AD-A051 073 TEXAS TECH UNIV LUBBOCK DEPT OF ELECTRICAL ENGINEERING F/G 9/3
A NYQUIST-LIKE TEST FOR THE STABILITY OF TWO-DIMENSIONAL DIGITA--ETC(U)
JUN 77 R DECARLO, R SAEKS, J MURRAY N00014-76-C-1136

UNCLASSIFIED AFOSR-TR-78-0229 NL

| OF |
ADA
051073

AFOSR-TR- 78 - 0229

AD A051073

A NYQUIST-LIKE TEST FOR THE STABILITY OF TWO-DIMENSIONAL DIGITAL FILTERS

Approved for public release;
distribution unlimited.

AD No. ____
DDC FILE COPY

DDC
RECEIVED
MAR 8 1978
F

R. Decarlo, R. Saeks, and J. Murray

Reprinted from Proceedings of the IEEE, Vol. 65, No. 6, June 1977

Copyright © 1977 by The Institute of Electrical and Electronics Engineers, Inc.
Printed in U.S.A. Annals No. 706PR020

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER: AFOSR-TR-78-0229	2. GOVT ACCESSION NO. / 3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): A Nyquist-like Test for the Stability of Two-Dimensional Digital Filters.	5. TYPE OF REPORT & PERIOD COVERED: Interim report
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): R. DeCarlo, R. Saeks, J. Murray	8. CONTRACT OR GRANT NUMBER(s): AFOSR 74-2631, N00014-76-C-1136
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Texas Tech University, Dept. of EE, Lubbock, Texas 79409	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F, 2304/A6
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Office of Scientific Research/NM, Bolling AFB, DC 20332	12. REPORT DATE: June 1977 (11 JUN 77)
	13. NUMBER OF PAGES: 12 (4 p.)
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release - Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

PROCEEDINGS OF THE IEEE, Vol 65, No 6, pp 978-979, June 1977

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Nyquist Criterion, Multidimensional Digital Filters, Stability, Homotopy Theory

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

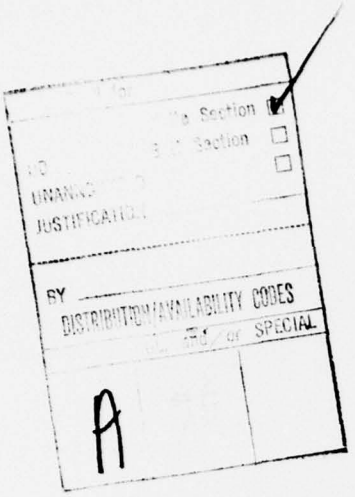
This paper constructs a Nyquist-like test for the stability of two-dimensional digital filters. The test takes the form of a continuum of classical one-variable Nyquist plots parameterized by the elements of the unit circle of the complex plane. Since the parameter space is compact, the test can be accurately approximated by a finite number of classical one-variable Nyquist plots and is therefore readily implemented on a computer.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

406 820



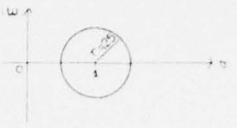


Fig. 1. Nyquist plot of $A(z_1, 0)$ for Example 1.

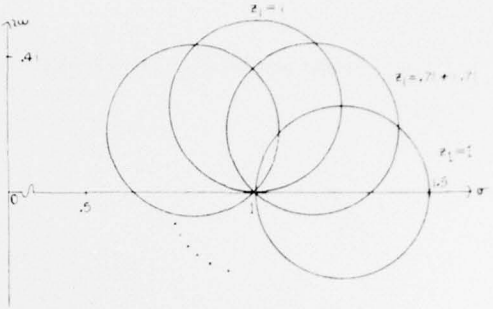


Fig. 2. Nyquist plots for $A(e^{i\alpha}, z_2)$ for Example 1.

A Nyquist-Like Test for the Stability of Two-Dimensional Digital Filters

R. DECARLO, R. SAEKS, AND J. MURRAY

Abstract—**This paper constructs a Nyquist-like test for the stability of two-dimensional digital filters. The test takes the form of a continuum of classical one-variable Nyquist plots parameterized by the elements of the unit circle of the complex plane. Since the parameter space is compact, the test can be accurately approximated by a finite number of classical one-variable Nyquist plots and is therefore readily implemented on a computer.**

Introduction

A two-dimensional digital filter is characterized by a rational transfer function in two complex variables

$$\frac{B(z_1, z_2)}{A(z_1, z_2)} \tag{1}$$

where $A(z_1, z_2)$ and $B(z_1, z_2)$ are relatively prime polynomials in z_1 and z_2. For the purpose of this paper we say that the digital filter is stable if $A(z_1, z_2) \neq 0$ for $|z_1| \leq 1$ and $|z_2| \leq 1$. This *structural stability* condition implies that the filter is bounded-input bounded-output stable, though as recently shown by Goodman [1] the condition is actually slightly stronger. Huang showed that this four-dimensional stability condition was actually equivalent to the three-dimensional condition that $A(z_1, z_2) \neq 0$ for $|z_1| = 1$ and $|z_2| \leq 1$ or $|z_1| \leq 1$ and $z_2 = 0$ which we use as the basis of our theory.

The key to the formulation of our Nyquist-like theory is the observation that from an abstract analytic function point of view the classical one-variable Nyquist plot is simply a method of determining whether or not an analytic function in one variable has zeros in an appropriate region by plotting the image of the function on the boundary of the region. To obtain a Nyquist theory in two variables we therefore decompose the region of C^2, in which $A(z_1, z_2)$ is forbidden to have zeros by Huang's theorem into the union of a family of one-variable regions to which the classical Nyquist theorem applies. More precisely, for real α, $0 \leq \alpha < 2\pi$, we define the disk D_α in C^2 by

$$D_\alpha = \{(e^{i\alpha}, z_2) \quad |z_2| \leq 1\} \tag{2}$$

and we define the disk D by

$$D = \{(z_1, 0) \quad |z_1| \leq 1\}. \tag{3}$$

Manuscript received January 6, 1977. This research was supported in part by AFOSR Grant 74-2631 and ONR Contract 76-C-1136.

The authors are with the Department of Electrical Engineering, Texas Tech. University, Lubbock, TX 79409.

Now, Huang's theorem may be restated as "the digital filter is stable if and only if $A(z_1, z_2)$ has no zeros in the disks D and D_α, $0 \leq \alpha < 2\pi$." Observing that each disk is fixed in one of its variables, the polynomial $A(z_1, z_2)$ (restricted to any of the above defined disks) is an analytic function of one variable and hence the classical Nyquist test can be used to check for zeros within the disk. In particular, $A(z_1, z_2)$ has zeros in the disk D_α if and only if the Nyquist plot for the one-variable function $A(e^{i\alpha}, z_2)$ does not equal or encircle zero. Similarly, $A(z_1, z_2)$ has no zeros in the disk D if and only if the Nyquist plot for the one-variable function $A(z_1, 0)$ does not equal or encircle zero. Combining these observations we obtain the following stability theorem.

Theorem

A digital filter characterized by the two-variable transfer function

$$\frac{B(z_1, z_2)}{A(z_1, z_2)}$$

where $A(z_1, z_2)$ and $B(z_1, z_2)$ are relatively prime polynomials in two variables, is stable (structurally stable) if and only if the Nyquist plots for the family of one-variable functions

$$A(e^{i\alpha}, z_2) \quad 0 \leq \alpha < 2\pi$$

and

$$A(z_1, 0)$$

do not equal or encircle zero.

Although the theorem formally implies that one check a continuum of Nyquist plots parameterized by the complex numbers of magnitude one, in fact, since this set of numbers is compact, one can obtain a test with arbitrarily good resolution using only a finite number of plots. Indeed, in a somewhat different context, the authors have shown that a similar continuum of Nyquist plots can actually be reduced to a single plot without inducing any error into the stability test [2]. The following examples are based on a finite approximation to the continuum of plots required by the theorem.

Example 1: Let the transfer function of a digital filter be

$$H(z_1, z_2) = \frac{1}{1 + 0.25z_1 + 0.25z_2} = \frac{B(z_1, z_2)}{A(z_1, z_2)}. \tag{4}$$

Step 1: Draw the Nyquist plot for $A(z_1, 0)$. This curve, shown in Fig. 1, does not encircle zero. So we proceed to the next step as outlined in the theorem.

Step 2: Now consider the family of Nyquist plots for the functions $A(e^{i\alpha}, z_2)$; $0 \leq \alpha < 2\pi$. This family of curves does not encircle "0" as indicated in Fig. 2. Thus the filter is stable.

Example 2: Now consider the filter whose transfer function is

$$H(z_1, z_2) = \frac{1}{1 + 0.5z_1 + 0.5z_2 + 1.2z_1z_2} = \frac{B(z_1, z_2)}{A(z_1, z_2)}. \tag{5}$$

N0014-76-C-1136

Copyright © 1977 by The Institute of Electrical and Electronics Engineers, Inc.
Printed in U.S.A. Annals No. 706PR020

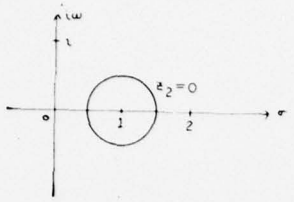


Fig. 3. Nyquist plot for $A(z_1, 0)$ for Example 2.

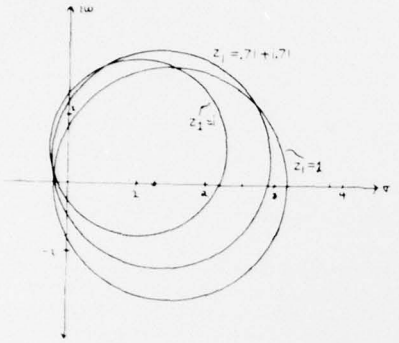


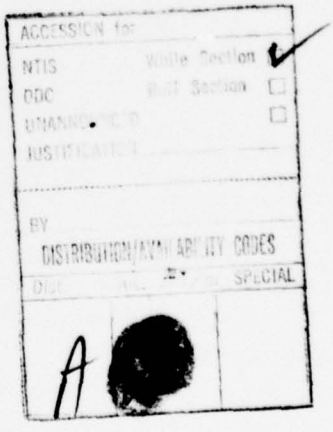
Fig. 4. Nyquist plots for $A(e^{j\alpha}, z_2)$ for Example 2.

Step 1: Consider $A(z_1, 0)$. This Nyquist plot is illustrated in Fig. 3 and does not encircle zero.

At this point no decision can be made so proceed to Step 2.

Step 2: Consider the family of functions $A(e^{j\alpha}, z_2)$; $0 \leq \alpha < 2\pi$. Nyquist plots for some of these functions are shown in Fig. 4. They indicate that the filter is indeed unstable.

References

[1] D. Goodman, "Some stability properties of two-dimensional linear shift-invariant digital filters," *IEEE Trans. Circuits Syst.*, to appear.

[2] R. A. DeCarlo, J. Murray, and R. Saeks, "Multivariable Nyquist theory," *Int. J. Contr.*, to be published.