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FINITE DIFFERENCE ENERGY TECHNIQUES FOR ARBITRARY MESHES APPLIE--ETC(U)
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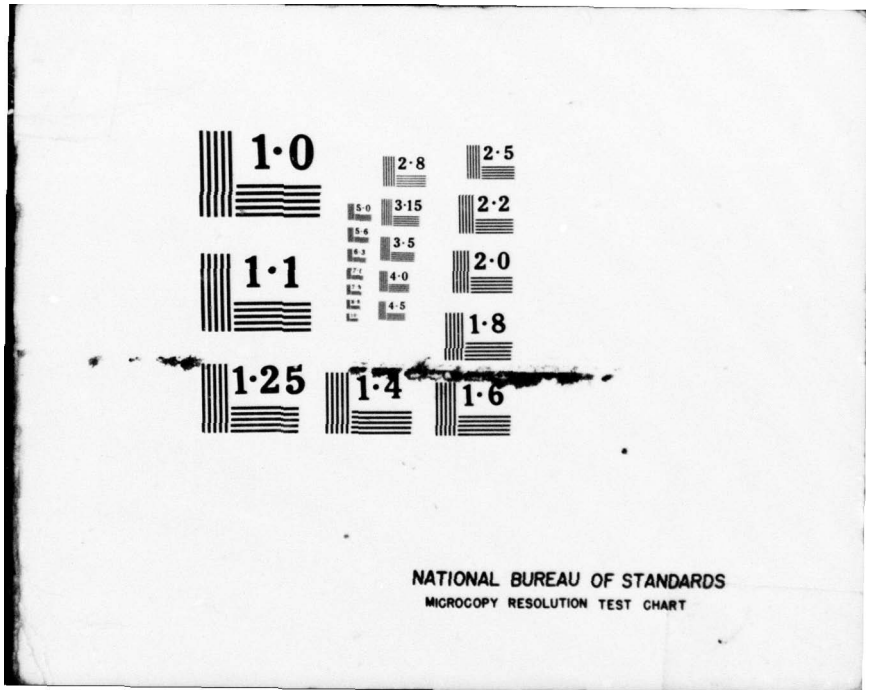
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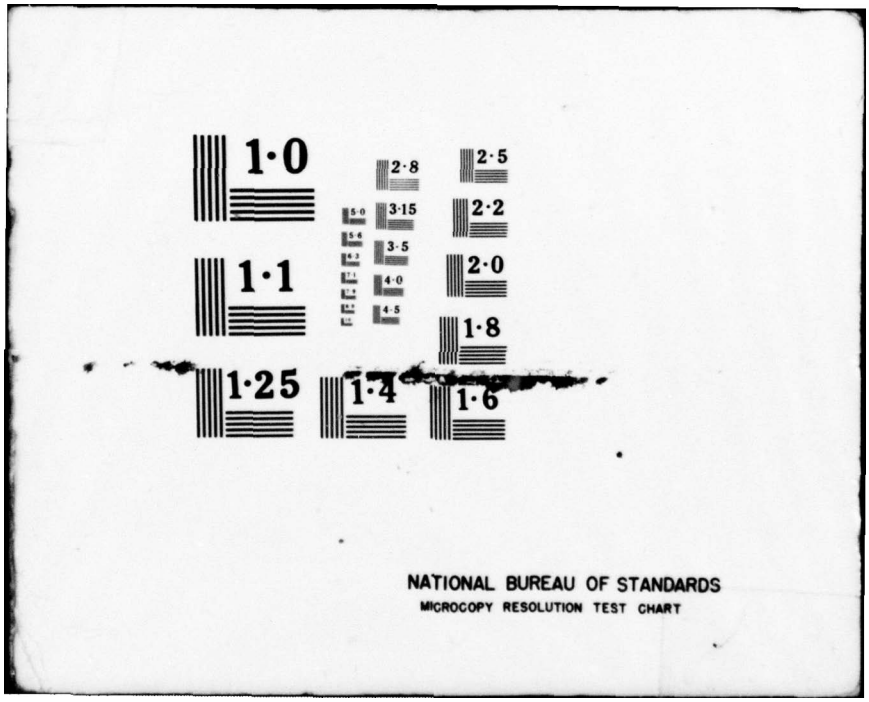
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FINITE DIFFERENCE ENERGY TECHNIQUES FOR ARBITRARY MESHES, APPLIED TO LINEAR PLATE PROBLEMS

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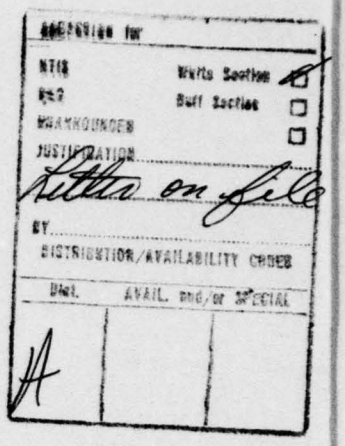

FINITE DIFFERENCE ENERGY TECHNIQUES FOR ARBITRARY MESHES, APPLIED TO LINEAR PLATE PROBLEMS[1]

BY

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and

N. Perrone[3]

ABSTRACT

A new energy based finite difference analytical technique is introduced. The method incorporates certain energy concepts and the ability to use arbitrary, irregular meshes within the framework of the Finite Difference Method. This formulation reduces any governing partial differential equations to a set of difference equations containing partial derivatives up to and including the second order. Further, certain strong simularities with the popular Finite Element Method are shown and the ability to solve problems with irregular boundaries is discussed. To demonstrate the Finite Difference Energy Method several plate bending problems are solved.

INTRODUCTION

The paper presents a new analytical method for two dimensional problems with irregular boundaries. The ability to handle any shaped boundaries and

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utilize arbitrary meshes is a direct result of this Finite Difference Energy Method. This method offers a promising, new approach to the solution of problems in applied mechanics.

The finite difference method approximates continuous derivatives by algebraic expressions relating discrete points in a domain; hence, governing relations may be formulated as a series of algebraic equations which are related to some finite difference mesh. A critical requirement in finite difference techniques is an accurate algebraic represenatation of derivatives. For the usual finite difference method with regularly spaced grids, these representations are satisfactory but results are generally quite sensitive to the grid size. For problems involving arbitrary meshes the question of a derivative accuracy becomes an essential problem. A major disadvantage of using the finite difference method as opposed to using finite element method is that one is usually required to use regular meshes with the former.

Recently there have been a number of research efforts directed at removing this restriction. These investigations progressed in two directions: The first, more usual one, focused on solving structural problems by using governing differential equations; the second more recent approach is based on an energy variational principle.

A finite difference approach for arbitrary meshes dealing with direct solution of governing differential equations was developed by Perrone and Kao [1]*. The accuracy of finite difference derivatives was addressed and a method for obtaining accurate approximations for derivatives up to the second order including mixed derivatives was presented. Fayed [2] introduced a so

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*Numbers in square brackets denote references which are collected at the end of the paper.

called variant finite-difference mesh and applied it to the solution of a problem with a complicated geometry. Other attempts have been made to develop finite difference techniques which used generalized or variable grids [3,4,5]. Using in part the approach for the calculation of derivatives described in Reference [1], Liszka and Orkisz [6] developed basic equations by minimization of derivative errors. They expressed the opinion that the finite difference method for arbitrary meshes is well enough developed to be competitive with the finite element method, especially in nonlinear cases.

A variational energy formulation has recently been used by Bushnell [7]. He demonstrated the usefulness of this approach and obtained some results which are very competitive with the finite element method. In Reference [7], he discusses recent applications of the finite difference energy method to stress, buckling and vibration problems. A two dimensional finite difference energy formulation for arbitrary meshes was introduced by Pavlin [8]. In his dissertation research he applied the method to solution of plate, membrane and shell problems.

Since the theory in this present effort is based on a variational energy approach, it might be useful to cite similarities with the Rayleigh-Ritz and finite element method. Unlike the usual Rayleigh-Ritz method, the entire domain of the treated structure is subdivided into finite difference type elements. Taylor series "displacement functions" are used to cover the domain of each finite difference element separately. As in the finite element method, one might interpret the Taylor series expressions used to determine derivative approximations as a sort of shape function. Similarities between the finite difference energy method and the finite element method were discussed in the paper of Key and Krieg [9].

The basic theory for the plate problem is presented in matrix form in the next section. Subsequently, finite difference approximation of the derivatives as well as nodal point arrangements for arbitrary meshes are discussed. Connection of the general finite difference procedure to a variational approach is treated in next section. Detailed mathematical conditions which must be enforced for various types of physical boundary conditions are then described. After presenting numerical results for a series of specific plate problems, a discussion and conclusions are provided in the final section.

BASIC THEORY FOR LINEAR PLATE PROBLEM

It is assumed that deformations are small implying simple bending of the plate and that no median surface stretch occurs. The Principle of Minimum Potential Energy requires that the variational of the total energy of system must vanish

$$\delta\Pi = \delta(U_E - W_P) = 0 \tag{1}$$

where

Π - total potential energy

U_E - strain energy of deformation

$(-W_P)$ - potential energy of all external forces

The total potential energy formula could be expressed in the following form:

$$\Pi = \iiint_{Vol} dU_T - \iiint_{Vol} (\bar{X}U + \bar{Y}V + \bar{Z}W)\, dVol - \iint_{S} (\bar{T}_X U + \bar{T}_Y V + \bar{T}_Z W)\, dS \tag{2}$$

where the terms on the right side are defined as follows:

$dU_T = \frac{1}{2}\{\varepsilon\}^T\{\sigma\}\, dVol$ - strain energy density

$\{\bar{X}\}^T = [\bar{X} \quad \bar{Y} \quad \bar{Z}]$ - external body forces

$\{\bar{T}\}^T = [\bar{T}_X \quad \bar{T}_Y \quad \bar{T}_Z]$ - external surface tractions

$\{\bar{U}\}^T = [U \quad V \quad W]$ - displacement components

According to simple bending theory, displacement components in the planar directions, are functions of the transverse deflection W, and therefore

$$\begin{aligned} U_B &= -Z\,W_{,X} \\ V_B &= -Z\,W_{,Y} \end{aligned} \tag{3}$$

As a consequence, the strain vector $\{\varepsilon\}$ will take the following form:

$$\{\varepsilon\} = \begin{Bmatrix} U_{B'X} \\ V_{B'Y} \\ U_{B'Y} + V_{B'X} \end{Bmatrix} = -Z \begin{Bmatrix} W_{,XX} \\ W_{,YY} \\ 2\,W_{,XY} \end{Bmatrix} \tag{4}$$

If the [C] matrix represents the material properties at each node, the stress vector will have the form:

$$\{\sigma\} = [C]\{\varepsilon\} \tag{5}$$

In the case where body forces are negligible and only the surface traction components in the Z direction remain, the total potential energy of element e will be:

$$\Pi_e = \frac{1}{2} \iiint_{Vol} \{\varepsilon\}^T [C] \{\varepsilon\}\, dVol - \iint_S \{\bar{U}\}^T \{\bar{T}\}\, dS_e \tag{6}$$

DERIVATIVE APPROXIMATIONS AND NODAL POINT TOPOLOGY

The governing equations, such as Equation (1) are based on an energy Minimal Principle. As a result, for most plate and shell problems to be treated with the energy approach, only second order derivatives are necessary to characterize the energy field. Unlike higher order derivatives, it is simpler to obtain accurate finite difference approximations for second order derivatives for an irregular mesh [10].

An arbitrary finite difference element is defined by central nodal point i and eight or less neighboring nodes j (Figure 1). The area of an element has

a polygonal shape obtained by cutting distances between node i and nodes j in half. The derivatives of a finite difference element i, obtained at node i, are taken as constant over the whole area of that finite difference element.

Elements along the edge of the structure (Figure 2) are called boundary finite difference elements. An internal type of node i uses an X-Y coordinate system, while boundary nodes use a T-N system, T being the tangential direction and N a normal direction relative to the boundary curve.

Five partial derivatives must be determined for the interior nodes specifically $W_{,X}$, $W_{,Y}$, $W_{,XX}$, $W_{,YY}$ and $W_{,XY}$. For the nodes along the boundary the derivatives $W_{,T}$, $W_{,N}$, $W_{,TT}$, $W_{,NN}$ and $W_{,TN}$ must be specified.

Depending on the mesh type different forms of elements will occur. The basic building block is a triangular shape. However, an element may have any polygonal form, from triangular to octagonal. The form of the element will greatly depend on whether the node is on the boundary or in the domain.

To define an element i the neighboring nodal points around node i must be chosen. The criteria suggested in Reference [1] will be used to select neighboring nodal points. This selection process is very important because it avoids an ill-conditioned coefficient matrix and results in better accuracy of partial derivatives. The domain in the vicinity of a given node i is broken into eight 45^{o} pie shaped segments (see Figure 3) and the closest neighboring nodal points j, in each segment to the center node i, is noted. Since the nodal mesh is arbitrary, it may happen that in some of the segments there is no close node, or no node at all (such as nodes along the boundary). In this case the scheme may become incomplete, which will have some effect on the accuracy of the derivatives; but this procedure is still acceptable because the minimum number of neighboring nodes is five. Should the closest node in a zonal pie segment be at a distance that is more than two average distances (between the nodes of

the entire mesh), such a mode will be considered as nonacceptable. For nodes along the boundary, neighboring nodes will be selected in the same way. Also, along the boundaries, the manner in which neighboring nodes are selected and their number, depends greatly on the boundary condition type.

As in Reference [1], connection between five unknown derivatives at node i and the values of the function in node i and neighboring nodes j will be established through a Taylor's series expansion; if we retain derivatives up to the second order, the expansion takes the following form:

$$W_j = W_i + (X_j - X_i)W_{i,X} + (Y_j - Y_i)W_{i,Y} + \frac{1}{2}(X_j - X_i)^2 W_{i,XX} + \frac{1}{2}(Y_j - Y_i)^2 W_{i,YY} + (X_j - X_i)(Y_j - Y_i)W_{i,XY} \tag{7}$$

A physical interpretation of this function is that when we know the values of the displacement function W and its five partial derivatives at node i, we may calculate values of the function at any other local field point. The associated surface is a nonsymmetric paraboloid, i.e. a surface of the second degree. However, we will invert the procedure and calculate the five unknown partial derivatives, starting with known values of the function W at node i and at least five adjacent nodes j. We then obtain a system of five equations with five unknown partial derivatives: by inverting this system the derivatives can be determined. The set of equations may be described in matrix form as follows:

$$[A]\{D_f\} = \{f\} \tag{8}$$

where

$[A]$ - is the coefficient matrix

$\{D_f\}$ - is the derivative vector

$\{f\}$ - is vector related to nodal function values at five selected nodes

By inverting the coefficient matrix the partial derivatives (D_f) may be obtained in an explicit form.

To achieve acceptable accuracies it is desirable to select more than five j nodes surrounding a given central i node. As shown in Reference [1] such a procedure could be established by adopting an averaging process. If we consider eight points surrounding a given central point as a typical case (such is the case for the usual, regular square mesh), then four points closest to i in the primary zones I-IV, Figure 3, are selected along with the closest node in each of zones V, VI, VII and VIII in turn. Derivative approximations are obtained by averaging the four sets of results [1].

It is useful to introduce a concept related to nodal point selection called mutual reciprocity. Two nodes are said to be reciprocal if the first selects the second as an adjacent node (using say scheme of Figure 3), and vice versa. Should the first select the second as an adjacent node, but the second does not select the first one then the nodes are said to be nonreciprocal. Better accuracy is achived when nodes are mutually reciprocal. Nonreciprocal nodes occur more often on the boundary than in the domain.

ARBITRARY FINITE DIFFERENCES COUPLED TO A VARIATIONAL APPROACH

If we define [B] as the inverse of the coefficient matrix, the derivative vector may be expressed as

$$\{D_f\} = [B]\{f\} \tag{9}$$

In the case of small deformations, the energy equation contains only second order partial derivatives (see Equation 4), it is convenient to use a reduced inverted coefficient matrix [B*], which is rectangular by its nature

$$[B^*] = b_{pq}, \qquad \begin{matrix} p = 3,4,5 \\ q = 1,\cdots,5 \end{matrix} \tag{10}$$

Substituting Equation (10) into Equation (4), we obtain the strain vector in the following form:

$$\{\varepsilon\} = -Z[B^*]\{f\} \tag{11}$$

Using Equations (11) and (6) and the fact that there are no U and V displacements in the middle plane, we find that the total potential energy of an element to be

$$\Pi_e = \frac{1}{2}\iiint_{Vol} Z^2\{f\}^T[B^*]^T[C][B^*]\{f\}\,dVol - \iint_S W\,\bar{T}_Z\,dS_e \tag{12}$$

To enforce equilibrium, a variational form of finite difference energy method will be used. The total potential energy over the entire structure area will be:

$$\Pi = \sum_{e=1}^{n} \Pi_e \tag{13}$$

where n is the total number of finite difference elements. The Principle of Minimum Potential Energy (see Equation 1) requires the variation of Π with respect to each nodal deflection vanish

$$\delta_e\{\Pi\} = 0 \quad , \quad e = 1, 2, \ldots, n \tag{14}$$

What evolves are n equations in terms of n unknown displacements at n nodal points.

Two different type of terms from the variation of a generic node i appear in the following equation:

$$\delta_i\{\Pi\} = \delta_i\{\Pi_i\} + \delta_i\Big\{\sum_{j=1}^{r_i} \Pi_j\Big\} = 0 \tag{15}$$

Applying operator (15) and using formula (12), where all matrices and vectors have averaged values, we obtain the following expression:

$$\left\{ \iiint_{Vol} Z^2 \frac{\partial[\{f\}^T]}{\partial W_i} [B^*]^T [C][B^*]\{f\}\, dVol - \iint_S \bar{T}_Z\, dS \right\}_i + \sum_{j=1}^{r_i} \left\{ \iiint_{Vol} Z^2 \frac{\partial[\{f\}^T]}{\partial W_i} [B^*]^T [C][B^*]\{f\}\, dVol \right\}_j = 0 \tag{16}$$

To transform Eq. (16) into a more convenient form, we may impose the following substitutions

$$\begin{aligned} [G^*]_{i,i} &= \frac{\partial[\{f\}^T]_i}{\partial W_i} [B^*]_i^T \\ [G^*]_{j,i} &= \frac{\partial[\{f\}^T]_j}{\partial W_i} [B^*]_j^T \\ q(i) &= \iint_S \bar{T}\, dS_i \end{aligned} \tag{17}$$

where the last item represents the load term.

The volume differential is obviously

$$dVol = dZ\, dS$$

and the thickness of plate is constant everywhere; after separating the variables of integration and combining Equations (17) and (16), the following result is obtained:

$$\int_{-h/2}^{h/2} Z^2 \left\{ \left[\iint_S [G^*]_{i,i} [C][B^*]\{f\}\, dS \right]_i + \sum_{j=1}^{r_i} \left[\iint_S [G^*]_{j,i} [C][B^*]\{f\}\, dS \right]_j \right\} dZ = q(i) \tag{18}$$

Integrating over the Z variable and replacing the area integrals with finite values of the areas, because all other matrices under the integral are assumed to be constant over the area of the elements, we may write Equation (18) in the form

$$\frac{h^3}{12} \left\{ [G^*]_{i,i} [C][B^*]_i \{f\}_i \Delta S_i + \sum_{j=1}^{r_i} [G^*]_{j,i} [C][B^*]_j \{f\}_j \Delta S_j \right\} = q(i) \tag{19}$$

Now, we introduce the stiffness coefficients vector of node i

$$[k]_{ii} = \frac{h^3}{12} \Delta S_i [G^*]_{i,i} [C][B^*]_i \tag{20}$$

and the off diagonal stiffness elements of the vector node of i:

$$[k]_{ji} = \frac{h^3}{12} \Delta S_j [G^*]_{j,i} [C][B^*]_j \tag{21}$$

Now, Equation (19) for node i assumes the following form:

$$[k]_{ii}\{f\}_i + \sum_{j=1}^{r_i} [k]_{ji}\{f\}_j = q(i) \tag{22}$$

By regrouping k and f elements of Equation (22) in node i and creating new vectors

$[k]_{(i)}$ - n dimensional stiffness coefficient vector for node i

$\{W\}$ - n dimensional displacement vector

we transform Equation (22) into the following form:

$$[k]_{(i)}\{W\} = q(i) \tag{23}$$

which for all n nodes produces the overall assembly matrix equation of the problem

$$KW = Q \tag{24}$$

A computer program, developed using Fortran V language, solves this system of n linear, nonhomogeneous equations with n unknown displacement, for any domain and boundary conditions.

BOUNDARY CONDITIONS

For most boundary conditions in solid mechanics, specification of displacement derivatives up to the second order are sufficient. When the boundary is irregular the tangent and normal at a boundary node usually have different directions from the X and Y global coordinate system (Figure 4).

IRREGULAR BOUNDARIES

As it is important to know geometrical characteristics of boundary in any boundary node, two classes of domains will be analyzed: Areas described by a known mathematical function and other shapes of irregular domains.

When the analytical function of a boundary is known it is quite easy to find out their characteristics using pure mathematical derivatives. For example, finding the tangential direction at a boundary node is straightforward.

The case where the boundary is not functionally definable is less trivial. To determine the tangent at the boundary at node N_1, relative to the X-Y system, part of the boundary N_o-N_1-N_2 will be approximated by interpolating a polynomial (Figure 4). It is convenient to pass a parabola through the three boundary points. Boundary derivatives may then be readily calculated.

The shape function Equation (7), applied to a boundary node, has the simplified form:

$$W_j = W_C + T_j W_{C,T} + N_j W_{C,N} + \frac{1}{2} T_j^2 W_{C,TT} + \frac{1}{2} N_j^2 W_{C,NN} + T_j N_j W_{C,TN} \qquad (25)$$

Generally, five nodes are selected through which the Taylor series is forced to pass in an N-T coordinate system. The fact that the nodes along the boundary are described via a local coordinate system does not cause any difficulty, because the energy stored depends on the distances between nodes, which doesn't vary during transformation.

SPECIFIC SUPPORT CONDITIONS

Boundary conditions for different supports are developed by exploring the form of shape function given through Equation (25). The following three boundary conditions are considered:

-- Simple supports

-- Clamped supports

-- Free edges

If we analyze different types of supports regarding their kinematic and natural boundary conditions it is obvious that a varying number of unknown boundary derivatives remain to be determined. Where more boundary conditions could be applied, less unknowns appear; whenever less unknown partial derivatives occur at a boundary node, less adjacent nodes have to be taken in combination. If more nodes than necessary are chosen, averaging is applied. For example, it will be shown that for clamped supports only one adjacent node is necessary, while in the case of free edge nodes, six points are needed.

Simply Supported Plate

If the edge of a plate is simply supported, the deflection as well as the bending moment along the edge must vanish. As the condition W = 0 is also applied in the vicinity of the node C (see Figure 5), it means that

$$W_{C,T} = W_{C,TT} = 0$$

and therefore the boundary conditions are

$$\begin{aligned} W_C &= 0 \\ W_{C,NN} &= 0 \end{aligned} \tag{26}$$

As a consequence, the boundary shape function given through Equation (25), for a simply supported boundary, reduces to:

$$W_j = N_j W_{C,N} + T_j N_j W_{C,TN} \tag{27}$$

It becomes obvious, that we need only two j nodes in order to define unknown derivatives; other nodes could be used by averaging to improve the solution accuracy.

Plate Boundary with Clamped Support

If the edge of the plate is clamped in node C the deflection as well as slope of the middle plane in N direction must be zero. The boundary conditions at such a node (see Figure 6), are:

$$\begin{aligned} W_C &= 0 \\ W_{C,N} &= 0 \end{aligned} \tag{28}$$

As a result, we find that

$$W_{C'T} = W_{C'TT} = W_{C'TN} = 0$$

The shape function at the nodes along the boundary, assumes the following form in this case:

$$W_j = \frac{1}{2} N_j^2 W_{C'NN} \tag{29}$$

Only one j node is required for satisfaction of the clamped support condition.

Plate Boundary with Free Edge

A free boundary (see Figure 7), requires that the edge moment and shear force vanish (or be equal to applied loads). As shear forces are related to third derivatives of the displacement, an expanded form of Equation (25), which includes those terms is necessary. The resulting form of Equation (25) is as follows:

$$W_j - W_C - \frac{1}{6D} N_j^3 q(C) = T_j W_{C'T} + N_j W_{C'N} + \frac{1}{2}(T_j^2 - \nu N_j^2) W_{C'TT}$$
$$+ T_j N_j W_{C'TN} + \frac{1}{6} N_j^3 (3\frac{T_j^2}{N_j^2} + \nu - 2) W_{C'TTN} + \frac{1}{2} T_j N_j^2 W_{C'TNN} \tag{30}$$

For this case, at least 6 j adjacent nodes on the boundary and inside the domain are required.

EXAMPLES

Obviously, accuracy of solutions obtained depends on the number of nodal points selected. If the number of nodal points is increased, the solution should become more accurate. As accuracy and the number of nodal points affects computer time, some compromise should be achieved.

Results obtained are correlated with existing solutions [14, 15, 16]. For illustrative purposes, the displacement function W in some characteristic cross-sections are presented.

SIMPLY SUPPORTED CIRCULAR PLATE

A simply supported circular plate (Figure 8), loaded by uniformly distributed load is solved using four different types of arbitrary meshes. The input data are

$$\nu = 0.3$$

$$r_o, E, h, p$$

with the flexural rigidity:

$$D = \frac{E h^3}{12(1-\nu^2)}$$

The theoretical solution in dimensionless form is

$$w = \frac{W}{r_o} = \frac{5+\nu-\varsigma^2(6+2\nu)+\varsigma^4(1+\nu)}{64(1+\nu)} \frac{p\, r_o^3}{D} = C_2 \frac{p r_o^3}{D}$$

where

$$\varsigma = \frac{r}{r_o}$$

All solutions obtained by the finite difference energy method for arbitrary meshes are shown in Figure 9.

Each type of mesh used D1, D2, D3 and D4 has in parenthesis the number of nodes covering the entire domain; solutions obtained show accuracy associated with distribution of mesh points and the number of the nodes.

Deflections along the radius of the plate obtained for mesh type D4, with 77 nodal points, very closely coincides with the exact, theoretical solution.

CLAMPED SQUARE PLATE

A square plate with clamped boundaries (Figure 10), loaded by a uniformly distributed load is solved using five different types of meshes. Input data are a, h, ν, E, p with flexural rigidity D.

All solutions (Figure 11) in dimensionless form are given in the following expression

$$w = \frac{W}{a} = C_3 \frac{p a^3}{D}$$

where W is the dimensionless deflection of plate.

Each type of mesh used D5, D6, D7, D8 and D9 has in parenthesis the number of nodes covering the entire domain of the plate. Solutions obtained, show accuracy being associated with the total number of used nodes. Results of case D9 with the largest number of nodes, almost coincides with the results from Reference [16]. It should be noted that mesh D6 represents an example with an aribtrary spread of 64 nodes totally.

KIDNEY SHAPED PLATE WITH MIXED BOUNDARY CONDITIONS

In this example, the structure with an irregular domain (see Figure 12), loaded with a uniformly distributed load p, is solved. Boundary conditions are mixed such that the plate is clamped along a portion of the boundary; the remaining part is free.

Figure 12 shows 107 nodes used and the results obtained for deflections along three marked, cross-sections.

Geometric and physical properties are as follows:

Dimensions: see scale of geometry

$E = 332\,0000$ kips/ft^2

$\nu = 0.3$

$h = 0.25$ ft

$p = 1.0$ kips/ft^2

DISCUSSION AND CONCLUSIONS

A finite difference energy method capable of handling irregular domains has been developed and applied to linear plate problems. The method incorporates the Principle of Minimum Potential Energy and the ability to use arbitrary meshes within the framework of the finite difference method.

The total area of a domain is subdivided into smaller portions, which we might call finite difference elements. A Taylor's series expansion is used in the spirit of a shape function to determine derivatives at a given node as function of surrounding node values. The entire potential energy in the system is

minimized with respect to each of n nodal displacements. An n x n system of equations arises which is solved for the displacement field. When calculating derivatives at each node a zonal criteria is setup to assist in selecting proper adjacent nodes for a given central node. Should node A select node B as a surrounding point and node B select node A, the two nodes are said to be mutually reciprocal. If many nodes are nonreciprocal appreciable error will arise.

To show the similarity between the finite difference energy method for arbitrary meshes and the finite element method, the current methodology has been developed in a way paralleling the finite element method. The final form of equilibrium equations, with assembled stiffness matrix, clearly demonstrates this point. Further comparisons with the finite element method reveal the finite difference elements to be esentially nonconforming; what results usually is a more "flexible" stiffness matrix.

In addition to the linear theory shown here, the theory has been extended to nonlinear problems and will be presented at a later date. The techniques and programs are developed for the solution of membranes and thin, shallow spherical shells [8].

Using basic elements and the essential approach of the finite difference energy method for arbitrary meshes described here, we should be able to solve any general thin shell. Only minor changes in theory would be necessary.

Since energy related techniques could be used in both the formulation of the finite element and arbitrary mesh finite difference methods, it should be possible to utilize a combined approach in a single problem. Indeed, Bushnell has done so for essentially one-dimensional problems, using Fourier series expansions in the second direction. In principle, there is no obstacle to such a general combination. Displacement components at the nodes are the fundamental unknowns

for both methods. The shape function will differ between the finite difference and finite element method and some attention will have to be focused on the interface regions. Significant economies in computer time may be possible with a combination method. Finite elements would probably be selected in zones where sharp load or stress gradients exist as well as at complex boundary regimes.

The theory of the arbitrary finite difference energy method for irregular domains could be extended to another large class of problems in which a structure, undergoes plastic deformations in addition to elastic deformations. To accomplish this, an extension of the theory and procedure is feasible.

It should be of interest to examine the accuracy of the energy finite difference/arbitrary mesh approach when concentrated or other sharp load gradients occur. From an energy viewpoint it should be feasible to obtain the work done as a result of deformations in the vicinity of these load systems.

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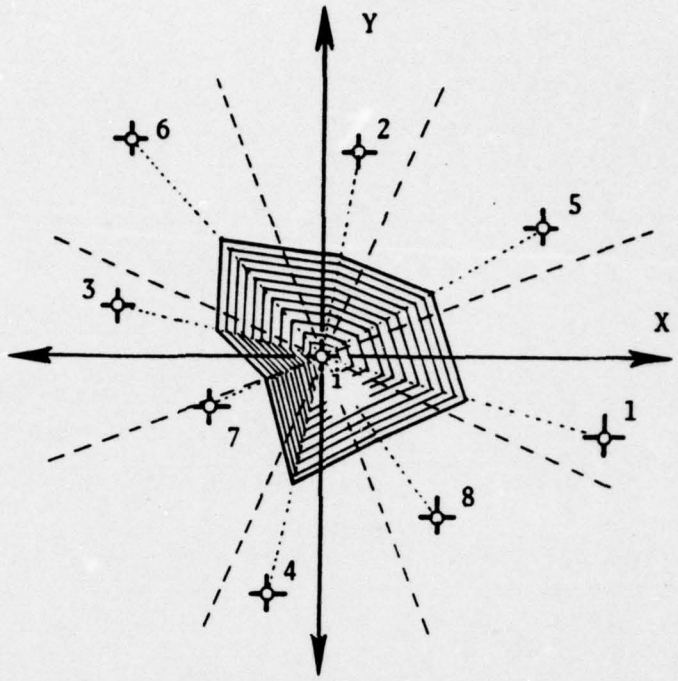

Fig. 1 Area Associated with 8 Arbitrary Finite Difference Nodes (by connecting mid point locations)

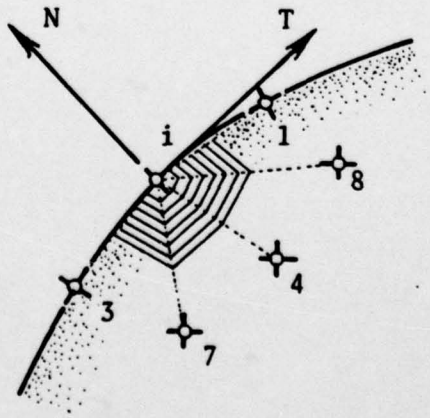

Fig. 2 N-T Coordinates at Boundary Node i

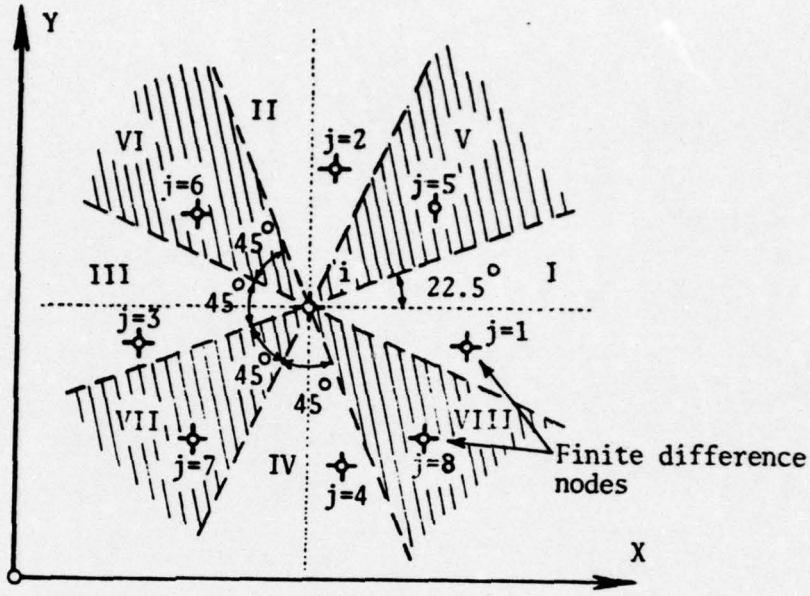

Fig. 3 Example of 8 FD Nodes. Primary in I to IV; Secondary in V to VIII

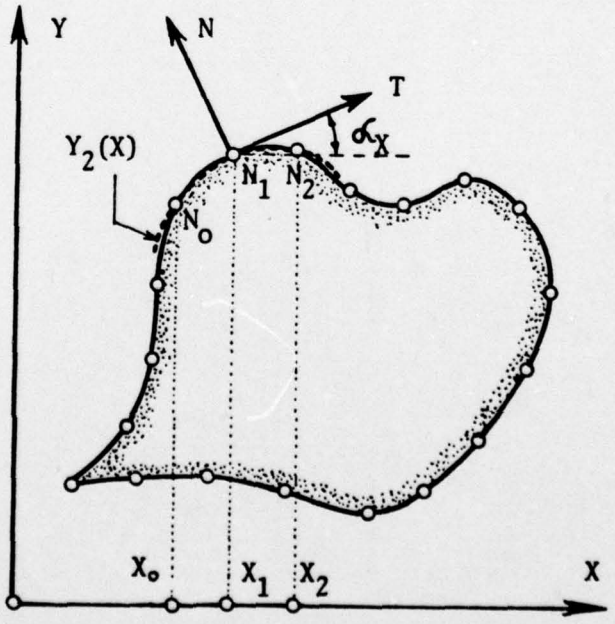

Fig. 4 Location of Boundary Coordinates Relative to Fixed Ones

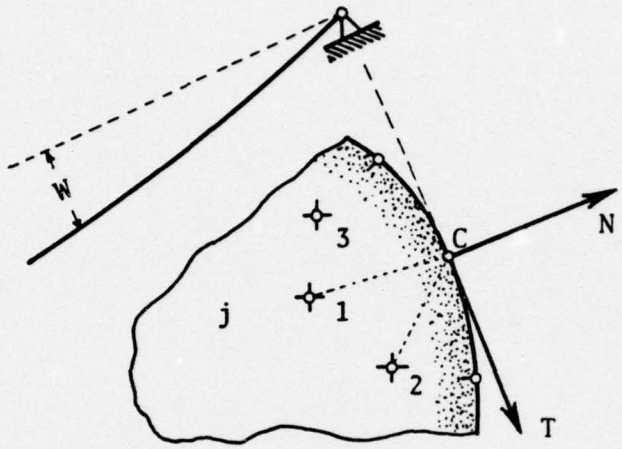

Fig. 5 Boundary Nodes for Simple Support

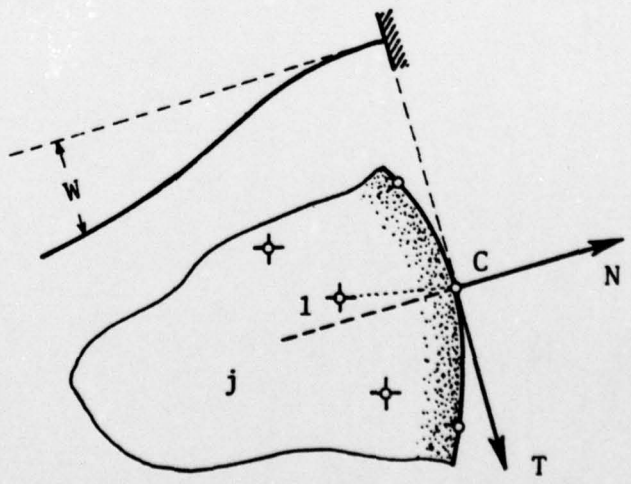

Fig. 6 Boundary Nodes for Clamped Support

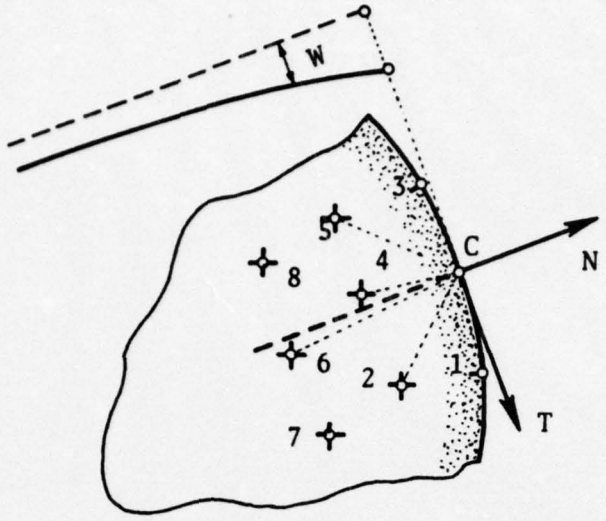

Fig. 7 Boundary Nodes for Free Edge

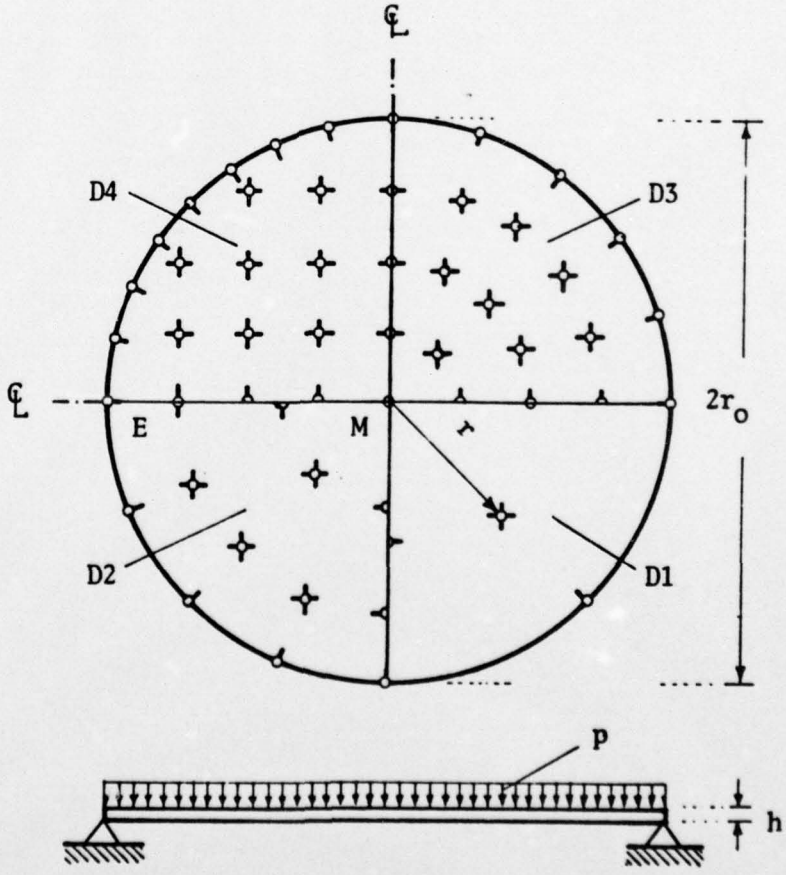

Fig. 8 Circular Plate with Various Nodal Distributions in Each Quadrant

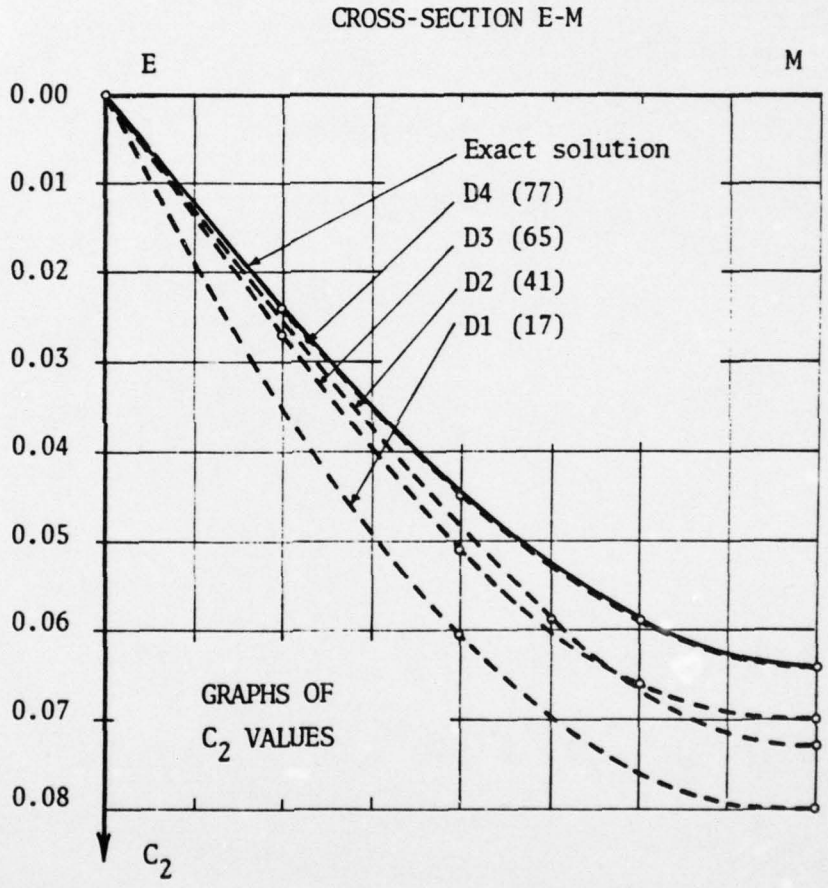

Fig. 9 Circular Plate Deflection Profiles for Different Solutions

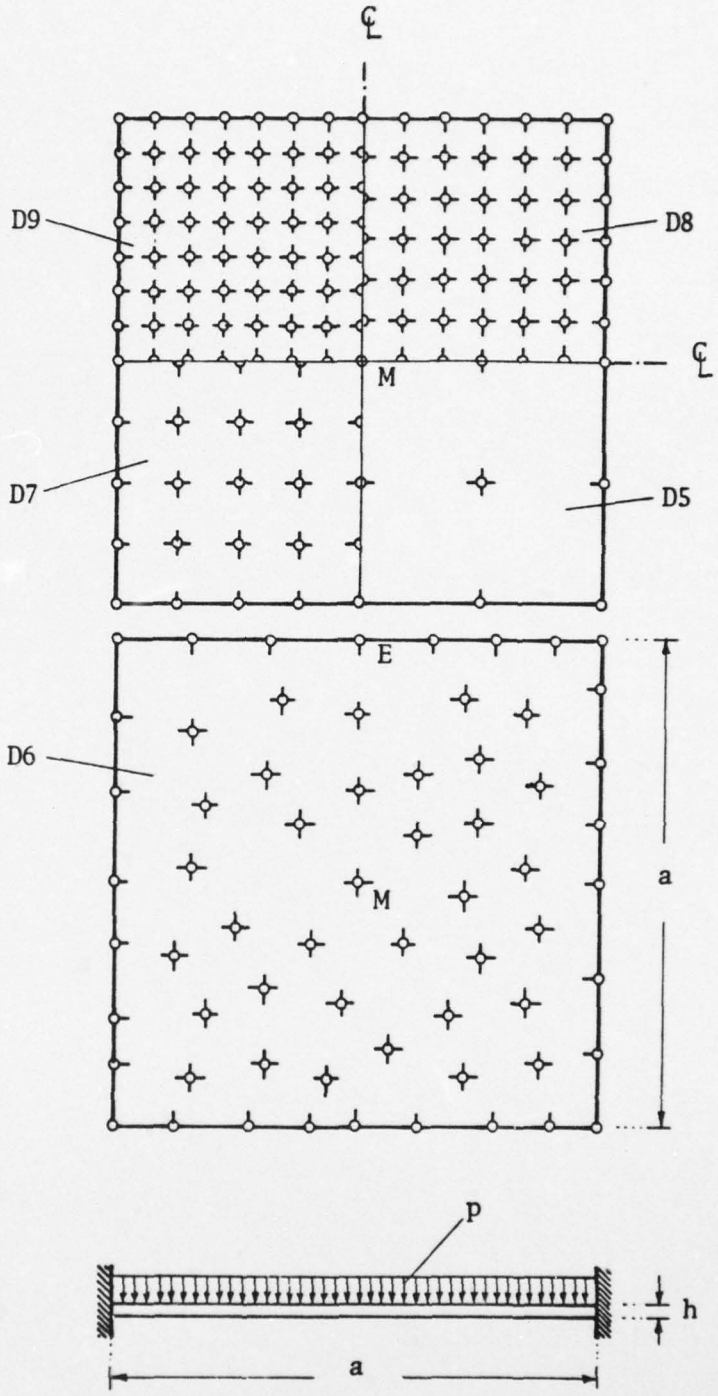

Fig. 10 Square Clamped Plate with Different Nodal Distribution Patterns

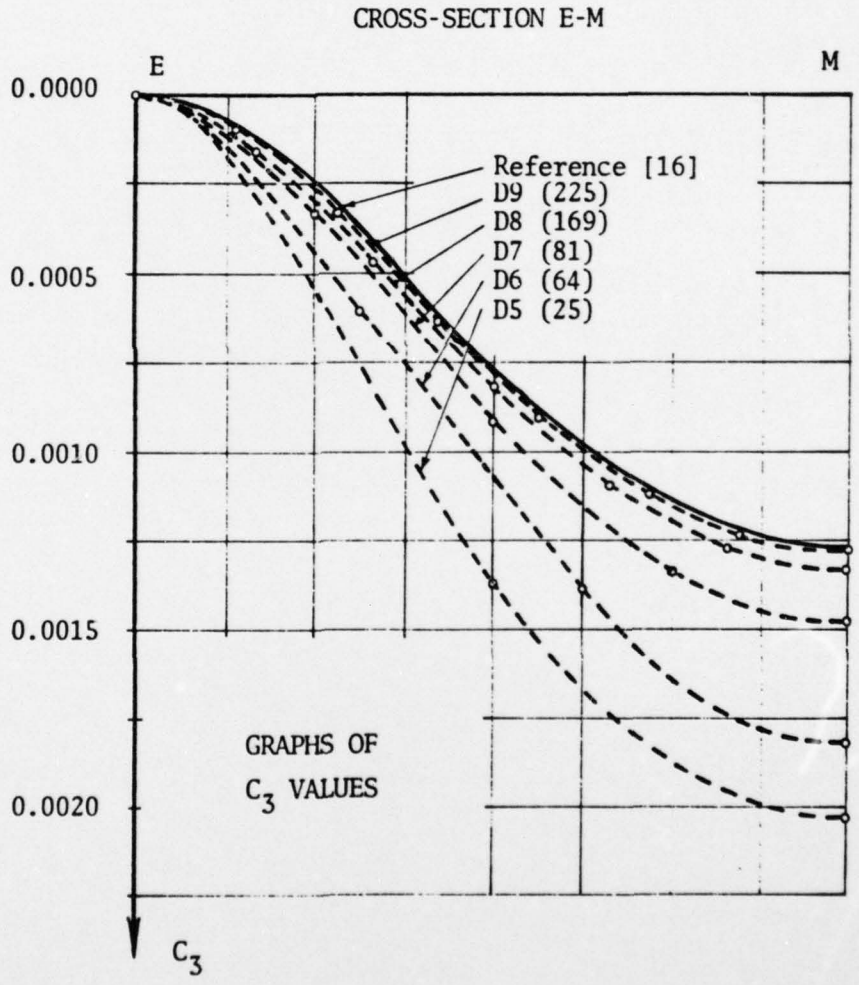

Fig. 11 Deflection Profiles for Clamped Square Plate

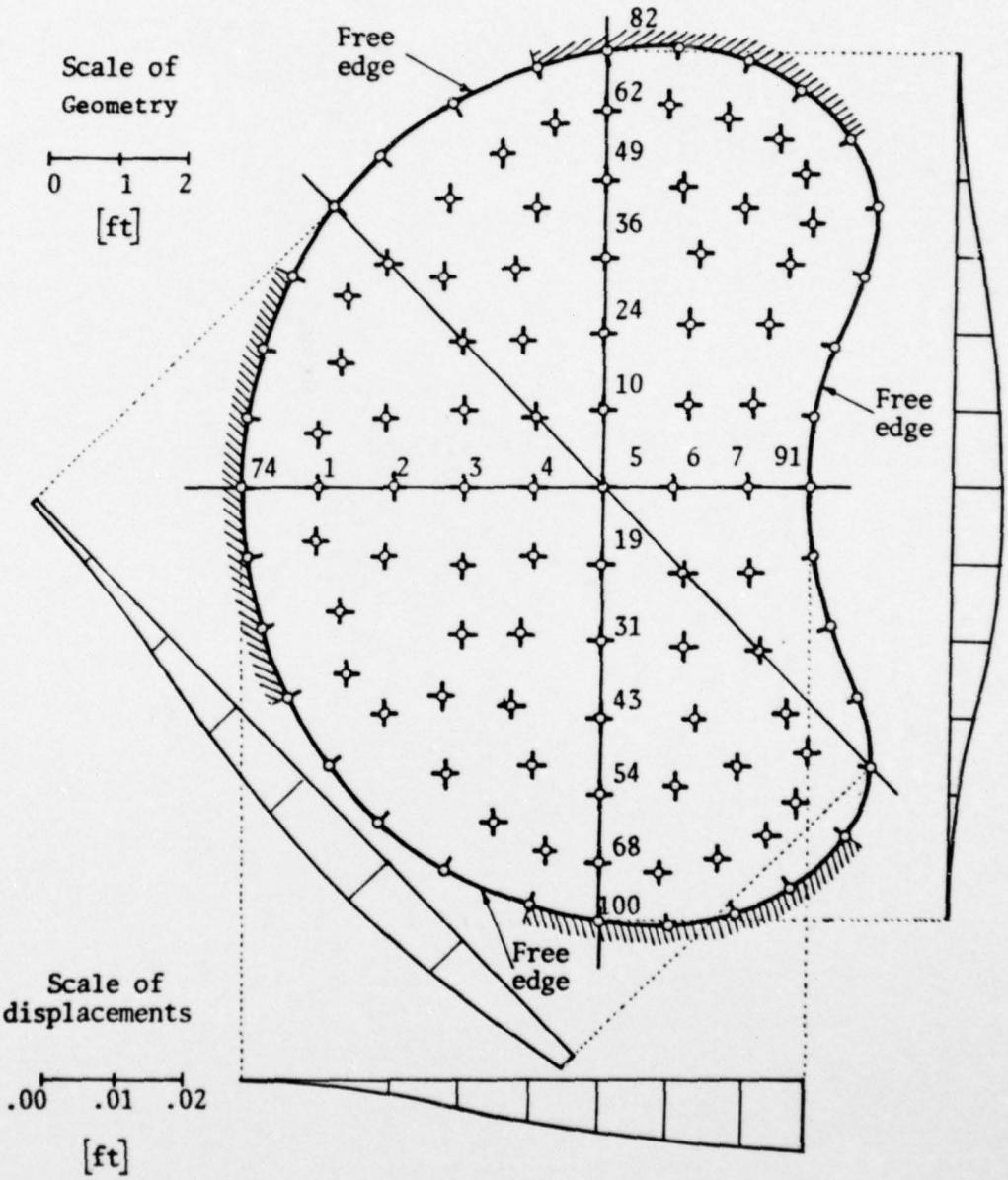

Fig. 12 Kidney Shaped Plate with Mixed Edge Supports.
Deflection Profiles for Selected Sections

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: TR-78-1
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER:

4. TITLE (and Subtitle): Finite Difference Energy Techniques for Arbitrary Meshes Applied to Linear Plate Problems.

5. TYPE OF REPORT & PERIOD COVERED: Technical Report

6. PERFORMING ORG. REPORT NUMBER:

7. AUTHOR(s): V. Pavlin and N. Perrone

8. CONTRACT OR GRANT NUMBER(s): N00014-76-C-0872 new
NR064-583

9. PERFORMING ORGANIZATION NAME AND ADDRESS:
University of Maryland
Dept. of Aerospace Engineering
College Park, Md. 20742

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS:

11. CONTROLLING OFFICE NAME AND ADDRESS:
Office of Naval Research
Arlington, VA 22217

12. REPORT DATE: January 1978

13. NUMBER OF PAGES: 35 72 30 pp.

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):

15. SECURITY CLASS. (of this report): Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:

16. DISTRIBUTION STATEMENT (of this Report)

DISTRIBUTION STATEMENT A
Approved for public release
Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
finite differences
Finite elements
arbitrary meshes
Energy techniques
kidney shaped
plate

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
A new energy based finite difference analytical technique is introduced. The method incorporates certain energy concepts and the ability to use arbitrary, irregular meshes within the framework of the Finite Difference Method. This formulation reduces any governing partial differential equations to a set of difference equations containing partial derivatives up to and including the second order. Further certain strong simularities with the popular Finite Element Method are shown and the ability to solve problems with irregular

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boundaries is discussed. To demonstrate the Finite Difference Energy Method several plate bending problems are solved.