AD-A051 076 WEAPONS RESEARCH ESTABLISHMENT SALISBURY (AUSTRALIA) F/G 20/4
THEORY OF TRANSONIC GAS GUNS.(U)
AUG 77 L M SHEPPARD
UNCLASSIFIED WRE-TR-1877(W) NL

1 OF 1
ADA
051076

END
DATE
FILMED
4-78
DDC

WRE-TR-1877 (W)

AR-000-936

AUSTRALIA

DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

WEAPONS RESEARCH ESTABLISHMENT

SALISBURY, SOUTH AUSTRALIA

TECHNICAL REPORT 1877 (W)

THEORY OF TRANSONIC GAS GUNS (U)

L.M. SHEPPARD

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

Approved for Public Release.

COPY No. 87

AR-000-936

DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

WEAPONS RESEARCH ESTABLISHMENT

TECHNICAL REPORT 1877 (W)

THEORY OF TRANSONIC GAS GUNS (U)

L.M. Sheppard

S U M M A R Y

A review is given of analytical methods for predicting the muzzle velocity of compressed gas guns. Steady adiabatic expansion theory, quasi-steady theory and unsteady theory are all examined, with emphasis placed on simplified analytical methods applicable to subsonic and transonic muzzle velocities. New formulae are obtained and compared with numerical results from a full theoretical treatment of the gas flow. The comparisons show that the new analytic results are remarkably effective and can therefore be used for gun design. The effects of atmospheric counter pressure are treated separately and a new method of correcting the predicted velocity is given. An analysis of data from actual subsonic guns highlights the importance of muzzle pressure reflections in reducing the retarding pressure at low subsonic speeds. The observed muzzle velocities are about 10% below the muzzle velocities predicted by theory. Finally, nitrogen, helium and hydrogen are compared as potential driver gases for transonic guns.

POSTAL ADDRESS: The Director, Weapons Research Establishment, Box 2151, G.P.O., Adelaide, South Australia, 5001.

DOCUMENT CONTROL DATA SHEET

Security classification of this page: UNCLASSIFIED

1 DOCUMENT NUMBERS		2 SECURITY CLASSIFICATION	
AR Number:	AR-000-936	a. Complete Document:	Unclassified
Report Number:	WRE-TR-1877 (W)	b. Title in Isolation:	Unclassified
Other Numbers:		c. Summary in Isolation:	Unclassified

3 TITLE

THEORY OF TRANSONIC GAS GUNS

4 PERSONAL AUTHOR(S):

L.M. Sheppard

5 DOCUMENT DATE:

August 1977

6

6.1 TOTAL NUMBER OF PAGES: 43

6.2 NUMBER OF REFERENCES: 13

7

7.1 CORPORATE AUTHOR(S):

Weapons Research Establishment

7.2 DOCUMENT (WING) SERIES AND NUMBER

8 REFERENCE NUMBERS

a. Task: DST 76/011

b. Sponsoring Agency:

9 COST CODE:

331252

10 IMPRINT (Publishing establishment):

Weapons Research and Development Wing
TR-1877

11 COMPUTER PROGRAM(S)
(Title(s) and language(s))

12 RELEASE LIMITATIONS (of the document):

Approved for public release.

12.0	OVERSEAS	NO		P.R.	1	A		B		C		D		E	

Security classification of this page: UNCLASSIFIED

Security classification of this page: UNCLASSIFIED

13 **ANNOUNCEMENT LIMITATIONS** (of the information on these pages):

No limitation.

14 **DESCRIPTORS:**

a. EJC Thesaurus Terms

Nitrogen helium hydrogen
Gas guns
Gas dynamics
One-dimensional flow
Guns (ordnance)
Flight tests
Predictions
Subsonic flow
Transonic flow
Analyzing

b. Non-Thesaurus Terms

Muzzle velocity
Driver gases

15 **COSATI CODES:**

1906
2004

16 **LIBRARY LOCATION CODES** (for libraries listed in the distribution):

SW SR SD AACA NL

17 **SUMMARY OR ABSTRACT:**
(if this is security classified, the announcement of this report will be similarly classified)

A review is given of analytical methods for predicting the muzzle velocity of compressed gas guns. Steady adiabatic expansion theory, quasi-steady theory and unsteady theory are all examined, with emphasis placed on simplified analytical methods applicable to subsonic and transonic muzzle velocities. New formulae are obtained and compared with numerical results from a full theoretical treatment of the gas flow. The comparisons show that the new analytic results are remarkably effective and can therefore be used for gun design. The effects of atmospheric counter pressure are treated separately and a new method of correcting the predicted velocity is given. An analysis of data from actual subsonic guns highlights the importance of muzzle pressure reflections in reducing the retarding pressure at low subsonic speeds. The observed muzzle velocities are about 10% below the muzzle velocities predicted by theory. Finally, nitrogen, helium and hydrogen are compared as potential driver gases for transonic guns.

Security classification of this page: UNCLASSIFIED

TABLE OF CONTENTS

LIST OF APPENDICES

1. INTRODUCTION

This report is concerned primarily with the gas dynamic theory underlying gas guns operating with muzzle velocities in the transonic range. A good reference to the fundamental theory of unsteady gas motions is the review of Seigel(ref.1) who has applied the theory to two-stage hypervelocity launchers in which a moving piston is used as a gas compressor. More than this, Seigel has given a comprehensive coverage of the theory of single-stage gas guns such as are considered in this report. The bulk of the design charts in reference 1 apply to single-stage guns and are therefore relevant to the present investigation. A few of the many papers concerned with gas dynamics, hypervelocity launchers and the use of moving pistons as gas compressors are given in references (2,3,4,5,6,7). Reference 2 is included as an example of a book providing an introduction to unsteady gas dynamics. References 3,4 and 5 contain additional bibliographies. Reference 6, on the other hand, is concerned solely with the use of a moving piston to compress gas to a very high pressure. Reference 7 describes the application of a piston compressor to drive a hypersonic wind tunnel.

Interest in gas guns began at Weapons Research Establishment ten years ago. During an investigation into the flight of lifting projectile shapes, it was suggested that a gas gun could fire single projectiles, mounted in sabots, and that measurements could be made of their subsequent flight. A gun of 127 mm bore and capable of operating at pressures up to 3000 kPa was constructed and has been in almost continuous use since its first firing late in 1967(ref.8). Sabot speed is measured at the muzzle of the gun. The gun has supported tasks ranging from the statistical study of projectile impact patterns to assisting in the development of a W.R.E. image synchronization camera. Early in 1973, a larger gas gun of 384 mm bore was commissioned to study the flight of larger projectiles and clusters of small projectiles(ref.9). This gun operates at pressures up to 1000 kPa. The two gas guns, together with associated instrumentation, provide a facility for aerodynamic research(ref.10,11) and the development of small munitions in a close-to-the-laboratory environment where remote and extensive equipments such as those at Woomera in South Australia are not necessary.

The gas guns, a control and recording building and pressurizing equipment are located at one end of an area approximately 1100 m by 600 m in size. The ground instrumentation includes some fixed wide-angle ballistic cameras which are suitable for trials at night to determine projectile position, attitude and velocity throughout flight. Two lights flashing at a frequency of about 50 Hz are carried in the projectile and the light flashes recorded on the ballistic camera plates. During day firings performance of the sabot and subsequent flight of the projectile may be recorded by high speed cameras.

The W.R.E. gas guns continue to be used extensively. For this reason, preliminary consideration has been given to the design of a gun capable of much higher speeds. Since aerodynamic problems of a varied nature frequently occur at transonic speeds, any such gun should be capable of testing projectiles at speeds up to at least a Mach number of 1.2, and preferably up to a Mach number of 1.5 or so. The purpose of the present report is to provide the necessary theoretical design basis. As will be seen, the report relies heavily on work carried out by other investigators.

We begin with the simple flow model described by Perfect(ref.12). This model is extended to higher subsonic speeds in Section 2. For supersonic gas speeds, a different model is necessary and one based on unsteady expansion waves is described in Section 3. Throughout, the emphasis in on simplified analytical methods which may be applied readily by gun designers. Section 4 introduces the retarding effect of atmospheric counter pressure acting on the front face of a projectile. The performance of existing subsonic gas guns is examined in detail in Section 5 in order to assess the precision with which muzzle velocities can be predicted. A summary of the recommended design procedures is given in Section 6. An example is presented in Section 7, to compare hydrogen, helium and nitrogen as driver gases for a transonic gas gun. Finally, conclusions are given in Section 8.

The reader interested solely in design application should begin by reading Section 1.1, followed by Sections 6 and 7.

1.1 Steady internal flow model

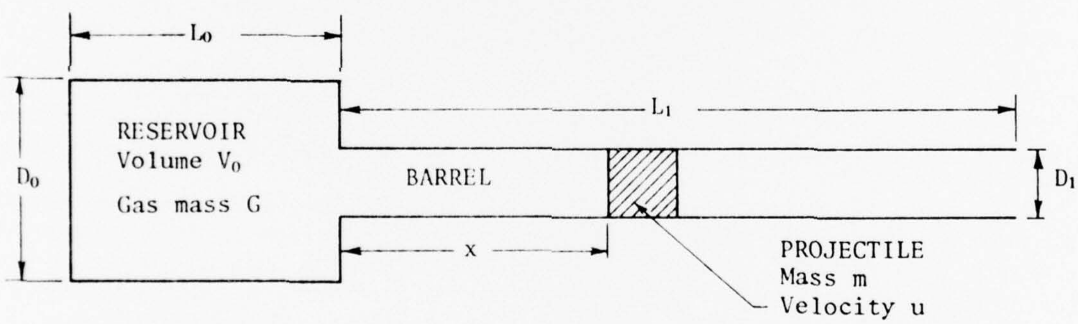
Notation for the analysis of gas gun performance is illustrated in figure 1. A projectile of mass m is accelerated by compressed gas along a barrel of length L_1. Initially, the gas pressure is applied to the projectile by opening a quick-acting valve or bursting a diaphragm. The equation of motion of the projectile is

$$m\frac{d^2x}{dt^2} = mu\frac{du}{dx} = (p_d - p_r).\ A_1 \tag{1}$$

where p_d is the accelerating or driving pressure and p_r is the retarding pressure.

Gas conditions before firing

	Reservoir	Barrel
Pressure	p_0	p_1
Speed of sound	a_0	a_1
Ratio of specific heats	γ_0	γ_1



Reservoir area $A_0 = (\pi/4)D_0^2$

Reservoir volume $V_0 = A_0 L_0$

Barrel area $A_1 = (\pi/4){D_1}^2$

Volume ratio $K = V_0/A_1 L_1$

Muzzle velocity $U = (u)_{x=L_1}$

Mass ratio $G/m = \dfrac{\rho_0 V_0}{m} = \gamma_0 . K . \dfrac{p_0 A_1 L_1}{m a_0^2}$

Figure 1. Notation

Perfect(ref.12) presents a simple solution to equation (1) by using a steady flow model. The driving pressure p_d is calculated from an adiabatic expansion of the reservoir gas from its initial volume V_0 to the volume $(V_0 + A_1 x)$. At the same time, the retarding pressure is taken to be the same as the initial pressure in the barrel. Therefore

$$p_d = p_0 \left[V_0/(V_0 + A_1 x) \right]^{\gamma_0} \tag{2a}$$

and

$$p_r = p_1. \tag{2b}$$

The solution to equation (1) is now found to be

$$U^2 = \frac{2p_0 V_0}{(\gamma_0 - 1)m} \left[1 - \left(1 + \frac{A_1 L_1}{V_0} \right)^{-(\gamma_0 - 1)} \right] - \frac{2p_1 A_1 L_1}{m}$$

or

$$U^2 = \frac{2p_0 A_1 L_1}{m} \left[\frac{K}{(\gamma_0 - 1)} \right] \cdot \left[1 - \left(\frac{K+1}{K} \right)^{-(\gamma_0 - 1)} \right] - \frac{2p_1 A_1 L_1}{m} \tag{3}$$

where $K = (V_0/A_1 L_1)$, the ratio of the reservoir volume to the barrel volume.

The analysis to follow will begin by ignoring the effect of the retarding pressure since, for transonic guns, the initial reservoir pressure is expected to be very much greater than the initial retarding pressure. With this in mind, it is convenient to write equation (3) in the alternative form

$$U^2 = (U^2)_{p_1 = 0} - \frac{2\, p_1 A_1 L_1}{m} . \tag{4}$$

Equation (4) will be used in subsequent analysis of atmospheric counter pressure effects.

2. QUASI-STEADY INTERNAL FLOW MODEL

The steady flow model of Section 1.1 above relies on an adiabatic expansion of the reservoir gas. This means that the Mach number of the flowing gas is always assumed to be small. It is, however, straightforward to derive a correction to equation (2a) and so develop a quasi-steady model. Such a model is discussed in reference 9.

2.1 General theory

Following reference 9, the basic idea is to regard equation (2a) as giving the stagnation pressure. A new expression for the driving pressure p_d is then found using standard compressible flow formulae such as are given in reference 2 for example. We find that

$$p_d = p_s \left[1 - \left(\frac{\gamma_0 - 1}{2}\right) \frac{u^2}{a_s^2} \right]^{\gamma_0/(\gamma_0 - 1)} \tag{5a}$$

where

$$p_s = p_0 \left[V_0/(V_0 + A_1 x) \right]^{\gamma_0} \tag{5b}$$

and a_s, the stagnation speed of sound, is given by the square root of the temperature ratio found from the adiabatic expansion. i.e. $(a_s/a_0) = (T_s/T_\theta)^{\frac{1}{2}}$

and so $$a_s^2 = a_0^2 \left[V_0/(V_0 + A_1 x) \right]^{(\gamma_0 - 1)} . \tag{5c}$$

Equation (1), with the retarding pressure p_r omitted, now leads to the equation of motion

$$mu \frac{du}{dx} = A_1 p_d . \tag{6}$$

This equation cannot be solved analytically as the variables u and x are not separated, because of the form of equation (5a). Numerical solution on a computer is simple and straightforward. An analytic solution can, however, be found by seeking a low Mach number solution. Then, from equation (5a) we write

$$p_d = p_s \left[1 - \frac{\gamma_0}{2} \cdot \frac{u^2}{a_s^2} \right]$$

or $$p_d = p_0 \left[V_0/(V_0 + A_1 x) \right]^{\gamma_0} - \frac{\gamma_0 p_0 u^2}{2a_0^2} \left[V_0/(V_0 + A_1 x) \right] ,$$

using equations (5b) and (5c).

The solution to equation (6) can be shown to be

$$u^2 = \frac{2\,(p_0 V_0/(\gamma_0 - 1)m)}{[1 - (\gamma_0 p_0 V_0/(\gamma_0 - 1)ma_0^2)]} \left[\left(\frac{V_0}{V_0 + A_1 x}\right)^{\gamma_0 p_0 V_0/ma_0^2} - \left(\frac{V_0}{V_0 + A_1 x}\right)^{(\gamma_0 - 1)} \right].$$

Hence the muzzle velocity is given by

$$U^2 = \frac{2\,(p_0 V_0/(\gamma_0 - 1)m)}{[1 - (\gamma_0 p_0 V_0/(\gamma_0 - 1)ma_0^2)]} \left[\left(\frac{K+1}{K}\right)^{-\gamma_0 p_0 V_0/ma_0^2} - \left(\frac{K+1}{K}\right)^{-(\gamma_0 - 1)} \right] \qquad (7)$$

where $K = (V_0/A_1 L_1)$. When V_0 is very large, equation (7) becomes

$$U^2 = \frac{2a_0^2}{\gamma_0} \left[1 - \exp\left(-\gamma_0 p_0 V_0/Kma_0^2 \right) \right]$$

in agreement with equation (3) when $p_1 = 0$ and $(p_0 A_1 L_1/ma_0^2)$ is small. Denoting the muzzle velocity from equation (3) by U_0, we have

$$(U_0/a_0)^2 = 2\,(p_0 A_1 L_1/ma_0^2).$$

Hence, when V_0 is very large, the result can be written as

$$U^2/a_0^2 = \left(\frac{2}{\gamma_0}\right) \cdot \left[1 - \exp\left(-\frac{\gamma_0}{2}(U_0^2/a_0^2) \right) \right]$$

or, since we have assumed that the Mach number is small,

$$U^2/a_0^2 = (U_0^2/a_0^2) \cdot \left[1 - \frac{\gamma_0}{4} \cdot (U_0^2/a_0^2) \right].$$

Hence we obtain

$$U/a_0 = (U_0/a_0) \cdot \left[1 - \frac{\gamma_0}{8} \cdot (U_0^2/a_0^2) \right]. \qquad (8)$$

This result shows in a very simple way the magnitude of the quasi-steady correction in terms of the muzzle velocity U_0 calculated from the steady flow model. In the general case, equation (3) shows that this is

$$U_0/a_0 = \left[\frac{2\,p_0 A_1 L_1}{ma_0^2}\right]^{\frac{1}{2}} \cdot \left[\frac{K}{\gamma_0 - 1}\right]^{\frac{1}{2}} \cdot \left[1 - \left(\frac{K+1}{K}\right)^{-(\gamma_0 - 1)} \right]^{\frac{1}{2}}. \qquad (9)$$

Equation (8) is compared with the exact result from equation (7) in table 1. The performance of equation (8) is surprisingly good even at high subsonic speeds. The effect of the parameter K is seen to be of secondary importance. Consequently, equation (8) is taken to be a satisfactory first approximation for all values of K and γ_0. Equation (8) is recommended for use in design calculations.

An alternative method of allowing for quasi-steady effects is to use a simplified form of the "Pidduck-Kent Special Solution" discussed in Appendix I.

TABLE 1. QUASI-STEADY MACH NUMBER CORRECTION

Uncorrected mach number (U_0/a_0), from equation (9).	Ratio of specific heats, γ_0.	Corrected mach number (U/a_0)				
		From equation (7)				From equation (8)
		K=1	K=2	K=∞	Mean	
0.5	1.1	0.483	0.482	0.483	0.483	0.483
	1.4	0.475	0.472	0.479	0.475	0.478
	5/3	0.467	0.461	0.475	0.468	0.474
0.75	1.1	0.693	0.692	0.696	0.694	0.692
	1.4	0.669	0.662	0.682	0.671	0.676
	5/3	0.645	0.629	0.670	0.648	0.662
1.0	1.1	0.872	0.869	0.877	0.873	0.863
	1.4	0.822	0.805	0.848	0.825	0.825
	5/3	0.773	0.740	0.824	0.779	0.792

2.2 Barrel density effect

An additional effect may be incorporated in computer versions of the quasi-steady model. By allowing for the decrease in gas density as Mach number increases a change in the mass of gas, and therefore the pressure, in the reservoir can be calculated. The starting point for our analysis is the compressible flow density relationship

$$\rho/\rho_s = \left[1 - \left(\frac{\gamma_0 - 1}{2}\right) \frac{u^2}{a_s^2} \right]^{1/(\gamma_0 - 1)} \tag{10a}$$

We now assume that the gas in the reservoir is at rest since, for gas guns, the reservoir area A_0 is considerably greater than the barrel area A_1. The gas in the barrel is taken to be moving with the projectile velocity u.

The mass of gas in the barrel, when the projectile has travelled a distance x, is $\rho A_1 x$ which may be written as $\rho_s A_1 [x(\rho/\rho_s)]$. Thus, in calculating the steady-state adiabatic expansion, the reservoir behaves as though the projectile had only moved a distance $\rho x/\rho_s$. i.e. the stagnation pressure in the reservoir is given by

$$p_s = p_0 \cdot \left[V_0/\{ V_0 + A_1 x(\rho/\rho_s)\} \right]^{\gamma_0} \tag{10b}$$

and the speed of sound is given by

$$a_s^2 = a_0^2 \cdot \left[V_0 / \{ V_0 + A_1 x(\rho/\rho_s) \} \right]^{(\gamma_0 - 1)} . \qquad (10c)$$

Equations (10b) and (10c) replace equations (5b) and (5c). Equation (5a) is now combined with equations (10) to give the driving pressure p_d required in the equation of motion (6).

Some computer calculations have been made for actual gas guns. The results show that the density effect gives rise to relatively small increases in muzzle velocity. The effect is, of course, more pronounced as the gas Mach number increases. Equation (8) therefore continues to provide a satisfactory analytic approximation for predictions of muzzle velocity.

3. UNSTEADY EXPANSION WAVE MODEL

As the gas Mach number increases to supersonic speeds, unsteady flow effects become more and more important. Put another way, the time taken for pressure waves to travel from the barrel to the end of the reservoir, and back again, is no longer small compared with the time taken for the projectile to move down and out of the barrel. The quasi-steady model of section 2 implicitly assumes that a lot of wave interaction takes place. By this means a nearly steady flow pattern is set up. We now turn to a situation where wave interactions are not important and the flow field is dominated by the simple unsteady expansion wave set up by the moving projectile.

3.1 Constant area case

The constant area case, $A_0 = A_1$, is considered first. Therefore, the standard equations of one-dimensional gas dynamics apply. In particular, the pressure behaviour in a centred expansion wave, often referred to as a simple wave, is of direct use. References 1 and 2, for example, provide further information. The formula relating pressure and velocity in an unsteady expansion is

$$p/p_0 = \left[1 - \left(\frac{\gamma_0 - 1}{2} \right) \frac{u}{a_0} \right]^{2\gamma_0/(\gamma_0 - 1)} . \qquad (11)$$

This formula is exact up to the time of arrival of the first pressure reflections from the back of the reservoir. Equation (11) shows that the maximum possible speed, or the escape speed, is given by $1 - ((\gamma_0 - 1)u/2a_0) = 0$. i.e. the maximum possible speed is $2a_0/(\gamma_0 - 1)$. Clearly, gas speeds are not restricted to a maximum of Mach 1. For $\gamma_0 = 1.4$ for example, the maximum possible speed based on the speed of sound a_0 is Mach 5.

Equation (11) is well known in analyses of shock tube performance. It describes the initial reduction in pressure in the high pressure section. When coupled with a shock wave in the low pressure section a complete description of the initial flow field is available. The motion of the contact surface separating the gas in the high pressure section from the gas in the low pressure section is, in some ways, analagous to the motion of a projectile in a gas gun. Equation (11) describes the pressure accelerating the contact surface while the corresponding shock wave formula describes the retarding pressure. The major difference is simply that the contact surface has zero mass (m=0) and is therefore accelerated up to maximum speed instantaneously. The present analysis, in contrast, is concerned with predicting the velocity/distance history of projectiles with non-zero mass.

Returning now to equation (11), this can be coupled with the equation of motion (6) to give

$$mu \frac{du}{dx} = A_1 p_0 \left[1 - \left(\frac{\gamma_0 - 1}{2}\right) \frac{u}{a_0} \right]^{2\gamma_0/(\gamma_0 - 1)}$$

whence the muzzle velocity U is found from

$$\frac{p_0 A_1 L_1}{m a_0^2} = \left[\frac{2}{\gamma_0 + 1} \right] + \left\{ \frac{(U/a_0) - (2/(\gamma_0+1))}{[1 - ((\gamma_0 - 1)U/2a_0)]^{(\gamma_0+1)/(\gamma_0-1)}} \right\} . \qquad (12)$$

The formula suffers from the disadvantage that U is given implicitly. For design purposes, an explicit formula showing the variation of U with the driving parameter $p_0 A_1 L_1 / m a_0^2$ is more convenient. Such a formula is investigated in the next section.

3.2 Logarithmic approximation for muzzle velocity

An empirical formula giving U explicitly at transonic and supersonic speeds is now sought. The behaviour of equation (12) over the range $U/a_0 = 1$ to $U/a_0 = 2$ immediately suggests a logarithmic or nearly logarithmic dependence of U on $p_0 A_1 L_1 / m a_0^2$. A number of logarithmic formulae were investigated. The simplest form was found to be

$$U/a_0 = \frac{1}{2\gamma_0} \ln [C \, . \, p_0 A_1 L_1 / m a_0^2]$$

with C proportional to γ_0^2 . The performance of this formula for $C = 5\gamma_0^2$ and $C = 6\gamma_0^2$ is shown in table 2. The general performance is surprisingly good both at supersonic speeds and over a very wide range of values of γ_0. Overall, $C = 6\gamma_0^2$ seems to be the best choice. The recommended design formula is therefore

$$U/a_0 = \frac{1}{2\gamma_0} \ln [6\gamma_0^2 p_0 A_1 L_1 / m a_0^2] . \qquad (13)$$

TABLE 2. UNSTEADY EXPANSION FORMULAE

Ratio of specific heats γ_0	Mach number U/a_0	$\frac{p_0 A_1 L_1}{m a_0^2}$ (from eq.(12))	Simplified formula for mach number $\frac{U}{a_0} = \left(\frac{1}{2\gamma_0}\right) \ln \, (C. \; p_0 A_1 L_1 / m a_0^2)$		Mach number U/a_0
			$C = 5\gamma_0^2$	$C = 6\gamma_0^2$	
	0.5	0.183	0.05	0.13	0.5
	0.75	0.501	0.50	0.59	0.75
1.1	1.0	1.09	0.86	0.94	1.0
	1.5	3.77	1.42	1.50	1.5
	2.0	10.53	1.89	1.97	2.0
	0.5	0.206	0.25	0.32	0.5
	0.75	0.612	0.64	0.71	0.75
1.4	1.0	1.47	0.95	1.02	1.0
	1.5	6.50	1.48	1.55	1.5
	2.0	25.84	1.98	2.04	2.0
	0.5	0.232	0.35	0.41	0.5
	0.75	0.750	0.70	0.76	0.75
5/3	1.0	2.02	1.00	1.05	1.0
	1.5	12.75	1.55	1.61	1.5
	2.0	102.0	2.18	2.23	2.0

3.3 Area change at barrel/reservoir junction

The question now is whether equation (13) can be generalized to the usual gas gun situation where $A_0 > A_1$ and gas velocities in the reservoir are much smaller than in the barrel. Fortunately, the barrel entry sonic approximation, as described in reference 1 for example, provides a basis for an analytic approximation. Because the local gas velocity at the beginning of the barrel approaches sonic speed with increasing time, the fundamental assumption made in calculating the muzzle velocity is that the flow is always sonic at the barrel entry. In order to show how the formula has been derived we begin by examining the infinite area reservoir in which the gas is at rest.

Since the unsteady expansion flow we are considering is a simple wave, the Riemann variable $a + (\gamma_0 - 1)u/2$ is constant throughout the wave. At the barrel entry, we take $u = a = (a)_{sonic}$ and the Riemann variable has the value $((\gamma_0 + 1)/2)\ (a)_{sonic}$ which can be expressed as $a_0 \sqrt{(\gamma_0 + 1)/2}$ since $(a)_{sonic} = a_0 \sqrt{2/(\gamma_0 + 1)}$ is an isentropic flow. Hence

$$a + (1/2)(\gamma_0 - 1)u = a_0 \sqrt{(\gamma_0 + 1)/2}$$

But, in isentropic flow, $p/p_0 = (a/a_0)^{2\gamma_0/(\gamma_0-1)}$ so that

$$p/p_0 = \left[\sqrt{(\gamma_0+1)/2} - ((\gamma_0-1)u/2a_0) \right]^{2\gamma_0/(\gamma_0-1)}$$

Writing the term $\sqrt{(\gamma_0+1)/2}$ as $1 + (\sqrt{(\gamma_0+1)/2} - 1)$ shows that the term $(\sqrt{(\gamma_0+1)/2} - 1)$ is the sole addition to equation (11) resulting from the use of the sonic entry approximation. Following Seigel (ref.1) an inverse area ratio interpolation is introduced in this additional term to account for arbitrary values of A_1/A_0. The pressure result is

$$p/p_0 = \left[1 + Z - \left(\frac{\gamma_0-1}{2}\right) \frac{u}{a_0} \right]^{2\gamma_0/(\gamma_0-1)}$$

where $$Z = \{1 - (A_1/A_0)\} . \{\sqrt{(\gamma_0+1)/2} - 1\} .$$

Comparison with equation (11) shows that the counterpart of equation (13) can now be found by replacing

$$p_0 \text{ by } p_0 [1 + Z]^{2\gamma_0/(\gamma_0-1)}$$

and a_0 by $a_0 [1 + Z]$. Hence the generalized form of equation (13) becomes

$$U/a_0 = \left(\frac{1+Z}{2\gamma_0}\right) \ln [6\gamma_0^2 p_0 A_1 L_1/ma_0^2] + \left(\frac{1+Z}{2\gamma_0}\right) . \left(\frac{2}{\gamma_0-1}\right) \ln [1+Z] .$$

This result can be simplified by taking $\sqrt{(\gamma_0+1)/2}$ to be equal to $1+((\gamma_0-1)/4)$ which strictly applies when (γ_0-1) is "small", and expanding the logarithm in a similar way. i.e. we take

$$\ln [1+Z] = \{(\gamma_0-1)/4\} . \{1 - (A_1/A_0)\} .$$

Omitting terms multiplied by (γ_0-1), the final result is therefore

$$U/a_0 = \frac{1}{4\gamma_0}\left[1 - \frac{A_1}{A_0}\right] + \frac{1}{2\gamma_0} \ln \left[6\gamma_0^2 p_0 A_1 L_1/ma_0^2\right] . \quad (14)$$

Equation (14) is the basic design formula that was being sought. In the next section, it will be shown that this formula works surprisingly well.

For future reference, we note that the driving pressure counterpart to equation (14) is

$$p/p_0 = \left[1 + \left(\frac{\gamma_0 - 1}{4} \right) \left(1 - \frac{A_1}{A_0} \right) - \left(\frac{\gamma_0 - 1}{2} \right) \frac{u}{a_0} \right]^{2\gamma_0/(\gamma_0 - 1)} . \qquad (15)$$

An alternative pressure formula due to Seigel(ref.1) is given in Appendix II.

3.4 Comparison with exact numerical results

Seigel(ref.1) has given a large number of design charts based on numerical integration of the gas dynamic differential equations that describe the unsteady gas flow. These charts provide exact results which can be used to assess the usefulness of equation (14). Figures 2, 3, 4 present results for γ_0 = 1.1, 1.4, 5/3. The broken straight lines are the results given by equation (14). The results for $A_0/A_1 = \infty$ can be compared directly with Seigel's results for $A_0/A_1 = 25$ since, from equation (14), the difference between the $A_0/A_1 = \infty$ case and the $A_0/A_1 = 25$ case must be extremely small.

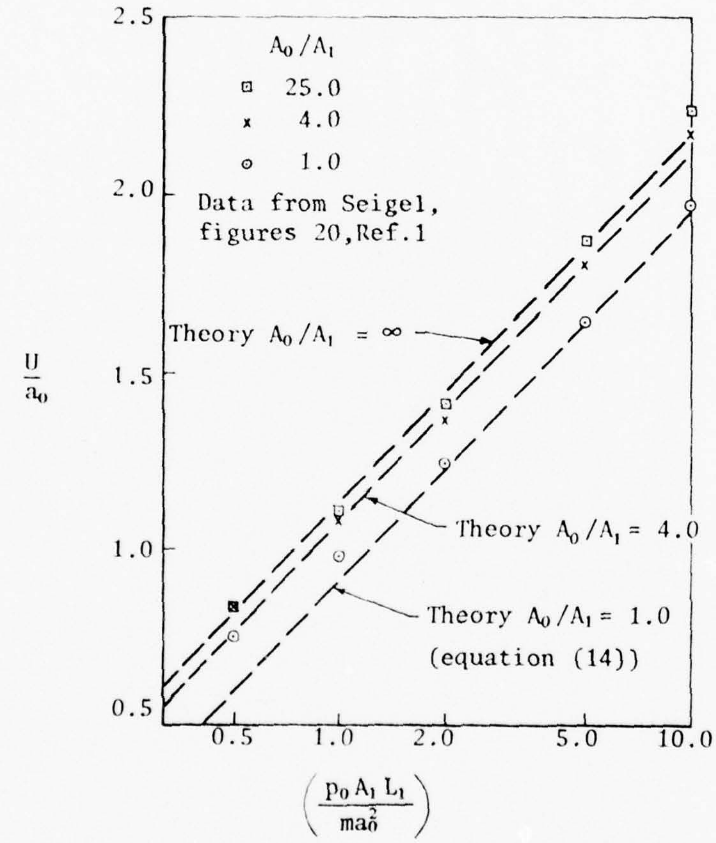


Figure 2. Theory for infinite reservoir gun, γ_0 = 1.1

For γ_0 = 1.1, figure 2 compares equation (14) with Seigel's exact results. The exact results apply to the case where the reservoir is effectively infinite in length i.e. it is long enough for wave reflections from the back surface of the reservoir not to influence the motion of the projectile. Under these conditions, equation (14) is every effective. For A_0/A_1 = 4.0 the maximum error in the predicted velocity is about 2% for muzzle velocities in the range 1.0 a_0 to 2.0 a_0. The performance for A_0/A_1 = 1.0 is much the same, except near U = 1.0 a_0 where the error in U is greater than 2%.

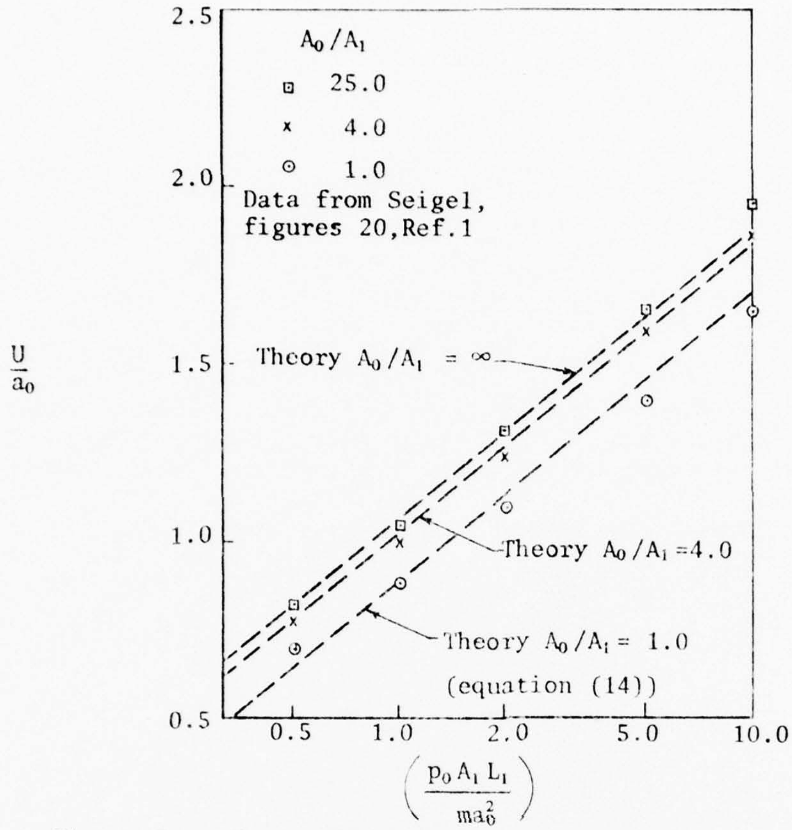


Figure 3. Theory for infinite reservoir gun, $\gamma_0 = 1.4$

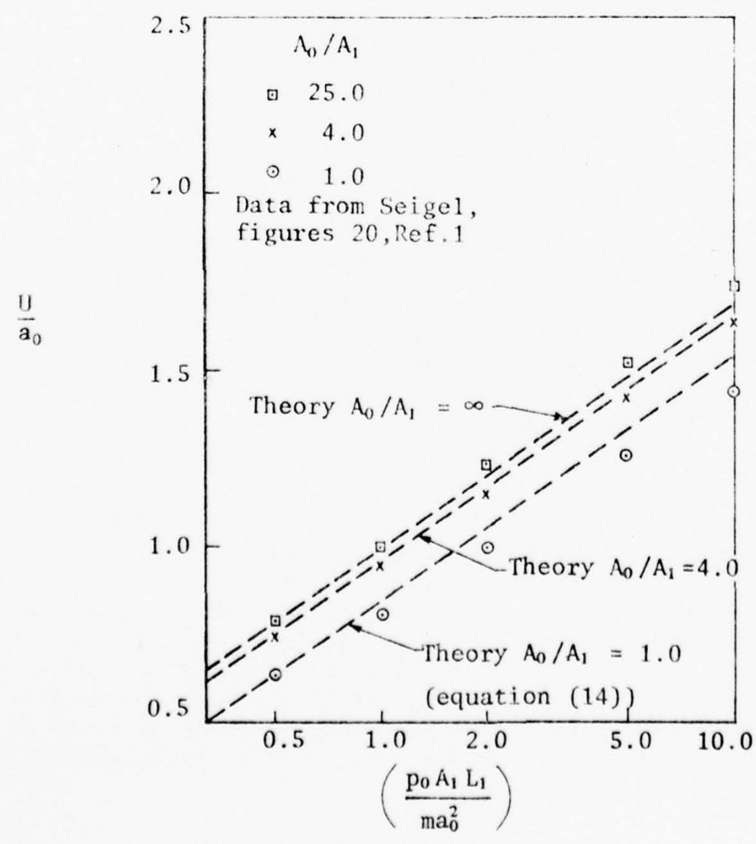


Figure 4. Theory for infinite reservoir gun, $\gamma_0 = 5/3$

Similar results apply for $\gamma_0 = 1.4$ and $\gamma_0 = 5/3$, as shown in figures 3 and 4. For $\gamma_0 = 1.4$, the muzzle velocity range for minimum error is $0.8\,a_0$ to $1.7\,a_0$ while, for $\gamma_0 = 5/3$, the range is $0.7\,a_0$ to $1.5\,a_0$. We conclude that equation (14) provides a most satisfactory performance when the driver gas velocities lie in the transonic or the low to moderate supersonic speed ranges. The major differences between figures 2, 3 and 4 may be removed by using the parameters $(\gamma_0 U/a_0)$ and $(\gamma_0 p_0 A_1 L_1/ma_0^2)$. Details are given in Appendix III. We turn now to the problem of deciding when the reservoir is long enough for equation (14) to be applicable.

3.5 Effect of reservoir length

Seigel(ref.1) presents some results for the minimum length of reservoir necessary to achieve maximum muzzle velocity. Under these conditions, the first wave reflection from the back surface of the reservoir reaches the projectile just as it leaves the barrel.

When $A_0 = A_1$, the analytic results vary only a little with γ_0 and, as shown by Seigel(ref.1), a satisfactory formula is

$$\left[\frac{L_1}{L_0}\right]^2_{\text{maximum}} \sim 2.5\ p_0 A_1 L_1/ma_0^2 \quad .$$

As $p_0 \to 0$, the gas Mach numbers approach zero and the minimum reservoir length approaches infinity. i.e., as expected, the reservoir volume $V_0 \to \infty$ for maximum gun performance.

Generalization of the $A_0 = A_1$ results to the case $A_0 > A_1$ presents some difficulties as Seigel(ref.1) has given numerical results for $\gamma_0 = 1.4$ only. We assume that these results, like those for $A_0 = A_1$, are only weakly dependent on γ_0. A simple representation of reasonable accuracy is then found to be

$$\left[\frac{L_1}{L_0}\right]^2_{\text{maximum}} \sim \{2.5 + 2(1-(A_1/A_0))\}\ p_0 A_1 L_1/ma_0^2 \ . \tag{16}$$

The value of L_0 satisfying equation (16) is denoted $(L_0)_{min}$. Until additional data become available, it is recommended that equation (16) be used to determine when the reservoir length L_0 is large enough for wave reflections not to influence the motion of the projectile.

The problem of how to treat cases when L_0 is less than $(L_0)_{min}$ is a very difficult one. In general, recourse to Seigel's design charts (1) is by far the most effective approach. When the gas velocities are subsonic, however, the quasi-steady theory of section 2 is applicable and should be used. The limit of applicability of this theory is hard to define precisely. However, for gas Mach numbers reaching 0.5 when the projectile leaves the barrel, the theory is clearly acceptable for actual guns in view of the good agreement reported in reference 9 between the theoretical and actual performance of two W.R.E. gas guns. The maximum muzzle velocity was 189 m/s, which corresponds to a maximum gas Mach number of 0.56. When the maximum gas Mach number reaches 1.0 in an actual gun the quasi-steady theory is likely not to be acceptable. A gas Mach number limit of 0.75 is therefore suggested for the quasi-steady theory, with no allowance made for L_0 effects. In order to provide some assistance to designers at higher gas Mach numbers, a brief analysis has been made of a representative range of Seigel's results. As expected, a simple description of the penalty of short reservoirs is just not possible. However, the following formula may be of some use. The purely empirical result is

$$U/[U]_{(L_0)_{min}} \sim \left[L_0/(L_0)_{min} \right]^{1/r} , \quad r = 4+(A_0/A_1) \tag{17}$$

where $(L_0)_{min}$ is the value of L_0 that satisfies equation (16). The ratio on the left hand side of equation (17) may contain errors up to about 0.1. For a given L_0, equation (17) shows that the velocity loss decreases as A_0 increases.

An extreme and unrealistic example of the limitations of equation (17) can be found by returning to the case when the steady internal flow model of section 1.1 applies and the gas Mach numbers are always very small. Equation (9) shows that $U_0 \rightarrow$ constant, when $L_0 \rightarrow \infty$. i.e. $K \rightarrow \infty$. In addition, $U_0 \propto L_0^{1/2}$ when L_0 is small, showing that the 1/5 power in equation (17) is misleading in this case. Nevertheless, equation (17) does provide the correct limit behaviour in that $U \rightarrow o$ as $L_0 \rightarrow o$ and $U \rightarrow [U]_{(L_0)_{min}}$ $L_0 \rightarrow (L_0)_{min}$.

4. CORRECTION FOR ATMOSPHERIC COUNTER PRESSURE

So far, the analysis has been concerned with the driving pressure force. Only in the case of the steady theory of section 1.1 was an atmospheric counter pressure considered. The purpose of the present section is to look at the physical basis of what happens in front of the projectile and then to derive suitable correction formulae for the effects of counter pressure.

4.1 Shock wave and unsteady compression wave effects

As the projectile accelerates, the gas in front of it is compressed. The central problem is simply how to determine the magnitude of this compression and how it varies as the projectile moves down the barrel. The starting point is similar to equation (11). A simple wave can be set up to describe an unsteady compression. The result (1) is

$$p/p_1 = \left[1 + \left(\frac{\gamma_1 - 1}{2} \right) \frac{u}{a_1} \right]^{2\gamma_1 (\gamma_1 - 1)} . \tag{18}$$

Provided that the projectile continues to accelerate, the compression waves will coalesce and form a shock wave. The pressure formula for a shock wave is

$$p/p_1 = 1 + \frac{\gamma_1 (\gamma_1 + 1)}{4} \left(\frac{u}{a_1} \right)^2 + \gamma_1 \frac{u}{a_1} \left[1 + \left(\frac{\gamma_1 + 1}{4} \right)^2 \frac{u^2}{a_1^2} \right]^{1/2} . \tag{19}$$

For Mach numbers u/a_1 less than 1, the numerical difference between equations (18) and (19) is very small. When u/a_1 is small, both equations behave like

$$p/p_1 = 1 + \gamma_1 (u/a_1) + 0\,(u^2/a_1^2).$$

In the case of air, $\gamma_1 = 1.4$ and the difference between equations (18) and (19) increases up to 3% at $u/a_1 = 1$. For $\gamma_1 = 1.1$ and $\gamma_1 = 5/3$ the performance is similar. The conclusion is that either equation (18) or equation (19) could be used at subsonic Mach numbers. Since, however, equation (19) must be used at higher Mach numbers it is convenient to use equation (19) at all Mach numbers.

Whether a shock wave forms in a particular case can, if desired, be found by numerical integration of the gas dynamic equations of motion. In the case of a projectile with constant acceleration (f), Seigel (1) quotes the result that the shock wave forms at a time

$$t_{shock} = 2a_1 / \{ (\gamma_1 + 1)\, f \}$$

after the projectile began to move and at a distance

$$x_{shock} = 2a_1^2 / \{ (\gamma_1 + 1)\, f \}$$

from the beginning of the barrel. At this time the projectile velocity will be $f\, t_{shock} = 2a_1/(\gamma_1+1)$ corresponding to a Mach number of $2/(\gamma_1+1)$. This Mach number is in the subsonic range, being at most 1 when $\gamma_1 = 1$. The justification for using the shock wave equation (19) for calculating the retarding pressure is now complete. Only if formation of the shock wave could be delayed to higher Mach numbers would the difference between equations (18) and (19) become significant.

For transonic gas guns, as distinct from supersonic guns, it follows that either equation (18) or equation (19) can be used to calculate the retarding pressure acting on the projectile. Once the muzzle velocity increases to supersonic speeds, equation (19) should be used.

4.2 Steady flow model

The steady flow model of section 1.1 already incorporates a counter pressure correction. Since the theory applies only for small gas Mach numbers, the additional assumption is made that the muzzle Mach number is also small. Hence equation (19) gives the retarding pressure as $p_r = p_1$. The end result is equation (3) or, equivalently, equation (4):

$$U^2 = (U^2)_{p_1 = o} - 2\, p_1 A_1 L_1 / m\,.$$

This formula may have wider applicability than the theory on which it is based.

4.3 Method of Seigel and velocity correction factor

Seigel(ref.1) has used a high Mach number approximation in equation (19) to obtain

$$p/p_1 = 1 + \frac{\gamma_1(\gamma_1+1)}{2}\,(u/a_1)^2.$$

This result is now combined with a constant driving pressure. i.e. constant driving acceleration. The simple differential equation for $\frac{du}{dx}$ leads directly to

$$U^2/(U^2)_{p_1=o} = \left[1 - \frac{p_1}{p_o}\right] \cdot \left[\frac{1-\exp(-y)}{y}\right] \quad (20a)$$

where $y = \gamma_1(\gamma_1+1)p_1 A_1 L_1/ma_1^2$. The form of equation (20a) is important. Substituting for $(U)_{p_1=o}$ produces a result which is inferior when applied to cases where the driving pressure is not constant.

The right hand side of equation (20a) provides a velocity correction factor. Seigel comments that equation (20a) works very well for high performance guns. However, for application to subsonic and transonic guns, it should be noted that Seigel's result does not agree with equation (4) at low Mach numbers.

4.4 Generalized velocity correction factor

The purpose of this section is to generalize Seigel's result, equation (20a), so that it will be more effective for the lower speed guns of direct interest in the present report. Combining equation (4) with equation (20a) leads to

$$U^2 = \left[(U^2)_{p_1=o} - (2p_1 A_1 L_1/m)\right] \cdot \left[\frac{1-\exp(-y)}{y}\right] \quad (20b)$$

with

$$y = \gamma_1(\gamma_1+1)\, p_1 A_1 L_1/ma_1^2. \quad (20c)$$

Equation (20b) is recommended for design calculations. It is exact when U/a_1 is small, or large. When U/a_1 is small, the first term in equation (20b) dominates and so the exact result from steady internal flow theory, with low gas Mach numbers, is obtained. When U/a_1 is large, the first term is small and the second term dominates. Consequently, Seigel's high Mach number result is recovered for U/a_1 large.

4.5 Terminal velocity

The purpose of this section is to derive simple estimates of either the terminal or the maximum muzzle velocity when barrel length is regarded as variable. When the pressure forces depend on projectile velocity (u) and not on distance travelled (x), a terminal velocity is reached eventually. In other cases, a maximum velocity is reached and the velocity then decreases owing to the dependence of reservoir pressure on x in the steady and quasi-steady internal flow models. Equation (20b) is not suitable for calculating maximum possible muzzle velocities because y depends on the distance travelled by the projectile. We begin by examining the steady flow model of section 1.1.

4.5.1 Quasi-steady internal flow model

The maximum velocity occurs when the driving pressure p_d is equal to the retarding pressure p_r. Hence, using equations (2) which describe the steady internal flow model,

$$p_1 = p_o[1 + (A_1 x/V_o)]^{-\gamma_o}$$

so that $K^{-1} \equiv A_1 x/V_0 = (p_0/p_1)^{1/\gamma_0} - 1$

and equation (3) gives the maximum velocity U_{max} as

$$U^2_{max} = \frac{2\,p_0 V_0}{(\gamma_0 - 1)m}\left[1 - \left(\frac{p_1}{p_0}\right)^{(\gamma_0-1)/\gamma_0}\right] - \frac{2\,p_1 V_0}{m}\left[\left(\frac{p_0}{p_1}\right)^{1/\gamma_0} - 1\right] . \qquad (21a)$$

When $p_1 = 0$, the maximum velocity occurs at $x = \infty$ and is $[2p_0 V_0/(\gamma_0 - 1)m]^{\frac{1}{2}}$.

Equation (21a) covers the situation where p_d varies with x and $p_r = p_1$. Another case where analysis is possible occurs when V_0 is large and $a_0 \gg a_1$ so that $p_d = p_0$, but we have p_r dependent on velocity. For this purpose, the generalized Seigel formula given in equation (20a) can be rearranged by removing $(U)_{p_1=o}$ and replacing it by $\sqrt{2p_0 A_1 x/m}$. The result obtained is

$$U^2_{max} = [2a_1^2 (p_0 - p_1) / \gamma_1(\gamma_1 + 1)p_1] \qquad (21b)$$

with the maximum velocity occurring at $x = \infty$. Equation (21b) must be rejected as unrealistic because p_d could not remain constant from $x = 0$ to $x = \infty$ in an actual gun. An alternative approach for improving equation (21a) is to use a constant, mean retarding pressure greater than p_1. This is denoted by $\lambda\, p_1$ and we assume that λ can be determined by evaluating the shock wave equation (19) at some mean velocity. Hence

$$\lambda = 1 + \frac{\gamma_1(\gamma_1+1)}{4}\left[\frac{U_{mean}}{a_1}\right]^2 + \gamma_1 \frac{U_{mean}}{a_1}\left[1 + \left(\frac{\gamma_1+1}{4}\right)^2 \frac{U^2_{mean}}{a_1^2}\right]^{\frac{1}{2}} \qquad (22a)$$

and, using a root mean square value for U_{mean} from equation (21a) as a first estimate, we obtain

$$U^2_{mean} = \frac{p_0 V_0}{(\gamma_0 - 1)m}\left[1 - \left(\frac{p_1}{p_0}\right)^{(\gamma_0-1)/\gamma_0}\right] - \frac{p_1 V_0}{m}\left[\left(\frac{p_0}{p_1}\right)^{1/\gamma_0} - 1\right] . \qquad (22b)$$

The final result is

$$U^2_{max} = \frac{2p_0 V_0}{(\gamma_0 - 1)m}\left[1 - \left(\frac{\lambda p_1}{p_0}\right)^{(\gamma_0-1)/\gamma_0}\right] - 2\,\frac{\lambda p_1 V_0}{m}\left[\left(\frac{p_0}{\lambda p_1}\right)^{1/\gamma_0} - 1\right] \qquad (22c)$$

Equations (22) provide an estimate of the maximum possible velocity when the steady model is valid. Compressibility effects in the driver gas can be estimated by using equation (8), derived from the quasi-steady internal flow model of section 2. The velocity U_0 calculated from the steady model is replaced by U_{max} calculated from equation (22c) to give

$$\left(U_{max}/a_0\right)_{q-s} = \left(U_{max}/a_0\right) . \left[1 - \frac{\gamma_0}{8}\left(U^2_{max}/a_0^2\right)\right] . \qquad (22d)$$

4.5.2 Unsteady expansion model

At transonic and supersonic driver gas speeds, the unsteady theory of section 3 is valid. This is combined with the counter pressure formulae of section 4.1 and, since the counter pressures now do not depend on x but only on u, a terminal velocity is reached. The retarding pressure given by equation (18) can be converted to an exponential by assuming that $(\gamma_1 - 1)$ is small. i.e.

$$(p/p_1)_{retard} = \exp\,(\gamma_1 u/a_1)\,.$$

Equation (11) for the driving pressure when $A_0 = A_1$ leads to a similar result. The case $A_0 > A_1$ is given in equation (15), which leads to the exponential approximation

$$(p/p_0)_{drive} = \exp\left[\frac{\gamma_0}{2}\left(1 - \frac{A_1}{A_0}\right) - \gamma_0\,(u/a_0)\right] .$$

Equating p_{retard} to p_{drive} now gives the terminal velocity U_{max} from

$$\ln\,(p_0/p_1) = \gamma_1\,(U_{max}/a_1) - (\gamma_0/2)(1-(A_1/A_0)) + \gamma_0\,(U_{max}/a_0)\,.$$

$$\text{i.e.} \quad U_{max} = \frac{(\gamma_0/2)\ (1 - (A_1/A_0)) + \ln\,(p_0/p_1)}{[\,(\gamma_0/a_0) + (\gamma_1/a_1)\,]} \qquad (23)$$

The noteworthy feature of this result is the logarithmic dependence on pressure ratio. As previously noted in discussion of the unsteady expansion model, equation (23) is valid when the gas Mach number U_{max}/a_0 is transonic or supersonic. In addition, the velocity given by equation (23) may not be realistic for all reservoir lengths. The minimum necessary reservoir length has been examined in section 3.5.

4.6 Wave reflections from muzzle

The shock wave pressure formula, equation (19), is valid up to the time at which the shock wave is modified by pressure reflections from the muzzle. At the muzzle, the gas pressure is dominated by the atmospheric pressure p_1. The effect of muzzle reflections can be allowed for in a simple, approximate way in computer calculations based on simplified analytic expressions for the driving and retarding pressures. A more complete treatment requires full numerical solution of the basic gas dynamics equations using the method of characteristics.

The first calculation to be made is the position reached by the projectile when the pressure pulse generated by the initial movement of the projectile down the barrel has had time to travel the full length of the barrel and then return to the projectile. The pressure pulse travels at the speed of sound a_1.

The second calculation introduces an approximation. We begin by noting that the pressure near the gun muzzle will be p_1, the initial barrel pressure. It is now assumed that a change in pressure formula must be introduced in order that the quasi-steady formula

$$p/p_1 = \left[1 + \frac{(\gamma_1 - 1)}{2} \frac{u^2}{a_1^2}\right]^{-\gamma_1/(\gamma_1 - 1)} \tag{24}$$

is used at the gun muzzle. Equation (24) describes the retarding pressure when many muzzle reflections have taken place.

Equation (19) is used in full when the first muzzle reflection arrives at the projectile and equation (24) is used in full when the projectile reaches the muzzle. In between, we assume a linear change in pressure formula based on distance travelled by the projectile down the barrel.

5. PERFORMANCE OF SUBSONIC GAS GUNS

We pass on now to a review of the performance of subsonic guns. The aim is to determine the value of the theoretical methods developed in the previous sections and, in particular, to determine the precision with which the muzzle velocities of actual guns can be predicted. Data are available for five guns, namely the W.R.E. 127 mm gun(ref.8) and the W.R.E. 384 mm gun(ref.9), the R.A.E. 152 mm gun(ref.12), the N.A.E. 254 mm gun(ref.13) and, as reported in reference 12, the CAATDC 152 mm gun. Different methods of firing projectiles are used. The two W.R.E. guns employ quick-acting valves and are used primarily for missile investigations, while the other guns employ bursting diaphragms and are used for investigating the effect of bird impacts on aircraft structures. A wide range of sabot designs is used in the different guns. Air or nitrogen is used as the driver gas so that $\gamma_0 = \gamma_1$ and $a_0 \doteq a_1$.

No data are available for transonic gas guns. The author is not aware of any gas guns operating at transonic or low supersonic speeds.

5.1 Predicted muzzle velocities

Figure 5 shows how the measured projectile speed compares with the predicted speed. In general terms, the measured speed is about 10% less than that predicted by theory. Throughout, the full computer version of the quasi-steady theory of section 2 is used, including a barrel density correction, and wave reflections from the muzzle are allowed for as discussed in section 4.6.

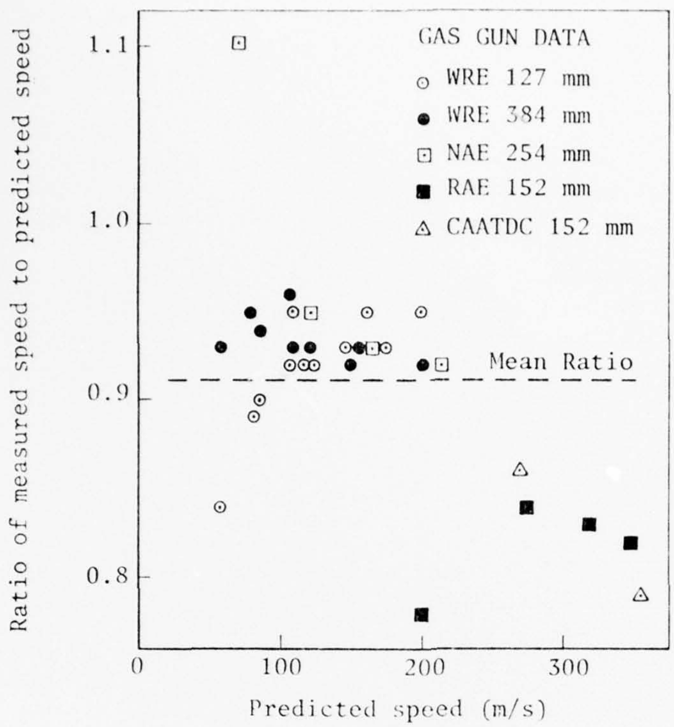


Figure 5. Performance of subsonic gas guns

Details of the data shown in figure 5 are given in table 3. Figure 6 shows how the ratio of measured projectile speed to predicted speed varies with reservoir pressure. No clear trend is apparent. The results show that the NAE gun at its lowest reservoir pressure has a measured speed about 20% greater than the average trend. In view of the fact that the net driving pressure was only about half an atmosphere (55 kPa) the relatively good agreement is perhaps surprising and confirms the value of the muzzle reflection corrections in an extreme case where the net driving pressure is equal to about half the barrel pressure. When the net driving pressure is about one atmosphere (105 kPa), the performance of the theory is excellent.

For predicted speeds around 300 m/s, the results in figure 5 suggest that the quasi-steady theory is breaking down. The number of data at these high subsonic speeds is rather small so definite conclusions cannot be drawn. We can say, however, that the results confirm the earlier suggestion that the quasi-steady theory can be used up to a driver gas Mach number of about 0.75 or so.

TABLE 3. MUZZLE VELOCITY PREDICTIONS

Gun	Mass (kg)	Pressure (k Pa gauge)	Muzzle velocity (m/s)		Ratio of measured velocity to predicted velocity	Mean value of ratio
			Theory	Measured		
WRE 127 mm	2.3	690	111	105	0.95	0.92
		1380	163	155	0.95	
		2070	200	189	0.95	
	4.5	690	81	72	0.89	
		1380	119	109	0.92	
		2070	147	136	0.93	
		2760	170	158	0.93	
	9.1	690	58	49	0.84	
		1380	86	77	0.90	
		2070	106	97	0.92	
		2760	123	113	0.92	
WRE 384 mm	14.5	345	107	103	0.96	0.93
		690	163	152	0.93	
		1030	201	184	0.92	
	29.0	345	79	75	0.95	
		690	121	112	0.93	
		1030	150	138	0.92	
	58.1	345	57	53	0.93	
		690	87	82	0.94	
		1030	109	101	0.93	
NAE 254 mm	3.2	55	68	75	1.10	0.98
		105	121	115	0.95	
		175	167	155	0.93	
		275	213	197	0.92	
RAE 152 mm	1.8	275	202	158	0.78	0.82
		550	274	229	0.84	
		830	318	265	0.83	
		1100	349	287	0.82	
CAATDC 152 mm	1.8	760	271	233	0.86	0.83
		1590	355	280	0.79	

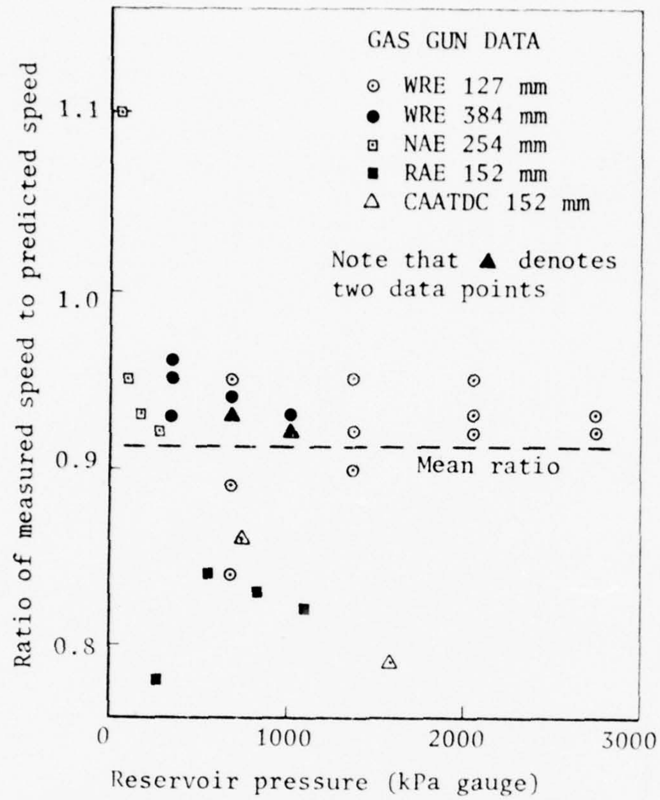


Figure 6. Influence of reservoir pressure on performance of subsonic gas guns

5.2 Counter pressure corrections

Section 4 was concerned with the calculation of atmospheric counter pressure corrections. The purpose of the present section is to examine the different corrections discussed and compare them with the full theoretical results of section 5.1. The difference between the muzzle velocity without any counter pressure and with counter pressure is the velocity correction to be examined.

An overview of the performance of the generalized velocity correction factor of section 4.4 is given in table 4. The maximum error in the mean values of table 4 is 25%, and the minimum is 3%.

TABLE 4. MODIFIED COUNTER PRESSURE CORRECTION

Gun	Mean pressure (k Pa gauge)	Mean measured muzzle velocity (m/s)	Mean value of (exact - estimated) velocity correction (m/s)	Mean value of (estimated/exact) velocity correction
WRE 127 mm	1630	115	+ 3	0.75
WRE 384 mm	690	111	+ 2	0.93
NAE 254 mm	150	136	-12	1.12
RAE 152 mm	690	235	-10	1.14
CAATDC 152 mm	1180	257	+ 2	0.97

The data on which table 4 is based are shown in tables 5 and 6. Table 5 compares the muzzle velocities for no counter pressure with a number of different predictions of the corrected muzzle velocities. In some cases, the simple theory for no counter pressure predicts velocities much larger than any of the corrected velocities. Comparison with the simple theory for the corrected velocity shows that the performance of Seigel's method (equation 20a) is encouraging and that of the modified method, equation (20b) and table 4, is even more so. The final column in table 5 gives the full theory with allowance for the barrel density effect described in section 2.2. The difference between the last two columns is small, which shows that the influence of the barrel density effect on the muzzle velocity is small. For this reason, the results in table 5 for the theory without counter pressure were not re-calculated using the full theory and the driver gas density correction.

TABLE 5. CORRECTED MUZZLE VELOCITIES

Gun	Mass (kg)	Pressure (k Pa gauge)	Muzzle velocity (m/s)				
			Simple theory for no counter pressure *	Corrected velocity			
				Seigel method	Modified method	Simple theory *	Full theory **
WRE 127 mm	2.3	690	134	123	112	109	111
		1380	180	170	162	158	163
		2070	213	204	197	192	200
	4.5	690	96	89	84	80	81
		1380	130	125	121	118	119
		2070	156	151	148	144	147
		2760	177	172	169	165	170
	9.1	690	69	64	60	58	58
		1380	93	90	87	85	86
		2070	112	109	107	105	106
		2760	128	125	123	121	123
WRE 384 mm	14.5	345	150	125	107	106	107
		690	194	174	161	160	163
		1030	227	208	197	195	201
	29.0	345	108	91	80	78	79
		690	142	129	120	119	121
		1030	167	156	149	147	150
	58.1	345	77	66	58	56	57
		690	102	93	88	86	87
		1030	121	114	109	108	109
NAE 254 mm	3.2	55	208	108	54	67	68
		105	233	145	107	118	121
		175	263	182	151	163	167
		275	298	222	194	208	213
RAE 152 mm	1.8	275	274	211	188	199	202
		550	336	277	256	267	274
		830	377	319	299	308	318
		1100	407	350	329	337	349
CAATDC 152 mm	1.8	760	325	279	258	259	271
		1590	404	357	338	332	355

* Excluding barrel density effect. i.e. gas density in barrel is less than that in reservoir.

** Including barrel density effect.

TABLE 6. COMPARISON OF COUNTER PRESSURE CORRECTIONS

Gun	Mass (kg)	Pressure (k Pa gauge)	Velocity correction (m/s)			Correction ratio	
			Seigel method	Modified method	Exact	Seigel/exact	Modified/exact
WRE 127 mm	2.3	690	11	22	25	0.44	0.88
		1380	10	18	22	0.45	0.82
		2070	9	16	21	0.43	0.76
	4.5	690	7	12	16	0.44	0.75
		1380	5	9	12	0.42	0.75
		2070	5	8	12	0.42	0.67
		2760	5	8	12	0.42	0.67
	9.1	690	5	9	11	0.45	0.82
		1380	3	6	8	0.38	0.75
		2070	3	5	7	0.43	0.71
		2760	3	5	7	0.43	0.71
	Mean value		6	11	14	0.43	0.75
WRE 384 mm	14.5	345	25	43	44	0.57	0.98
		690	20	33	34	0.59	0.97
		1030	19	30	32	0.59	0.94
	29.0	345	17	28	30	0.57	0.93
		690	13	22	23	0.57	0.96
		1030	11	18	20	0.55	0.90
	58.1	345	11	19	21	0.52	0.90
		690	9	14	16	0.56	0.88
		1030	7	12	13	0.54	0.92
	Mean value		15	24	26	0.56	0.93
NAE 254 mm	3.2	55	100	154	141	0.71	1.09
		105	88	126	115	0.77	1.10
		175	81	112	100	0.81	1.12
		275	76	104	90	0.84	1.15
	Mean value		86	124	112	0.78	1.12
RAE 152 mm	1.8	275	63	86	75	0.84	1.15
		550	59	80	69	0.86	1.16
		830	58	78	69	0.84	1.13
		1100	57	78	70	0.81	1.11
	Mean value		59	81	71	0.84	1.14
CAATDC 152 mm	1.8	760	46	67	66	0.70	1.02
		1590	47	66	72	0.65	0.92
	Mean value		47	67	69	0.68	0.97

Note: Modified correction ~ (3/2). (Seigel correction).

Table 6 compares the velocity corrections instead of the muzzle velocities shown in table 5. The performance of the Seigel method and the modified method are now shown in detail. As a rough generalization, the modified method gives velocity corrections about 50% larger than does the Seigel method. The very large velocity correction for the low pressure NAE gun is noteworthy.

We conclude that the modified velocity correction method of section 4.4, and equation (20b), is satisfactory when account is taken of the precision with which gun performance can be estimated, as shown in figures 5 and 6. The generalized velocity correction factor is therefore recommended for design purposes. For high performance transonic guns, the reservoir pressure must be very much greater than the barrel pressure, so that the velocity correction from counter pressure is relatively small and the magnitude of the error in the velocity correction must be a lot less than the likely error in any theoretical muzzle velocity prediction.

5.3 Friction and boundary layer corrections used for hypervelocity guns

Seigel(ref.1) has compared the performance of hypervelocity guns with theoretical predictions and so determined a correction. The correction increases as the driver gas Mach number increases, and has been put down to the effects of projectile friction and of boundary layer growth in the driver gas. Some careful tests reported by Seigel(ref.1) show that variations in muzzle velocity occur as friction between the barrel and the projectile is changed. Seigel also reports one boundary layer calculation showing that the effect on muzzle velocity increases with speed. Both these observations are consistent with the velocity decrements observed in the hypersonic gun data. The data give a velocity decrement of about 2% when $\gamma_0 U/a_0 = 1.0$ and of about 7% when $\gamma_0 U/a_0 = 2.0$. Consequently, when U/a_0 is less than 1.0 this effect is very much less than the effects observed for subsonic guns in figures 5 and 6.

The hypervelocity gun correction is therefore not helpful in attempting to explain the average velocity loss of about 10% observed for subsonic gas guns. However, ballistic gas compressors (6) are a possible source of relevant information and they are examined in the next section.

5.4 Corrections used for ballistic gas compressors

In ballistic gas compressors, a moving piston is used to compress gas in a sealed tube. The kinetic energy of the piston is utilised to heat the gas. Alkidas, Plett and Summerfield(ref.6) have identified a number of different effects that must be included in any method of predicting the performance of ballistic compressors. The important effects are

(a) real gas effects (equation of state),

(b) valve losses at entrance to barrel,

(c) gas leakage between piston and barrel,

(d) generation of shock waves in the compressed gas,

(e) heat losses, and

(f) friction between piston and barrel.

Items (c) and (f) are the only items relevant to transonic gas guns. All the other items arise either because of the very high pressures and temperatures generated in the ballistic compressor or because of special features in the ballistic compressor.

The major causes of the discrepancy observed between theory and experiment in figures 5 and 6 are taken to be

(i) gas leakage between barrel and sabot, and

(ii) friction between barrel and sabot.

Effect (i) is always present to some extent and a method for calculating it is given in reference 6. The method is based on incompressible, viscous flow and requires as input data the size of the gap between the sabot and barrel, and the pressure difference across the length of the sabot. Effect (ii) cannot be calculated and, if necessary, may be best determined empirically as it will probably vary a lot from sabot to sabot and from gun to gun. Some tests on the W.R.E. 384 mm gun show that effect (ii) was not important for that gun. However, this gun did not have a machined barrel so that effect (i) may have been significant.

5.5 Empirical assessment of subsonic gun performance

Figures 5 and 6 show that the actual projectile speed is, in general terms, about 10% less than predicted. There seems to be no simple way of improving the theoretical prediction methods. The discussion in section 5.3 and 5.4 has led only to the qualitative conclusion that gas leakage and friction between the sabot and barrel are the main causes of the 10% or so loss in performance. Consequently, for design purposes, we make the empirical assumption that all theoretical velocities should be reduced by 10% as a final step in predicting projectile speeds.

6. SUMMARY OF DESIGN PROCEDURE

When the Mach number in the driver gas is subsonic, less than 0.75 or so, the quasi-steady theory applies. The calculation steps are the muzzle velocity U when there is no gas in the barrel initially, a counter pressure correction for gas in the barrel and, finally, the empirical 10% correction of section 5. The simple analytical procedure is summarized in table 7 below.

TABLE 7. SUBSONIC GUN DESIGN, $U/a_0 < 1$

Step number	Velocity	Formula	Comment
1	U_0	$\left(U_0/a_0\right)^2 = \dfrac{2p_0 A_1 L_1}{m a_0^2} . \dfrac{K}{(\gamma_0 - 1)} . \left[1 - \left(\dfrac{K+1}{K}\right)^{-(\gamma_0 - 1)}\right]$	equations (3), (9)
2	$(U)_{p_1=o}$	$\left(U/a_0\right)_{p_1=o} = (U_0/a_0) . \left[1 - (\gamma_0/8)(U_0^2/a_0^2)\right]$	equation (8)
3	U	$U^2 = \left[(U^2)_{p_1=o} - (2p_1 A_1 L_1/m)\right] . \left[\dfrac{1-\exp(-y)}{y}\right]$	equation (20b)
		where $y = \gamma_1(\gamma_1+1)p_1 A_1 L_1/m a_1^2$	equation (20c)
4	$U_{estimate}$	$U_{estimate} = 0.9\ U$	empirical correction, section 5.5

When the Mach number in the driver gas reaches transonic or supersonic speeds a different procedure is used to calculate the muzzle velocity U when no gas is in the barrel. The entire procedure is summarized in table 8 in a form suitable for immediate use by a designer.

TABLE 8. TRANSONIC OR SUPERSONIC GUN DESIGN, $U/a_0 \gtrsim 1$

Step number	Velocity	Formula	Comment
1	$(U)_{p_1=o}$	$(U/a_0)_{p_1=o} = \frac{1}{4\gamma_0}\left(1-\frac{A_1}{A_0}\right)+\frac{1}{2\gamma_0}\ln\left[6\gamma_0^2(p_0A_1L_1/ma_0^2)\right]$	equation (14)
		provided that the value of (L_1/L_0) satisfies	
		$\left(\frac{L_1}{L_0}\right)^2 \leqslant \left[2.5+2\left(1-\frac{A_1}{A_0}\right)\right]\left(\frac{p_0A_1L_1}{ma_0^2}\right)$	equation (16)
		(If L_0 is too small, refer to section 3.5)	
2	U	$U^2 = \left[(U^2)_{p_1=o} - (2p_1A_1L_1/m)\right].\left[\frac{1-\exp(-y)}{y}\right]$	equation (20b)
		where $y = \gamma_1(\gamma_1+1)\,p_1A_1L_1/ma_1^2$	equation (20c)
3	$U_{estimate}$	$U_{estimate} = 0.9\,U$	empirical correction, section 5.5

7. EXAMPLE

In order to illustrate the use of the analytic design procedures summarized in section 6, the performance of a hypothetical transonic gas gun has been calculated. Figure 7 and table 9 compare the performance of hydrogen, helium and nitrogen as driver gases. Nitrogen is markedly inferior to hydrogen or helium because the driver gas Mach numbers are transonic and, as a direct consequence, there is a substantial drop in driving pressure as the projectile accelerates. On figure 7, assuming a design Mach number of 1.32, a nitrogen gun with a 23 m barrel has the same performance as a helium gun with a barrel 10 m in length. For the same performance a hydrogen gun would have a slightly shorter barrel, 8.8 m long. For the 23 m barrel with nitrogen, the reservoir length (and volume) have been increased slightly in order to satisfy the length requirement of table 8.

Table 9 compares the mass (G) of gas in the reservoir with the projectile mass (m). For the cases considered in the table, the ratio G/m is always a little less than unity for helium and hydrogen, but is markedly greater than unity for nitrogen. Hence, if using nitrogen as a driver gas, transonic muzzle velocities demand that the mass of driver gas be greater than the mass of the projectile.

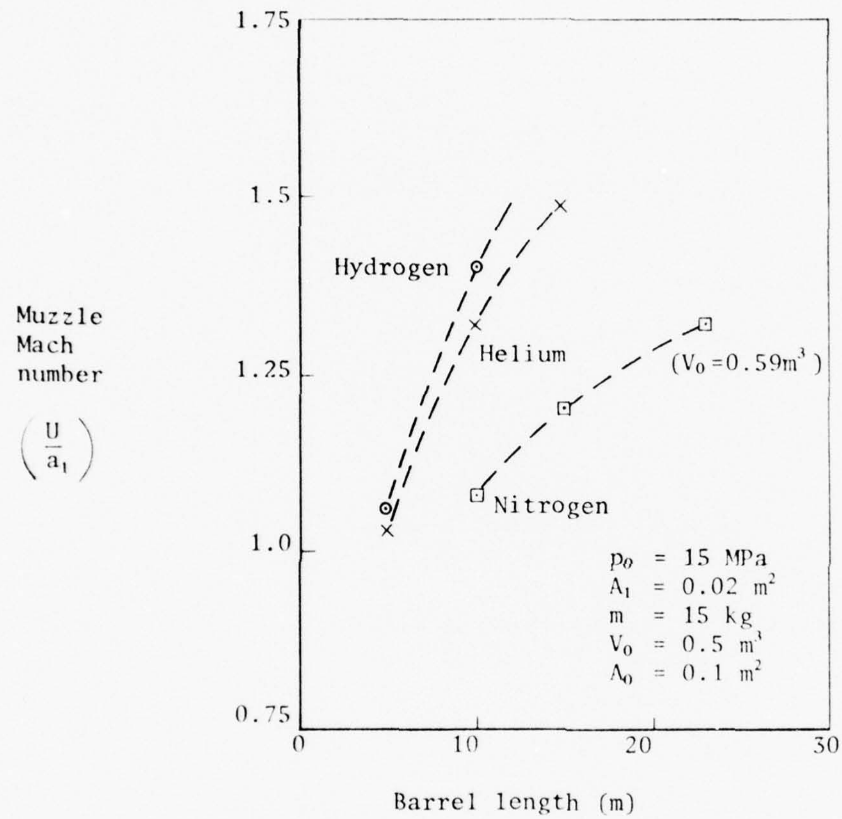


Figure 7. Example of transonic gun design

TABLE 9. GUN DESIGN EXAMPLE

Basic gun parameters	Driver gas	Driving parameter $\left(\frac{p_0 A_1 L_1}{m a_0^2}\right)$	Retarding parameter $\frac{\gamma_1(\gamma_1+1)p_1 A_1 L_1}{m a_1^2}$	Muzzle velocity* (m/s)	Muzzle mach number*
Reservoir pressure, $p_0=15$ MPa Barrel area, $A_1=0.02\ m^2$	Nitrogen $a_0=350$ m/s $\gamma_0=1.4$ G/m=5.72	1.633	0.038	371	1.08
Barrel length, $L_1=10$ m	Helium $a_0=1000$ m/s $\gamma_0=5/3$ G/m=0.83	0.200	0.038	456	1.32
Total mass, m=15 kg	Hydrogen $a_0=1300$ m/s $\gamma_0=1.4$ G/m=0.41	0.118	0.038	482	1.40
Reservoir volume, $V_0=0.5\ m^3$ Reservoir area, $A_0=0.1\ m^2$	Hydrogen $L_1=5$ m G/m=0.41	0.059	0.019	367	1.06
Air pressure, $p_1=0.1$ MPa	Helium $L_1=5$ m G/m=0.83	0.100	0.019	356	1.03
Air speed of sound $a_1=345$ m/s	Helium $L_1=15$ m G/m=0.83	0.300	0.057	513	1.49
Air ratio of specific heats, $\gamma_1=1.4$	Nitrogen $L_1=15$ m G/m=5.72	2.450	0.057	413	1.20
	Nitrogen $L_1=23$ m** G/m=5.72	3.756	0.087	456	1.32

* Empirical loss factor of 10% is included.

** For nitrogen, $L_1=23$ m is equivalent to either $p_0=32$ MPa or m=7.0 kg. The case $L_1=23$ m requires $V_0=0.59\ m^3$ so that the reservoir will be long enough (5.9 m) to stop reflections from reaching the projectile.

Figure 8 and table 10 show how gun performance varies with reservoir pressure. The extreme cases of hydrogen and nitrogen are compared and show that there is a marked increase in the performance advantage of hydrogen as the reservoir pressure increases. The performance of helium would fall a little below that of hydrogen, being very much better than that of nitrogen. Figure 8 shows that the high Mach numbers in the case of nitrogen do indeed have a dramatic effect on muzzle velocity. As the pressure drops, the performance of nitrogen and hydrogen becomes much less different. In the limit of low pressures, the nitrogen and hydrogen guns must have identical muzzle velocities as compressibility effects in the driver gas are no longer significant.

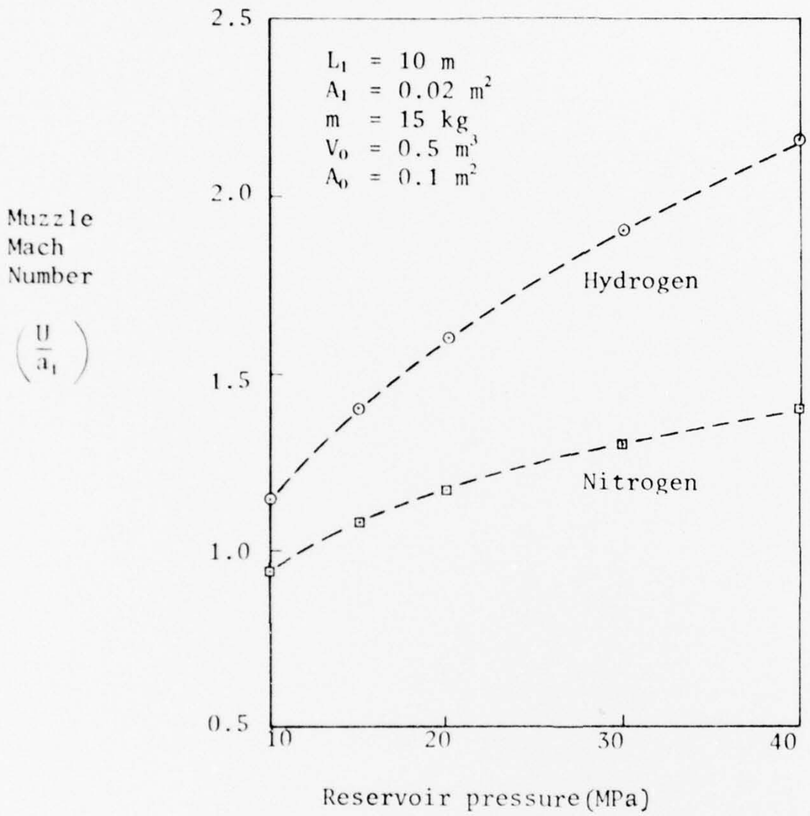


Figure 8. Example showing performance variation with reservoir pressure

TABLE 10. EFFECT OF RESERVOIR PRESSURE

Driver gas	Reservoir pressure (MPa)	Driving parameter $\left(\frac{p_0 A_1 L_1}{m a_0^2}\right)$	Retarding parameter $\frac{\gamma_1 (\gamma_1 + 1) p_1 A_1 L_1}{m a_1^2}$	Muzzle velocity * (m/s)	Muzzle mach number * (U/a_1)
Hydrogen	10	0.079	0.038	397	1.15
	15	0.118		482	1.40
	20	0.158		551	1.60
	30	0.237		660	1.91
	40	0.316		744	2.16
Nitrogen	10	1.088	0.038	326	0.94
	15	1.633		371	1.08
	20	2.177		403	1.17
	30	3.265		449	1.30
	32	3.483		456	1.32
	40	4.354		481	1.40

*Empirical loss factor of 10% is included

Note: Gun parameters are taken from gun design example, with L_1 = 10 m and V_0 = 0.5 m throughout.

8. CONCLUSIONS

The primary aim of this report has been to derive simple analytic design formulae in forms suitable for the use of gas gun designers. The main conclusions are listed below.

(1) The case of subsonic flow by the driver gas can be treated analytically. A simple analytic formula based on a quasi-steady theory has been derived. Muzzle velocity results from gas guns show that the theory is applicable up to a gas Mach number of 0.75 or so.

(2) When the driver gas moves at transonic or supersonic speeds a simple design formula can be derived. The range of usefulness extends up to a Mach number of 2. The basis of the formula is an unsteady simple wave expansion. The effect of an area change at the barrel/reservoir junction is included in the general formula.

(3) There is a minimum reservoir length for the validity of the unsteady expansion wave model. A simple, empirical expression for the minimum length has been derived. The unsteady theory applies when the reservoir is sufficiently long to prevent the expansion wave reflected by the back of the reservoir from reaching the projectile before it leaves the gun barrel.

(4) It is very difficult to estimate the muzzle velocity when the reservoir is too short for the simple unsteady theory to be valid. A simple, analytic formula of wide applicability cannot be found. For this reason, a formula of limited value has been derived empirically to give some indication of the loss in performance with short reservoirs.

(5) The effects of counter pressure can be treated analytically. A simple correction formula has been deduced. The correction is recommended for general use and has been derived so that it agrees both with Seigel's correction formula when the muzzle velocity is large and with the simple steady theory when the Mach number is low.

(6) Estimates can be made of the greatest velocity possible in a particular gun, when barrel length is treated as a variable quantity. A maximum velocity formula has been derived by allowing barrel length to vary in the quasi-steady theory. Similarly, a terminal velocity formula has been derived for higher speeds where both the driving and retarding pressure are dependent on velocity only.

(7) The performance of actual gas guns is less than that predicted by theory. An analysis has been made of the performance of subsonic gas guns in order to obtain guidelines for the likely errors in theoretical muzzle velocity predictions. Overall, the velocity is about 10% less than predicted. The loss in performance is attributed to gas leakage between the sabot and the barrel, and to friction between the barrel and the sabot.

(8) Either hydrogen or helium is superior to nitrogen as a driver gas for transonic guns. A design example shows that, for similar muzzle velocities, the reservoir pressure in the case of nitrogen is about twice that required for hydrogen or helium. Because the speed of sound in hydrogen is greater than that in helium, hydrogen always gives slightly better performance than helium.

NOTATION

a	speed of sound
$(a)_{sonic}$	value of 'a' where the gas velocity is equal to 'a'
A	area of cross-section
C	constant
D	diameter
f	acceleration of projectile, when constant
G	mass of gas in reservoir
K	$= V_0/A_1 L_1$
L	length of reservoir or barrel
$(L_0)_{min}$	minimum length of reservoir to avoid wave reflections at projectile
m	mass of projectile, including sabot
p	pressure of gas
p_d	driving pressure
p_r	retarding pressure
r	$= 4 + (A_0/A_1)$
t	time, measured from when projectile first begins to move
t_{shock}	time t at which shock wave is first formed
T	temperature of gas
u	velocity of projectile in barrel
U	muzzle velocity
U_0	value of U when $(U/a_0) \to 0$ (steady theory)
$U_{estimate}$	estimated value of U for a gas run
U_{max}	maximum value of U when L_1 varies
U_{mean}	mean value of U, equation (22b)
$(U)_{p_1=0}$	value of U when $p_1=0$
$[U]_{(L_0)_{min}}$	value of U when $L_0 = (L_0)_{min}$
V_0	volume of reservoir
x	distance travelled by projectile
x_{shock}	position of shock wave when first formed
y	retarding parameter $\gamma_1(\gamma_1+1)p_1 A_1 L_1/ma_1^2$
Z	$= (\sqrt{(\gamma_0+1)/2} - 1)(1 - (A_1/A_0))$

γ	ratio of specific heats
λ	ratio of mean retarding pressure to initial retarding pressure p_1
ρ	density of gas

Subscripts

$_0$	initial reservoir conditions
$_1$	initial barrel conditions
$_s$	gas stagnation conditions
$_{q-s}$	quasi-steady model

REFERENCES

No.	Author	Title
1	Seigel, A.E.	"The theory of high speed guns" AGARDograph 91, May 1965.
2	Liepmann, H.W. and Roshko, A.	"Elements of gasdynamics" Wiley, New York, 1957.
3	Riddell, F.R. (ed.)	"Hypersonic flow research" Academic Press, New York, 1962.
4	Krill, A.M. (ed.)	"Advances in hypervelocity techniques" Plenum Press, New York, 1962.
5	Canning, T.N., Seiff, A. and James, S.C.	"Ballistic range technology" AGARDograph No.138, 1970. (AGARD-AG-138-70).
6	Alkidas, A.C., Plett, E.G. and Summerfield, M.	"Performance study of a ballistic compressor" AIAA Journal, Vol.14, No.12, p. 1752-58, December 1976.
7	Stalker, R.J.	"Development of a hypervelocity wind tunnel" The Aeronautical Journal, June 1972, p. 374-384.
8	Dudley, R.E.	"The development of a gas gun to study the subsonic flight of projectiles" WRE-TN-639(WR&D), April 1972.
9	Sheppard, L.M.	"Gas guns for aerodynamic testings at subsonic speeds" Proceedings, Fifth Australiasian Conference on Hydraulics and Fluid Mechanics, Christchurch, New Zealand, p. 514-521, December 1974.
10	Pope, R.L.	"Use of precision trajectories to determine missile flight behaviour" WRE-TN-1704(WR&D), October 1976.
11	Pope, R.L.	"A new technique for obtaining the aerodynamics of missiles from flight trials on a gas gun range" WRE-TN-1719(WR&D), November 1976.
12	Perfect, D.A.	"The development of a smooth bore gun for the projection of bird carcasses" RAE-TR-66008, January 1966.
13	Noonan, J.W. and Heath, J.B.R.	"NAE Flight Impact Simulator" Quarterly Bulletin, Report No. DME/NAE 1969(4), p. 47-68, December 1969.

APPENDIX I

THE PIDDUCK-KENT SPECIAL SOLUTION

When the mass (G) of gas in the reservoir is very small compared with the projectile mass (m), the "Pidduck-Kent Special Solution", often used in internal ballistics and described in reference 1 for example, predicts the same muzzle velocity as the steady theory of equation (3). This is not surprising because the "Pidduck-Kent Special Solution" is a most useful approximation of wide applicability (1). However, in order to obtain an analytic solution, it does assume that there is a pressure gradient of a particular form and this assumption could lead to a different quasi-steady theory. The muzzle velocity correction for higher speeds can be found from the series expansion (1) in powers of (G/m). The result for small values of (G/m) is a velocity correction factor equal to $1 - \{(3\gamma_0 - 1)/12\gamma_0\}(G/m)$, which is different from the quasi-steady correction of equation (8). In both cases, the small correcting term is proportional to U_0^2/a_0^2. From equation (9),

$$U_0^2/a_0^2 = \left(\frac{G}{m}\right) \cdot \left[\frac{2}{\gamma_0(\gamma_0 - 1)}\right] \cdot \left[1 - \left(\frac{K+1}{K}\right)^{-(\gamma_0 - 1)}\right]$$

and the velocity correction factor of equation (8) can be written as

$$\left[1 - \frac{(G/m)}{4(\gamma_0 - 1)}\left\{1 - \left(\frac{K+1}{K}\right)^{-(\gamma_0 - 1)}\right\}\right]$$

Therefore the quasi-steady correction depends on K as well as γ_0 and (G/m) whereas the correction from the "Pidduck-Kent Special Solution" depends solely on γ_0 and (G/m). The coefficients of (G/m) are respectively

$$\frac{1}{4(\gamma_0 - 1)}\left\{1 - \left(\frac{K+1}{K}\right)^{-(\gamma_0 - 1)}\right\} \quad \text{and} \quad \frac{(3\gamma_0 - 1)}{12\gamma_0}$$

APPENDIX II

SEIGEL PRESSURE FORMULA FOR $A_0 > A_1$

The barrel entry sonic approximation is described in section 3.3. Equation (15) gives

$$p/p_0 = \left[1 + Z - \left(\frac{\gamma_0 - 1}{2}\right) \frac{u}{a_0} \right]^{2\gamma_0/(\gamma_0 - 1)} \qquad (II.1)$$

where
$$Z = \left\{ \sqrt{(\gamma_0 + 1)/2} - 1 \right\} \left\{ 1 - (A_1/A_0) \right\}.$$

Seigel(ref.1) has introduced an improved formula by taking equation (II.1) to apply only for $\gamma_0 u/a_0 \geq 1.5$. When $\gamma_0 u/a_0 \leq 1.5$, Z is replaced by $(\gamma_0 u/1.5a_0).Z$ which leads to the correct behaviour in the limit as $u \to 0$. Seigel's improved formula leads to a muzzle velocity formula more complex than equation (14). However, in cases where the distance travelled for a given muzzle velocity is required, Seigel's improved formula is straightforward to use in the equation of motion (6). The inversion to give velocity explicitly is not so straightforward and must be restricted to transonic and supersonic speeds if a logarithmic formula is desired. In view of the good performance of equation (14) at transonic and supersonic speeds, as described in section 3.4, Seigel's more complex pressure formula has not been used. The simplicity of equation (14) is worthwhile preserving.

The case $\gamma_0 U/a_0 \leq 1.5$ will now be examined in more detail. The factor Z in equation (II.1) is multiplied by $(\gamma_0 u/1.5a_0)$. The resulting pressure formula therefore looks like equation (11) with p_0 unchanged and a_0 replaced by

$$a_0 \left[1 - \frac{2\gamma_0 Z}{1.5(\gamma_0 - 1)} \right]^{-1}.$$

The a_0 multiplying factor becomes

$$\left[1 - \frac{\gamma_0}{3} \left(1 - \frac{A_1}{A_0}\right) \right]^{-1}$$

by introducing an expansion based on $(\gamma_0 - 1)$ being small.

Hence equation (13) for the muzzle velocity becomes

$$U/a_0 = \left[\frac{1}{2\gamma_0 [1 - (\gamma_0/3)(1 - (A_1/A_0))]} \right] . \ln \left[6\gamma_0^2 . \left\{ 1 - \frac{\gamma_0}{3}\left(1 - \frac{A_1}{A_0}\right) \right\}^2 . p_0 A_1 L_1 / m a_0^2 \right],$$

$$1.0 \leq \gamma_0 U/a_0 \leq 1.5.$$

When $\gamma_0 U/a_0 \geq 1.5$, even more complexity will be introduced in the formula. Thus Seigel's improved pressure formula is best used to calculate distance travelled for a prescribed muzzle velocity. The result for $\gamma_0 U/a_0 \geq 1.5$ can be shown to be

$$U/a_0 = \frac{1}{4\gamma_0}\left(1 - \frac{A_1}{A_0}\right) + \frac{1}{2\gamma_0} \ln \left[E + (6\gamma_0^2 p_0 A_1 L_1 / m a_0^2) \right], \quad \gamma_0 U/a_0 \geq 1.5,$$

where $E = \exp[3 - (1/2)(1 - (A_1/A_0))] - \{1 - (\gamma_0/3)(1 - (A_1/A_0))\}^{-2} \exp[3 - \gamma_0 (A_1/A_0))]$.

APPENDIX III

SIMILARITY PARAMETERS FOR UNSTEADY EXPANSION MODEL

The pressure formula in an unsteady expansion is given by equation (11), namely

$$p/p_0 = \left[1 - \left(\frac{\gamma_0 - 1}{2} \right) \frac{u}{a_0} \right]^{2\gamma_0/(\gamma_0 - 1)} .$$

Treating $(\gamma_0 - 1)$ as small, this formula becomes

$$p/p_0 = \exp\,(-\gamma_0 u/a_0) . \qquad \text{(III.1)}$$

Seigel(ref.1) comments that equation (III.1) is a useful simplification. Furthermore, it shows that $\gamma_0 u/a_0$ is likely to be an important similarity parameter. We can now combine equation (III.1) with the equation of motion (6) to obtain

$$m\, u \frac{du}{dx} = A_1 p_0 \exp\,(-\gamma_0 u/a_0) .$$

i.e.

$$(\gamma_0 u/a_0) \frac{d(\gamma_0 u/a_0)}{d(x/l_1)} = \frac{\gamma_0^2\, p_0 A_1 L_1}{m a_0^2} \exp\,(-\gamma_0 u/a_0) .$$

Hence we expect that the only similarity parameters of importance in determining muzzle velocity are likely to be $\gamma_0 U/a_0$ and $\gamma_0^2 p_0 A_1 L_1/m a_0^2$. The area ratio A_0/A_1, or the diameter ratio D_0/D_1, completes the set of parameters.

The usefulness of the similarity parameters $\gamma_0 U/a_0$ and $\gamma_0^2 p_0 A_1 L_1/m a_0^2$ can best be determined by re-plotting the data given in figures 2, 3 and 4. The results are given in figure III.1 and show that the similarity parameters are remarkably effective.

The predictions of the simple logarithmic formula, equation (14), are shown in figure III.1 together with data for $A_0/A_1 = 1{,}25$ taken from Seigel's exact results(ref.1). The slope of the straight lines representing equation (14) is a good average representation of the data. For $A_0/A_1 = 1$, the slope is a little larger than optimum while, for $A_0/A_1 = 25$, the slope is a little less than the optimum. Figure III.1 therefore confirms the usefulness of equation (14) as a simple, effective design formula.

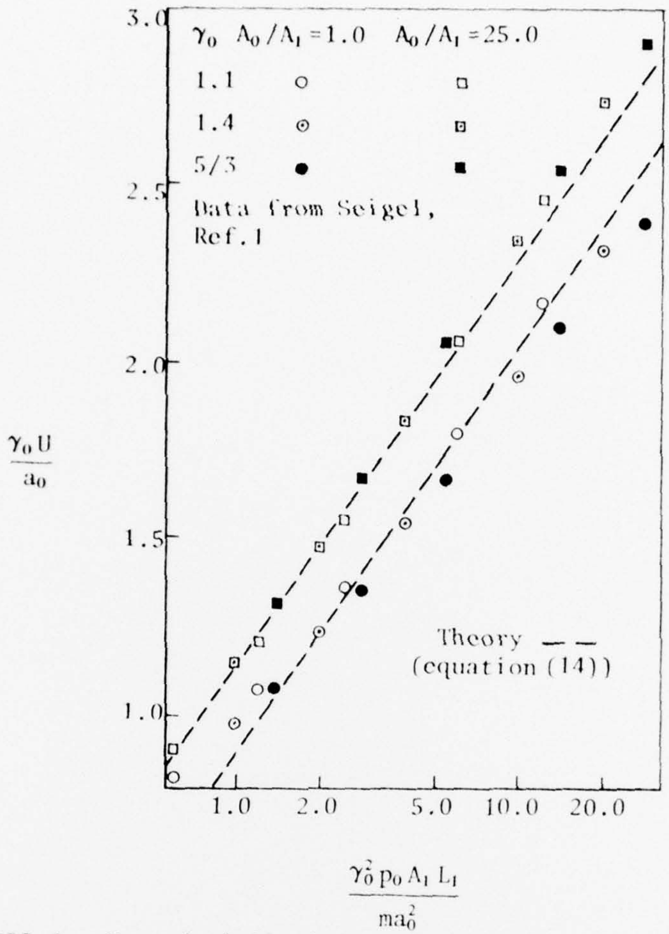


Figure III.1 Use of similarity parameters for infinite reservoir gun, γ_0 = 1.1, 1.4, 5/3

Another feature of figure III.1 is the very good agreement between the simple logarithmic formula and Seigel's data for $A_0/A_1 = 25$, with $\gamma_0 U/a_0 < 2$. This is significant, because it covers the region of most interest for guns operating at transonic speeds. When $A_0/A_1 = 1$, the logarithmic formula is very effective, but not quite so good as in the $A_0/A_1 = 25$ case.

DISTRIBUTION

EXTERNAL	Copy No.
In United Kingdom	
Defence Scientific and Technical Representative, London	1
Royal Aircraft Establishment	
Aero Department	2 - 3
Space Department	4
Structures Department	5 - 6
Weapons Department	7 - 8
E.G. Cane	9 - 10
Bedford	11
Library	12
R.A.R.D.E.	13
T.T.C.P., U.K. National Leader Panel W-2	14 - 17
Aeronautical Research Council	18 - 19
Aircraft Research Association, Bedford	20
C.A.A.R.C. Secretary	21
National Lending Library of Science and Technology	22
Royal Aeronautical Society, Library	23
In U.S.A.	
Counsellor, Defense Science, Washington	24
Defense Research and Development Attache, Washington	25
Applied Physics Laboratory, Johns Hopkins University	26
Air Force Armament Testing Laboratory	27
Ballistics Research Laboratories	28
Edgewood Arsenal	29
Eglin Air Force Base	30
N.A.S.A.	31 - 34
Naval Surface Weapons Center	
Dahlgren	35
White Oak	36
Dr. A.E. Seigel	37
Naval Weapons Center	38
Naval Ship Research and Development Center	39
Picatinny Arsenal	40
Redstone Arsenal	41
T.T.C.P. U.S. National Leader Panel W-2	42 - 45
Wright-Patterson Air Force Base, Library	46

	Copy No.
American Institute of Aeronautics and Astronautics, Library	47
Pacific Technical Information Services, Northrop Institute of Technology	48
Applied Mechanics Reviews	49
Arnold Engineering Development Center	50
A.R.O. Inc.	51
Sandia Corporation, Library	52
In Canada	
Defence Research Establishment, Valcartier	53
N.A.E., Ottawa	54
Structures and Materials Laboratory	55 - 56
T.T.C.P., Canadian National Leader Panel W-2	57 - 60
University of Toronto, Institute of Aerospace Studies	61
In Europe	
A.G.A.R.D., Brussels	62 - 67
In India	
Aeronautical Development Establishment, Bangalore	68
Indian Institute of Science, Bangalore (Department of Aeronautical Engineering)	69
Indian Institute of Technology, Madras, (Department of Aeronautical Engineering	70
Hindustan Aeronautics Ltd., Bangalore	71
National Aeronautical Laboratory, Bangalore	72 - 73
Space Science and Technology Centre, Trivandrum	74
In Australia	
Chief Defence Scientist	75
Executive Controller, Australian Defence Scientific Service	76
Controller, Programme, Planning and Policy	77
Superintendent, Defence Science Administration	78
Central Studies Establishment	79
Assistant Secretary, Defence and Information Services (for microfilming)	80
For Australian National Library	81
United Kingdom, for Ministry of Defence, Defence Research Information Centre (DRIC)	82
United States, for Department of Defense, Defense Documentation Center	83 - 94
Canada, for Ministry of Defence, Defence Science Information Service	95
New Zealand, for Ministry of Defence	96

	Copy No.
Director, Joint Intelligence Organization (DDSTI)	97
Aeronautical Research Laboratories	
Chief Superintendent	98
Superintendent, Aerodynamics Division	99
D.A. Secomb, for data exchange agreement	100
Superintendent, Mechanical Engineering Division	101
Defence Library, Campbell Park	102
Library, Aeronautical Research Laboratories	103
Library, Materials Research Laboratories	104
B.D.R.S.S., Canberra	105
Australian National University	
Dr. H.G. Hornung, Department of Physics	106
Air Office	
Air Force Scientific Adviser	107 - 108
Army Office	
Army Scientific Adviser	109 - 110
Navy Office	
Navy Scientific Adviser	111 - 112
Aircraft Research and Development Unit, Edinburgh	113
Department of Productivity, Melbourne	
Government Aircraft Factories	114
R.A.A.F. Academy, Point Cook	115
C.A.C.	116
Institution of Engineers, Australia	117
INTERNAL	
Director	118
Chief Superintendent, Weapons Research and Development Wing	119
Superintendent, Aerospace Division	120
Superintendent, Propulsion and Marine Physics Division	121
Head, Ballistics Composite	122
Principal Officer, Dynamics Group	123
Principal Officer, Aerodynamic Research Group	124 - 125
Principal Officer, Ballistic Studies Group	126
Principal Officer, Combustion and Explosives Group	127
Principal Officer, Field Experiments Group	128 - 129
Principal Officer, Flight Research Group	130

	Copy No.
Senior Engineer, Aerospace Engineering Services Section	131
Author	132
W.R.E. Library	133 - 134
Spare Copies	135 - 158
A.D. Library	159 - 160