MOST Project-3 (1) sc p

NE 051458

SUBTASK 1

PROBLEM NEL J1-2

technical memorandum no. TM-123

4041

(14) NEL-TM-123

(11) 14 Jul[redacted]55
date

(12) 7p.

AD A051370

(6) GRAVITATIONAL ATTRACTION OF A FLOATING, HOLLOW,
title
RECTANGULAR PARALLELOPIPED.

(10) A. N./Smith
author

2822h
code

(9) Technical memo.

123

U.S. Navy Electronics Laboratory, San Diego 52, California

253 55∅

7

TECHNICAL MEMORANDUM

GRAVITATIONAL ATTRACTION OF A FLOATING, HOLLOW, RECTANGULAR PARALLELOPIPED

In studies of detection techniques, the question has arisen of the effectiveness of gravitational methods for determining the presence of underwater objects. Since such objects are complex structures, an analytical approach is possible only through drastic simplification. As the simplest object resembling such structures, there was chosen a hollow rectangular parallelopiped. Other geometrical figures might have been selected, but the one picked has the advantage that its gravitational field may be described at all points by elementary functions.

MacMillan[1] gives the formula for the gravitational potential of the rectangular box. It is too long to be quoted here. By straightforward differentiation the force components can be found. The general case is cumbersome, but the special case of interest here is relatively simple. This is the force of attraction due to the object at a point on the perpendicular bisector of the diagonals of one of the faces.

[1] MacMillan, William Duncan, A.M., Ph.D., Sc.D., Theoretical Mechanics: The Theory of the Potential, McGraw-Hill Book Company, Inc., New York, London, 1930, First Edition, pp 42--80.

Coordinates are chosen as indicated in the figure. Let the dimensions of the box be related by

$$mnpa=mnb=mc=d \tag{1}$$

Then, upon carrying out the required partial differentiations, the components of the gravitational force at point P, are:

$$Y_p=Z_p=0, \quad X_p=\alpha Q d \tag{2}$$

in which $\alpha=2G\sigma/mn$, and Q is the following:

$$Q=\log\frac{(A+n)(B-n)}{(A-n)(B+n)}+n\log\frac{(A+1)(B-1)}{(A-1)(B+1)}-2(mn+1/p)\tan^{-1}\frac{n}{(mn+1/p)A}+2(mn-1/p)\tan^{-1}\frac{n}{(mn-1/p)B} \tag{3}$$

where

$$A = \left[(mn+1/p)^2 + (1+n^2)\right]^{\frac{1}{2}}, \quad B = \left[(mn-1/p)^2 + (1+n^2)\right]^{\frac{1}{2}} \tag{4}$$

The density of the box, σ, is constant, cgs units are used throughout, logarithms are taken to the base e, and the gravitational constant, G, is (6.66) x 10^{-8} cgs units.

If instead of a solid object one considers a hollow box of wall thickness t, i.e., $a-a' = b-b' = c-c' = t$, where $2a'$, $2b'$, $2c'$ are the interior dimensions, the net force at the exterior point P due to the shell can be computed by application of the superposition principle. Thus, if X'_p is the force at P due to the material that was contained within the interior dimensions, then the force X due to the shell alone is just

$$X = X_p - X'_p \tag{5}$$

Using $c = rt$, it is found that

$$X'_p = \sigma \Phi' d \tag{6}$$

where now

$$\Phi' = (1-n/r) \log \frac{[A'+n(1-1/r)][B'-n(1-1/r)]}{[A'-n(1-1/r)][B'+n(1-1/r)]} + n(1-1/r) \log \frac{[A'+(1-n/r)][B'-(1-n/r)]}{[A'-(1-n/r)][B'+(1-n/r)]}$$

$$-2[nm+(r-np)/pr]\tan^{-1}\frac{n(1-1/r)(1-n/r)}{[nm+(r-np)/pr]A'} + 2[nm-(r-np)/pr]\tan^{-1}\frac{n(1-1/r)(1-n/r)}{[nm-(r-np)/pr]B'} \tag{7}$$

and

$$A' = \left\{[nm+(r-np)/pr]^2 + (1-n/r)^2 + n^2(1-1/r)^2\right\}^{\frac{1}{2}}, \quad B' = \left\{[nm-(r-np)/pr]^2 + (1-n/r)^2 + n^2(1-1/r)^2\right\}^{\frac{1}{2}} \tag{8}$$

The above applies to the object by itself. Suppose now that it displaces a fluid of uniform density ρ, that the bc face is parallel to the surface of the fluid, and that 2e is the distance from the surface of the fluid to the bottom of the object, $e \leq a$. The weight of the fluid displaced is $8ebc\rho$, the weight of the object is $8(abc-a'b'c')\sigma$; since these are equal, $e = df/mr$ where

$$f = \frac{\sigma}{\rho}\left[1 + \frac{1}{p} - \frac{n}{r} + \frac{1}{np} - \frac{1}{pr} - \frac{1}{r} + \frac{n}{r^2}\right] \tag{9}$$

The gravitational attraction of the block of fluid by itself would be

$$X''_f = \beta Q''d \tag{10}$$

in which $\beta = 2G\rho/mn$ and Q'' is the following:

$$Q'' = \log\frac{(A''+n)(B''-n)}{(A''-n)(B''+n)} + n\log\frac{(A''+1)(B''-1)}{(A''-1)(B''+1)} - (2/pr)(mnpr-r+2fnp)\tan^{-1}\frac{npr}{(mnpr-r+2fnp)A''}$$

$$+(2/p)(mnp-1)\tan^{-1}\frac{np}{(mnp-1)B''} \tag{11}$$

with

$$B'' = \left[(mnp-1)^2p^{-2}+(1+n^2)\right]^{\frac{1}{2}},\quad A'' = \left[(mnpr+2fnp-r)^2/p^2r^2 + (1+n^2)\right]^{\frac{1}{2}} \tag{12}$$

If the object does not float, but is either neutral and of indeterminate depth or is heavier than the fluid, so that no definite relationship exists between e and the other dimensions, then the attraction of the fluid displaced is simply

$$X''_h = \frac{\rho}{\sigma}X_p \tag{13}$$

Now the difference between the attraction at P due to the fluid to be displaced and that due to the earth, including all the rest of the undisplaced fluid, is $X'' - F$, F being 980 dynes. The difference between the attraction at P due to the object and that due to the earth is $X - F$. The fractional change in the net force at P when the object is present compared to that when it is not is

$$S = \left[(X''-F)-(X-F)\right]/(X''-F) \cong (X-X'')/F \qquad (14)$$

since $X'' \ll F$. If S is the fractional sensitivity of a gravimeter to be used to detect the change, then eq. (14) is the condition that the object just be detected by the instrument.

It follows that for a floating object to be detected

$$S = d\left[\lambda(Q-Q') - \rho Q''\right]/F \qquad (15a)$$

while for an object heavier than the fluid or one of neutral bouyancy completely submerged

$$S = \lambda d\left[Q(1-\frac{\rho}{\sigma}) - Q'\right]/F \qquad (15\text{-}b)$$

Computations were made of the fractional sensitivity S necessary to detect an object under the conditions listed in the table below. The material of the box was taken as steel, $\sigma = 8.00$, and the fluid was supposed to be seawater, $\rho = 1.024$. In all cases S turned out to have to be better than one part in 10^8. According to Heiland[2], most portable gravimeters

2. Heiland, C. A., Sc.D., Geophysical Exploration, Prentice-Hall, Inc., New York, 1940, p 105 & pp 124--125.

(there are some types available for submerged off-coast prospecting) are capable of a maximum fractional sensitivity of 10^{-7}, or one part in 10^7. One possible exception is a trifilar gravimeter (not portable)[3] which may

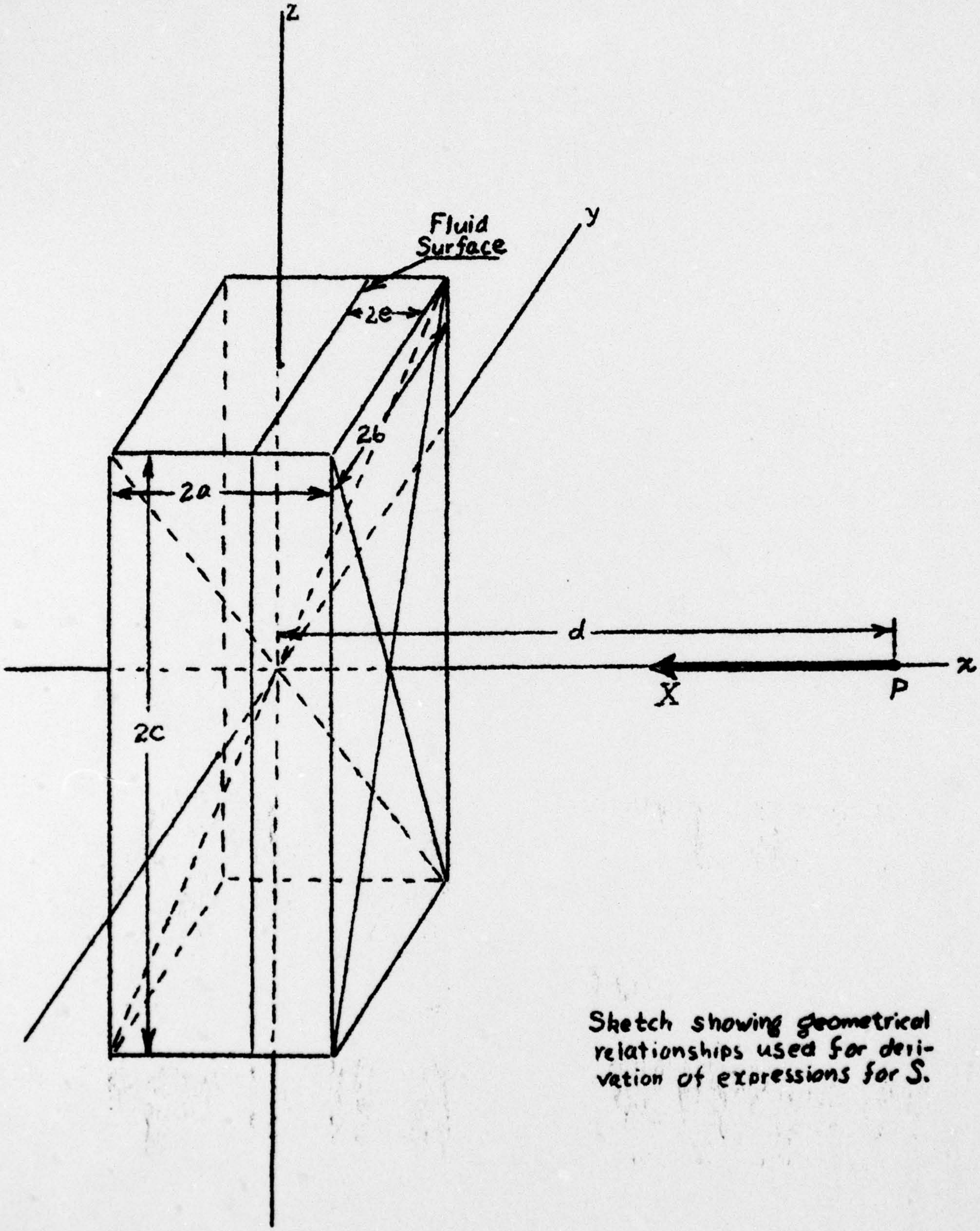
3. Heiland, loc. cit., pp 131--132.

go down to 10^{-9}. It is seen that in no case considered above are the objects detectable by field, or portable, type gravimeters at the distances specified. This should not be taken as complete condemnation of gravimetric methods in general, since a second major class of instruments, the gradiometers (torsion balance instruments), has not as yet been considered.

Ex.	a ft.	b ft.	c ft.	d ft.	t ft.	f	Wt. Metric Tons	Eq.
1	2	2	10	50	0.05	17.16	4.08	15-2
2	4	4	20	50	0.20	16.66	60.9	15-a
3	4	4	20	50	0.40	---	119.8	15-b
4	4	4	20	50	0.24	---	74.4	15-b
5	10	10	150	200	0.50	15.71	2760	15-a
6	5	5	100	100	0.25	15.61	444	15-a
7	5	5	75	100	0.25	---	337	15-b
8	10	10	200	100	0.50	15.59	3640	15-a

ANDREW N. SMITH

3 May 1955



Sketch showing geometrical relationships used for derivation of expressions for S.