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NONLINEAR THEORY OF A BEARING SURFACE OF ARBITRARY EXTENT

by

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Nonlinear Theory of a Bearing Surface of Arbitrary Extent by A. N. Panchenkov

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The nonlinear problem of a wing of small extent in liquid and gas is one of the difficult problems of fluid mechanics. These difficulties are primarily related to the complexity of the phenomena arising in a fluid when the motion of a wing of small extent moves with a large angle of attack. The formation of strong turbulent eddies at the leading and lateral wing edges, the large stream angle of taper, the formation of a turbulent region beyong the wing--all result in the hydromechanical characteristics of the wing being nonlinear with respect to the angle of attack so that the nonlinearity, with its attendant increase in the lift on the wing, is substantial even at small angles of attack. This severely handicaps the mathematical analysis of the phenomenonsince the fundamental set of assumptions in the linear theory of a bearing surface are not even applicable here.

In conjunction with the mathematical difficulties, the well-known results from the nonlinear theory of a wing of small extent are obtained on the basis of certain physical pictures of the flow, based on experimental facts. Many investigators have assumed various flow models and have obtained, within the framework of their models, results in agreement with experiment (2, 3, 8, 9, 11). These results are definitely of interest from the point of view of a basic explanation of the phenomena

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involved, the pattern of these phenomena, and many technical applications. However, a lag in mathematical investigations has lead to a restriction in the class of problems studied and to a lack of rigor and consistency in our concepts.

The purpose of this article is to get theoretical results from the nonlinear theory of a bearing surface by mathematical analysis. In [7] are formulated two conditions for the "complete" approximation of a certain boundary-value problem by means of a new, different boundary-value problem. Now it is easy to show that the linear problem of a bearing surface, derived from the nonlinear problem by the classical perturbation technique, does not yield a uniformly convergent approximation. It is know that where a bearing surface moves in a fluid a semi-infinite region is formed beyong it, in which the velocity potential and its derivatives possess discontinuities. If  $B_h$  is the boundary of this region, then  $\frac{\partial B_h}{\partial B_h} = O(B_h)$  (region)

$$\frac{\partial B_{\lambda}}{\partial x} = Q(\beta) \quad (x \to -\infty),$$

where  $\beta$  is some avergaed stream taper angle at a point at infinity beyond the wing.

In all known linear theories (see, for example,  $\begin{bmatrix} 4 \end{bmatrix}$ ,  $\begin{bmatrix} 5 \end{bmatrix}$ ) the turbulent region is reckoned from the surface and  $\frac{\partial B_{\lambda}}{\partial x} = 0$   $(x \to -\infty)$ ,

but then for the metric in R<sup>3</sup> space we have

 $d(B_0; B_{01}) = 0(\beta) |x|$ (1)

and the condition in [7] for large |x| is not fulfilled. The stream taper angle of the wing has the following asymptotic values:

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$$\beta = Q\left(\frac{\alpha}{\lambda}\right) \quad (\lambda \to \infty);$$

$$\beta = Q(\alpha) \qquad (\lambda \to 0),$$
(2)

where  $\alpha$  is the wing angle of attack and  $\lambda$  is the relative span of the wing.

In this problem, it is sufficient to consider the region  $\Omega_3$ ,  $x \in [+\infty \div C]$  where C is some constant and then, for  $\alpha \ll 1$ ; L and  $\frac{\alpha}{\lambda} \ll 1$ the classical linear theories give a uniformly convergent approximation.

It must be noted that perturbation theory gives an "incomplete" approximation. In the well-known linear theories, the operator  $P \equiv 1$  and the operator  $T = T_0$ . The "complete" approximation has a supplementary freedom of choice in the operators P and  $T_0$  and can yield a better solution. In this sense, the method of Poincare-Lighthill-Go [10] is quite interesting since it is based on the idea of a "complete" approximation.

Let us consider the motion of a bearing surface of arbitrary extent with a constant velocity v in an unbounded fluid. Late x,y, and z be the Cartesian space coordinats. The Ox axis is aligned in the direction opposite to the stream flow. For the velocity potential of the perturbed motion, the boundary-value problem can be formulated in the following form:

$$\Delta \phi = 0; \quad Q \in \mathfrak{S}_{\infty} \setminus \mathfrak{S}_{\lambda}; \tag{3}$$

$$\varphi_n = v_0 \sin \alpha \quad Q \in s_1; \tag{1}$$

$$\begin{array}{l} (\varphi \to 0) \\ (x \to \infty) \end{array}; \quad \overline{\Omega}_{\lambda} = \Omega_{\lambda} \setminus s_{1}; \end{array}$$

$$(4)$$

$$\Delta \varphi = 4\pi \varrho; \qquad Q \in \Omega_{\lambda}.$$

Here s, is the surface of the bearing wing and  $\Omega_{\infty}$  is the infinite region in the R<sup>3</sup> space.

The velocity potential  $\Phi$  cannot be a continuous function throughout the region outside s, since there will exist beyond the bearing wing a semi-infinite region  $\Omega_1$ , in which the velocity potential and some of its derivatives will undergo discontinuities. Indeed, the presence of a semi-infinite region  $\Omega_{\lambda}$  determines all the difficulties in this problem. In the region  $\Omega_{\lambda}$ , the velocity potential will even satisfy the Poisson equation (5) but with an unknown density  $\rho$ . To get a solution with the required properties, we need supplementary conditions determining the flow structure in the region  $\Omega_{\lambda}$ . As one such condition, we can take the physical condition of pressure continuity in the transition of any surface to  $\Omega_{\infty}$ . As examples of such surfaces we can take surfaces  $s_1$ , on which the density  $\rho$  is constant.

If  $\varphi_{s_i}$  is a tangential derivative of the potential, then the condition of pressure continuity during transition of surface s<sub>i</sub> can be written in the form

$$\varphi_{s+} - \varphi_{s-} = 0; \quad Q \in s_{s}. \tag{6}$$

Moreover, the solution of the problem must satisfy the supplementary conditions related to satisfying the Zhukovskiy-Chaplygin postulate which can be written as:

$$\varphi_{s} - \varphi_{s} = 0; \quad Q \in L. \tag{7}$$

(8)

where L is the equation for the trailing edge of the wing.

The difficulties of the problem formulated are clear. The region  $\Omega_{\lambda}$  is not known beforehand and also unknown is the density in the Poisson equation (5). The region  $\Omega_{\lambda}$ , in dimensionless coordinates such that the wing span is equal to 1, has a thickness  $\delta = 0\left(\frac{a}{\lambda}\right)$  and thus we obtain one of the required conditions for the linearization of the problem:

 $\frac{\alpha}{\lambda} \ll 1$ 

Under condition (8), the region  $\Omega_{\lambda}$  can be approximately replaced by a certain surface  $s_{\lambda}$ . Well known linear theories [4, 5] start out from the assumption that the surface  $s_{\lambda}$  is inclined only slightly from the xOy plane and condition (7), having the form:

$$\varphi_x + - \varphi_{x-} = 0,$$

is satisfied on the plane  $s_{n\lambda}$  lying in the xOy plane.

We shall solve the problem posed by using the idea of a "complete" approximation. Let us introduce a new three-dimensional space  $R_1^3$  with coordinates  $x_1$ ,  $y_1$ ,  $z_1$ . We take the relation between the coordinates in the  $R^3$  and  $R_1^3$  space in the following form:

$$\begin{aligned} x &= \dot{x_1} + F_1(x_1; y_1; z_1); \\ y &= y_1 + F_2(x_1; y_1; z_1); \\ z &= z_1 + F_3(x_1; y_1; z_1), \end{aligned}$$
(9)

where the functions  $F_j$  belong to the class  $F_j \in C^2(\Omega)$ . Let the norms of the functions  $F_j$  and their first and second derivatives in the class  $C^2$  have the values:

$$\|F_i\|_k = 0 (\varepsilon), \tag{10}$$

where  $\varepsilon$  is some small parameter in the problem. Then, when the conditions (10) are fulfilled, the potentials  $\varphi$  and space coordinates can be written as a series:

$$\varphi = \varphi_0 (x_1, y_1, z_1) + \varepsilon \varphi_1 (x_1, y_1, z_1) + \varepsilon^2 \varphi_2 (x_1, y_1, z_1) + \dots;$$

$$x_j = x_j^0 + \varepsilon x_j^1 (x_1, y_1, z_1) + \varepsilon^2 x_j^2 (x_1, y_1, z_1);$$

$$x_1 = x; \quad y_2 = y; \quad x_3 = z.$$

In the zero'th approximation in the  $R_1^3$  space, the potential  $s_0$  will also satisfy the Laplace equation. In the usual formalism, the problems for the functions  $\varphi_i(\xi, \eta, \zeta)$ , can also be treated, wherein they will even satisfy the Poisson equation. Since it is

clear from the physical nature of the problem considered that the greatest influence on the wing characteristic will be exerted by the presence of the stream taper angle at infinity beyond the wing, we shall limit ourselves in what follows to the problem involving deformation of only the y coordinate. Thus, we shall set:

$$\begin{array}{c} x = x_{1}; \\ y = y_{1} + \varepsilon y_{1}^{1}(x_{1}, y_{1}, z_{1}) + \varepsilon^{2} y_{1}^{2}(x_{1}, y_{1}, z_{1}); \\ z = z_{1}. \end{array} \right\}$$
(11)

In this method, there is a freedom of choice of the functions  $\varphi_{(j)}(\xi,\eta,\zeta)$ , which should be so chosen that they meet the approximation conditions. According to the second approximation condition (b) in reference [7]:

$$\zeta = z + \int_{0}^{x} \varphi_{\zeta} d\tau \qquad (12)$$

and to a first approximation:

$$\zeta = z + \int_{0}^{z_{1}} \varphi_{0z} d\tau. \qquad (13)$$

Let us introduce into consideration the function  $Q = -\varphi_{01}$ . For the function  $\Theta$  in the  $R_1^3$  space we have the following problem:

$$\Delta \Theta = 0;$$
  

$$\Theta_{n1} = P_1(g_1); \quad g_1 \in s_{11};$$
  

$$\Theta \to 0; \quad x \to +\infty;$$
  

$$\Theta_+ - \Theta_- = 0 \text{ Ha } L_1;$$
  

$$\Theta = 0; \quad g_1 \in S_{\lambda 1}.$$
(14)

Let us take the solution of the problem to be in the form:

$$Q(p) = \int_{m} \gamma(g) K(p,g) dg,$$
 (15)

where K(p, g) is the fundamental solution of problem (14) corresponding to a dipole.

In the linear theory, in (15) they take  $s_{lp}$  instead of  $s_{ll}$ . In this theory we are developing, it is sufficient to limit ourselves to the approximation

$$Q(p) \approx \int_{s_{1p}} \gamma(g_1) K_1(p_1g_1) dg_1;$$
  

$$K(p_1g_1) = K[(x_1 - \xi_1); (y_1 - \eta); (z_1 - \zeta)];$$
  

$$K_1(p_1g_1) = K[(x - \xi_1)(y - \eta_1)(z - \zeta_1) + F(\xi_1)].$$
(16)

where  $s_{lp}$  is the projection of the bearing surface onto the  $x_{l}Qy_{l}$  plane. Continuing on, we have

$$Q_{n1} \approx Q_{z} = P_{1}(g_{1}); \quad g_{1} \in s_{11};$$
(17)  
$$Q(p) = \int_{s_{1p}} \gamma_{1}(g_{1}) \frac{\partial}{\partial \zeta} \frac{1}{|P_{1} - g_{1}|} dg_{1}.$$
(18)

Going over to the potential  $\varphi_0$  and carrying out the computations, we get from condition (17) the following integral equation for the problem:

$$\varphi_{0z_{1}} = \frac{1}{4\pi} \int_{s_{1p}} \gamma g_{1} \frac{\partial}{\partial y_{1}} \left\{ \frac{y - \eta}{(y - \eta)^{2} + F^{2}(\xi)} - \left[ 1 - \frac{[(y_{1} - \eta)^{2} + (x_{1} - \xi)^{2} + 2F^{2}(\xi)]}{(x - \xi) \sqrt{(y - \eta)^{2} + (x - \xi)^{2} + F^{2}(\xi)}} \right] dg_{1}. \quad (19)$$

For further work it is convenient to switch to dimensionless coordinates

$$\overline{x} = \frac{x}{C(\eta)}$$
,  $\overline{y} = \frac{y}{B}$ ;  $\lambda(\eta) = \frac{B}{C(\eta)}$ ,

where  $C(\eta)$  is the wing's semi-chord in cross section  $\eta$  and B is the wing's semi-span. Then

$$\varphi_{z_{i}} = \frac{1}{4\pi} \int_{s_{1p}} \frac{\gamma(g_{1})}{\lambda(\eta)} \cdot \frac{\partial}{\partial \bar{y}} \left\{ \frac{\bar{y} - \bar{\eta}}{(\bar{y} - \bar{\eta})^{2} + \left(\frac{\bar{F}(\xi)}{\lambda(\eta)}\right)^{2}} \left[ 1 - \frac{1}{(x - \xi)} \cdot \frac{[\lambda^{2}(\eta)(\bar{y} - \bar{\eta})^{2} + (\bar{x} - \bar{\xi})^{2} + 2F^{2}(\xi)]}{\sqrt{\lambda^{2}(\eta)(\bar{y} - \bar{\eta})^{2} + (\bar{x} - \bar{\xi})^{2} + \bar{F}^{2}(\xi)}} \right] \right\} dg.$$
 (20)

Equation (20) solves the problem of the motion of the bearing surface of any arbitrary shape for average attack angles  $\alpha$ . The function  $\overline{F(\xi)}$  is determined by the nature of the transformation in (12) in the region of the bearing surface. For  $F(\xi) = 0$ , equation (20 yields well known results. The function  $F(\xi)$  from the point of view of the flow mechanics determines the distance of the vortices from the bearing surface at any point  $s_{1p}$ .

Since the nonlinear effects are most substantial on wings of small extent, we shall study more closely the problem of a wing of small extent. For  $\lambda \rightarrow 0$ , equation (20) takes on the following correct limiting form:

$$\varphi_{z} = -\frac{1}{2\pi\lambda(x)} \int_{-1}^{+1} \frac{\Phi_{\eta}(x)(y-\eta)}{(y-\eta)^{2} + \left(\frac{F(\bar{x})}{\lambda(\bar{x})}\right)^{2}} d\eta; \qquad (21)$$
$$\varphi_{\eta}(x) = \int_{r}^{1} \gamma_{\eta}(\xi, \eta) d\xi.$$

Let us consider a solution for the function  $F(\xi)$ :

$$F(\xi) = (\xi - 1)\beta(\eta).$$
 (22)

Fulfilling condition (4) on the trailing edge of the wing, we have, for the function  $\Phi_{\rm c}$ , the equation

$$\frac{1}{2\pi\lambda}\int_{-1}^{+1}\frac{\Phi_{\eta}(y-\eta)}{(y-\eta)^{2}+\left(\frac{2\beta(\eta)}{\lambda}\right)^{2}}d\eta=\sin\alpha.$$
 (23)

Function  $F(\xi)$  in the form (22) and equation (23) encompass a large group of nonlinear physical theories. In connection with this, we note one important fact related to the choice of angle  $\beta$ . It is easy to show that for the angle  $\beta$ , the correct asymptotic value is:

 $\beta = 0 (\lambda^{\varkappa}) \quad (\varkappa > 0). \quad (24)$ 

The value in (24) is obtained from equation (23) under the condition that the norm of  $\mathbf{\Phi}$  is bounded as  $\lambda \neq 0$ . If  $\|\mathbf{\Phi}\|_{\lambda=0} = M$  (where M is some number), then  $\mathbf{x} = 0.5$ . In all known theories [2, 3, 5, 7, 10], the angle  $\beta$  is taken as finite. In a series of works on this problem, it is taken as equal to half the flow taper angle and; in other works, various other means are employed for its computation. Of course, it is basic that for finite values of the angle  $\beta$  ( $\lambda \neq 0$ ), the results of the theory must give infinite values for the lift on the wing - a prediction which does not agree with experimentally observed results.

Equation (21) makes it possible to obtain detailed information on wings of small extent of arbitrary shape. The determination of the function  $F(\xi)$  is quite important. Since it is not evidently possible to completely describe, within the framework of the theory of an ideal fluid, the physical phenomena during motion of a wing of small extent, the function  $F(\xi)$  makes possible a more precise reckoning of the influence of viscosity, if in determining this function certain supplemental hypothesis are introduced concerning the flow pattern. However, in this regard it must be noted that even without the attraction of supplementary data on the influence of viscosity within the framework of an ideal fluid results are obtained agreeing satisfactorily with experimental data. To get

further results, let us take the angle  $\beta$  in the form

$$\beta = \alpha - a_i, \tag{25}$$

where  $\alpha_i$  is the flow taper angle at infinity beyond the wing.

In connection with definition (25) and the evaluation in (24), still one more important assumption must be noted: formulas (24) and (25) assumed the presence of more information than that given by the linear theory for a wing of small extent. The basic result of the theory of a wing of small extent (Namely that the flow taper angle equals the angle of attack) leads to the result that  $\beta=0$  whereas we need information of the type in (24).

If we make the Laidlow approximation for a wing of arbitrary extent (11) and determine the flow taper angle from the theory with arbitrary extension, then we get

$$\beta = \alpha \frac{\lambda C_2(\lambda) + 2[C_1(\lambda) - 1]}{\lambda C_2(\lambda) + 2[C_1(\lambda) + 1]}; \qquad (26)$$
$$C_1(\lambda) = C_2\left(\frac{1}{\lambda}\right),$$

where  $C_1(\lambda)$  can be determined from the work of Nishiyama:

$$C_1(\lambda) = 1 - \frac{\sqrt{\lambda}}{\sqrt[4]{\lambda^2 + 4}}; \qquad (27)$$

for  $\lambda \rightarrow 0$ , we have:

$$\beta \approx \frac{\alpha \sqrt{\lambda}}{2\sqrt{2}}.$$
(28)

The results in (28) coincides with the value in (24) for  $\varkappa = 0.5$ . Equations (21) and (23) can be solved by known techniques [5]. To get finite results for the lift coefficient, the solution to equation (23) is obtained via a variational approximation. The lift coefficient is obtained in the form:

$$C_{\mu} = \frac{\pi \lambda}{2\Psi_{\beta} \alpha}, \qquad (29)$$

$$\Psi_{\beta} = 2 \int_{0}^{\infty} e^{-\frac{2\beta}{\lambda} k} \frac{J_{1}^{2}(k)}{k} dk.$$

The function  $\, \Psi_{\!_{B}} \,$  is calculated in the form:

$$\begin{aligned} \mathbf{F}_{\beta} &= 0, \, 5\tau_{\beta}^{2} + 0, \, 25\tau_{\beta}^{4} + 0, \, 0625\tau_{\beta}^{6} + 0, \, 0469\tau_{\beta}^{8} + \\ &+ 0, \, 0237\tau_{\beta}^{'0} + 0, \, 0188\tau_{\beta}^{'2} + 0, \, 0981\tau_{\beta}^{'4}; \\ \tau_{\beta} &= \sqrt{\left(\frac{\beta}{2\lambda}\right)^{2} + 1} - \frac{\beta}{2\lambda}. \end{aligned}$$
(30)

In conclusion, let us consider the problem of a wing of arbitrary extent. In this problem, we shall introduce into equation (20) the approximation:

$$\frac{[\lambda^{2}(y)(y-\eta)^{2} + (x-\xi)^{2} + 2F^{2}(\xi)]}{\sqrt{\lambda^{2}(y)(y-\eta)^{2} + (x-\xi)^{2} + F^{2}(\xi)}} \approx (y-\eta)^{2} + (\frac{F(\eta)}{\lambda(\eta)})^{2}$$
$$\approx C_{1}(\lambda)|x-\xi| + C_{2}(\lambda)\lambda(\eta) \operatorname{sign}|y-\eta| \frac{(y-\eta)^{2} + (\frac{F(\eta)}{\lambda(\eta)})^{2}}{(y-\eta)}. \quad (31)$$

Then equation (20) goes over into the following form:

$$\varphi_{z_{1}} = -\frac{C_{1}(\lambda)}{2\pi} \int_{-1}^{+1} \frac{\partial}{\partial \eta} \int_{-1}^{1} \gamma(g_{1}) d\xi(y-\eta) \\ \frac{\partial}{\partial y} \int_{-1}^{1} \frac{\partial}{\partial y} \int_{-1}^{1} \frac{\gamma(g_{1})}{(y-\eta)} d\eta + C_{2}(\lambda) \frac{\lambda(y)}{2\pi} \int_{-1}^{+1} \frac{\gamma(g_{1})}{(x-\xi)} d\xi. \quad (32)$$

The approximation equation (32) is significantly simpler than equation (20) and can be effectively solved. The following step is a transition from equation (32) to an equation of the Prandtl type:

$$\Gamma(y) = \frac{a_{\infty}}{2\lambda(y)C_2(\lambda)} \left[ \alpha(y) - \frac{C_3(\lambda)}{2\pi} \int_{-1}^{+1} \frac{\Gamma'(\eta)(y-\eta)}{(y-\eta)^2 + \left(\frac{F^2}{\lambda(\eta)}\right)^2} d\eta \right], \quad (33)$$

where

$$C_{3}(\lambda) = 1 + C_{1}(\lambda);$$
  

$$F = F(-1); \quad \Gamma(\eta) = \frac{1}{2} \int_{-1}^{+1} \gamma(g_{1}) d\xi.$$

For F=0 and  $C_2(\lambda)=C_3(\lambda)=1$ , equation (33) becomes the well-known Prandtl equation for a wing of large relative span.

The functions  $C_1(\lambda)$  and  $C_2(\lambda)$  should be selected from the condition of best approximation in (31). Equation (33) makes it possible to study a wide class of problems posed by the Prandtl equation in linear theory.

As a first result we obtained formulas for the lift force and parasitic drag of a wing with an elliptic circulation distribution:

$$C_{y} = \frac{a_{\infty}}{C_{2} (\lambda) \left[ 1 + \frac{a_{\infty}C_{3}(\lambda)}{C_{2} (\lambda)\pi\lambda} \Psi_{\beta} \right]} a_{g}; \qquad (34)$$
$$C_{x_{i}} = \frac{C_{y}^{2}}{\pi\lambda} \Psi_{\beta}. \qquad (35)$$

As follows from (35), the wing's parasitic drag, obtained from the nonlinear theory, has turned out to be less than that obtained from the linear theory. Formulas (34) and (35) are exact in the linear theory, but in the theory being considered they are obtained via a variational approximation, since an elliptical distribution does not even allow an exact solution of equation (33).

As is known from experiments [11], for large attack angles, some flow regimes for a wing of small extension are possible depending on the shape of the leading and lateral edges of the wing. For a wing with rounded edges, there is observed a more uniform distribution of the rising vortices along the wing surfaces, while for sharp edges there is observed strong concentrated vortices at the lateral edge, with the result that the lift in the latter case is significantly higher than that for a wing with rounded edges. Function  $F(\xi)$  from (22) and angle  $\beta$ from (26) and (28) correspond to a wing with rounded edges.

Fore the second regime, one must take another distribution for the function  $F(\xi, \eta)$  along the wing surface. From experiment it is known [11] that the lift with sharp edges is approximately two times larger than that for a wing with rounded edges. To a first approximation, we shall characterize this regime by the angle:

$$\beta = \frac{\alpha \sqrt{\lambda}}{\sqrt{2}}.$$
 (36)

Calculation of the functions  $\Psi_{\beta}$  and  $C_{y}$  from formulas (29), (30), and (34) with angle  $\beta$  from (36) gives good correspondence with known experimental data [8, 11].

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