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METHODS OF OBTAINING MAXIMUM ELECTRICAL POWER IN SHORT PULSES,(U)

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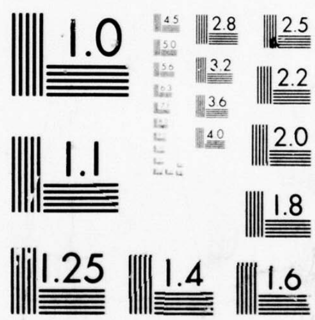

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METHODS OF OBTAINING MAXIMUM ELECTRICAL POWER IN SHORT PULSES

by

M. V. Babykin, A. V. Bartov

FOREIGN TECHNOLOGY DIVISION

EDITED TRANSLATION

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ë in Russian, transliterate as yë or ë. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

GREEK ALPHABET

Alpha	Α	α	α	Nu	Ν	ν	
Beta	Β	β		Xi	Ξ	ξ	
Gamma	Γ	γ		Omicron	Ο	ο	
Delta	Δ	δ		Pi	Π	π	
Epsilon	Ε	ε	ϵ	Rho	Ρ	ρ	ϱ
Zeta	Ζ	ζ		Sigma	Σ	σ	ς
Eta	Η	η		Tau	Τ	τ	
Theta	Θ	θ	ϑ	Upsilon	Υ	υ	
Iota	Ι	ι		Phi	Φ	φ	ϕ
Kappa	Κ	ϰ κ	ϰ	Chi	Χ	χ	
Lambda	Λ	λ		Psi	Ψ	ψ	
Mu	Μ	μ		Omega	Ω	ω	

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}

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rot	curl
lg	log

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METHODS OF OBTAINING MAXIMUM ELECTRICAL POWER IN SHORT PULSES

M. V. Babykin, A. V. Bartov

[Key words: thermonuclear studies, maximum powers, methods of obtaining, short pulses, energy accumulators.]

I. INTRODUCTION

In 1964-1967, electron accelerators were developed for large currents (100-200 kA) with an electron energy of more than 10 MeV and a total energy in the pulse of more than 100 kJ which use cold emission from a point [1-6]. This gave rise to hopes of implementing the idea of Harrison [7] who proposed initiating a thermonuclear reaction by heating a small volume of solid or liquid DT-mixture by the impact of a macroscopic particle of a solid ("macron") accelerated to a velocity greater than 10^8 cm/s. Harrison's idea remained unexecuted due to the absence of macron accelerators. With the appearance of powerful electron beams, a more realistic possibility appeared, utilizing the effect of the collective interaction of an electron beam with plasma and accomplishing the heating of some critical volume of thermonuclear fuel to the necessary temperature. This was indicated independently by Ye. K. Zavoysk in the USSR [8] and F. Winterberg in the United States [9].

The critical parameters necessary for the initiation of a thermonuclear reaction were calculated by Harrison [7] and also by other authors [10-13]. Harrison showed that some optimum temperature which is independent of volume exists at which the ratio of the energy which is released due to thermonuclear reactions to the energy lost to expansion is maximum. For the DT-mixture this temperature is 18.25 keV, and for DD-36.8 keV. With a temperature higher than the optimum the power expended on expansion begins to grow with an increase in temperature more rapidly than the yield of the thermonuclear reaction. Harrison also showed that some critical radius exists and, consequently, a critical volume in which the power which is released in this volume at a given temperature begins to exceed the power which is expended on the expansion of the critical volume. Obviously, the critical radius has a minimum with optimum temperature.

For the DT-mixture it equals $\sim$2 mm, i.e., for the initiation of a thermonuclear reaction it is necessary to heat approximately 30 mm^3 of DT-mixture to a temperature on the order of 20 keV. For this it is necessary to apply an energy on the order of 5-10 MJ to the critical volume during a time which is less than the time for the scattering of this volume and which, for order of magnitude, is equal to one or several nanoseconds. This means that a source of energy with an energy reserve $Q_0 = 10^6-10^7$ J and time of action $\tau = 10^{-9}-10^{-8}$ s is necessary.

There is no difficulty in accumulating the necessary amount of energy. Even now, accumulators with an energy of up to 100 MJ or more are known. It proves to be more difficult to contribute this energy to a load during a very short time, i.e., to create an energy source of great power and to contribute this power quickly. In order to illustrate how short the necessary time is, it is sufficient to point out that during a time of $3 \cdot 10^{-9}$ s the electrical pulse is propagated along a line or cable over a distance on the order of 1 m.

As M. V. Babykin and V. V. Starykh showed [13], it is possible to find the optimum temperatures at which the energy or power necessary for the initiation of a thermonuclear reaction have a minimum. For the DT-mixture this temperature equals respectively 15.4 and 12.5 keV. It is interesting to note that the most important power optimization is obtained with the least of all temperature cases.

The power necessary to initiate a thermonuclear reaction, according to estimates, should be on the order of 10^{14}-10^{16} W and the rate of increase in power on the order of 10^{23}-10^{25} W/s. This means that the basic problems are the creation of an energy source of great power and rapid commutation of this power.

The rate of increase of the electrical accumulators in the case where it is not restricted by the processes of current development in the commutator is inversely proportional to the total inductance of the commutator and the load (if we consider the accumulator non-inductive or if an accumulator is employed in the form of a matched electrical line). To ensure the necessary rate of power increase it is necessary that the inductance be less than 10^{-1}-10^{-2} cm, i.e., a single concentrated commutator is not suitable and it is necessary to employ either a very broad commutator or a set of commutators which operate in parallel. Hence it follows that one of the basic problems is the control of a broad distributed commutator or the synchronization of a large number of commutators with an accuracy on the order of 0.1 ns. For the same reason, a concentrated load is not suitable either since it would have introduced too great an inductance; furthermore, the surface of the load should be large for another reason - because the energy flow across a unit of load surface is limited by the break-through strength of the dielectric gap.

The purpose of this work is to examine the question, what determines the maximum powers and rate of increase in the power of various systems and under what conditions can parameters be obtained which are necessary for the initiation of a thermonuclear reaction. Here, we will consider only the final stages of energy accumulators which are able to create power pulses shorter than 10^{-7}-10^{-8} s. All the slower sources of energy such as, for example, explosion-magnetic generators which can ensure a current pulse no shorter than 5-10 μs will not be examined here although they, along with regular capacitor batteries and inductive and electromagnetic accumulators, can be used as preliminary sources of energy for fast systems and their comparative evaluation unquestionably will be necessary in the future.

The problem of rapid commutation of large powers and the precise synchronization of parallel discharges is still poorly developed and special studies are required for its solution. Therefore, examination of the commutation problems is not included in the goal of this work. In the subsequent presentation, we will proceed on the assumption that the commutation problem can be solved in some way. For example, one of the solution methods may be the employment of a controlled plasma cathode in which the functions of the commutator and electron emitter can be combined while another is the use of dischargers with multiple breakthrough [14].

2. ELECTROMAGNETIC ENERGY ACCUMULATORS

The energy which is stored in a unit volume occupied by a field equals

$$q_e = \frac{\varepsilon E^2}{8\pi}; \qquad q_m = \frac{\mu H^2}{8\pi}$$

It is interesting to note that the maximum energy density in a magnetic field may be considerably higher than in an electrical field since the maximum permissible fields are determined by different physical limitations. In the case of an electrical accumulator the maximum energy density is determined by the electrical strength of the selected dielectric (or the strength of the vacuum in the case of vacuum capacitors).

In contemporary paper-and-oil capacitors the electrical field which has been maintained for a long time by a dielectric attains 10^6 V/cm and the energy density in a unit of volume is 0.1 J/cm^3. The same field value is attained in the gas gaps in Van de Graaf accelerators. The strength of the vacuum gaps reaches 500 kV/cm. Fields on the order of 200-300 kV/cm were attained in water capacitors. There is a hope of obtaining a field on the order of 750-1000 kV/cm in the water of the field in pulses shorter than 0.3 μs. (V. M. Lagunov. Dissertation. Institute of Nuclear Physics, Siberian Branch of the Academy of Sciences USSR, Novosibirsk, 1969.) It can be hoped that the employment of a pulsed charge as well as operation in the line mode with a traveling wave will permit increasing the electrical field 8-10 times in the case of paper-oil or gas insulation, i.e., achieving field intensities on the order of 10^7 V/cm and energy density on the order of 5-10 J/cm^3.

It should be noted that autoelectronic emission begins to appear with fields greater than 10^7-10^8 V/cm; therefore, fields $\sim 10^8$ V/cm apparently are limiting.

The maximum permissible magnetic fields are determined by the magnetic strength of the coil material. It is possible to obtain fields on the order of 500-600 kOe without destroying the coil. With such fields, an energy of more than 1 kJ is contained in 1 cm^3. Unfortunately, the obtaining of these advantages is connected with the necessity to use an explosive current breaker which does not permit obtaining a commutation time shorter than 10^{-7} s and, as a rule, is not self-restoring, i.e., it requires disassembly and replacement after triggering.

The total volume and size of the accumulator system are determined by the permissible energy density, i.e., in the final analysis by the permissible field. For example, with an electrical field of 10^6 V/cm the energy density is 0.1 J/cm^3 and for the accumulation of 10^7 J 100 m^3 of dielectric are necessary. With a field of 10^7 W/cm, the necessary volume will be only 1 m^3. For comparison, it is interesting to note that in a magnetic accumulator with a field of $5 \cdot 10^5$ oersteds an energy of 10^7 J will be contained in a volume equal to only 10 liters but, unfortunately, the magnetic accumulator cannot release energy with the required speed.

Both types of accumulators (both electrical and magnetic) can be made both in the form of an oscillatory circuit as well as in the form of a line segment with distributed or concentrated parameters. Let us first examine accumulators in the form of lines with distributed parameters.

3. ELECTRICAL LINES WITH A CONSTANT WAVE IMPENDANCE

First, coaxial electric lines whose external conductor is the tank of a Van de Graaf accelerator and whose central conductor is the internal shield of the accelerator were used to obtain powerful electron beams. Subsequently, special lines including double lines (Bloomline lines) were constructed; however, these lines retained the characteristic features of high-voltage electrostatic accelerators. The basic parameters of the electron beams which have been obtained down to the present with the use of these lines are presented in Table 1.

The capacitance and inductance of a unit of length of coaxial line equal respecively

$$C = \frac{\varepsilon}{2 \ln \frac{R}{r}}; \qquad L = 2\mu \ln \frac{R}{r}.$$

The wave impendance of the line equals

$$\rho = \sqrt{\frac{L}{C}} = \frac{2\sqrt{\mu}}{\sqrt{\varepsilon}} \ln \frac{R}{r}.$$

The electrical field on the surface of an internal conductor equals

$$E = \frac{V}{r \ln \frac{R}{r}} .$$

With fixed V and R this field is minimum when $\ln \frac{R}{r} = 1$, i.e., when $\frac{R}{r} = 2{,}72$:

$$E_{min} = \frac{V}{r} .$$

In the case of a vacuum or air line a wave impedance of 60 Ω corresponds to this relationship of the radii. This relationship of the radii is selected in those cases where it is desirable for the line to withstand the maximum voltage (for example, for a transmission line or in accelerators when it is necessary to obtain the greatest energy of the particles; see [1]).

The energy per 1 centimeter of charged line equals

$$q \frac{\varepsilon E^2 \cdot r^2}{4} \ln \frac{R}{r} .$$

Here E - the electrical field on the surface of the internal conductor. With a given R and fixed electrical field (with the given strength of the dielectric $E = E_{max}$) the energy on 1 centimeter of line has a maximum width $\ln \frac{R}{r} = 1/2$, i.e., with $\frac{R}{r} = 1{,}65$. The power which is transmitted over the line is also maximum with $\ln \frac{R}{r} = 1/2$. With $\varepsilon = 1$ a wave impedance of 30 Ω corresponds to this ratio of the radii.

The maximum power developed by the line with a given strength of the dielectric E_{max} and fixed dimensions equals

$$W_{max} = \frac{c\sqrt{\varepsilon}\,\tau^2 E_{max}^2}{4}\ \frac{\text{эрг}}{\cancel{\text{сек}}\ \text{s}} = \frac{\sqrt{\varepsilon}\,\tau^2 E_{max}^2\,(\text{V/cm})}{120}\ W.$$

For example with τ = 100 cm, $E_{max} = 10^6$ W/cm the maximum power reaches 10^{14} W.

In the case of a matched line in the traveling wave mode the current and power equal

$$J = \frac{V}{\rho}; \qquad W = \frac{V^2}{\rho}.$$

Here V - the pulse amplitude.

If a charged line is connected to a load with resitance R = ρ using a commutator, the current and power equal

$$J = \frac{V_0}{\rho + R} = \frac{V_0}{2\rho}; \qquad W = \frac{V_0^2 R}{(\rho + R)^2} = \frac{V_0^2}{4\rho}.$$

If the matched load possesses inductance, then in the traveling wave mode the current and power in it increase in accordance with the laws

$$J = \frac{V}{\rho}\left(1 - e^{-\frac{2\rho}{L}t}\right); \quad W = \frac{V^2}{\rho}\left(1 - e^{-\frac{2\rho}{L}t}\right)^2.$$

The rate of power increase depends on the time as:

$$\frac{dW}{dt} = \frac{4V^2}{L}\left(1 - e^{-\frac{2\rho}{L}t}\right)e^{-\frac{2\rho}{L}t}$$

The rate of power increase has a maximum at the point of time when

$$e^{-\frac{2\rho}{L}t} = \frac{1}{2}; \quad t = \frac{L}{2\rho}\ln 2 = 0{,}693\,\frac{L}{2\rho}.$$

With this time value, the product of the exponential terms equals 1/4 and the maximum rate of increase in the power equals

$$\left.\frac{dW}{dt}\right|_{max} = \frac{V^2}{L}.$$

The power increases from zero to 0.9 W_{max} during time t = 1.5 L/ρ.

In the case of a line which is charged with a constant voltage and which is connected through a commutator to resistor R ≈ ρ, the voltage, current, and power in the load are determined by expressions

$$V_R = \frac{V_0 R}{R+\rho}\left(1-e^{-\frac{R+\rho}{L}t}\right); \quad J_R = \frac{V_0}{R+\rho}\left(1-e^{-\frac{R+\rho}{L}t}\right);$$

$$W = \frac{V_0^2 R}{(R+\rho)^2}\left(1-e^{-\frac{R+\rho}{L}t}\right)^2; \quad \frac{dW}{dt} = \frac{2RV_0^2}{(R+\rho)L}\left(1-e^{-\frac{R+\rho}{L}t}\right)e^{-\frac{R+\rho}{L}t}$$

With R = ρ the maximum rate of power increase

$$\left.\frac{dW}{dt}\right|_{max} = \frac{V_0^2}{4L},$$

i.e., four times less than in the traveling wave mode. The rate of power increase has a maximum at point in time

$$t = \frac{L}{R+\rho}\ln 2.$$

The power, just as in the traveling wave mode, increases from zero to 0.9 W_{max} during time t = 1.5 L/ρ.

If the line operates as an inductive accumulator, i.e., if a constant current J_0 flows through it, and then the current is broken by a breaker (explosive element) and the line is connected to resistance R ≈ ρ, then the voltage, current, power, and rate of power increase are given by the expressions:

$$V_R = \frac{\rho R J_0}{R+\rho}\left(1-e^{-\frac{R+\rho}{R\rho C}t}\right); \quad J = \frac{\rho}{R+\rho} J_0\left(1-e^{-\frac{R+\rho}{R\rho C}t}\right);$$

$$W = \frac{R\rho^2 J_0^2}{(R+\rho)^2}\left(1-e^{-\frac{R+\rho}{R\rho C}t}\right)^2; \quad \frac{dW}{dt} = \frac{2\rho J_0^2}{R+\rho}\left(1-e^{-\frac{R+\rho}{R\rho C}t}\right)e^{-\frac{R+\rho}{R\rho C}t},$$

where C - the spurious capacitance of the breaker and the load. The maximum rate of power increase in the case $R = \rho$ is attained at point in time $t = 0.693\ RC/2$ and equals

$$\left.\frac{dW}{dt}\right|_{max} = \frac{J_0^2}{4C}.$$

The power increase from zero to 0.9 W_{max} during time $t = 1.5\ \rho s$ (with $R = \rho$).

In the technique for obtaining powerful pulsed beams great propagation has been received by double lines which, in foreign literature, are most often called Bloomline lines. The advantage of double lines is convenience in commutation with a grounded end of the line and also the possibility of obtaining a double voltage in the interrupted mode as a result of the consecutive connection of the lines, and on a matched load a voltage which is equal to the initial charge voltage while in the case of a single line only half the charge voltage is obtained on a matched load.

A shortcoming of the Bloomline scheme is the necessity of employing charging inductances or inductors when charging lines from one source. These inductors shunt the load; therefore, their inductance cannot be made low and this interferes with obtaining a short time for charging the line. Two charging sources can be employed; however, this increases the cost of the device and complicates it due to the necessity for the precise synchronization of the sources.

In connection with the wide employment of the Bloomline scheme which uses coaxial lines arranged one inside the other, it is interesting to consider the question of selecting the relationships of radii and wave impedances of the lines with which it is possible to obtain the maximum voltages and powers with a given external diameter. Calculations show that with the placement of one line inside another higher voltages and powers are obtained with unequal wave impedances of the lines.

Let us first examine the problem of obtaining maximum voltage under the condition where the voltages and intensities of the field are the same in both lines:

$$E r \ln \frac{R}{r} = E R \ln \frac{R_{вн}}{R};$$

$$\ln \frac{R_{вн}}{R} = \frac{r}{R} \ln \frac{R}{r}; \qquad \frac{R_{вн}}{R} = e^{\frac{r}{R} \ln \frac{R}{r}}.$$

Here $R_{вн}$ is the external radius, r - the small radius of the interior line, R - the mean radius. Let us take, for example, the voltage on the external line

$$U = E_0 R \ln \frac{R_{вн}}{R}$$

(here E_0 - the permissible load) and we express it by the ratio $\frac{R}{r}$

$$U = E_0 R_{вн} \frac{r}{R} \ln \frac{R}{r} e^{-\frac{r}{R} \ln \frac{R}{r}}.$$

The voltage has a maximum as in the case of a single line with

$$\ln \frac{R}{r} = 1, \text{ i.e., with } \frac{R}{r} = e \approx 2{,}72.$$

The power of an asymmetrical double shaping line which is calculated with consideration of the pulse reflection coefficient is maximum with the load resistance $R_H = \rho_1 + \rho_2$ and equals

$$W = \frac{U^2}{\rho_1+\rho_2} \quad , \quad \text{or} \quad W = \frac{\sqrt{\varepsilon}\, E_0^2 r^2 \ln^2 \frac{R}{r}}{60\left(\ln\frac{R}{r} + \ln\frac{R_{вн}}{R}\right)} =$$

$$= \frac{\sqrt{\varepsilon}\, E_0^2 R_{вн}^2 \ln^2\frac{R}{r}}{60\left(\ln\frac{R}{r} + \frac{r}{R}\ln\frac{R}{r}\right)} \cdot \frac{r^2}{R^2} \cdot \frac{R^2}{R_{вн}^2}.$$

Expressing everything through $X = \frac{R}{r} = \frac{\rho_1}{\rho_2}$, we obtain

$$W = \frac{\sqrt{\varepsilon}\, E_0^2 R_{вн}^2}{60} \cdot \frac{\ln x\, e^{-\frac{2}{x}\ln x}}{x^2 + x} .$$

The power which is developed by an asymmetrical double line on a matched load R = $\rho_1 + \rho_2$ has a maximum with x = ρ_1/ρ_2 = 1.52 and, accordingly, wave impedances of the internal line $\rho_1 \approx \frac{25}{\sqrt{\varepsilon}}$ Ω and external line $\rho_2 \approx \frac{16.4}{\sqrt{\varepsilon}}$ Ω and equals

$$W_{max} = 1{,}05 \cdot 10^{-3} \sqrt{\varepsilon}\, E_0^2 R_{вн}^2 .$$

As calculations show, the power utilization coefficient in the main pulse comprises

$$K = \frac{4\rho_1\rho_2}{(\rho_1+\rho_2)^2}$$

and with ρ_1/ρ_2 = 1.52 = 0.95, i.e., is close to unity while with ρ_1/ρ_2 = 2 it loses about 11% of the power.

In order to have the opportunity to estimate the difference of the power from the maximum value with the deviation $X = \frac{\rho_1}{\rho_2} = \frac{R}{r}$ from the optimum value and also to select compromise versions, where it is desirable to have both high voltage and great power, calculations were conducted and graphs of the dependence of voltage and power on R/r were constructed both for a double and for single lines. From Table 2 and the graphs in Fig. 1 which were constructed according to the data from this table it can be concluded that the compromise versions, where the voltage and the power are less than the maximum values altogether by only 5-6%, are accomplished for single lines with $\frac{R}{\tau} \approx 2$ and $\rho = \frac{41.5}{\sqrt{\varepsilon}}$ Ω and for Bloomline lines with $\frac{R}{\tau} \approx 1.9$ and $\rho_1 = \frac{33.5}{\sqrt{\varepsilon'}}$ Ω and $\rho_2 = \frac{20.2}{\sqrt{\varepsilon}}$ Ω.

Table 2 presents the calculated data.

The voltage on the load of a single line depending on $x = \frac{R}{\tau}$

$$U_{RI} = E_0 R_{BH} y_1; \quad y_1 = \frac{1}{2} \frac{\ln x}{x}.$$

The voltage on the load of an unsymmetrical double line

$$U_{R\,II} = E_0 R_{BH} y_2; \quad y_2 = \frac{\ln x}{x} e^{-\frac{\ln x}{x}}.$$

The power on the load of a single line

$$W_{RI} = \frac{\sqrt{\varepsilon'} E_0^2 R_{BH}^2}{60} y_3; \quad y_3 = \frac{1}{4} \frac{\ln x}{x^2}.$$

The power on the matched load of an asymmetrical double line

$$W_{R\,II} = \frac{\sqrt{\varepsilon'} E_0^2 R_{BH}^2}{60} y_4; \quad y_4 = \frac{\ln x}{x} \cdot \frac{e^{-2\frac{\ln x}{x}}}{x+1}.$$

In the calculations a practical system of units was utilized (voltage in volts, power in watts).

4. ELECTRICAL LINES WITH VARIABLE WAVE IMPENDANCE

Let us now examine non-uniform lines, i.e., lines whose parameters (linear inductance L'_x and capacitance C'_x) vary along the length of the line, in which regard we will limit ourselves here to the case where the wave impedance of the line $\rho(x) = \frac{L'_x}{C'_x}$ increases monotonically in the direction of propagation of the pulse.

This case is most interesting for the problem being examined here since it permits increasing the pulse voltage (which would be very important since it is rather difficult to make a commuting device for a voltage V ~ 10^7).

As is known, a line with variable wave impedance (Fig. 2) possesses the property of transforming the pulse which is fed to it; in the case where the initial pulse has the shape of a voltage step with infinitely steep leading edge and height V_0, on the output of the line which is loaded for matched impedance (equal to the wave impedance at the end of the line $R = \rho(\ell)$, the pulse also has the form of a step (see Fig. 2) with height $V_1 = V_0 \sqrt{\frac{\rho(\ell)}{\rho(0)}}$ with a subsequent drop in accordance with some law which depends on the function $\rho(x)$ and the length of the line ℓ: $V_1 = V_0 \cdot \sqrt{\frac{\rho(\ell)}{\rho(0)}} \cdot f(t) = K \cdot V_0 \cdot f(t)$,

$\sqrt{\frac{\rho(\ell)}{\rho(0)}} = K$ - the pulse transformation coefficient.

Thus, in a non-uniform line (without losses) the pulse peak is distorted as a result of reflections (in an actual line, of course, the steep slope of the pulse front will also occur due to losses at high freqeuncy), i.e., an increase in pulse amplitude is accompanied by the reflection of part of the transmitted energy which is low only in the case of a sufficiently short pulse duration τ and becomes noticeable with $\tau \sim \frac{1}{C} \cdot \frac{\rho}{d\rho/dx} \sim \frac{\ell}{C}$ with $\frac{\Delta\rho}{\rho} \sim 1$, where C - the rate of pulse propagation along the line. For a quantitative estimate of the fraction of energy which is transmitted to the load, it is necessary to know the function $f(t) = \frac{1}{K} \frac{V_1(t)}{V_0}$ (usually called the transient function); its calculation for a non-uniform line is extremely difficult in the majority of cases since the system of basic equations of the line

$$-\frac{\partial u}{\partial x} = L'(x) \frac{\partial i}{\partial t},$$

$$-\frac{\partial i}{\partial x} = C'(x) \frac{\partial u}{\partial t}$$

does not lead to a wave equation as for a uniform line.

For one of the simplest cases of the so-called exponential line in which $L'(x) = L_0 e^{\gamma x}$, $C'(x) = C_0 \cdot e^{-\gamma x}$ and accordingly $\rho(x) = \rho(0) \cdot e^{\gamma x}$, detailed calculations were accomplished for the first time in work [15] and exact solutions were obtained. Let us present as an example one of the graphs from this work (Fig. 3) which presents the time dependence of the calculated value of a line's efficiency which is equal to the ratio of the energy transmitted to the load to the energy which is fed by a given point in time to the input of the line (for times less than the duration of the applied pulse).

From these curves it is evident that if we want to accomplish the transmission of energy with efficiency n = 0.6 and transformation coefficient $K = \sqrt{\frac{\rho(\ell)}{\rho(0)}} = e^{\frac{\gamma \ell}{2}} = 6$, we need: $\frac{\gamma ct}{2} \approx 0{,}5$, i.e., $\ell = 2ct \ln 6 \sim 3{,}6ct$. With $t = 10^{-8}$ s, $C = 2 \cdot 10^{10}$ cm/s (as in a regular high-frequency cable) $\ell = 7.2$ m*.

5. LIMITING POWER VALUES OF ELECTRIC LINES

The limiting power flux which flows through 1 cm^2 of line cross section as well as through a unit of load surface in the case where it is matched with the line is given by the Umov-Pointing formula if we substitute in it the maximum electrical field which is permissible for a given dielectric

$$W_S = c\sqrt{\varepsilon}\,\frac{[\bar{E} \times \bar{H}]}{4\pi}.$$

*As follows from the theory of nonuniform lines, for the case in Fig. 2, i.e., in the case of $R_i = \rho(0)$, the exponential line is optimum, i.e., with a change in wave impedance $\rho(x)$, in accordance with another law, with assigned value of transformation coefficient and efficiency a greater line length is needed.

In a traveling wave E = H, consequently

$$W_S = c\sqrt{\varepsilon} \cdot \frac{E^2}{4\pi}.$$

The limiting values of the power flux for different values of E with $\varepsilon = 1$ are presented in Table 3.

For real dielectrics this value must be multiplied by $\sqrt{\varepsilon}$. It is interesting to note that the power flux per unit of area depends considerably more strongly on the electrical strength than on the dielectrical constant ε.

Therefore, the dielectric should be compared for the product $\sqrt{\varepsilon}E^2$ or $\sqrt[4]{\varepsilon}E$. It is interesting to compare, as an example, such dielectrics as transformer oil ($\varepsilon = 2.2$) and water ($\varepsilon = 81$). The water may become more advantageous than oil when

$$E_B > \sqrt[4]{\frac{\varepsilon_M}{\varepsilon_B}}\, E_M = 0{,}4 E_M.$$

In a comparison with castor oil ($\varepsilon = 4.5$) the electrical strength of the water should be greater than half the strength of the oil

$$E_B > 0{,}5 E_M.$$

Since the power flux is limited by the strength of the dielectric, this means that the area of the line's cross section and the area of the load surface cannot be lower than some value

$$S \geqslant \frac{W}{W_S}.$$

Table 4 presents the minimum permissible values of radii of line and load with an intensity of the electrical field equal to 10^7 V/cm and a distance between the electrodes of 1 cm ($V = 10^7$ V). From Table 4 it is evident that the dimensions of the load turn out to be impermissibly large.

However, it is possible to reduce the dimensions of the load through an increase in the power flux which, in the general case, is determined by the product [E x H] and, with the same E, can be increased in comparison with the traveling wave mode if H > E. (This, for example, occurs in a regular oscillatory circuit.)

Such a situation can also be accomplished if, for example, we replace a load of annular shape with radius r_1 and resistance R which has been matched with a line having wave impedance $\rho(r_1) = R$, with a load which is less than the radius $r_2 < r_1$, but with the same resistance R and we connect it with the former line by a segment of a disk line (see Fig. 4) which, of course, introduces additional inductance and leads to an elongation of the pulse front. This permits rasing the density of the energy flux and obtaining the necessary total power on a load of smaller radius if, of course, we can permit some elongation of the pulse front.

Actually, in this case, as is evident from Fig. 4, we are dealing with a non-uniform line which has not been matched at the point where the load is connected, since $R = \rho(r_1) < \rho(r_2)$. Therefore, a pulse having the shape of a step with height V_0 at point r_1 will have amplitude $V = V_0\sqrt{\rho_2/\rho_1}$ in reaching r_2.

As a result of reflection (as a result of the line's mismatch) we obtain on load R $V_R = V(1+\alpha)$, where $\alpha = \frac{R-\rho(r_2)}{R+\rho(r_2)} = \frac{\rho(r_1)-\rho(r_2)}{\rho(r_1)+\rho(r_2)}$ - the reflection coefficient of the pulse from load R. As a result, we obtain

$$V_R = V_0\sqrt{\frac{\rho_2}{\rho_1}}\cdot\frac{2\rho_1}{\rho_2+\rho_1} = V_0\sqrt{\frac{\rho_2}{\rho_1}}\cdot\frac{2}{\rho_2/\rho_1+1}.$$

Accordingly $$\eta_0 = \frac{W_2}{W_0} = \left(\frac{V_2}{V_0}\right)^2 = \frac{\rho_2}{\rho_1}\cdot\frac{4}{(\rho_2/\rho_1+1)^2}.$$

Thus, with $\rho_2/\rho_1 = 5$ $\eta_0 = \frac{20}{36} \approx 0{,}56$, i.e., at the moment of the pulse's arrival the power equals 56% of the pulse power; then during a time on the order of $\tau \sim \frac{z_1 - z_2}{c} \approx \frac{z_1}{c}$ the value of η reaches 1 (see Fig. 4) and the density of the energy flux increases during time τ from $P_0 = \eta_0 P_1 \cdot \frac{z_1}{z_2}$ to $P_2 = P_1 \cdot \frac{z_1}{z_2}$ and exceeds the initial density of energy flux 5 times in the example under consideration with the same field intensity at the place where the load is connected. It should be noted that at any intermediate point z between z and z_2 a traveling wave mode exist during time $\tau \simeq \frac{2(z - z_2)}{c}$ during which the increase in density of the energy flux is accompanied by an increase in field intensity to $E > E_0$, assigned initially.

However, if the pulsed (for time $\tau \sim \frac{2(z_1 - z_2)}{c}$) electrical strength of the line's dielectric $E_{имп}$ noticeably exceeds E_0, the described procedure can be used to attain an increase in the density of energy flux. Furthermore, the value of the transformation coefficient of the segment of non-uniform line under consideration for an actual pulse with a build-up front $\tau_ф$ less than the value presented here for a stepped pulse is approximately $K = \frac{V}{V_0} = \frac{E}{E_0} \approx \sqrt{\rho_2/\rho_1} \cdot \left(1 - \frac{\tau_{ф}}{\tau_{сп}}\right)$, where $\tau_{сп} \sim \left(\frac{\Delta f}{\Delta t}\right)^{-1}$ a constant voltage drop on the output of the non-uniform line (f - the line's transition function).

If $\tau_{сп}$ is small, i.e., in the case where $\frac{z_1 - z_2}{c} < \tau_{фр}$ (more precisely $\frac{\rho}{d\rho/dz} \cdot \frac{1}{c} < \tau_{фр}$), the segment of line (from z_1 to z_2) does not possess transforming properties and it can be replaced in an approximate examination by the concentrated inductance L which is equal to the inductance of this line segment $L = 2d\, \ln \frac{z_1}{z_2}$ (Fig. 5). The presence of inductance L leads to an increase in the front of the initial pulse by the value $\tau \sim \frac{L}{R} = \frac{L}{\rho(z_1)}$.

Here, also, an increase in the energy flux density is accompanied by an increase in the time during which the pulse power achieves maximum value and by the emergence of a reflected wave (in point τ_1 as a result of mismatch), and consequently overvoltage $E/E_0 > 1$ for the time $\tau \sim L/R$. Here, $E/E_0 \leq 2$ and in the case of an actual pulse with front τ_ϕ this value may prove to be a little greater than one.

With the operation of short lines in the mode of connection to an inductive load the system can degenerate into a regular oscillatory circuit with concentrated parameters. Systems with concentrated parameters, evidently, may also be of interest; however they require a specific calculation.

6. SOME PROPERTIES OF SYSTEMS WITH CONCENTRATED PARAMETERS. AN APERIODIC INDUCTIVE ACCUMULATOR

Of the general systems with concentrated parameters most interesting for the problem being examined here is the dependence of the extreme power $P_{\text{экст}}$ which is released in the load of the oscillatory circuit in the current maximum on the relationship between the resistance of the load R and the characteristic resistance of the circuit $\rho_0 = \sqrt{\frac{L}{C}}$, i.e., on the values of the circuit's attenuation $\delta = 1/Q = \frac{R}{\rho}$ (or $\delta = \frac{\rho}{R}$ for the parallel connection of the load).

In the case of connecting a capacitive accumulator which has been charged to a voltage V_0 to an active load which possesses some spurious inductance, an oscillatory circuit is obtained with the series connection of resistance R (6). In this case, the current varies in accordance with the law $J(t) = \frac{V_0}{\omega_1 L} e^{-\frac{R}{2L}t} \sin \omega_1 t_1$, where $\omega_1 = \sqrt{\frac{1}{LC} - \left(\frac{\rho}{2L}\right)^2}$.

In the case of a break in the current in the circuit of the inductive accumulator an oscillatory circuit is obtained from the inductance and spurious capacitance which is parallel to the load. In this case the voltage on the load varies in accordance with the following law

$$U(t) = \frac{J_0}{\omega_1 C} e^{-\frac{1}{2RC}t} \sin \omega_1 t \, ,$$

where

$$\omega_1 = \sqrt{1/LC - 1/(2RC)^2}.$$

Combining the condition of the maximum power for time and for R, we can obtain the condition of maximum peak power in the load (matching condition)

$$\sqrt{\frac{1}{x^2} - 1} = tg \frac{(1+x^2)}{2} \sqrt{\frac{1}{x^2} - 1} \, ,$$

where $x = \frac{R}{2\omega_0 L}$ in the case of the series connection of R and $x = \frac{1}{2R\omega_0 C}$ for the parallel connection of the load.

This equation has two solutions: x = 0.552 and x = 1. The first of them corresponds to the power maximum and the second - to the bending point on the curve which corresponds to the transition to an aperiodic mode.

The maximum value of $P_{экст}$ (Fig. 6) equals 0.68 P_0, where

$P_0 = \frac{CV^2}{2}\omega_0 = \frac{LJ^2}{2}\omega_0$ - the initial reactive power of an unloaded circuit. At the bending point the power which is developed on the load is 0.54 P_0. With x = 0.55 the minimum power value is attained at point in time $t = \frac{1.23}{\omega_0}$, and with x = 1 at point in time $t = \frac{1}{\omega_0}$.

These calculations determine the optimum (to obtain maximum power) value of the characteristic resistance of the circuit if the load resistance R is considered given. Thus, for example, if the load of a circuit is the accelerating gap of an accelerator, its resistance with given geometry may not be less than some value due to the space charge created by the electrons.

It is of interest to examine separately the properties of an inductive accumulator which is discharged to the load resistance aperiodically with a rapid break in the current. This accumulator may prove to be more compact and cheaper, especially under the condition where the electrical strength of the medium which is formed during the transition of the breaking element to the non-conducting state and, namely, the burst products of the metal foil or plasma which has been converted to a poorly conducting state as a result of some instability, proves to be greater than the strength of the dielectrics which are used in the output devices in systems with capacitive accumulation. Questions of obtaining large powers with the aid of magnetic accumulators during their operation on an inductive load to obtain strong rapidly increasing magnetic fields were considered earlier by several authors [16], [17]. We will not be interested in questions of the speed and effectiveness in the transmission of energy from an inductive accumulator to an active load. The basic factor which limits the power and rate of increase in power in the case of inductive accumulation is the rate of current cutoff.

The minimum current cutoff times obtained in the experiment reach $\sim 10^{-7}$ s and the maximum rates of current change approach 10^{13} A/s [16, 17].

The maximum power developed by an inductive accumulator equals

$$W = J \cdot V = L J \frac{dJ}{dt},$$

and the rate of increase in power is proportional to $\left(\frac{dJ}{dt}\right)^2$

$$\frac{dW}{dt} \sim L \left(\frac{dJ}{dt}\right)^2.$$

To attain powers of 10^{14}-10^{16} W and a rate of power increase of 10^{23}-10^{25} W/s, currents of 10^{6}-10^{7} A and a rate of current cutoff $\frac{dJ}{dt} \sim 10^{14}$-$10^{16}$ A/s are necessary.

A schematic diagram of an inductive accumulator which operates on an active load is presented in Fig. 7. To simplify the problem, we assume that the current in the breaking element varies according to the exponential law

$$J_p = J_0 \cdot e^{-t/\tau_1}$$

with characteristic time τ_1 which, in the general case, is not equal to the characteristic time for the discharge of the accumulator to the load

$$\tau_0 = \frac{L}{R}.$$

The equations of the electrical circuit are written as follows:

$$J_p + J_H = J_n,$$

$$\frac{dJ_p}{dt} + \frac{dJ_H}{dt} = -\frac{R}{L} J_H,$$

$$\frac{dJ_H}{dt} + \frac{1}{\tau_0} J_H = \frac{J_0}{\tau_1} e^{-t/\tau_1}.$$

The solution of this system is

$$J_H = J_0 \frac{\tau_0}{\tau_1 - \tau}\left(e^{-t/\tau_1} - e^{-t/\tau_0}\right).$$

With $\tau_1 = \tau_0$ the solution of the initial equation has a different form

$$J_H = J_0 \frac{t}{\tau_0} e^{-t/\tau_0}.$$

In the case ($\tau_1 \neq \tau_0$) the current achieves a maximum with t

$$t = \frac{\tau_0 \tau_1}{\tau_0 - \tau_1} \ln \frac{\tau_0}{\tau_1},$$

$$J_{max} = J_0 \frac{\tau_0}{\tau_0 - \tau_1} \left(e^{-t/\tau_0} - e^{-t/\tau_1} \right).$$

With $\tau_1 \ll \tau_0, t = \tau_1 \ln \frac{\tau_0}{\tau_1}$ and $J_{max} \approx J_0$.

In the second case the current maximum occurs with $t = \tau_0$ and equals

$$J_{max} = \frac{J_0}{e}.$$

The maximum power $W_{max} = J_{max}^2 R$ and with $\tau_1 = \tau_0$ equals

$$W_{max} = \frac{J_0^2 R}{e^2} \approx 0{,}135\, J_0^2 R,$$

$$W_{max} = \frac{L J_0^2}{e^2 \tau_0} = 0{,}27 \frac{L J_0^2}{2\tau_0} \approx 0{,}135\, L J \frac{dJ}{dt}.$$

The maximum rate of power increase $\left(\frac{dW}{dt}\right)_{max}$ is attained in the case of $\tau_1 \ll \tau_0$ with $t \approx \tau_1 \cdot \ln 2$, and in the case $\tau_1 = \tau_0$ at point in time $t = \left(1 - \frac{1}{\sqrt{2}}\right)\tau_0$ and equals

$$\left(\frac{dW}{dt}\right)_{max} = \frac{J_0^2 R}{\tau_0} \left(\sqrt{2} - 1\right) e^{-2+\sqrt{2}} \approx 0{,}23 \frac{J_0^2 R}{\tau_0}$$

or

$$\left(\frac{dW}{dt}\right)_{max} = 0{,}46 \frac{L J_0^2}{2\tau_0^2} = 0{,}23\, L \left(\frac{dJ}{dt}\right)_0^2.$$

The total energy which is released in a load in the case of $\tau_1 \neq \tau_0$ equals

$$Q = J_0^2 R \frac{\tau_0^2}{2(\tau_0 + \tau_1)} = \frac{L J_0^2}{2} \frac{\tau_0}{\tau_0 + \tau_1}.$$

With $\tau_1 = \tau_0$ this energy equals half the stored energy

$$Q = J_0^2 R \cdot \frac{\tau_0}{4} = \frac{L J_0^2}{4}.$$

Giving energy Q which should be distinguished by the accumulator, the time for liberation of the energy τ_0 and voltage V_{max} which it is desirable to obtain may comprise a system of equations which determine the necessary current value $\mathcal{J}_0$, accumulator inductance L, and load resistance R. With $\tau_1 \ll \tau_0$ this system of equations

$$\frac{L\mathcal{J}_0^2}{2} \doteq Q_0 \approx Q\,; \quad \frac{L}{R} = \tau_0\,, \quad \mathcal{J}_0 R \doteq V_{max}$$

has the following solutions:

$$\mathcal{J}_0 = \frac{2Q_0}{V_{max}\tau_0}\,, \quad L = \frac{V_{max}^2 \tau_0^2}{2Q_0}\,, \quad R = \frac{V_{max}^2 \tau_0}{2Q_0}\,.$$

With $\tau_1 = \tau_0$ the system has the form

$$\frac{L\mathcal{J}_0^2}{4} = \bar{Q}\,, \quad \frac{L}{R} = \tau_0\,, \quad \frac{\mathcal{J}_0 R}{e} \approx V_{max}$$

(here e = 2.72 - the base of natural logarithms) and gives the following solutions:

$$\mathcal{J}_0 = \frac{4Q}{eV_{max}\tau_0}\,, \quad L = \frac{e^2 V_{max}^2 \tau_0^2}{4Q_4}\,, \quad R = \frac{e^2 V_{max}^2 \tau_0^2}{4Q_4}\,.$$

To illustrate the possibilities of the method of inductive accumulation Table 5 is presented below and provides $\mathcal{J}_0$, L, R, and W at different Q, τ_0, and V_{max} for the case $\tau_1 = \tau_0$.

In Table 5 we accepted $\tau_1 = 10^{-7}$ s everywhere, which is the shortest time obtained in the experiment. If we assume that in the future this time will be decreased to 10^{-8} s, then here the necessary current increases by an order, the inductance which is needed will be 100 times less, the resistance decreases 10-fold, and the power increases 10-fold.

From this table we can draw the conclusion that the inductive accumulators unquestionably are of interest for this problem and their capabilities must be studied experimentally.

In particular, perhaps, we will succeed in obtaining shorter current cutoff times using the instabilities of plasma with current or the instabilities of the current in plasma which lead to the turbulent state of the plasma and a sharp decrease in its conductance. Such phenomena have already been observed in the experiment of N. V. Filippov and others [18]. With the collapse of a non-cylindrical plasma shell a very fine current channel is formed and a discontinuity is formed on the oscillograms of the current and voltage which testifies to a certain change in current. The authors explain this phenomenon by the explosion of the material of the anode through which a current of very great density flows. However, it is not excluded that this occurs as a result of the development of turbulence in the plasma and a strong decrease in its conductance. At the moment of the emergence of the discontinuity on the oscillograms, the appearance of X-ray radiation from the anode with an energy which exceeds by several-fold the applied voltage and an intensity which corresponds to the flux of accelerated electrons on the order of half the complete discharge current $\sim 0.5 \cdot 10^6$ A is observed. The total energy of the electron beam in the experiment of N. V. Filippov and others reaches 10^5 J, the energy of the electrons - 100 keV, and the duration of the current of accelerated electrons on the order of 10^{-7} s. These data, evidently, tell us that an inductive accumulator can be created with a plasma current interrupter for great powers; however, special experiments are necessary for this.

Thus, the data which have been presented show that inductive accumulators can be of interest for obtaining powers up to 10^{14} W and higher under the condition where the problem of the breaking of large currents $\gtrsim 10^7$ A during a time shorter than 10^{-7} s will be solved. This method of accumulating energy would be especially convenient when using explosive-magnetic energy generators which utilize the energy of explosives to generate experimental magnetic fields.

7. LIMITATION OF CURRENT BY A SPACE CHARGE AND THE MATCHING OF THE ENERGY SOURCE WITH THE LOAD

In contrast to current in a metal of plasma, with the emission of electrons into a vacuum there is no space compensation for the electric charge and the space charge can change the electrical field at the cathode so much that this affects the emission current. It is known, for example, that with a thermoelectronic emission the field can even change direction, forcing a large portion of the electrons to return to the cathode. The current of auto-electronic emission strongly depends on the electrical field; therefore, some self-consistant value of electrical field is established at the cathode which ensures the emission current, the space charge of which leads to a reduction in the initial electrical field at the cathode right up to this value. Limiting will be a current which decreases the field at the cathode to zero since with a zero field both the autoelectronic emission as well as the emission from the plasma cathode knowingly cease.

The distribution of the potential, the field at the cathode, and the total current depending on the voltage can be found by integrating the Poisson equation

$$\Delta\varphi = -4\pi q, \text{ where } q = \frac{j}{v},$$

with consideration of the dependence of emission current on the electrical field (the Fowler-Nordheim law in the case of auto-electronic emission).

The limiting current which is calculated with consideration of the effect of the space charge alone will be overstated and will be the upper limit of the attainable current. Such an examination provides the opportunity for the approximate determination of the nature of the dependence of J on v and the lower limit of the accelerating gap's resistance, which is necessary for the calculation of the generator's matching with the load.

If a voltage is applied to the gap which is much greater than 0.5 MV, the electron will pass through the major portion of the path at a rate close to c. Therefore, for an approximate solution we can disregard the dependence of v on the potential difference which has been passed through, i.e., of v on x, and consider v = c. Then

$$\frac{d^2\varphi}{dx^2} = \frac{4\pi j}{c}.$$

The "minus" sign in the right side is replaced by a "plus," since the speed of the electrons is directed opposite to the current voltage. Integrating this equation once, we obtain

$$\frac{d\varphi}{dx} = \frac{4\pi j x}{c} + C_1.$$

From the condition with X = 0 we obtain $c_1 = E_k$, where E_k - the electrical field which is established at the cathode. Integrating for x from 0 to d, where d - the distance between the electrodes, we obtain

$$U = \frac{2\pi j d^2}{c} + E_K d.$$

Here U - the total voltage.

With large U, where $\frac{U}{d} \gg E_k$, the last term can be disregarded and we introduce the specific resistance of the gap:

$$U = \frac{2\pi j d^2}{c}; \quad \rho = \frac{U}{j} = \frac{2\pi d^2}{c}.$$

This means that with large current voltages the space charge is determined and is weakly dependent on E_k and on the properties of the emitter in general. This simple result was known earlier. It also is presented as the limiting case by V. A. Godyak, L. V. Dubovoy, and G. R. Zablotskaya [19].

For a more precise examination we can introduce the dependence of current density on E_k. If a current is determined by auto-electronic emission, the current density is determined by the

Fowler-Nordheim law and the corresponding system of equations is reduced to a transcendental equation which is easily solved graphically. It follows from the calculations that for the effective use of a high voltage, i.e., to obtain the maximum possible with the given current voltage and, consequently, the maximum power, most advantageous is the mode where the current is limited by the space charge and is weakly dependent on the intensity of the field at the cathode, which corresponds to the approximation examined above.

Thus, in disregarding the dependence of the electron speed on the potential difference which has been passed through, the current is proportional to the voltage between the electrodes and we can introduce the resistance of the gap which determines the current density in the limiting case with $E_k \ll E_0$, i.e., with very large voltages where the current is limited only by the space charge and does not depend on the emission mechanism. In the plane case this resistance per cm^2 equals

$$\rho_{\text{пл}} = \frac{U}{j} = \frac{2\pi d^2}{c},$$

or in a practical system of units

$$\rho_{\text{пл}} = 60\pi d^2 \text{ ом.см}^2.$$

In the cylindrical and spherical cases the following are obtained respectively

$$\rho_{\text{ц}} = \frac{2}{c}\left[R - r\left(\ln\frac{R}{r} + 1\right)\right]; \quad \rho_{\text{ц}} = 60\left[R - r\left(\ln\frac{R}{r} + 1\right)\right] \text{ ом}\cdot\text{см},$$

$$\rho_{\text{сф}} = \frac{1}{c}\left(\frac{r}{R} + \ln\frac{R}{r} - 1\right); \quad \rho_{\text{сф}} = 30\left(\frac{r}{R} + \ln\frac{R}{r} - 1\right) \text{ ом}.$$

From these expressions, it is possible to make an approximate estimate of the lower boundary of resistance which the gap possesses. The best approximation to the actual geometry of a single point or band emitter is a point in the form of a hyperboloid of rotation or hyperbolic cylinder arranged opposite a flat anode. The case of the hyperbolic point with a plane provides a resistance value on the order of 50 Ω.

The resistance which is introduced in the ultrarelativistic approximation determines the upper limit of the current which can be obtained in a vacuum gap without compensation of the charge. A more exact calculation with consideration of the dependence of v on x provides a smaller current value. Such a calculation for the plane case was done using electronic computers by V. A. Godyak, L. V. Dubovoy, and G. R. Zablotskaya [19]. This calculation showed that the ultrarelativistic approximation in the plane case can be used only with voltages greater than 5 MV. Unfortunately, the equations of the cylindrical and spherical cases could not be reduced to such a form that the results of the numerical calculations which have been mentioned could be used; therefore, now we cannot indicate a voltage beginning with which the ultrarelativistic approximation is valid in these cases. We can only express the assumption that this voltage does not differ greatly from the value for the plane case.

In connection with the presence of the limiting resistance of the diode, the limiting power of the beam which the diode can create also exists.

In a practical system of units the specific powers are expressed as:

plane diode $$\frac{W}{S} = \frac{E_0 (E_0 - E_K)}{60\pi} \approx \frac{E_0^2}{60\pi} \ W/cm^2$$

cylindrical $$\frac{W}{l} = \frac{E_{x_0} U}{60\left(\frac{R-r}{r \ln \frac{R}{r}} - 1\right)} \ W/cm^2$$

spherical $W = \dfrac{U^2}{30\left(\frac{r}{R} + \ln\frac{R}{r} - 1\right)}$ W.

In the plane case the specific power from 1 cm^2 is determined by the square of the intensity of the electrical field, and in the cylindrical case the power per unit of length is proportional to U^2 and inversely proportional to the radius of the cathode, i.e., the intensity of the field on the cathode is determined in the absence of current E_{k_0} also by the total voltage. In the spherical case the power from one point is proportional to the square of the voltage and is weakly dependent on the cathode parameters.

Let us examine briefly the questions of the matching of lines with a load. As the calculations which have been presented as well as the experimental data show, the gap with one point has a resistance on the order of 50-100 Ω and it can be matched only with a line having the same resistance. If the line has a lesser resistance, we can employ several points or use a distributed ring emitter. The number of points can be determined from the following approximate equality:

$$\rho = \frac{60 d_1}{\sqrt{\varepsilon} R} \approx \frac{60}{N}\ \Omega .$$

Here d_1 - the distance between the conductors of the line, R - the radius of the line.

Hence

$$N \approx \frac{\sqrt{\varepsilon} R}{d_1},$$

i.e., the points should be placed at a distance from each other on the order of d_1.

In the case of employment of a ring emitter we can utilize calculations of specific resistance per centimeter of length for the cylindrical case, considering that the current fills an angle on the order of 1 radian in the cross section of the cylinder. In this case, the resistance of the ring load with radius R equals

$$\rho_H = \frac{60d}{R},$$

where d - the distance between the cathode and the anode.

Equalizing the resistances

$$\frac{60d_1}{\sqrt{\varepsilon}R} \approx \frac{60d}{R};$$

we obtain the approximate matching condition

$$\frac{d_1}{d} \approx \sqrt{\varepsilon}.$$

Thus, employing a ring emitter, it can be matched with a line, selecting the ε of the line and the distance between the electrodes of the accelerating gap. Here it should be stressed once again that these matching calculations should be considered only as estimates since approximate expressions for the resistance of the accelerating gap are used in them.

8. CONCLUSION

From everything which has been stated above, we can draw the conclusion that the problem of obtaining electrical power of 10^{14}-10^{16} W in a short pulse is very difficult but not hopeless. To obtain such powers using the direct acceleration of an electron beam, megavolt electrical accumulators with a large cross sectional area are necessary to ensure the necessary power flux which is determined by the Umov-Pointing vector. This requirement is in certain contradiction to the requirement for a strong focusing of the beam; however, in accordance with theoretical concepts, we can also hope to focus tubular beams with a rather large diameter.

In conclusion, the authors consider it their duty to thank Ye. K. Zavoyskiy for his constant interest in this work and numerous discussions and L. I. Gudakov and I. N. Slivkov for useful discussions.

Table 1.

Фирма (1)	(2) Тип	(3) Энергия частиц, Мэв	(4) Ток пучка, ка	Длит. имп., нсек (5)	Энергия пучка, кдж (6)	Мощность, вт (7)	Скорость нараст. мощн., вт/сек (8)	Удельная энергия, кдж/см^2 (9)	Удельная мощность, вт/см^2 (10)	Примечание (11)
Айон физикс корпорейшен, Бурлингтон, Массачусетс (12)	FX-15	0,25	50	30	0,4	$1,3.10^{10}$	2.10^{18}	0,125	4.10^{9}	
	FX-25	1,6	25	30	1,1	4.10^{10}	5.10^{13}	0,33	$1,1.10^{10}$	линии на основе генератора Ван-де-Граафа (28)
	FX-35	2,5	50	30	3,7	$1,2.10^{11}$	2.10^{19}	0,9	3.10^{10}	
Филд эмиссион корпорейшен, Мак-Миннвилл, Орегон (13)		0,6	80 ка/см^2 kA/	3	0,15	5.10^{10}	5.10^{19} вт/сек.см^2 W/s,		5.10^{10}	генератор Аркадьева - Маркса (29)
		2	6	50	0,45	1.10^{10}	5.10^{17}	0,125	3.10^{10}	
Физикс интернейшнл компани, Сан-Леандро, Калифорния (14)	модель 730 (22)	3	50	30	5	10^{11}-10^{12}	10^{20}	2	10^{11}	(30) линия Блюмлейна
Гос. университет Сев. Каролины, Галей, Сев. Каролина (15)		3,5	30-50	30	3	10^{11}	10^{19}	10÷15	$(3÷5).10^{12}$	(30) линия Блюмлейна
Корнельский университет (16)		0,2-0,5	30-100	10		5.10^{10}				(30) линия Блюмлейна
Фирма Сандиа, Альбукерк, Нью-Мексико (17)	(23) Гермес II	12	170	85	175	2.10^{12}	7.10^{19}	2	2.10^{10}	(30) линия Блюмлейна
Физ.инт.комп. (18)	Снарк (24)	1	1 ма mA	60	30	10^{12}	5.10^{19}	-	-	(31) изоляция-майлар
Нэйвал рисёрч лаб. Вашингтон (19)	Гэмбл (25)	1	1 ма mA	50	55	10^{12}	6.10^{19}	-	-	(32) изоляция-вода
Максвелл лаб., Сан-Диего, Калиф. (20)	Блэк Джек (26)	0,8	0,8 ма mA	70	50	$6,5.10^{11}$	3.10^{18}	-	-	-"-
Генри Даймонд (21)	Аврора (27)	15	1,7 ма mA	100	2 Мдж MJ	2.10^{13}	5.10^{20}	-	-	(33) изоляция-масло

Key: (1) Firm; (2) Type; (3) Energy of particles, MeV; (4) Beam current, kA; (5) Pulse duration, ns; (6) Beam energy, kJ; (7) Power, W; (8) Rate of increase in circuit, W/s; (9) Specific energy, kJ/cm^2; (10) Specific power, W/cm^2; (11) Remarks; (12) Ion Physics Corporation, Burlington, Massachusetts; (13) Field Emission Corporation, McMinnville, Oregon; (14) Physics International Company, San Leandro, California; (15) North Carolina State University, Raleigh, North Carolina; (16) Cornell University; (17) Sandia Firm, Albequerque, New Mexico; (18) Physics International Company; (19) [word illegible] Research Laboratory, Washington; (20) Maxwell Laboratory, San Diego, Cal.; (21) Henry Diamond; (22) Model; (23) Hermes II; (24) Snark; (25) Gamble; (26) Black Jack; (27) Aurora; (28) Line on the basis of a Van de Graaf generator; (29) Ardad'yeva-Marks generator; (30) Bloomline line; (31) Insulation - mylar; (32) Insulation - water; (33) Insulation - oil.

Table 2.

X	y_I	y_2	y_3	y_4
I	0	0	0	0
$.II_{IO}I$	$4.33204_{IO}-2$	$7.94539_{IO}-2$	$I.9691I_{IO}-2$	$3.46957_{IO}-2$
$.I2_{IO}I$	$.75962_{IO}-I$	$I.30519_{IO}-I$	$.3I65I_{IO}-I$	$.50864_{IO}-I$
$.I3_{IO}I$	$I.009_{IO}-I$	$I.6493I_{IO}-I$	$.38807_{IO}-I$	$.56802_{IO}-I$
$.I4_{IO}I$	$I.20I57_{IO}-I$	$I.89990_{IO}-I$	$.42913_{IO}-I$	$.6I924_{IO}-I$
$.I5_{IO}I$	$I.35I46_{IO}-I$	$2.06208_{IO}-I$	$.45048_{IO}-I$	$.62969_{IO}-I$
$.I6_{IO}I$	$I.46832_{IO}-I$	$2.I8973_{IO}-I$	$4.58945_{IO}-2$	$.62766_{IO}-I$
$.I7_{IO}I$	$I.56052_{IO}-I$	$2.28452_{IO}-I$	$4.53979_{IO}-2$	$.6I928_{IO}-I$
$.I8_{IO}I$	$I.63263_{IO}-I$	$2.35550_{IO}-I$	$4.535I0_{IO}-2$	$.60597_{IO}-I$
$.I9_{IO}I$	$I.669_{IO}-I$	$2.40977_{IO}-I$	$4.44473_{IO}-2$	$5.92795_{IO}-2$
$.2_{IO}I$	$I.73272_{IO}-I$	$2.45063_{IO}-I$	$4.33I9I_{IO}-2$	$5.7767I_{IO}-2$
$.2I_{IO}I$	$I.76538_{IO}-I$	$2.48I45_{IO}-I$	$4.20566_{IO}-2$	$5.62243_{IO}-2$
$.22_{IO}I$	$I.79I86_{IO}-I$	$2.50459_{IO}-I$	$4.0724I_{IO}-2$	$5.46944_{IO}-2$
$.23_{IO}I$	$I.8I056_{IO}-I$	$2.52I22_{IO}-I$	$3.9360I_{IO}-2$	$5.3I907_{IO}-2$
$.24_{IO}I$	$I.8236I_{IO}-I$	$2.53292_{IO}-I$	$3.79960_{IO}-2$	$5.I733I_{IO}-2$
$.25_{IO}I$	$I.8325_{IO}-I$	$2.54065_{IO}-I$	$3.665_{IO}-2$	$5.03I29_{IO}-2$
$.26_{IO}I$	$I.83742_{IO}-I$	$2.54490_{IO}-I$	$3.53350_{IO}-2$	$4.89627_{IO}-2$
$.27_{IO}I$	$I.83925_{IO}-I$	$2.54640_{IO}-I$	$3.40603_{IO}-2$	$4.76400_{IO}-2$
$.28_{IO}I$	.I8333	.25457	$.32828_{IO}-I$	$.46380_{IO}-I$
$.29_{IO}I$	.I8355	.2543I	$.3I646_{IO}-I$	$.45I73_{IO}-I$
$.3_{IO}I$	.I8308	.2539I	$.305I3_{IO}-I$	$.440I3_{IO}-I$
$.3I_{IO}I$	.I8246	.25335	$.29430_{IO}-I$	$.4290I_{IO}-I$
$.32_{IO}I$	.I8I7I	.25270	$.28393_{IO}-I$	$.4I834_{IO}-I$
$.33_{IO}I$	.I8087	.25I95	$.27405_{IO}-I$	$.40808_{IO}-I$
$.34_{IO}I$	.I7995	.25II4	$.26464_{IO}-I$	$.39824_{IO}-I$
$.35_{IO}I$	.I7894	.25023	$.25563_{IO}-I$	$.38877_{IO}-I$
$.36_{IO}I$	.I7783	.24928	$.24706_{IO}-I$	$.37967_{IO}-I$
$.37_{IO}I$	.I7678	.24837	$.23869_{IO}-I$	$.37093_{IO}-I$
$.38_{IO}I$	.I7564	.24724	$.23III_{IO}-I$	$.36250_{IO}-I$
$.39_{IO}I$	.I7447	.246I7	$.22368_{IO}-I$	$.35439_{IO}-I$
$.4_{IO}I$	.I7327	.24506	$.2I659_{IO}-I$	$.34656_{IO}-I$
$.4I_{IO}I$	.I7206	.24393	$.20983_{IO}-I$	$.33903_{IO}-I$
$.42_{IO}I$	.I7083	.24280	$.20337_{IO}-I$	$.33I78_{IO}-I$
$.43_{IO}I$	.I6959	.24I63	$.I9720_{IO}-I$	$.32476_{IO}-I$
$.44_{IO}I$	.I6835	.24045	$.I9I30_{IO}-I$	$.3I800_{IO}-I$
$.45_{IO}I$	.I67II	.23927	$.I8567_{IO}-I$	$.3II45_{IO}-I$
$.46_{IO}I$	.I6585	.23808	$.I8028_{IO}-I$	$.305II_{IO}-I$
$.47_{IO}I$	.I646I	.23689	$.I75I2_{IO}-I$	$.29900_{IO}-I$
$.48_{IO}I$	.I6338	.23569	$.I70I9_{IO}-I$	$.29308_{IO}-I$
$.49_{IO}I$	.I62I5	.23449	$.I6546_{IO}-I$	$.28736_{IO}-I$
$.5_{IO}I$	$I.6093_{IO}-I$	.23329	$I.6093_{IO}-2$	$.28I8I_{IO}-I$

Table 3.

E, ~~в/см~~ W/cm	W_s, ~~вт/см²~~ W/cm²
10^{6}	$2,4.10^{9}$
10^{7}	$2,4.10^{11}$
10^{8}	$2,4.10^{13}$

Table 4.

W, ~~вт~~ W	τ
10^{13}	6 см
10^{14}	60 см
10^{15}	6 м
10^{16}	60 м

Table 5.

Q, ~~дж~~ J	τ_1, ~~сек~~ s	V_{max}, ~~в~~ V	J_0, ~~а~~ A	L, ~~генри~~ Henry	R, ~~ом~~ Ω	W, ~~вт~~ W
10^{7}	10^{-7}	10^{7}	$1,5.10^{7}$	$1,9.10^{-7}$	1,9	$1,5.10^{14}$
10^{6}	10^{-7}	10^{7}	$1,5.10^{6}$	$1,9.10^{-6}$	1 9	$1,5.10^{13}$
10^{5}	10^{-7}	10^{7}	$1,5.10^{5}$	$1,9.10^{-5}$	190	$1,5.10^{12}$
10^{6}	10^{-7}	10^{6}	$1,5.10^{7}$	$1,9.10^{-8}$	0,2	$1,5.10^{13}$
10^{5}	10^{-7}	10^{6}	$1,5.10^{6}$	$1,9.10^{-7}$	1,9	$1,5.10^{12}$

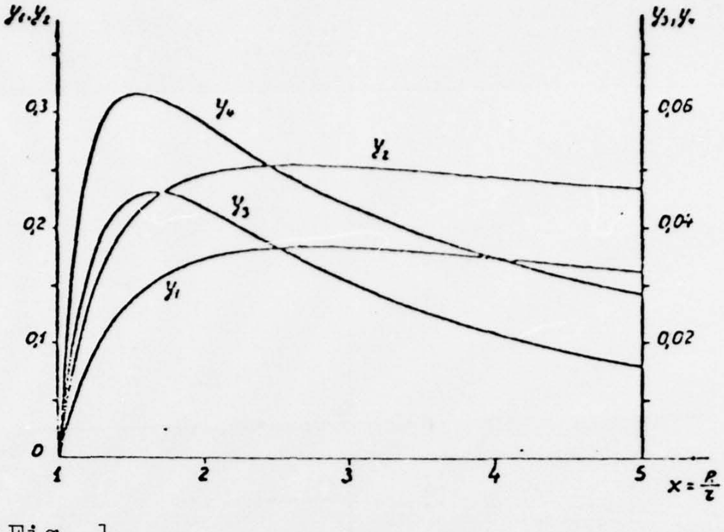

Fig. 1.

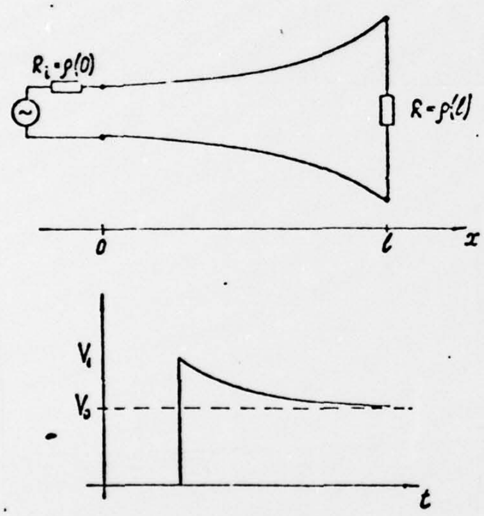

Fig. 2.

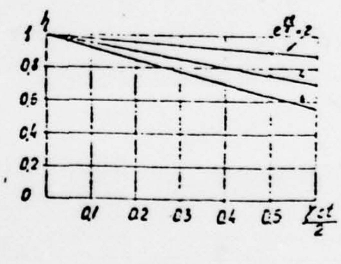

Fig. 3.

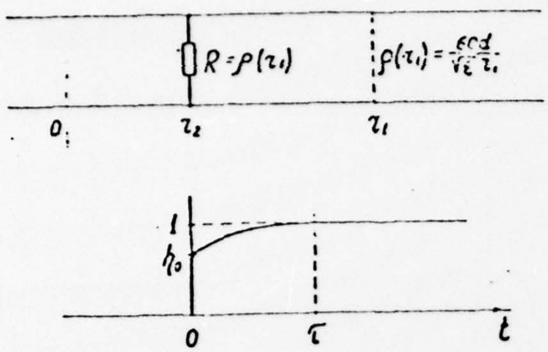

Fig. 4.

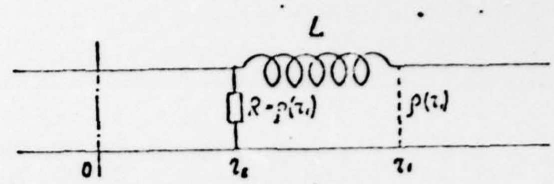

Fig. 5.

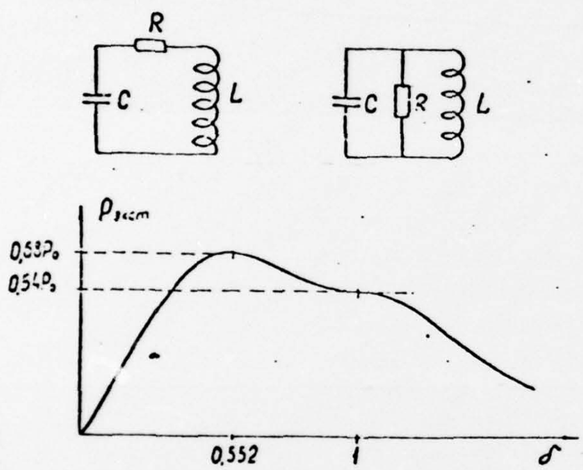

Fig. 6.

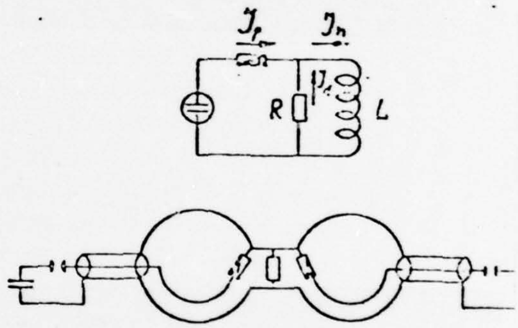

Fig. 7.

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