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A CONTINUOUS-SLOPE CURVE FITTING ROUTINE FOR USE IN SONAR PERFO--ETC(U).  
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## A CONTINUOUS-SLOPE CURVE FITTING ROUTINE FOR USE IN SONAR PERFORMANCE PREDICTION.

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✓ This technical note describes a prediction technique presently in use at the Naval Undersea Warfare Center, San Diego Division. This note is not to be considered an official NUWC report. Its purpose is to document the prediction technique for the benefit of those who may have need to use it.

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A CONTINUOUS-SLOPE CURVE FITTING ROUTINE  
FOR USE IN  
SONAR PERFORMANCE PREDICTION

Ray tracing techniques have long been used to estimate the propagation loss of sound in the ocean. The availability of high-speed digital computers has made practical the application of such techniques to increasingly complex environmental conditions. The "velocity profile", which specifies the dependence of sound velocity  $C$  on depth below the surface of the ocean  $Z$ , is the most critical environmental input to any ray-tracing routine. The velocity profile is specified as a set of points  $(C_i, Z_i)$  for input to a digital computer program. A profile for which ray-path solutions can be obtained in closed form is then fit to the set of points. A relatively simple solution can be obtained for a profile consisting of consecutively joined straight-line segments passing through all the points  $(C_i, Z_i)$ . However, the slope discontinuities which inevitably occur in such straight-line-segment curves lead to the prediction of non-physical singularities in the sound intensity. More satisfactory predictions have been obtained by Pedersen, Gordon, and Keith,<sup>1,2</sup> using a continuous slope curve to fit a given velocity profile. The fitted curve consists of joined segments, each of which satisfies an equation of the form

$$a + bZ + cZ^2 = C^{-2}.$$

Pedersen, Gordon, and Keith<sup>1</sup> have worked out the ray-trace theory in detail for this case and have written flow charts for a computer program to map sound intensities. The entire ray trace package is being programmed

in ETRAN\* by NUWC Code D556. This report will describe the curve-fitting part of the overall ray trace package. The remainder of the ray trace package will be described in a subsequent report.

The "computer notation" used in the balance of this report will treat as independent variables quantities that, in mathematical notation, would normally be written as subscripts, e.g.,  $C_i$  and  $Z_i$  will be written  $C(I)$  and  $Z(I)$ .

The curve-fitting routine divides the ocean between the shallowest point and the deepest point of the input velocity profile into layers bounded by horizontal planes, as indicated in Figure 1. Depths of the bounding planes will be denoted by  $ZU$ . The layer numbered  $IL$  extends from  $ZU(IL)$  downward to  $ZU(IL+1)$ . Parameters  $AA$ ,  $BB$ , and  $CC$  are computed for each layer such that the condition

$$|AA(IL) + BB(IL) \times ZS + CC(IL) \times ZS^2 - CS^{-2}| < EM$$

is satisfied for each point  $(CS, ZS)$  of the input profile for which  $ZU(IL) \leq ZS \leq ZU(IL+1)$ . Here,  $EM$  is an input parameter specifying the maximum distance in the  $(C^{-2}, Z)$  plane that the fitted curve is allowed to deviate from any point of the input profile. The coefficients  $AA(IL)$ ,  $BB(IL)$ ,  $CC(IL)$ , and  $AA(IL+1)$ ,  $BB(IL+1)$ ,  $CC(IL+1)$  must satisfy the condition that the slope of the segment numbered  $IL$  equals that of the segment numbered  $IL+1$  at the junction depth  $ZU(IL+1)$ .

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\*ETTRAN is a scientific programming language developed by NUWC Code D556 (formerly NEL Code 3110D) used in Sonar Performance Prediction.

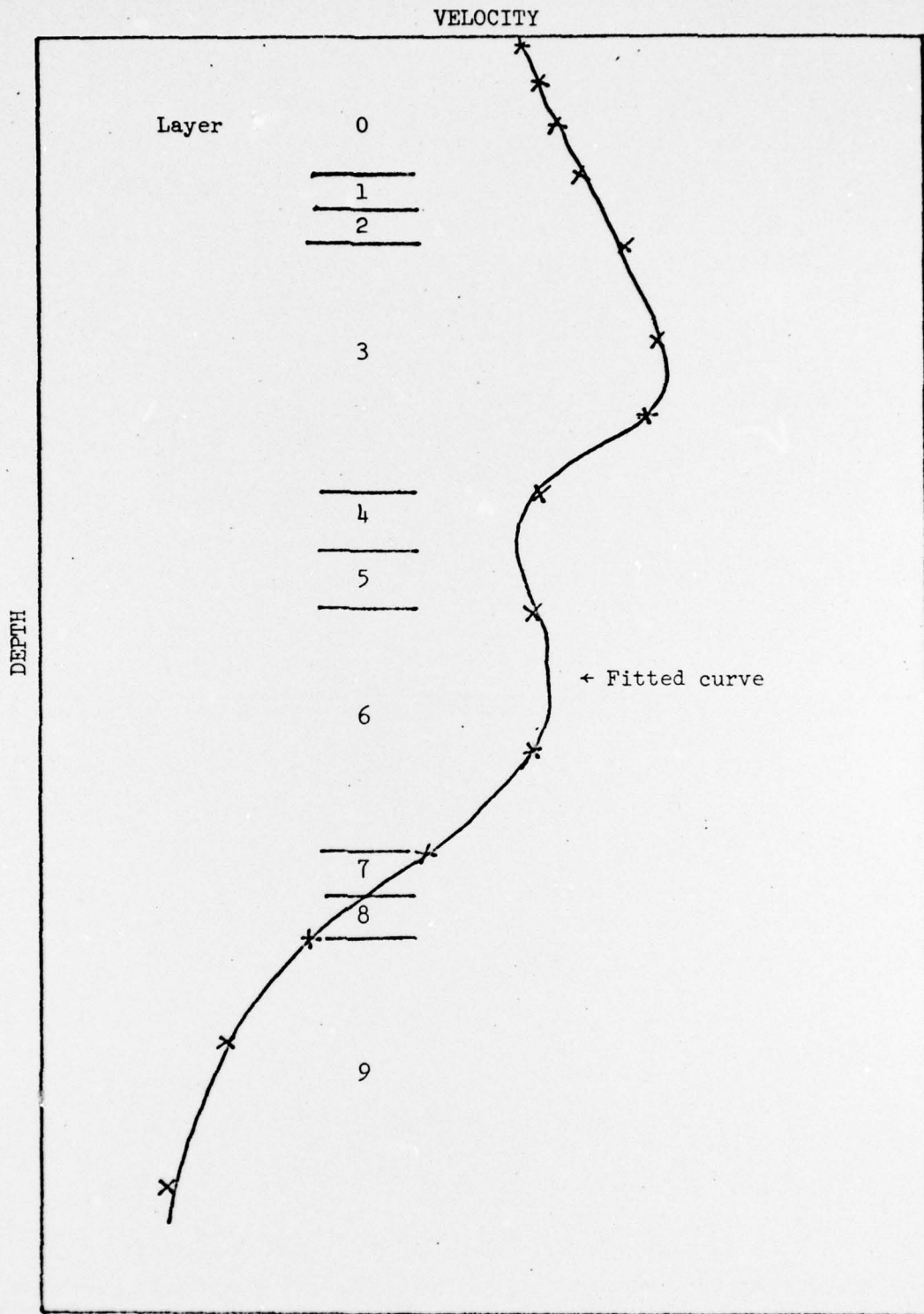


Figure 1

The primary task of the curve-fitting program, then, is to determine layer interface depths ZU and the coefficients AA, BB, and CC for each layer. The first step is to make a least-squares fit of the equation

$$a + bZ + cZ^2 = C^{-2}$$

to the first four points of the velocity profile.

If the condition

$$|a + b \times ZS + c \times ZS^2 - CS^{-2}| < EM \quad (1)$$

is not satisfied for each of the four points, then an exact fit is made to the first three points. If the four-point fit is satisfactory, then a least-squares fit is made to the first five points and each point is tested against equation (1). The number of points is increased by one until an unsatisfactory fit is obtained, or until all points have been fitted satisfactorily. The coefficients of the last last satisfactory fit are stored as AA(0), BB(0), CC(0). The depth of the deepest point used in the last satisfactory fit is stored as ZU(3). The depth of the next shallower point is stored as ZU(1). Layer 0 thus extends from the depth of the shallowest point, ZU(0), to a depth ZU(1). Layers 1 and 2 are contained between depths ZU(1) and ZU(3). The program returns later to compute ZU(2) and the coefficients AA, BB, and CC for layers 1 and 2.

Starting with the point at depth ZU(3), the program fits another group of points using the same procedure that was used to fit the first group. The quantities ZU(4), ZU(6), AA(3), BB(3), and CC(3) are computed.

This procedure is continued until the deepest point has been fit. A special situation arises if a fit including the next-to-deepest point is satisfactory, but a fit including the deepest point as well is unsatisfactory. In this case, a curve is constructed which passes through the deepest of the input points and which meets the preceding curve with matching slope at a depth equal that of the next-to-deepest point.

After all the points have been fitted, the program goes back and fills in all the gaps with bridging curves. The first bridging curve extends through layers 1 and 2. Recall that  $ZU(1)$ ,  $ZU(3)$ , and the coefficients  $AA$ ,  $BB$ , and  $CC$  for layers 0 and 3 have already been determined. It is necessary to compute  $ZU(2)$  and  $AA$ ,  $BB$ , and  $CC$  for layers 1 and 2. The depth  $ZU(2)$  is taken as the depth of intersection of the tangent to curve 0 at depth  $ZU(1)$  and the tangent to curve 3 at depth  $ZU(3)$ , provided that this depth of intersection occurs at a depth greater than  $ZU(3)$ , then  $ZU(2) = 1/3 ZU(1) + 2/3 ZU(3)$ . Coefficients are now computed for the bridging curves 1 and 2. The conditions which must be satisfied are that curves 0 and 1 meet and have equal slopes at depth  $ZU(1)$ , that curves 2 and 3 meet and have equal slopes at depth  $ZU(3)$ , and that curves 1 and 2 meet with equal slopes at depth  $ZU(2)$ .

Other necessary bridging curves are constructed in the same manner.

Using values determined for  $ZU$ ,  $AA$ ,  $BB$ , and  $CC$ , additional parameters  $P2$ ,  $P3$ ,  $P4$ , and  $P5$ , which are required as inputs to the ray tracing program, are computed for each layer. If a positive-gradient surface channel is present, the channel depth and average velocity gradient in the channel are determined. Finally, points on the fitted velocity profile are printed out at closely-spaced depth intervals.



A complete list of inputs and the units of each is:

CS (velocity)

ZS (length)

NM

EM (velocity<sup>-2</sup>)

CH (length),

where

CS and ZS are arrays giving the points of the input velocity of profile in order of increasing depth.

NM is the number of points in the arrays ZS and CS.

EM is the maximum distance in the (C<sup>-2</sup>, Z) plane that the fitted curve is allowed to deviate from any of the given points. It is desirable to determine EM in terms of deviation allowed in the (C, Z) plane.

Note that

$$d(C^{-2}) = - \frac{2 \times dC}{C^3} .$$

Let  $C_0$  be a point in the velocity profile. If  $\frac{\Delta C}{C_0} \ll 1$ , then

$$|\Delta(C^{-2})| \hat{=} \left| \frac{2 \times \Delta C}{C_0^3} \right|$$

Letting  $\Delta C$  = maximum deviation allowed in the (C, Z) plane,

$$EM \hat{=} \frac{2 \times \Delta C}{C_0^3}$$

If the units for C are ft/sec, and we take  $C_0 = 4950$  ft/sec, then

$$EM \hat{=} 1.67 \times 10^{-11} \Delta C.$$

If the units are yds/sec,

$$EM = 4.5 \times 10^{-11} \Delta C.$$

Pedersen, Gordon, and Keith<sup>1</sup> have suggested the value .2 yds/sec, or .6 ft/sec, for  $\Delta C$ . These numbers give

$$EM = 1.00 \times 10^{-11} \text{ (units of } C = \text{ft/sec)}$$

$$EM = 9.0 \times 10^{-11} \text{ (units of } C = \text{yds/sec).}$$

CH is the depth interval at which points from the fitted profile are printed out. Typically specified values are 10 ft/sec or 5 yds/sec.

The outputs are:

ILM	ZP
IL	CP
ZUI	CL
CI	ZL
P2I	GRD
P3I	VA
P4I	VB
P5I	VC

"Profile items" (one value for each profile) are:

ILM - number of layers in profile

CL - absolute maximum velocity of fitted profile

ZL - depth at which velocity is absolute maximum

GRD - average velocity gradient in surface channel.

"Layer items" (one value for each layer) are:

IL = layer number

VA = coefficient AA

VB = coefficient BB

VC = coefficient CC

ZUI = depth of upper boundary of layer

CI = velocity at upper boundary of layer

P2I

P3I

P4I

P5I

} special parameters for input to ray tracing program

"Profile print-out items"

ZP

CP

} points taken from fitted profile at depth intervals of CH

The dotted line in Figure 2A represents a curve fitted to a set of points denoted by X's. The points are:

<u>Point No.</u>	<u>ZS(ft)</u>	<u>CS(ft/sec)</u>
0	0	4945.5
1	32.8	4946.1
2	65.6	4946.6
3	98.4	4947.1
4	164	4951.9
5	246	4951.5
6	328	4989.8
7	492	4905.8
8	656	4888.0

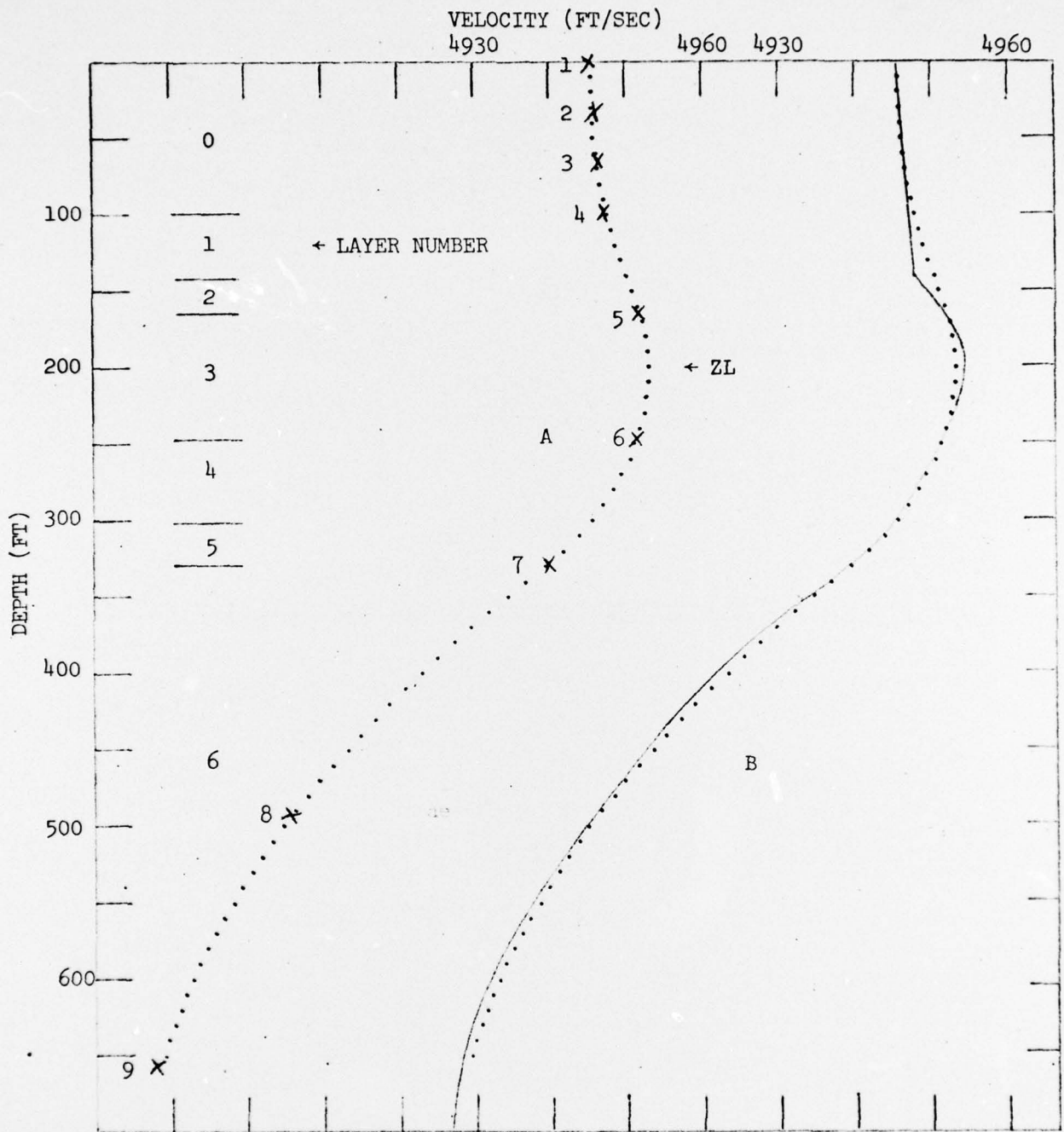


Figure 2

A least squares fit to the set of points (0,1,2,3) was first made. This fit was satisfactory (with  $EM = 1.00 \times 10^{-11}$ ), so a fit was made to the set (0,1,2,3,4). This fit was also satisfactory. A fit to (0,1,2,3,4,5) was unsatisfactory. The values  $ZU(0) = 0$ ,  $ZU(1) = 98.4$ , and  $ZU(3) = 164$  were stored. Coefficients determined for the fit to (0,1,2,3,4) were stored as  $AA(0)$ ,  $BB(0)$ ,  $CC(0)$ . A four-point fit was made to (4,5,6,7). This fit was unsatisfactory, so an exact fit was made to (4,5,6). The values  $ZU(4) = 246$  and  $ZU(6) = 328$  were stored. A three-point fit was made to (6,7,8), the last three input points. Next, the bridging curves were computed. The quantity  $ZU(2) = 142.1$  was determined; then, the coefficients  $AA$ ,  $BB$ ,  $CC$  for layers 1 and 2 were computed. Similarly  $ZU(5) = 300.7$  was determined; then,  $AA$ ,  $BB$ , and  $CC$  for layers 4 and 5 were computed.

The X's in Figure 2A represent points taken at standard Nansen depths from a continuous velocity profile determined from oceanographic data. Figure 2B compares this profile, represented by a solid line, with the fitted curve shown in Figure 2A, represented by a dotted line. The fact that there are noticeable differences of up to 3 ft/sec in the two curves is due to the large depth intervals between the points to which the dotted curve was fitted. This illustrates the importance of using bathythermograph or other continuously sampled data to supplement Nansen Cast data, since Nansen sampling intervals are normally too gross to be used in reproducing profile fine structure. A curve fit to a set of points taken from the original profile at depth intervals of 25' did not deviate from the original profile by more than 0.6 ft/sec. It is evident that, if, the depth interval between points is taken to be small enough, the fitted curve can be made to correspond closely the true velocity profile of the ocean.

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1. Navy Electronics Laboratory Report 1105, A New Ray Intensity Procedure for Underwater Sound Based on a Profile Consisting of Curvilinear Segments, by M. A. Pedersen, D. F. Gordon, and Alice Joy Keith, CONFIDENTIAL, 30 Mar 1962.
2. Navy Electronics Laboratory Report 1217, Extensions of the Ray Intensity for Underwater Sound Based on a Profile Consisting of Curvilinear Segments, by D. F. Gordon, CONFIDENTIAL, 3 Apr 1964.