

AD-A050 824

MASSACHUSETTS INST OF TECH CAMBRIDGE OPERATIONS RESE--ETC F/G 5/3  
A NOTE OF THE DETERMINISTIC DEMAND MULTI-PRODUCT SINGLE-MACHINE--ETC(U)  
JAN. 78 S C GRAVES N00014-75-C-0556  
TR-140 NL

UNCLASSIFIED

1 of 1

AD  
A050824



END  
DATE  
FILMED  
4 - 78  
DDC

AD No. \_\_\_\_\_  
DDC FILE COPY

AD A 050824

THE HISTORY OF THE UNITED STATES  
FROM 1789 TO 1861  
BY  
MORTIMER C. STAVES

Library No. 10  
OVERSEAS RESEARCH CENTER

DDC  
RESEARCH CENTER  
COLUMBIA UNIVERSITY

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM	
1. REPORT NUMBER Technical Report No. 140	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER 9	
4. TITLE (and Subtitle) A NOTE OF THE DETERMINISTIC DEMAND MULTI-PRODUCT SINGLE-MACHINE LOT SCHEDULING PROBLEM		5. TYPE OF REPORT & PERIOD COVERED Technical Report, January 1978	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Stephen C. Graves	8. CONTRACT OR GRANT NUMBER(s) N00014-75-C-0556	9. PERFORMING ORGANIZATION NAME AND ADDRESS M.I.T. Operations Research Center 77 Massachusetts Avenue Cambridge, MA 02139	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR 347-027
11. CONTROLLING OFFICE NAME AND ADDRESS O.R. Branch, ONR Navy Dept. 800 North Quincy Street Arlington, VA 22217	12. REPORT DATE January 1978	13. NUMBER OF PAGES 9 pages	14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 12 13 p.
15. SECURITY CLASS. (of this report)		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report) Releasable without limitation on dissemination.			
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)			
18. SUPPLEMENTARY NOTES			
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Multi-product Scheduling Problems Economic Lot Scheduling Problem Joint Replenishment Problem			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) See page ii.			

DISTRIBUTION STATEMENT A  
Approved for public release  
Distribution Unlimited  
DDC  
FEB 9 1978  
F

270 720

Handwritten initials/signature



A NOTE OF THE DETERMINISTIC DEMAND MULTI-PRODUCT  
SINGLE-MACHINE LOT SCHEDULING PROBLEM

by

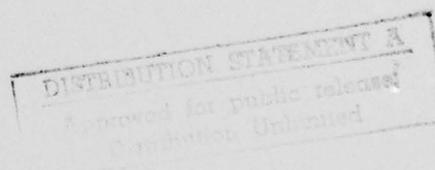
STEPHEN C. GRAVES

Technical Report No. 140

Work Performed Under  
Contract N00014-75-C-0556, Office of Naval Research  
Project No. 347-027

Operations Research Center  
Massachusetts Institute of Technology  
Cambridge, Massachusetts 02139

January 1978



Reproduction in whole or in part is permitted for any purpose  
of the United States Government.



## 1. INTRODUCTION

The purpose of this note is to try to unify and extend current theory on production lot sizing for multiple products sharing the same production resource and having known constant demand rates. In particular, two classes of problems, the economic lot scheduling problem (ELSP) and the joint replenishment problem (JRP), are shown to be equivalent. Furthermore, these two problems are shown to be equivalent to the problem of finding single cycle continuous review policies for a one-warehouse, several-retailer inventory system, for which an efficient branch and bound solution procedure exists.

The next two sections define and formulate the economic lot scheduling problem and the joint replenishment problem, respectively. Section 4 shows the equivalence of these two problems, while section 5 equates the two problems to the multi-stage inventory problem.

## 2. THE ECONOMIC LOT SCHEDULING PROBLEM

The economic lot scheduling problem (ELSP), also known as the single-machine, multi-product lot scheduling problem, assumes a single production facility on which  $n$  products are individually produced. The product  $i$  is demanded at a constant rate  $r_i$ , and, when in production, is produced at a constant rate  $p_i$ . A setup cost  $s_i$  is incurred whenever production of product  $i$  is initiated, and inventory holding costs for product  $i$  are linear with respect to the inventory level at rate  $h_i$ . All demand is met from inventory with no backordering. The problem is to find a feasible production schedule so to minimize the total setup and inventory holding costs per time unit.

This problem has been considered by several authors. The papers of Rogers [9], Bomberger [1], Madigan [7], Stankard and Gupta [11], and Doll and Whybark [3] are representative of the development of improved theory for this problem. In particular, Doll and Whybark [3], whose procedure is best on a set of test problems, consider policies of the form  $(k_1, \dots, k_n; T)$ , where  $k_i$  is integer for  $i=1, 2, \dots, n$ . This policy is a cycling policy with a fundamental period of length  $T$ ; product  $i$  is produced once every  $k_i$  periods or every  $k_i T$  time units. Note that since production capacity is finite, a given policy  $(k_1, \dots, k_n; T)$  may not be feasible; that is, a production schedule cannot be found such that the production requirements in each period are less than  $T$ , the length of the period.

Doll and Whybark present a two-step heuristic procedure for finding a good feasible schedule of the form  $(k_1, \dots, k_n; T)$ . The first step is to find a good policy with respect to setup and inventory holding costs, ignoring the question of feasibility. The second step is needed only if

the first step produces an infeasible schedule; in this case, the schedule is adjusted until a feasible schedule is found. The problem associated with the first step may be written as the following non-linear, mixed integer math program:

$$\begin{aligned} \text{ELSP: (1) } & \min \sum_{i=1}^n c_i \\ & \left. \begin{aligned} \text{(2) s.t. } c_i &= s_i/T_i + \hat{h}_i r_i T_i / 2 \\ \text{(3) } T_i &= k_i T \\ \text{(4) } \hat{h}_i &= h_i (1 - r_i / p_i) \\ \text{(5) } k_i &\geq 1, \text{ integer} \end{aligned} \right\} i=1, \dots, n \\ \text{(6) } & T \geq 0 \end{aligned}$$

where  $c_i$  is the average setup cost and holding cost for product  $i$ . Doll and Whybark find a heuristic solution to (1) - (6) by means of an iterative procedure. Given values for the vector  $(k_1, \dots, k_n)$ , the optimal period length  $T$  is easily found; given a value for  $T$ , optimal integer values for  $k_i$ ,  $i=1, \dots, n$ , can be found. The procedure starts with  $k_i = 1$ , for  $i=1, \dots, n$ , and stops upon convergence.

### 3. THE JOINT REPLENISHMENT PROBLEM

The joint replenishment problem (JRP), or packaging problem is identical to the ELSP except in the assumptions concerning the setup costs. For the JRP, a major setup cost  $S$  is incurred whenever the production facility is setup after having been idle or shutdown. In addition to this setup cost, a minor setup cost  $s_i$  is incurred prior to the processing of product  $i$ . The typical scenario for this problem is a facility dedicated to one product which may be packaged into several different containers. The major setup corresponds to the preparing of the facility for production, while the minor setups correspond to the cost of changing the packaging mechanism. Note here that "multiple" products may be the same product in different packages.

Representative of the development of work on this problem are the papers of Shu [10], Nocturne [8], and Goyal [4], [5]. Goyal considers policies of the form  $(k_1, \dots, k_n; N)$  where  $N$  is the frequency of major setups for manufacturing runs, and item  $i$  is packaged once every  $k_i$  manufacturing runs where  $k_i$  is integer. This policy class is identical to that considered for the ELSP where  $T = 1/N$ . Consequently, taking  $T = 1/N$ , the JRP may be written as

$$\begin{aligned} \text{JRP: (1')} \quad \min \quad & \sum_{i=1}^n c_i + S/T \\ \text{s.t.} \quad & (2) - (6) \end{aligned}$$

That is, the JRP is identical to (1) - (6) with only the addition to the objective function of a term for the major setup cost.

Goyal [5] solves (1') - (6) by means of a limited explicit enumeration. Bounds are placed on the optimal value of  $N$  (or equivalently  $T$ ), which yield bounds on the values of  $k_i$ . Each possible schedule is then evaluated,

with the lowest cost policy being optimal. Goyal assumes that the production rate is such that any policy can be made feasible; hence there is no need to adjust the schedule generated from (1') - (6).

4. EQUIVALENCE OF ELSP AND JRP

To show that the ELSP is equivalent to the JRP it is convenient to rewrite (1) - (6) by substituting (3) into (2), and (2) into (1) to obtain

$$\begin{aligned}
 \text{ELSP: (7) } \min \quad & \sum_{i=1}^n \left( \frac{s_i}{k_i T} + \frac{\hat{h}_i r_i k_i T}{2} \right) \\
 \text{(8) s.t. } \quad & \hat{h}_i = h_i (1 - r_i / p_i) \\
 \text{(9) } \quad & k_i \geq 1, \text{ integer} \\
 \text{(10) } \quad & T \geq 0
 \end{aligned}
 \left. \vphantom{\begin{aligned} \text{(7)} \\ \text{(8)} \\ \text{(9)} \end{aligned}} \right\} i=1,2,\dots,n$$

Similarly, by regarding the major setup as an additional product with no demand or production requirements, the JRP is rewritten as

$$\begin{aligned}
 \text{JRP: (7')} \quad \min \quad & \sum_{i=1}^{n+1} \left( \frac{s_i}{k_i T} + \frac{\hat{h}_i r_i k_i T}{2} \right) \\
 \text{s.t. (8), (9), (10)} \\
 \text{(11')} \quad & k_{n+1} = 1 \\
 \text{(12')} \quad & s_{n+1} = S \\
 \text{(13')} \quad & \hat{h}_{n+1} r_{n+1} = 0
 \end{aligned}$$

That is, product  $n+1$  is setup at cost  $S$  during each production period, but has no production requirements. Formulated in this manner, these two problems are identical except that the JRP requires the definition of an additional surrogate product by (11') - (13') to reflect the major setup cost.

## 5. EQUIVALENCE TO A MULTI-STAGE INVENTORY PROBLEM

Consider a multi-stage inventory problem (MSIP) consisting of one central warehouse which supplies  $n-1$  independent retailers. Demand occurs only at the retailers and is deterministic and constant over time; all delivery lead times are assumed to be zero. A fixed ordering cost  $K_i$  is incurred whenever retailer  $i$  places an order on the warehouse for  $i=1, \dots, n-1$ , and an ordering cost  $K_n$  is incurred whenever the warehouse (stage  $n$ ) orders from its external supplier. At each stage, a linear inventory holding cost  $c_i$  is charged against the echelon inventory at that stage, as defined by Clark and Scarf [2]. All demand is met from inventory with no backordering. The problem is to find a continuous review inventory policy to minimize total setup and inventory holding costs per unit time.

Graves and Schwarz [6] have analyzed the MSIP over the class of stationary single-cycle policies. A stationary policy assumes that each stage orders the same lot size whenever it orders, with lot sizes allowed to vary across stages. A single-cycle policy assumes that, ignoring any fixed lead times or delivery times, whenever the warehouse orders, all retailers also order. When lead times are present, this class of policies may be restated to reflect the time offset created by the deterministic lead times. To determine the optimal stationary single-cycle policy, the following problem is solved:

$$\text{MSIP: (14) } \min \sum_{i=1}^n \left( \frac{K_i D_i}{Q_i} + \frac{c_i Q_i}{2} \right)$$

$$(15) \quad \text{s.t.} \quad \frac{Q_n}{D_n} = \frac{k_i Q_i}{D_i}, \quad i=1, 2, \dots, n-1$$

$$(16) \quad k_i \geq 1, \text{ integer}, \quad i=1, 2, \dots, n-1$$

$$(17) \quad Q_n \geq 0$$

where  $D_i$  is the demand rate at retailer  $i$  for  $i=1, \dots, n-1$ ,  $D_n = \sum_{i=1}^{n-1} D_i$  is the demand rate at the warehouse,  $Q_i$  is lot size at stage  $i$ , and  $k_i$  is the frequency of orders at retailer  $i$  relative to orders by the warehouse. Hence the optimal solution to (14) - (17) may be characterized by the vector  $(k_1, \dots, k_{n-1}; Q_n)$ , which implies that the warehouse orders a lot size of  $Q_n$  every time it orders, and retailer  $i$  orders  $k_i$  times between successive orders by the warehouse.

To show the equivalence of the MSIP to both the ELSP and the JRP, it is necessary to define  $T = D_n/Q_n$ , the frequency with which the warehouse orders. By substitution, (14) - (17) may be restated as

$$\text{MSIP: (18) } \min \sum_{i=1}^n (K_i k_i T + \frac{c_i D_i}{2k_i T})$$

$$(19) \text{ s.t. } k_i \geq 1, \text{ integer} \quad i=1, 2, \dots, n-1$$

$$(20) \quad k_n = 1$$

$$(21) \quad T \geq 0$$

The problem (18) - (21) is equivalent to both (7) - (10) and (7') - (13') where  $K_i = \hat{h}_i r_i / 2$  and  $c_i D_i / 2 = s_i$ .

In [6], Graves and Schwarz develop and demonstrate a very efficient branch and bound procedure for optimally solving (18) - (21). Clearly this procedure is now applicable to both the ELSP and the JRP. In comparison with the procedure of Doll and Whybark for the ELSP, the branch and bound procedure finds optimal solutions; in comparison with Goyal's procedure for the JRP, the branch and bound procedure is implicit enumeration, rather than explicit, and hence is more efficient.

REFERENCES

1. Bomberger, E., "A Dynamic Programming Approach to a Lot Size Scheduling Problem", Management Science, Vol. 12, No. 11, July 1966, pp. 778-784.
2. Clark, A. J. and H. Scarf, "Optimal Policies for a Multi-Echelon Inventory Problem", Management Science, Vol. 6, No. 4, July 1960, pp. 475-490.
3. Doll, C. L. and D. C. Whybark, "An Iterative Procedure for the Single-Machine Multi-Product Lot Scheduling Problem", Management Science, Vol. 20, No. 1, September 1973, pp. 50-55.
4. Goyal, S. K., "Scheduling a Multi-Product Single-Machine Systems", Operations Research Quarterly, Vol. 24, No. 2, June 1973, pp. 261-269.
5. Goyal, S. K., "Determination of Optimum Packaging Frequency of Items Jointly Replenished", Management Science, Vol. 21, No. 4, December 1974, pp. 436-443.
6. Graves, S. C. and L. B. Schwarz, "Single Cycle Continuous Review Policies for Arborescent Production/Inventory Systems", Management Science, Vol. 23, No. 5, January 1977, pp. 529-540.
7. Madigan, J. G., "Scheduling a Multi-Product Single-Machine System for an Infinite Planning Period", Management Science, Vol. 14, No. 11, July 1968, pp. 713-719.
8. Nocturne, D. J., "Economic Ordering Frequency for Several Items Jointly Replenished", Management Science, Vol. 19, No. 9, May 1973, pp. 1093-1096.
9. Rogers, J., "A Computational Approach to the Economic Lot Scheduling Problem", Management Science, Vol. 4, No. 3, April 1958, pp. 264-291.
10. Shu, F. T., "Economic Ordering Frequency for Two Items Jointly Replenished", Management Science, Vol. 17, No. 6, February 1971, pp. B-406-410.
11. Stankard, M. F. and S. K. Gupta, "A Note on Bomberger's Approach to Lot Size Scheduling: Heuristic Proposed", Management Science, Vol. 15, No. 7, March 1969, pp. 449-452.