{\sim}{x},(r/1+r)I}(\underset{\sim}{\theta})d\Phi_{\underset{\sim}{0},(1+r)I}(\underset{\sim}{x}).$$

where b and g are appropriate constants.

Since both maximum type and average type procedures are translation invariant, we can reduce the set of parameters to just k,r and c_2/c_1.

Monte Carlo comparisons of Bayes, maximum type and average type procedures were carried out for k = 3 and k = 8. The range of r was $r^{\frac{1}{2}} = (1.8)^i$, i = -4,...,4 for both k = 3 and k = 8. The range of c_1/c_2 was $c_1/c_2 = 3^{i/2}$, i = 2,...,8 for k = 3 and $c_1/c_2 = 4{,}6{,}8^{i/3}$, i = 3,...,9 for k = 8. For k = 3, 400 simulations were performed at each $(r,c_1/c_2)$ pair, while for k = 8, the number of simulations was 200 each. For each simulation, the random vector $\underset{\sim}{X}$ is generated according to its marginal distribution. By numerically integrating the expression (4.1) for each i, the action taken by the Bayes procedure and the associated posterior risk are obtained. The average of these posterior risks then serve as an estimate of the Bayes risk. The best maximum type and average type procedures and the regrets incurred by using them are estimated by examining the average regrets corresponding to two sufficiently fine grids of the constants d_{max} and d_{avg}, where the two grids are determined from the result of a preliminary study. Tables IA and IB give for each $(r,c_1/c_2)$ pair the estimated Bayes risk, the estimated regrets incurred by the best maximum type procedure and the best average type procedure, and the estimated standard deviations of these estimates. Tables IIA and IIB list for each $(r, c_1/c_2)$ pair the estimated constants d_{max} and d_{avg} and their associated P* corresponding to the best maximum type and the best average type procedures. As can be seen from Tables IA and IB, both maximum type and average type procedures do almost as well as the Bayes procedures when the prior is concentrated (i.e. variance r is small), with the average type procedures having a slight edge. However, when the prior is diffuse (r large), the maximum type procedures continue to do almost as well as the Bayes procedures, while the average type procedures can do very badly. In this sense then, the maximum type procedures are safe to use. As by-products, Tables IIIA and IIIB give for each $(r,c_1/c_2)$ the average subset size and probability of a correct selection for the Bayes, the best maximum type, and the best average type procedure. They also give the proportions of times the best maximum type and the best average type procedure coincide with the Bayes procedure.

5. Concluding remarks

The Monte Carlo result of Chernoff and Yahav (1977) indicates that, with respect to the loss combination (4) and exchangeable normal priors, maximum type procedures do almost as well as the Bayes procedures. The result of the present Monte Carlo study indicates that, with respect to the loss combination (1) and exchangeable normal priors, maximum type procedures do almost as well as the Bayes procedures. From these results, it seems reasonable to expect the maximum type procedure to do well with respect to the loss combinations (2) and (3) and exchangeable normal priors also. One final point worth mentioning is that since the loss function $c_1\ ICS(\underset{\sim}{\theta},a) + c_2|a|$ depends only on the relative ranking of the θ_i, the results of this study extends (approximately) to problems that can be transformed monotonically (approximately) to the normal populations problem.

Thus the result extends to the problem of selecting a subset of log-normal populations in terms of the means. The result also sheds light, for example, on the problem of selecting a subset of binomial populations since the problem can be transformed approximately into the normal populations problem.

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Table IA, k = 3

This table gives the estimated Bayes risk, the regret incurred by the best maximum type procedure, and the regret incurred by the best average type procedure in that order. The numbers in the parenthesis are the estimated standard deviations of the estimates.

$\sqrt{r}$ \ c_1/c_2	3^1	$3^{1.5}$	3^2	$3^{2.5}$	3^3	$3^{3.5}$	3^4
$(1.8)^{-4}$	.7217(.0008) .0012(.0001) .0000(.0000)						
$(1.8)^{-3}$	.6985(.0014) .0025(.0003) .0000(.0000)	.4841(.0001) .0000(.0000) .0000(.0000)					
$(1.8)^{-2}$	.6597(.0024) .0035(.0004) .0003(.0001)	.4779(.0008) .0012(.0002) .0000(.0000)	.2995(.0002) .0000(.0000) .0000(.0000)				
$(1.8)^{-1}$	.5930(.0039) .0034(.0005) .0017(.0003)	.4467(.0024) .0026(.0004) .0011(.0003)	.2910(.0010) .0009(.0002) .0002(.0001)	.1793(.0003) .0003(.0001) .0000(.0000)	.1070(.0001) .0001(.0000) .0000(.0000)		
$(1.8)^{0}$	.5133(.0054) .0030(.0005) .0066(.0011)	.3765(.0040) .0028(.0004) .0041(.0008)	.2563(.0023) .0019(.0003) .0023(.0005)	.1657(.0012) .0011(.0002) .0007(.0002)	.1007(.0006) .0006(.0001) .0007(.0002)	.0605(.0003) .0002(.0001) .0003(.0001)	.0359(.0001) .0001(.0000) .0001(.0000)
$(1.8)^{1}$	.4196(.0059) .0012(.0004) .0087(.0012)	.3017(.0045) .0015(.0003) .0116(.0015)	.2089(.0030) .0016(.0003) .0084(.0010)	.1288(.0018) .0009(.0002) .0053(.0006)	.0827(.0010) .0009(.0002) .0024(.0003)	.0515(.0006) .0003(.0001) .0012(.0002)	.0312(.0003) .0002(.0000) .0008(.0001)
$(1.8)^{2}$	.3521(.0054) .0003(.0001) .0075(.0012)	.2496(.0044) .0010(.0002) .0262(.0025)	.1580(.0029) .0004(.0002) .0149(.0015)	.1011(.0018) .0003(.0001) .0102(.0010)	.0662(.0012) .0002(.0001) .0068(.0006)	.0396(.0007) .0002(.0001) .0036(.0004)	.0232(.0004) .0001(.0000) .0022(.0002)
$(1.8)^{3}$	.3016(.0042) .0000(.0000) .0047(.0009)	.2043(.0034) .0001(.0001) .0162(.0023)	.1343(.0025) .0005(.0002) .0192(.0019)	.0827(.0016) .0000(.0000) .0142(.0014)	.0528(.0010) .0001(.0001) .0092(.0008)	.0307(.0006) .0000(.0000) .0054(.0005)	.0193(.0004) .0001(.0000) .0029(.0003)
$(1.8)^{4}$	.2785(.0039) .0000(.0000) .0023(.0007)	.1886(.0029) .0001(.0001) .0129(.0022)	.1186(.0019) .0001(.0001) .0153(.0024)	.0738(.0013) .0001(.0001) .0171(.0019)	.0455(.0008) .0000(.0000) .0103(.0012)	.0268(.0005) .0001(.0000) .0073(.0005)	.0152(.0003) .0000(.0000) .0044(.0003)

Table IIA, k = 3

This table gives the estimated constants d_{max} and d_{avg} corresponding to the best maximum type and the best average type procedure in that order. The numbers in the parenthesis are their associated P*.

$\sqrt{r}$ \ c_1/c_2	3^1	$3^{1.5}$	3^2	$3^{2.5}$	3^3	$3^{3.5}$	3^4
$(1.8)^{-4}$	0.42(.458) 0.00(.500)						
$(1.8)^{-3}$	0.38(.446) -0.05(.484)	4.47(.998) 3.31(.997)					
$(1.8)^{-2}$	0.44(.464) -0.10(.467)	2.58(.939) 1.86(.936)	4.43(.998) 3.31(.997)				
$(1.8)^{-1}$	0.42(.458) -0.22(.429)	1.67(.804) 1.03(.800)	2.84(.959) 1.99(.948)	3.49(.987) 2.76(.988)	4.36(.998) 3.42(.997)		
$(1.8)^{0}$	0.44(.464) -0.37(.381)	1.42(.747) 0.59(.685)	2.06(.875) 1.26(.848)	2.69(.948) 1.73(.921)	3.18(.977) 2.25(.967)	3.49(.987) 2.55(.981)	4.08(.996) 2.96(.992)
$(1.8)^{1}$	0.60(.513) -2.23(.034)	1.22(.695) 0.21(.568)	1.77(.824) 0.85(.756)	2.20(.896) 1.05(.804)	2.75(.953) 1.58(.901)	3.05(.971) 1.89(.939)	3.40(.985) 2.38(.974)
$(1.8)^{2}$	0.50(.483) -3.82(.001)	1.03(.642) 0.02(.507)	1.71(.812) 0.35(.612)	2.20(.896) 0.63(.697)	2.49(.930) 1.37(.868)	2.93(.965) 1.55(.897)	3.17(.977) 1.86(.936)
$(1.8)^{3}$	0.62(.520) -9.54(.000)	1.21(.693) -5.72(.000)	1.66(.802) -0.34(.391)	2.13(.886) 0.08(.526)	2.54(.935) 0.57(.679)	2.92(.964) 1.03(.800)	3.07(.972) 1.41(.875)
$(1.8)^{4}$	0.63(.523) -15.07(.000)	1.23(.698) -14.15(.000)	1.67(.804) -5.41(.000)	2.15(.889) -1.68(.085)	2.55(.936) -1.06(.193)	2.87(.961) 0.82(.748)	3.17(.977) 1.13(.822)

Table IIIA, k = 3

The rows in each box from top to bottom correspond to the Bayes, the best maximum type and the best average type procedure. The columns from left to right correspond to average subset size, probability of a correct selection, and proportions of times that the best maximum type procedure and the best average type procedure coincided with the Bayes procedure.

$\sqrt{r}$ \ c_1/c_2	3^1	$3^{1.5}$	3^2	$3^{2.5}$	3^3	$3^{3.5}$	3^4
$(1.8)^{-4}$	1.47 .5261						
	1.40 .5012 .73						
	1.48 .5311 .99						
$(1.8)^{-3}$	1.45 .5511	2.99 .9982					
	1.36 .5194 .74	3.00 .9991 1.00					
	1.43 .5436 .96	2.99 .9982 1.00					
$(1.8)^{-2}$	1.40 .5871	2.74 .9580	2.98 .9977				
	1.40 .5825 .77	2.77 .9614 .91	2.98 .9982 1.00				
	1.39 .5817 .94	2.75 .9598 .98	2.97 .9972 1.00				
$(1.8)^{-1}$	1.32 .6502	2.30 .9095	2.72 .9786	2.93 .9968	2.98 .9994		
	1.31 .6415 .82	2.26 .8987 .85	2.74 .9798 .92	2.93 .9968 .96	2.98 .9993 .99		
	1.30 .6413 .88	2.32 .9125 .91	2.74 .9806 .98	2.92 .9964 .99	2.98 .9995 1.00		
$(1.8)^{0}$	1.30 .7481	1.86 .9095	2.23 .9628	2.55 .9873	2.69 .9950	2.78 .9977	2.90 .9995
	1.29 .7424 .87	1.95 .9224 .86	2.26 .9642 .88	2.57 .9871 .88	2.71 .9952 .94	2.75 .9968 .94	2.91 .9995 .96
	1.31 .7435 .82	1.91 .9127 .86	2.29 .9669 .90	2.55 .9862 .94	2.72 .9957 .93	2.79 .9976 .95	2.88 .9991 .97
$(1.8)^{1}$	1.20 .8414	1.51 .9304	1.78 .9659	1.84 .9810	2.10 .9919	2.28 .9962	2.40 .9980
	1.25 .8540 .94	1.51 .9296 .92	1.80 .9660 .91	1.85 .9803 .91	2.13 .9922 .91	2.29 .9961 .93	2.41 .9979 .92
	1.01 .7648 .81	1.58 .9305 .81	1.89 .9679 .78	1.92 .9802 .79	2.17 .9919 .84	2.31 .9957 .84	2.49 .9983 .85
$(1.8)^{2}$	1.14 .9105	1.30 .9520	1.41 .9809	1.52 .9898	1.69 .9940	1.75 .9971	1.78 .9984
	1.13 .9051 .97	1.27 .9455 .95	1.42 .9813 .98	1.53 .9902 .97	1.70 .9941 .96	1.78 .9974 .97	1.78 .9983 .96
	1.01 .8579 .87	1.53 .9650 .71	1.55 .9802 .75	1.67 .9884 .74	1.93 .9956 .72	1.95 .9975 .76	1.99 .9988 .74
$(1.8)^{3}$	1.08 .9578	1.16 .9796	1.25 .9894	1.30 .9951	1.37 .9961	1.40 .9986	1.52 .9992
	1.08 .9586 1.00	1.16 .9790 .99	1.25 .9891 .97	1.30 .9950 .99	1.38 .9963 .99	1.41 .9988 .99	1.52 .9991 .98
	1.00 .9250 .92	1.04 .9377 .85	1.42 .9873 .76	1.48 .9917 .74	1.61 .9955 .69	1.65 .9983 .71	1.75 .9990 .72
$(1.8)^{4}$	1.04 .9753	1.12 .9897	1.13 .9941	1.18 .9968	1.22 .9981	1.24 .9991	1.21 .9996
	1.04 .9753 1.00	1.12 .9901 1.00	1.13 .9937 .99	1.18 .9971 1.00	1.22 .9981 1.00	1.25 .9992 .99	1.21 .9996 1.00
	1.00 .9597 .96	1.01 .9532 .89	1.11 .9749 .87	1.36 .9907 .73	1.42 .9947 .73	1.58 .9989 .63	1.59 .9997 .60

Table IB, k = 8

This table gives the estimated Bayes risk, the regret incurred by the best maximum type procedure, and the regret incurred by the best average type procedure in that order. The numbers in the parenthesis are the estimated standard deviations of the estimates.

$\sqrt{r}$ \ c_1/c_2	4	6	8	16	32	64	128	256	512
$(1.8)^{-4}$			.8372(.0010) .0038(.0004) .0000(.0000)						
$(1.8)^{-3}$		.8443(.0015) .0007(.0002) .0000(.0000)	.7929(.0018) .0069(.0007) .0001(.0000)	.4702(.0001) .0002(.0001) .0000(.0000)					
$(1.8)^{-2}$		.7866(.0032) .0059(.0008) .0003(.0001)	.7333(.0031) .0087(.0010) .0005(.0001)	.4597(.0009) .0045(.0007) .0001(.0000)	.2420(.0001) .0001(.0001) .0000(.0000)				
$(1.8)^{-1}$	.7553(.0047) .0022(.0006) .0006(.0002)	.6820(.0061) .0107(.0015) .0023(.0004)	.6202(.0050) .0116(.0014) .0026(.0004)	.4052(.0030) .0112(.0014) .0015(.0003)	.2313(.0008) .0029(.0004) .0005(.0001)	.1208(.0003) .0009(.0001) .0002(.0001)	.0618(.0001) .0001(.0000) .0000(.0000)		
$(1.8)^{0}$	.6128(.0083) .0063(.0011) .0034(.0007)	.5164(.0084) .0119(.0016) .0164(.0020)	.4668(.0072) .0110(.0014) .0115(.0015)	.3091(.0041) .0083(.0011) .0063(.0009)	.1836(.0023) .0059(.0006) .0038(.0005)	.1042(.0010) .0029(.0004) .0017(.0003)	.0546(.0005) .0015(.0002) .0010(.0002)	.0289(.0002) .0004(.0001) .0004(.0001)	.0147(.0001) .0002(.0000) .0001(.0000)
$(1.8)^{1}$	.4572(.0105) .0065(.0012) .0195(.0024)	.3868(.0086) .0045(.0009) .0264(.0031)	.3178(.0077) .0067(.0012) .0272(.0036)	.2064(.0051) .0062(.0009) .0208(.0032)	.1276(.0027) .0028(.0004) .0107(.0014)	.0710(.0016) .0022(.0003) .0082(.0010)	.0386(.0007) .0013(.0002) .0028(.0004)	.0212(.0004) .0008(.0001) .0017(.0002)	.0112(.0002) .0004(.0001) .0009(.0001)
$(1.8)^{2}$	.3443(.0093) .0030(.0009) .0210(.0031)	.2766(.0079) .0031(.0010) .0362(.0042)	.2352(.0073) .0029(.0007) .0507(.0054)	.1383(.0039) .0017(.0004) .0270(.0030)	.0841(.0025) .0016(.0003) .0183(.0016)	.0460(.0013) .0011(.0002) .0103(.0011)	.0257(.0007) .0007(.0001) .0055(.0006)	.0133(.0004) .0004(.0001) .0041(.0003)	.0071(.0002) .0002(.0000) .0020(.0003)
$(1.8)^{3}$	.3012(.0086) .0018(.0007) .0174(.0029)	.2182(.0061) .0010(.0004) .0299(.0046)	.1856(.0060) .0007(.0003) .0430(.0056)	.1008(.0035) .0007(.0003) .0387(.0040)	.0577(.0019) .0005(.0002) .0245(.0021)	.0307(.0010) .0003(.0001) .0156(.0024)	.0158(.0005) .0001(.0000) .0075(.0005)	.0085(.0003) .0001(.0000) .0036(.0003)	.0047(.0002) .0001(.0000) .0027(.0002)
$(1.8)^{4}$	.2487(.0060) .0002(.0001) .0086(.0021)	.1813(.0050) .0008(.0005) .0164(.0037)	.1466(.0045) .0010(.0007) .0299(.0049)	.0801(.0022) .0001(.0001) .0284(.0047)	.0453(.0014) .0001(.0001) .0241(.0028)	.0239(.0008) .0001(.0000) .0169(.0033)	.0125(.0004) .0000(.0000) .0107(.0007)	.0066(.0002) .0001(.0000) .0049(.0005)	.0030(.0001) .0000(.0000) .0021(.0001)

Table IIB, k = 8

This table gives the estimated constants d_{max} and d_{avg} corresponding to the best maximum type and the best average type procedure in that order. The numbers in the parenthesis are their associated P*.

$\sqrt{r}$ \ c_1/c_2	4	6	8	16	32	64	128	256	512
$(1.8)^{-4}$			1.24(.465) -0.05(.481)						
$(1.8)^{-3}$		0.00(.125) -1.40(.095)	1.24(.465) -0.09(.466)	4.82(.998) 1.78(.952)					
$(1.8)^{-2}$		0.46(.226) -0.94(.190)	1.22(.459) -0.17(.437)	3.17(.937) 1.33(.893)	4.56(.996) 2.62(.993)				
$(1.8)^{-1}$	0.00(.125) -1.69(.057)	0.70(.292) -0.78(.233)	1.13(.428) -0.35(.372)	2.32(.795) 0.60(.713)	3.16(.936) 1.44(.911)	4.23(.992) 2.10(.975)	4.82(.998) 2.75(.995)		
$(1.8)^{0}$	0.21(.166) -1.73(.053)	0.91(.356) -1.06(.161)	1.25(.469) -0.68(.262)	1.83(.660) -0.03(.489)	2.62(.859) 0.51(.683)	3.03(.921) 1.06(.839)	3.68(.974) 1.43(.909)	4.08(.988) 1.89(.961)	4.79(.998) 2.27(.983)
$(1.8)^{1}$	0.50(.236) -3.72(.000)	0.97(.376) -1.89(.039)	1.28(.479) -1.35(.103)	1.94(.694) -0.76(.239)	2.38(.809) -0.34(.375)	2.94(.910) 0.22(.582)	3.24(.944) 0.51(.683)	3.42(.959) 0.97(.818)	3.92(.984) 1.36(.898)
$(1.8)^{2}$	0.74(.304) -6.85(.000)	1.11(.422) -3.57(.000)	1.25(.469) -3.14(.002)	1.85(.666) -1.88(.039)	2.33(.798) -1.11(.150)	2.83(.894) -0.85(.213)	3.16(.936) -0.41(.351)	3.50(.964) 0.01(.504)	3.81(.980) 0.16(.559)
$(1.8)^{3}$	0.85(.338) -10.50(.000)	1.20(.452) -8.33(.000)	1.43(.530) -7.91(.000)	2.02(.717) -4.42(.000)	2.38(.809) -3.12(.002)	2.96(.913) -2.89(.003)	3.24(.944) -2.59(.008)	3.35(.953) -1.90(.038)	3.71(.976) -0.92(.195)
$(1.8)^{4}$	0.84(.334) -20.24(.000)	1.28(.479) -19.10(.000)	1.58(.580) -19.59(.000)	2.14(.750) -13.62(.000)	2.64(.862) -8.02(.000)	2.86(.899) -7.68(.000)	3.39(.956) -4.17(.000)	3.59(.970) -4.49(.000)	3.82(.980) -4.83(.000)

Table IIIB, k = 8

The rows in each box from top to bottom correspond to the Bayes, the best maximum type and the best average type procedure. The columns from left to right correspond to average subset size, probability of a correct selection, and proportions of times that the best maximum type procedure and the best average type procedure coincided with the Bayes procedure.

$\sqrt{r}$ \ c_1/c_2	4	6	8	16	32	64	128	256	512
$(1.8)^{-4}$			3.90 .5451						
			3.68 .5133 .48						
			3.92 .5475 .95						
$(1.8)^{-3}$		1.12 .2008	3.74 .5755	7.92 .9954					
		1.00 .1808 .89	3.59 .5484 .44	7.97 .9984 .94					
		1.12 .2107 1.00	3.75 .5760 .92	7.93 .9961 .99					
$(1.8)^{-2}$		1.72 .3690	3.46 .6069	7.18 .9600	7.92 .9978				
		1.71 .3596 .59	3.57 .6115 .48	7.42 .9706 .62	7.96 .9990 .96				
		1.70 .3645 .91	3.42 .6019 .81	7.17 .9595 .91	7.91 .9975 .99				
$(1.8)^{-1}$	1.11 .3334	2.11 .5551	3.12 .6916	5.46 .9104	7.03 .9810	7.63 .9965	7.93 .9996		
	1.00 .3031 .89	2.12 .5444 .62	3.11 .6773 .46	5.72 .9148 .45	7.28 .9858 .65	7.76 .9977 .81	7.94 .9996 .96		
	1.09 .3276 .94	2.06 .5449 .76	3.11 .6874 .73	5.49 .9110 .81	7.09 .9824 .84	7.66 .9968 .94	7.92 .9996 .99		
$(1.8)^{0}$	1.31 .5602	2.08 .7434	2.55 .7930	3.97 .9194	5.02 .9673	6.03 .9884	6.51 .9958	7.09 .9987	7.29 .9995
	1.15 .5199 .79	2.01 .7187 .66	2.75 .8056 .63	3.93 .9084 .56	5.25 .9686 .53	6.18 .9878 .60	6.67 .9955 .63	7.21 .9987 .76	7.47 .9996 .75
	1.23 .5360 .83	2.02 .7152 .61	2.54 .7788 .57	4.07 .9190 .62	5.01 .9633 .62	6.16 .9886 .71	6.54 .9950 .69	7.19 .9987 .76	7.40 .9996 .80
$(1.8)^{1}$	1.39 .7748	1.81 .8496	2.01 .8937	2.67 .9472	3.44 .9758	4.01 .9904	4.40 .9954	4.93 .9980	5.30 .9992
	1.37 .7617 .81	1.79 .8411 .84	2.02 .8875 .76	2.81 .9497 .69	3.56 .9767 .72	4.22 .9914 .69	4.48 .9948 .69	4.91 .9970 .65	5.53 .9992 .63
	1.01 .6566 .64	1.61 .7864 .60	2.18 .8837 .61	2.84 .9361 .51	3.55 .9684 .55	4.42 .9885 .50	4.60 .9942 .54	5.39 .9980 .55	5.85 .9993 .60
$(1.8)^{2}$	1.29 .8908	1.49 .9248	1.71 .9492	1.90 .9715	2.41 .9886	2.64 .9945	2.92 .9969	3.07 .9986	3.34 .9994
	1.31 .8920 .90	1.54 .9304 .89	1.67 .9409 .88	1.88 .9684 .84	2.45 .9880 .84	2.71 .9944 .82	2.96 .9966 .76	3.12 .9984 .79	3.44 .9994 .78
	1.00 .7933 .72	1.47 .8801 .64	1.58 .8753 .55	2.31 .9688 .51	3.04 .9893 .43	3.29 .9942 .45	3.58 .9965 .41	4.03 .9982 .32	4.14 .9989 .41
$(1.8)^{3}$	1.22 .9285	1.30 .9613	1.42 .9687	1.56 .9901	1.72 .9941	1.84 .9976	1.87 .9986	2.01 .9993	2.27 .9997
	1.27 .9375 .94	1.29 .9594 .96	1.42 .9679 .96	1.59 .9913 .95	1.72 .9936 .94	1.88 .9979 .95	1.91 .9988 .95	2.00 .9991 .92	2.26 .9996 .91
	1.02 .8556 .79	1.13 .8981 .75	1.16 .8872 .66	1.90 .9705 .45	2.43 .9913 .41	2.70 .9951 .36	2.81 .9984 .31	2.95 .9993 .39	3.60 .9996 .22
$(1.8)^{4}$	1.14 .9729	1.15 .9802	1.21 .9857	1.26 .9937	1.39 .9966	1.48 .9989	1.52 .9993	1.62 .9996	1.50 .9999
	1.12 .9689 .93	1.16 .9809 .98	1.24 .9883 .99	1.26 .9932 .99	1.41 .9971 .99	1.46 .9984 .96	1.53 .9994 .99	1.63 .9996 .95	1.48 .9998 .97
	1.02 .9321 .88	1.02 .9394 .88	1.02 .9282 .81	1.24 .9623 .72	1.97 .9898 .47	1.96 .9891 .49	2.84 .9988 .19	2.75 .9992 .25	2.60 .9999 .24

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~~In~~ this paper, ~~we~~ restricts attention to the problem of selecting a subset of normal populations. The approaches and results of some previous studies of subset selection procedures are discussed briefly. And then the result of a new Monte Carlo study is presented.

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