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NONLINEAR PROBLEMS IN CONTINUUM MECHANICS.(U)  
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NONLINEAR PROBLEMS IN  
CONTINUUM MECHANICS

FINAL REPORT COVERING  
1 Jan. 1975 - 31 Dec. 1977

H. B. KELLER

JANUARY 26, 1978

U. S. ARMY RESEARCH OFFICE  
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CALIFORNIA INSTITUTE OF TECHNOLOGY

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Brief Summary of Research Program

During the past three years of Army Research Office sponsorship of this program we have made significant contributions in several basic areas. At least three students who have worked on various aspects of our program have obtained Ph.D. degrees during this period and one MS degree was granted. In addition five more Ph.D's are about to be granted. Thus many bright young applied mathematicians have been trained in part on difficult problems of interest to the Army.

The main areas of research were as follows: development and analysis of numerical methods for two-point boundary value problems, for boundary layer flows, and for bifurcation and nonlinear eigenvalue problems; the study of diffusion-reaction systems with particular attention to oscillatory solutions and their bifurcation; the development and testing of a turbulence model, the theory of vortex dynamics.

A number of very significant accomplishments have been made during this brief three year period. For example the existence of rotating spiral waves in chemically reacting systems has been demonstrated for the first time [12]. The spirals are slightly different (logarithmic) from what had been assumed in the literature (Archimedian). This work has just been submitted for publication and finally answers a question of great current interest and activity. The turbulence model of Saffman has been

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extended to turbulent shear flows and successful calculations show that it is a rather complete model [23]. This is perhaps the first such model to be severely tested on shear flows and to give such good agreement with the experiments. Finally a powerful general method for computing solutions of bifurcation and nonlinear eigenvalue problems has been developed [17]. It has numerous applications and extensions to a host of seemingly unrelated topics (i. e. global homotopies or Newton-like methods). The power of this new method has been demonstrated in the first rather complete numerical study of multiple solutions for the Navier-Stokes equations for the flows between rotating plane disks (manuscripts in preparation).

As in most healthy research programs we feel that we have raised more questions than we have answered. We hope to continue all of these studies in the future.

More details of these results can be obtained from the resulting publications and preprints which number about 27. They are listed in the following Bibliography.



USARO Bibliography

The main work of this project from 1 Jan. 1975

- 31 Dec. 1977 is contained in the following papers.

Publications of D.S. Cohen:

- [1] Oscillatory processes in the theory of particulate formation in super-saturated chemical solutions, SIAM J. Appl. Math., 28 (1975) 307-318; (with J. P. Keener).
- [2] Nonlinear oscillations in a reactor with two temperature coefficients, Nuclear Sci. and Eng., 56 (1975) 354-359; (with J. P. Keener).
- [3] Nonlinear problems in chemically reacting diffusive systems, Transactions of the Twenty-First Conference of Army Mathematicians, El Paso, Texas, 1975.
- [4] Instabilities in chemically reacting mixtures, Rocky Mountain J. of Math., 6 (1976) 551-559.
- [5] Bifurcation of localized disturbances in a model biochemical reaction, SIAM J. Appl. Math., 30 (1976) 123-135; (with J. A. Boa).
- [6] Multiplicity and stability of oscillatory states in a continuous stirred tank reactor with exothermic consecutive reactions  $A \rightarrow B \rightarrow C$ , Chem. Eng. Sci., 31 (1976) 115-122; (with J. P. Keener).
- [7] Bifurcation from multiple complex eigenvalues, J. Math. Anal. and Appl., 57 (1977) 505-521.
- [8] Perturbation Theory, Lectures in Modern Modeling of Continuum Phenomena, Lectures in Applied Mathematics, Vol. 16, American Mathematical Society, Prov., R.I., 1977.
- [9] Slowly varying oscillations in nonlinear diffusion processes SIAM J. Appl. Math. 33 (1977) 217-229; (with F. C. Hoppensteadt and R. M. Miura).
- [10] Interacting oscillatory chemical reactors, N. Y. Acad. Sci. Proceedings, 1978, N. Y. Acad. Sci. Proceedings, 1978 (with J. C. Neu).

- [11] On inhibiting runaway in catalytic reactors, *SIAM J. Appl. Math.* (with B. J. Matkowsky).
- [12] Rotating spiral wave solutions of reaction diffusion equations, submitted to *SIAM J. Appl. Math.* (with J.C. Neu and R. R. Rosales).

Publications of H. B. Keller:

- [13] Some computational problems in boundary layer flows; in *Lect. Notes in Phys. Vol. 35*, Springer-Verlag 1975 (pp.1-21). (Invited address to: 4th Int. Conf. on Num. Meth. in Fluid Dynamics).
- [14] Multiple eigenvalues lead to secondary bifurcation, *SIAM Review*, 17, 1975 (pp.101-122)(with L. Bauer & E. L. Reiss).
- [15] Turbulent boundary layers with assigned wall shear, *Computers & Fluids*, 3, 1975 (pp.37-49)(with T. Cebeci, N. Berkant, I. Silivri).
- [16] Numerical Solution of Two Point Boundary Value Problems, *CBMS/NBS Regional Conf. Series on Applied Math; SIAM*, Vol. 24, 1976, (61 pp + v). (Principal Lecturer at Regional Res. Conf. on Num. Solutions of Two Point Boundary Value Problems, Texas Tech, July 7-11, 1975).
- [17] Numerical solution of bifurcation and nonlinear eigenvalue problems, in *Applications of Bifurcation Theory* (P. Rabinowitz, ed.) Academic Press, New York 1977 (pp.359-384). (Invited address to M. R. C. Symposium, Madison, Wisconsin 1976).
- [18] Numerical methods in boundary-layer theory, in *Ann. Rev. Fluid Mech.*, 10, 1978 (pp.417-433), Annual Reviews Inc., Palo Alto.
- [19] Symbolic generation of finite difference formulae, to appear in *Math. of Comp.* 1978 (with V. Pereyra).
- [20] Difference methods and deferred corrections for ordinary boundary value problems submitted to *SIAM J. Num. Anal.* (with V. Pereyra).
- [21] Constructive methods for bifurcation and nonlinear eigenvalue problems, to appear in: *Proc. 3rd Int. Symp. on Comp. Meth. in Appl. Sci. and Engr.*, Springer-Verlag, 1978.

Publications of P. G. Saffman:

- [22] Turbulence model predictions for the inhomogeneous mixing layer. *Stud. App. Math.* 55, pp. 45-63 (1976) (with F. Milinazzo).
- [23] Development of a complete model for the calculation of turbulent shear flows. Statistical Mechanics and Dynamical Systems and papers from the first Duke turbulence conference. *Duke Univ. Math. Series* 3 (1977).

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- [ 24 ] Structure of a linear array of hollow vortices of finite cross-section. J. Fluid Mech. 74, pp. 469-476 (1976) (with G. R. Baker & J. S. Sheffield).
- [ 25 ] Flow over a wing with an attached free vortex. Stud. Appl. Math. 57, pp. 107-117 (1977) (with J. S. Sheffield).
- [ 26 ] The number of waves on unstable vortex rings. J. Fluid Mech. to appear.
- [ 27 ] Problems and progress in the theory of turbulence. Symposium on Turbulence, Berlin 1977. Springer-Verlag. to appear.



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The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  
The main areas of research summarized here were as follows: development and analysis of numerical methods for two-point boundary value problems, for boundary layer flows, and for bifurcation and nonlinear eigenvalue problems; the study of diffusion-reaction systems with particular attention to oscillatory solutions and their bifurcations; the development and testing of a turbulence model, the theory of vortex dynamics. Twenty-seven papers generated by this research are listed.