



A TECHNIQUE FOR SONAR DATA PROCESSING

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by

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PREFACE

This memorandum describes a novel approach to sonar data processing growing out of study by the author of the sonar requirements assumed for the ASW Hydrofoil during work on the command and control subsystems for this craft type. This memorandum has been prepared because it is felt that the information may be of use to other Leboratory employees in its present form. Very limited distribution outside the Laboratory is contemplated. This memorandum should not be construed as a report, as its only function is to present for the information of others a small portion of the work being done on this problem (NEL Problem J6-1; BUSHIPS SS 600 000, Task 1723).

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INTRODUCTION

In the command and control portion of the ASW Hydrofoil feasibility study (NEL Problem J6-1) requirements for sonar detection and tracking were postulated (NEL Research Reports 1118, 1119). The problem of extracting information from the sonar signal returns for tracking purposes were intensified by the high target closing rates, the modest sonar ranges and attendant high ping rates, and the need to provide multipletarget-tracking capability. The data processing rate took incremental steps as these requirements took form. Although certain aspects of the LORAD, SPADE and ASIAC developmental sonar data processing systems are probably applicable, as further study may show, none of these as a system fully satisfy the ASW Hydrofoil sonar data processing requirements.

It has been established that a coherent detector (matched filter) will be used in the sonar system, and its characteristics are well known. Consequently, beginning with its output the subsequent sonar data processing specifications are to be determined, as well as the design approach required to meet these specifications. The following discussion will attempt to give some insight into this particular problem.

Problem

The design of a sonar data processor specifically to perform the functions of detection, classification and tracking is of present concern in the ASW Hydrofoil feasibility study. Though present developmental processing systems have been somewhat successful, none is directly applicable as a solution to the AHC sonar processing problem. Some specific assumptions which prompt new approaches to processing development are:

 The data (ping) rate of the sonar is extremely high (range is relatively short, about 3000 yards).

2. Detection must be automatic or semi-automatic and it is to be completed in three pings.

 Classification is to be completed in five pings (or in 20 secs at a 4 sec. ping rate).

4. Tracking accuracy for fire control use will have reached its maximum in twelve pings (48 secs until a weapon can be fired).

5. The advanced hydrofoil craft will be space limited, and the techniques mentioned above suffer from complexity and size limitations.

The basic approach that will be taken here is as follows: . A model of the target, environment, and sonar are to be developed which will relate all observations (or clues) into one model of a random process. This model will be a time dependent random process, that is, some, or possibly all, observations will be a function of time. The model will be non-linear but can be linearized for purposes of simplifying computation.

This model will then be employed in calculating the maximum-likelihood ratio for best estimates of the sonar data or information that can be extracted from the clues or observations. It will be developed in a manner which gives a recursive computer algorithm.

Statistical Estimation of Parameters -- Likelihood Function

The theory of the estimation of parameters is part of the mathematical theory of statistics and is utilized frequently in the communications and information processing fields. In sonar data processing, a number of observations, χ_{i} are available at discrete time increments. These time dependent observations are clues which must be interpreted to evaluate whether a target is sub or non-sub, and whether it is hostile or friendly. This interpretation of parameters from given observations can be treated by the method of statistical estimation of time-dependent parameters.

This treatment requires one major assumption which permits the problem to be solved by analog as well as digital equipment. It is assumed that a complete statistical description of the error or noise is known. For most purposes, the noise or error may be assumed Gaussian. Gaussian noise is representative of many practical noise processes and, in addition, makes the mathematical analysis less complicated than do other distributions. This assumption is added to assure that, should the real time solution on a digital computer be too time consuming, an analog implementation is feasible.

The observations are represented in vector-matrix notations by

 $y(t) = \le (t_j \, \alpha_{i_j} \alpha_{2_j} \alpha_{3_j}, \dots, \alpha_m) + n(t)$ where $\dot{y}(t)$ is the sum of the noise and a signal, $S(t_j \, \alpha_{j_j}, \alpha_{2_j}, \alpha_{3_j}, \dots, \alpha_m) + n(t)$. The signal or parameters to be estimated are related and this functional relation is assumed linear. The observations are made at discrete intervals, or else continuously, for a finite time interval. The discrete form is represented by $y_i = S_i + n_i$, $\dot{t} = 1, 2, \dots, N$

These N values of $\mathcal{U}(\mathcal{E})$ are used to make estimates of the parameters.

Since the complete statistical description of the noise is assumed known, the joint probability-density function of noise $\mathcal{P}_{\mathcal{H}}(\mathcal{N}_{1}, \mathcal{N}_{2}, ..., \mathcal{N}_{\mathcal{M}})$ is available to the observer. And, since $\mathcal{N}_{i} = \mathcal{G}_{i} - \mathcal{G}_{i}$, the joint probability-density function for the N observed values of the

signal U(t) is $P_{s}(Y_{1}, Y_{2}, ..., Y_{n}; a_{1}, a_{2}, ..., a_{m})$ = $P_{n}(Y_{1}-S_{1}, Y_{2}-S_{2}, ..., Y_{n}-S_{n})$

Therefore, the function of the estimator is to form estimates of the α 's on the basis of the N observations $\mathcal{Y}_1, \mathcal{Y}_2, \dots \mathcal{Y}_n$ described

by a probability-density function P_S whose form is known and depends on the a's .

Let \bar{a} , be any estimate of a single parameter a, based on the N observations y_1, y_2, \dots, y_N . The variance of \bar{a} , obeys the inequality $[\partial E\bar{a}]^2$

Variance
$$\bar{a} \ge \frac{1}{E\left[\left(\frac{\partial \log p_s}{\partial a}\right)^2\right]}$$

where E denotes the expected or mean value of a random variable. If the expected value is equal to the parameter of interest [that is, $E(\overline{a})=a$ for all a's] the estimates are called unbiased and

ariance
$$\overline{a} \ge E\left[\left(\frac{\partial \log p_s}{\partial a}\right)^2\right]$$

Model of a Random Process

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The development of a model of a random process to represent the submarine target in a changing ensonified environment has been the subject of extensive sonar studies. Though an accurate model is highly desirable for increased accuracy, the approach to follow allows a crude linear model to be postulated, and then interprets it as a nonstationary model. This interpretation permits the model uncertainties to be attributed to this stochastic, non-stationary, property.

Consider the following simple example. The model is a first-order difference equation,

$$\chi_{k+1} = \alpha_1 \chi_k + \mathcal{V}_k$$

where x is a parameter or variable of interest. The random variable, $\mathcal{V}_{\mathcal{R}}$ is the course of the non-stationary property of the model. The variance and mean of $\mathcal{V}_{\mathcal{R}}$ are known or can be approximated from experimental work.

This linear model is quite versatile and can be used satisfactorily in most problems. Should experimental use of this proposed system prove inadequate, a non-linear or time-varying process model can be utilized without undue complexity.

Observations are given by $U_k = MX_k + W_k$

such that they are linear combination of the process state χ_k and measurement or equipment error, ω_k . Here again the statistics of these random variables are experimentally determined.

This model is now used in the statistical estimator for determining the maximum likelihood of the states (clues) for classification. Discrete Versus Continuous

The computation time as well as the computational storage requirements indicate that the complex system should be both discrete and continuous with a binary decision as an output. Such an output is sub(1) or non-sub (0) and it should automatically activate an alarm. The fact that a hybrid system is required is not a limitation but may actually be an asset. Such decision techniqueshave been sadly neglected and they hold much promise, at least mathematically.

A disadvantage to modeling the random process as a hybrid system is that the system designers frequently are skilled in either discrete

or continuous design but not both. This should not be a limitation, for those who attempt such hybrid modeling are rewarded with some interesting results. Practically, it may be the only way the problem can be implemented.

A Linear System as a Likelihood Decision Device

Having applied some smoothing to a set of clues (the maximum-likelihood estimates), it is necessary to evaluate whether the target is sub or non-sub given the above clues or observations. It is desired to perform this decision-making continuously, as along a given sonar beam. To show how this is accomplished, consider the simplest hypothesis testing problem. Let the observed signal $\chi(t)$ be due to noise only or to a precisely known signal and noise. The former hypothesis, noise only, is denoted by H_o and the latter hypothesis, signal and noise, is denoted by H_1 . It is desired to devise a test for deciding in favor of H_0 or H_1 .

If one lets $P_0(X)$ be the probability density that if H_0 is true, the observed waveform, X(t) could have arisen; and if $P_1(X)$ is the probability density that if H_1 is true, X(t) could have arisen: then the test has the form $P_1(X)$

CEBC HAB	one r	C	P(X)
accept	н,	it	$\frac{P_{i}(x)}{P_{i}(x)} > \lambda$
			$\frac{P_i(\mathbf{x})}{P_0(\mathbf{x})} \leq \lambda$

Here λ is a constant, dependent on "a priori" probabilities and costs, if these are known, or on the predetermined value of α , the false alarm probability. The test asks us to examine the possible causes of what we have observed.

Let us assume now that the noise n(t) is additive, gaussian and white with spectral density ($\frac{N_C}{C}$) and further that the signal, if present, has the known form $S(t-t_o)$, $t_o \le t = t_o + T$ where the delay to and the signal duration T are assumed known. Then, on observing $\mathcal{X}(\mathcal{H})$ in some observation interval, \mathbf{T} , which includes the interval $t_0 \le t \le t_0 + T$, the two hypotheses concerning its origin are: $H_0: \chi(t) = n(t)$ t in I $H_{i}: \chi(t) = S(t-t_{o}) + n(t) \quad t \in I$

Now, the probability density of a sample, $\eta_{L}(t)$, of white gaussian noise lasting from a to b may be expressed as $p(n) = k \exp\left[-\frac{1}{N_0} \int^b n^2(t) dt\right]$

where $\frac{N_{B}}{2}$ is the double-ended spectral density of the noise, and k is a constant not dependent on $\mathcal{N}(\mathcal{X})$. Hence the likelihood that, if is true, the observation $\chi(t)$ could arise is simply the H. probability density that the noise waveform can assume the form of $\chi(t)$ $P_o(X) = k \exp\left[-\frac{1}{N_0}\int X^2(t)dt\right]$ i.e.

the region of integration being, as indicated, the observation interval, I. Similarly, the likelihood that, if H, is true, $\chi(\mathcal{L})$ could arise is the probability density that the noise can assume the form $n(t) = \gamma(t) - S(t - t_{0})$ $P(X) = k \exp\left[-\frac{1}{N_0} \int \left[\frac{1}{\lambda(t)} - s(t-t_0) \right]^2 dt \right]$ = k exp $\left[\frac{1}{N_0}\int_T \chi^2(t)dt + \frac{2}{N_0}\int_T s(t-t_0)\kappa(t)dt - \frac{E}{N_0}\right]$

where we have denoted $\int S'(t-t_c)dt$, the energy of the signal, by E. On substituting the latter expression and taking the logarithm of both sides of the inequality, the hypothesis-testing criterion becomes accept H, if $y(t_c) > \lambda'$ accept Ho if $y(t_o) \leq \lambda'$