

ARO 12515.6-E AD A 0 5 0 7 9 3 RESEARCH LABORATORIES FOR THE ENGINEERING SCIENCES SCHOOL OF ENGINEERING AND APPLIED SCIENCE University of Virginia Charlottesville, Virginia 22901 HIGH SPEED ROTOR BALANCING. 9 Final Kepert. DAAG-29-Grant No. DAAG29 75 G 0055; 77 G 0027 15 - DAAG29-75-G-0055 Submitted to: DAAG 29 - 77. G - 9627 U. S. Army Research Office P. O. Box 12211 **Research Triangle Park** 4*RC* North Carolina, 27709 12515.6-E Attn: DRXRO-IP-L-12515-E Richard O. Ulsh, Chief Information Processing Office Submitted by: W. D. Pilkey MAR 1978 2 Report No. UVA/525037/MAE78/101 5 SIL Feb D 12 104 APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED 366 870 JOB

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FINAL REPORT

WALTER D. PILKEY

1977

U. S. ARMY RESEARCH OFFICE GRANT NO. DAAG29 75 G 0055; 77 G 0027 UNIVERSITY OF VIRGINIA

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#### PROBLEM STATEMENT AND ACCOMPLISHMENTS

The goal of the program is to explore new techniques for balancing, analyzing, and designing rotating shafts. This study began with the assumptions that the balancing problem is rightfully classified as an identification/optimization problem and that modern computational techniques hold great promise in aiding in the solution of the problem. Throughout the study emphasis is given to computational optimization formulations.

The achievements of this study, which was performed during the period 21 October 1974 - 20 October 1977, can be grouped into five categories:

Development of Computational Balancing Techniques Sensitivity Study of Rotors to Errors in Balancing Efficient Optimal Design of Rotor Bearing Systems Transient Analysis of Rotating Shafts Numerical Stability of Rotor Analyses These efforts are summarized below.

I. Development of Computational Balancing Techniques

A variety of computational methods have been developed for identifying the unbalance deflection of a flexible rotating shaft. These differ from previous techniques in that the new methods permit the magnitude of the balance weights to be controlled. In addition to permitting regulation of the balance weight magnitudes, one of the approaches leads to balance weights that reduce rotor runouts at speeds above the level at which the shaft can be safely run without balancing. Another technique selects balance weights of the least possible magnitude and thereby can sometimes avoid stress concentration problems. The most recent development is a method that in addition to selecting balance weights also finds the optimum axial location (balance planes) for positioning the weights. This method is based on an iterative computational scheme. Numerical simulations indicate that the technique is indeed effective. Most of these new methods are highly efficient computationally. This is due to the fact that the balancing problem can often be handled as a linear programming identification problem. See papers 1, 2, 8, and 9. The optimal balancing plane work has not yet appeared in print.

## II. Sensitivity Study of Rotors to Errors in Balancing

A method was developed for solving the problem of determining the maximum deflection of a shaft resulting from the worst combination of errors in placing balance weights along the axis of a rotor. This analysis, called a worst balance analysis, provides an index of sensitivity of the rotor to errors in balancing. The solution is formulated in general as a nonlinear programming problem and in particular cases as a linear programming problem.

This work is extended beyond rotors in the form of response bound analyses for elastic and plastic structures with incompletely prescribed loading. As in the case of shafts, these analyses serve to provide sensitivity measures for structures to uncertainties in loading. See papers 3, 4, 5, 7 and 8.

# III. Efficient Optimal Design of Rotor Bearing Systems

A new method for the optimal design of bearing systems was developed. With the new method, the conventional (automated) trial and error search for optimal parameter values for a prescribed design configuration is replaced by an efficient two-stage process. In the first stage absolute optimum (or limiting) performance characteristics of the shaft are computed. In the second stage using a chosen bearing system configuration, parameter identification techniques are applied to find the design parameters so that the suspension system will respond as close as possible to the absolute optimal performance. In this approach, the repetitive shaft analyses required in the conventional search techniques are avoided. Hence, the new technique is extremely efficient computationally and appears suitable for large systems.

Another less efficient, computational design technique is also proposed. This is a procedure for designing shafts or structures to be as insensitive as possible to uncertainties in the loading environment. In the case of rotating shafts this means the design is relatively insensitive to errors made in placing balance weights. See papers 5,6,7, and 8.

## IV. Transient Analysis of Rotating Shafts

A general modal solution for the transient response is formulated for a rotating shaft subjected to arbitrary loading functions. The shaft can be modeled with lumped or continuous parameters. The bearing systems can be arbitrarily complex, containing pedestal mass, bearing and pedestal springs and dampers. Transient or aperiodic loadings can be placed anywhere along the shaft, including displacement or velocity inputs at the base of the bearing systems. This would permit, for example, earthquake motions to be simulated. The formulation involves the sum of complex mode shapes. As an additional benefit the rotor stability can be studied by observing the behavior of the eigenvalues. A computer program implementing the formulation was prepared. See

## paper 11.

#### V. Numerical Stability of Rotor Analyses

A long-standing problem in rotor dynamics is that the use of the relatively efficient line-solution method, the transfer matrix method, for analyses is encumbered by the grave problem of numerical inaccuracies as soon as a complex rotor model, e.g. many lumped masses, is employed or a difficult problem, e.g. high critical speed, is to be solved. A new technique, the Riccati transfer matrix method, has been developed which not only eliminates the numerical instabilities but is about twice as efficient as the usual transfer matrix method. The method which employs Riccati transformations (Invariant Imbedding) still makes use of the existing large catalogs of transfer matrices for rotating shafts. A computer program using this method for the computation of damped critical speeds and mode shapes has been prepared. The program includes some novel methods for efficient eigenvalue computations. Another method has been developed for stabilizing the analysis of long shafts resting on stiff but non-rigid support systems. See paper 11, 12, and 13.

#### LIST OF PUBLICATIONS

- "A Linear Programming Approach for Balancing Flexible Rotors", R. M. Little and W. D. Pilkey, <u>The Journal of Engineering for</u> <u>Industry</u>, Vol. 94, 1976
- "Constrained Balancing Techniques for Flexible Rotors", W. D. Pilkey and J. T. Bailey, <u>The Journal of Engineering for Indusrty</u>, 1978.
- 3. "Identification of Shock and Vibration Forces", W. D. Pilkey and A. J. Kalinowski, in ASME Monograph, <u>Identification of Vibrating</u> Structures, 1972
- 4. "A Worst Balance Analysis for Rotating Shafts," W. D. Pilkey and J. T. Bailey, <u>J. of Sound and Vibration</u>, 1974
- "Response Bounds for Structures with Incompletely Prescribed Loadings," W. D. Pilkey and A. J. Kalinowski, <u>Shock and Vibration</u> <u>Bulletin</u>, Vol. 43, 1972
- "Efficient Optimal Design of Suspension Systems for Rotating Shafts," W. D. Pilkey, B. P. Wang, and D. Vannoy, <u>The Journal of Engineering</u> for Industry, Vol. 98, 1976.
- "Structural Design for Incompletely Prescribed Loading," W. D. Pilkey and A. J. Kalinowski, <u>Engineering Mechanics Journal</u>, ASCE, 1975
- "Limiting Performance, Identification, and Optimal Design of Vibromechanisms," <u>Proc. of 3rd International Conference on</u> <u>Vibromechanisms</u>, 1974
- "Rotating Machinery, A Review of Computational Capabilities,"
   H. Shapiro, G. Horner, and W. D. Pilkey, in <u>Shock and Vibration</u> Computer Programs, SVIC, 1975
- 10. "Rotating Shafts", W. D. Pilkey and P. Y. Chang, in <u>Modern Formulas</u> for Statics, Dynamics, McGraw Hill, 1978
- 11. "Transient Response of a Rotor in Damped Bearings," J. Strenkowski and W. D. Pilkey, ASME Paper No. 77-DET-21, to appear in <u>J. of</u> <u>Engineering for Industry</u>
- "The Riccati Transfer Matrix Method," G. C. Horner and W. D. Pilkey ASME Paper No. 77-DET-32, to appear in <u>J. of Engineering for</u> Industry, 1978.
- "Complex Infinite Beams on an Elastic Foundation," P. Y. Chang, D. Hsu, and W. Pilkey <u>J. of the Structural Division</u>, ASCE, Vol. 103, No. ST11, November, 2277-2282, 1977

Other papers are being considered for publication.

# PARTICIPATING PERSONNEL

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J.S. Strenkowski, Received Ph.D. degree

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#### UNIVERSITY OF VIRGINIA

#### School of Engineering and Applied Science

The University of Virginia's School of Engineering and Applied Science has an undergraduate enrollment of approximately 1,000 students with a graduate enrollment of 350. There are approximately 120 faculty members, a majority of whom conduct research in addition to teaching.

Research is an integral part of the educational program and interests parallel academic specialties. These range from the classical engineering departments of Chemical, Civil, Electrical, and Mechanical to departments of Biomedical Engineering, Engineering Science and Systems, Materials Science, Nuclear Engineering, and Applied Mathematics and Computer Science. In addition to these departments, there are interdepartmental groups in the areas of Automatic Controls and Applied Mechanics. All departments offer the doctorate; the Biomedical and Materials Science Departments grant only graduate degrees.

The School of Engineering and Applied Science is an integral part of the University (approximately 1,400 full-time faculty with a total enrollment of about 14,000 full-time students), which also has professional schools of Architecture, Law, Medicine, Commerce, and Business Administration. In addition, the College of Arts and Sciences houses departments of Mathematics, Physics, Chemistry and others relevant to the engineering research program. This University community provides opportunities for interdisciplinary work in pursuit of the basic goals of education, research, and public service.