AD-A050 474 DEFENSE COMMUNICATIONS ENGINEERING CENTER RESTON VA F/G 17/2
AN ARCHITECTURAL MODEL FOR ANALYZING AVERAGE STRUCTURAL PROPERT--ETC(U)
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DEFENSE COMMUNICATIONS ENGINEERING CENTER

TECHNICAL NOTE NO. 21-77

AN ARCHITECTURAL MODEL FOR ANALYZING AVERAGE STRUCTURAL PROPERTIES OF THE FUTURE DCS

NOVEMBER 1977

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER (9)

4. TITLE (and Subtitle) (6)
AN ARCHITECTURAL MODEL FOR ANALYZING AVERAGE STRUCTURAL PROPERTIES OF THE FUTURE DCS.

5. TYPE OF REPORT & PERIOD COVERED
Technical Note,

6. PERFORMING ORG. REPORT NUMBER (14)
DCEC-TN-21-77

7. AUTHOR(s) (10)
Major W. P. Dotson

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS
DCEC, R740, Systems Engineering Division, R700
1860 Wiehle Avenue
Reston, VA 22090
407519

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS (11)

11. CONTROLLING OFFICE NAME AND ADDRESS
Same as 9

12. REPORT DATE
November 1977

13. NUMBER OF PAGES (12)
54p.

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
N/A (18) SBIE
(19) AD-E100 022

15. SECURITY CLASS. (of this report)
Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
N/A

16. DISTRIBUTION STATEMENT (of this Report)
A. Approved for public release; distribution unlimited.

DDC RECEIVED FEB 27 1978 B

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
N/A

18. SUPPLEMENTARY NOTES
Review relevance 5 years from submission date.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
Approximation Techniques
Network Performance
Survivability
Grade of Service
Average Properties

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
This report develops some approximation techniques for use in communications network design problems which deal with "average properties" of the network rather than the discrete network structure. Only the architectural issue of backbone network topology - its effect on network Grade of Service and survivability under constrained cost are treated. A new way of thinking about network performance is developed so that survivability-capacity tradeoffs can be made early in the design process. The report is written in a problem solving sequence and results are presented and discussed at each step in the solution process.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

407519

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SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

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NOVEMBER 1977

Prepared by:

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FOREWORD

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I. INTRODUCTION

This report develops some approximation techniques for use in communications network design problems. The techniques deal with "average properties" (to be defined later) of networks and thus lose sight of the discrete structure; hence, they are not suitable for such problems as efficiently sizing <u>each</u> link in, say, the AUTOVON network. The techniques can, however, tell us something about the <u>average</u> link size, among other things. Average rather than precise parameters are already widely utilized in the Erlang formulas and in queueing theory. This report is useful for looking at average properties at a higher level of system issues in order to provide a better feel for the gross characteristics of various architectural issues.

Since the techniques developed work with average properties of the network, they require only that the designer know average quantities associated with the exogenous variables in network design. These too will be defined later. At this point we merely set the tone of this paper; viz., we wish to find analytical relationships between basic structural properties (on the average) of a communications network and the external variables which impact the network.

The development of these techniques was motivated by the need for a mechanism to assess quantitatively various future DCS architectural alternatives. There are two fundamental ways of approaching this problem. One is to attempt the construction of network design models in the traditional sense. Such models require detailed predictions concerning the values of external variables at some point in future time. For example, one needs to predict the location and associated traffic volume, by type, of each future DCS

user. This approach is discussed in reference [1] and will not be considered herein. The second approach is the subject of this paper. Here we take a macroscopic look at the network and its environment. Only one architectural issue is dealt with in this paper. This is network topology - i.e., its effect on Network Grade of Service (GOS) and survivability under constrained costs in a terrestrial backbone network.

1. PROBLEM DISCUSSION

A communications network is to be designed to service a rectangular area whose dimensions are A and B miles, as shown in Figure 1. Very little is known about the requirements of the users. Consequently, using the Laplacian assumption of rationality * [2], we assume that:

A1. The users are uniformly distributed in the area and all users are equally likely to generate traffic.

A2. A randomly selected user is equally likely to call any other user in the area.

A3. The total offered traffic for the network is E erlangs.

For our purposes it is reasonable to assume that the network is constructed from two basic elements; nodes and links. A cost model for each element is given by the assumptions:

A4. Link costs, D_L, are a function of the number of channels, c, in the link; and the length in miles, ℓ, of the link:

$$D_L = kc\ell$$

where k is the cost per channel mile. All links are assumed to be full duplex; where a "link" is taken to comprise the total interconnection of two nodes.

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* This, in essence, means that when we do not know the value of a parameter we assume all cases to occur with equal probability.

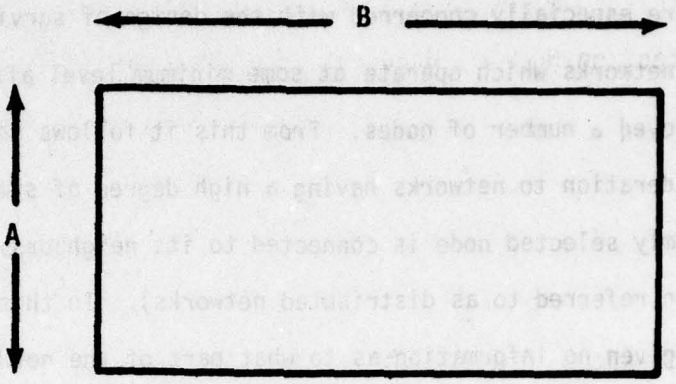


Figure 1. The Rectangular Area to be Serviced

A5. Node costs, D_N, comprise a fixed cost, a, and a cost per channel termination, b:

$$D_N = a + bt$$

where t is the number of channels terminating on the node.

We are especially concerned with the design of survivable networks; that is, networks which operate at some minimum level after an adversary has destroyed a number of nodes. From this it follows that we restrict our consideration to networks having a high degree of symmetry; i.e., any randomly selected node is connected to its neighbors in a uniform way (often referred to as distributed networks). In this fashion the enemy is given no information as to what part of the network is most sensitive - in fact, ideally no one part is more sensitive than any other. As a consequence:

A6. When the network is attacked, nodes are removed by a random process.

Our approach to this problem requires that we impose some geometric regularity on potential network designs while leaving as design variables the number of nodes and the number of links. The necessary geometric regularity consists of postulating a square grid arrangement of nodes in the area. Thus, if we were to investigate a design consisting of M·N nodes they would be arranged in a uniform grid inside the area, as shown in Figure 2. We assume, for the sake of definiteness:

A7. $M \leq N$

The number of nodes can vary as well as the number, length, and capacity of the links. Our problem is to find an optimum set in these parameters. By optimum we mean that design, costing equal to or less than a fixed amount, D, which maximizes the network throughput (or minimizes network GOS, G) subject to a minimum acceptable throughput after an attack at some fixed level.

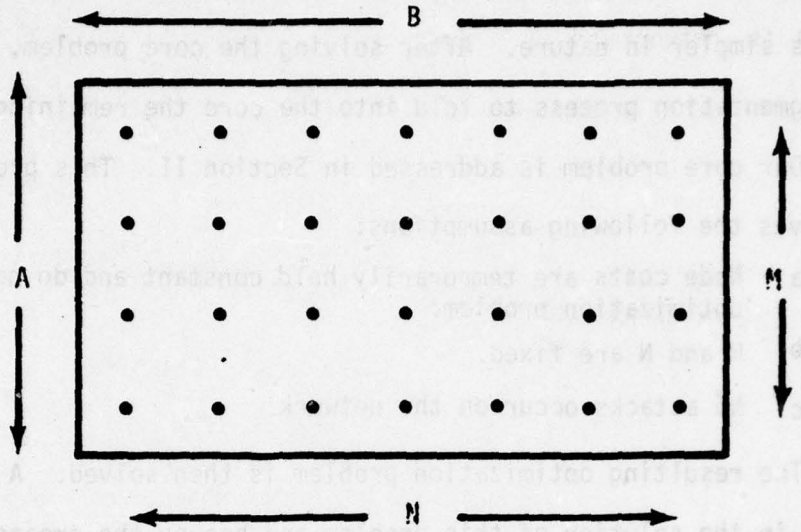


Figure 2. Representative Node Array

2. REPORT ORGANIZATION

This report is organized in a problem solving sequence. That is, we select a "core" problem from the previous subsection which contains the essential mathematical features of the overall problem, but is simpler in nature. After solving the core problem, we begin an augmentation process to fold into the core the remaining features.

Our core problem is addressed in Section II. This problem involves the following assumptions:

a. Node costs are temporarily held constant and do not enter into the optimization problem.

b. M and N are fixed.

c. No attacks occur on the network.

The resulting optimization problem is then solved. A number of subissues occur in the solution of this problem and occupy the greatest part of section II. The reader may occasionally have to refer to the end of the section, where the subissues are tied together, to understand why a particular subissue is being addressed.

Section III builds on section II by folding in the node cost model and allowing M and N to vary. Results are presented along the way to show how the behavior of the model changes in response to the added considerations.

Section IV concludes the building process by folding in attack considerations on the model of section III. In order to develop these considerations additional assumptions are required, which are given in section IV.

Section V discusses the results of some typical problems, and potential applications and extensions of the model.

II. THE CORE PROBLEM

This section considers the problem of Section I, 2 under the simplifing assumptions:

a. Node costs are zero.

b. M and N are fixed.

c. No attacks occur on the network.

A number of subissues are involved in this problem. One is the number of tandem links used in placing the average call over the most direct route. This quantity will be called L. A second subissue is the length, in miles, of the average link in the network. This quantity will be called ℓ. These quantities are somewhat difficult to solve for and occupy subsections 1 and 2 respectively.

Subsection 3 develops the equations for a number of simpler variables and also presents a (so far as we know) new model for computing network GOS. It also ties all the pieces together to solve the optimization problem of this section. Subsection 4 addresses the issue of the optimum routing strategy for our type of network through the parameter L.

1. SOLUTION FOR L

L will depend on a parameter called the average incidence degree, I, of nodes in the network as well as M and N. Let:

I = average number of links connected to a node in the network.

This parameter tells us the total number of links, T, in the network. Since each link is connected to two nodes, and there are M N nodes,

$$T = MNI/2.$$

To find L(I,M,N) we must consider particular cases. In fact we consider the cases:

$$I = \{2, 4, 8, (MN-1)\}. \tag{2}$$

After finding these we assume that L(I,M,N) can be calculated by log-Polynomial interpolation for the remaining values of I. Note that I cannot be greater than (MN-1) since at this value the network is completely connected; i.e., every node is directly connected to every other node.

a. Solution for L (2,M,N). Figure 3 is used to calculate L(2,M,N). The problem is simplified by redrawing Figure 3 as a "loop" in Figure 4.

From assumptions A1 and A2 we can look at any node in Figure 4 and calculate the average

$$L(2,M,N) = \sum_{i=2}^{MN} \left(\frac{1}{MN-1}\right) \; MIN\{(i-1),(MN+1-i)\}. \tag{3}$$

The MIN (·) operator denotes selection of the shortest path. Note that for computational purposes we are assuming that the node originating the call is labeled 1. From the symmetry of the problem, (3) can be rewritten as:

$$L(2,M,N) = \begin{cases} \sum_{i=2}^{\frac{MN+1}{2}} \left(\frac{2}{MN-1}\right) \; (i-1) & : MN \text{ odd} \\ \left[\sum_{i=2}^{\frac{MN}{2}} \left(\frac{2}{MN-1}\right) \; (i-1)\right] + \frac{MN}{2(MN-1)} & : MN \text{ even}. \end{cases} \tag{4}$$

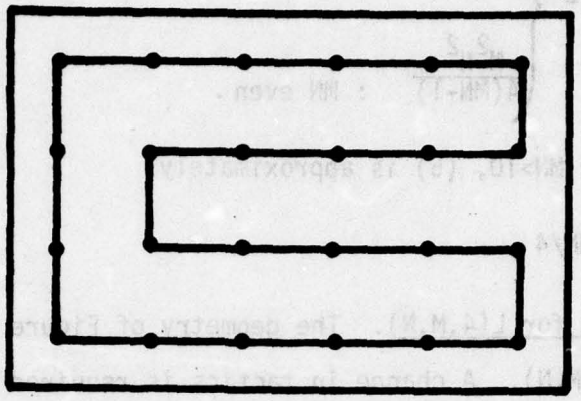


Figure 3. A Possible Geometry for I=2

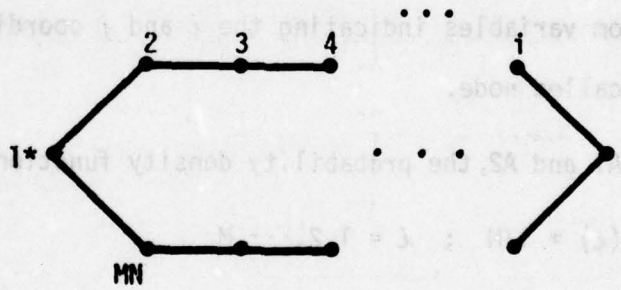


Figure 4. Loop Representation for I=2

Solving (4) results in

$$L(2,M,N) = \begin{cases} \frac{MN+1}{4} & : MN \text{ odd} \\ \frac{M^2N^2}{4(MN-1)} & : MN \text{ even}. \end{cases} \quad (5)$$

For large MN, say MN>10, (5) is approximately

$$L(2,M,N) = MN/4. \quad (6)$$

b. Solution for L(4,M,N). The geometry of Figure 5 applies in the solution for L(4,M,N). A change in tactics is required to solve this problem. First, we place an i,j discrete coordinate system on the nodes in the area. Let

$i1$ ≡ the random variable indicating the i coordinate of the calling node. $i = 1,2,\cdots,M$

$j1$ ≡ the random variable indicating the j coordinate of the calling node. $j = 1,2,\cdots,N$

$i2,j2$ ≡ random variables indicating the i and j coordinates of the called node.

From assumptions A1 and A2, the probability density functions are:

$$f_{i1}(i) = f_{i2}(i) = 1/M \quad ; \quad i = 1,2,\cdots,M \quad (7)$$

$$f_{j1}(j) = f_{j2}(j) = 1/N \quad ; \quad j = 1,2,\cdots,N. \quad (8)$$

The coordinate differences between calling pairs of nodes are defined as

$$x = |j1 - j2|. \quad (9)$$

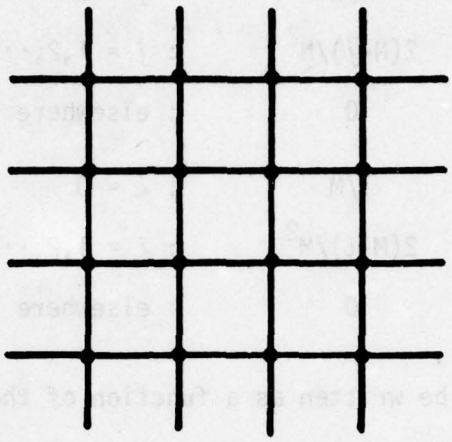


Figure 5. Geometry for Calculating L(4,M,N)

$$y \equiv |i1 - i2| . \tag{10}$$

Assuming for the moment that a node may also call itself (we correct for this later), we find by convolution and a transformation the probability density functions

$$f_x(j) = \begin{cases} 1/N & : j = 0 \\ 2(N-j)/N^2 & : j = 1,2,\cdots,N-1 \\ 0 & : \text{elsewhere} . \end{cases} \tag{11}$$

$$f_y(i) = \begin{cases} 1/M & : i = 0 \\ 2(M-i)/M^2 & : i = 1,2,\cdots,M-1 \\ 0 & : \text{elsewhere} \end{cases} \tag{12}$$

L(4,M,N) can be written as a function of the random variables x and y. These assume, however, that a node may call itself. This event occurs with probability 1/MN and implies a path length of zero. To see the effect, we write the equation

$$\frac{1}{MN}(0) + \frac{MN-1}{MN}(L(4,M,N)) = E\{x\} + E\{y\} .$$

Thus, L(4,M,N), corrected to exclude the assumption that a node may call itself, is

$$L(4,M,N) = \frac{MN}{MN-1}\ (E\{x\} + E\{y\}).$$

The sum of the expectations of x and y symbolize the accounting scheme associated with finding path length in the geometry of Figure 5. Using (11) and (12) we find

$$L(4,M,N) = \frac{MN}{MN-1}\left[\sum_{i=1}^{M-1} \frac{2(M-i)}{M^2}\, i + \sum_{j=1}^{N-1} \frac{2(N-j)}{N^2}\, j\right] . \tag{13}$$

By using some identities in [3], [4], Eq. (13) simplifies to

$$L(4,M,N) = \frac{MN}{MN-1}\left[\frac{M^2+3M-1}{3M} + \frac{N^2+3N-1}{3N} - 2\right] . \qquad (14)$$

c. Solution for L(8,M,N). The case for L(8,M,N) is slightly more difficult. Figure 6 applies. Again the distributions of (11) and (12) are useful. In considering the correction factor (for a node calling itself) and the accounting scheme for paths implied by Figure 6, we can see that

$$L(8,M,N) = \frac{MN}{MN-1} E\{z\} \qquad (15)$$

where

$$z = MAX\ (x,y) . \qquad (16)$$

Our problem is to find E{z}, but first we must find the probability density for z. From (16)

$$f_z(i) = Pr\{z=i\} = Pr\{x=i,\ y=i\} + Pr\{x=i,\ y<i\} + Pr\{x<i,\ y=i\} . \qquad (17)$$

Since we assume $M \leq N$

$$f_z(i) = \begin{cases} Pr\{x=0,\ y=0\} & : i = 0 \\ Pr\{x=i,\ y=i\} + PR\{x=i,\ y<i\} + Pr\{x<i,\ y=i\} & \\ & : i = 1,2,\cdots,M-1 \\ Pr\{x=i,\ y<i\} & : i = M,M+1,\cdots,N-1 \\ 0 & : \text{elsewhere} \quad . \end{cases} \qquad (18)$$

From (11), (12), and (18)

$$Pr\{x=0,\ y=0\} = 1/MN \qquad : i = 0$$

$$Pr\{x=i,\ y=i\} = 4(M-i)\ (N-i)/M^2N^2 \qquad : i = 1,2,\cdots,M-1$$

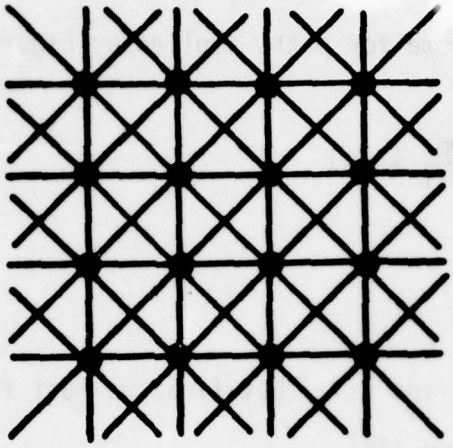


Figure 6. Geometry for Calculating L(8,M,N)

$$\Pr\{x=i,\ y<i\} = (2(N-i)/N^2) \sum_{j=0}^{i-1} f_y(j) = 2(N-i)M/M^2N^2 + \tag{19}$$

$$(2(N-i)/N^2) \sum_{j=1}^{i-1} \frac{2(M-j)}{M^2} \qquad : i = 1,2,\cdots,M-1$$

$$\Pr\{x<i,\ y=i\} = 2(M-i)N/M^2N^2 + (2(M-i)/M^2) \sum_{j=1}^{i-1} \frac{2(N-j)}{N^2}$$

$$: i = 1,2,\cdots,M-1$$

$$\Pr\{x=i,\ y<i\} = 2(N-i)/N^2 \qquad : i = M,M+1,\cdots,N-1 \ .$$

But

$$\sum_{j=1}^{i-1} \frac{2(M-j)}{M^2} = \frac{2}{M^2} \sum_{j=1}^{i-1} M - \sum_{j=1}^{i-1} j = \frac{2M(i-1) - i(i-1)}{M^2} \ .$$

Therefore

$$f_z(i) = \begin{cases} 1/MN & : i = 0 \\ 4(M-i)\ (N-i)/M^2N^2 \\ + 2(N-i)M/M^2N^2 \\ + [2(N-i)/N^2]\ [2M(i-1) - i(i-1)]/M^2 \\ + 2(M-i)N/M^2N^2 \\ + [2(M-i)/M^2]\ [2N(i-1) - i(i-1)]/N^2 & \\ & : i = 1,2,\cdots,M-1 \\ 2(N-i)/N^2 & : i = M,M+1,\cdots,N-1 \ . \end{cases} \tag{20}$$

Now from (15) and (20)

$$L(8,M,N) = \frac{NM}{MN-1} \sum_{i=0}^{N-1} i f_z(i) \ . \tag{21}$$

Solving (21) requires use of the identities [4]

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n(n+1)}{2}^2$$

$$\sum_{i=1}^{n} i^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n+1)$$

and a lot of algebra which we leave to the ambitious reader. The solution we found is

$$L(8,M,N) = \frac{MN}{MN-1}\left\{\frac{1}{6N}(7M^2-6M-1) - \frac{1}{30MN^2}(21M^4-30M^3+5M^2+4)\right.$$
$$\left. + \frac{1}{N^2}\left[N^2(N-1)-MN(M-1)+\frac{1}{3}(2M^3-3M^2+M)-\frac{1}{3}(2N^2-3N^2+N)\right]\right\}. \qquad (22)$$

d. Solution for L((MN-1),M,N). This case is simple. Since every node is directly connected to every other node

$$L((MN-1),M,N) = 1.0 \ . \qquad (23)$$

e. Summary of Solutions for L(I,M,N). So far we have obtained the number of tandem links, for the average call, on the shortest route for an M by N node grid network for various node incidence degrees, assuming a

uniform traffic distribution. We assume intermediate node incidence degrees are adequately approximated by a log-polynomial interpolation+. Several interpolation schemes were tried. We finally settled on

$$L(I,M,N) = a_0 + a_1 \left[\text{Log } \frac{MN-1}{I}\right] + a_2\left[\text{Log } \frac{MN-1}{I}\right]^2 \qquad (24)$$

$$: I = 3,4,\cdots,MN-1 \ .$$

The results so far are summarized below and an example plot for M=6, N=9 is given in Figure 7.

$$L(2,M,N) = MN/4 \qquad \text{(large MN)} \qquad (25)$$

$$L(4,M,N) = \frac{MN}{MN-1}\left[\frac{M^2+3M-1}{3M} + \frac{N^2+3N-1}{3N} - 2\right] \qquad (26)$$

$$L(8,M,N) = \frac{MN}{MN-1}\left\{\frac{1}{6N}(7M^2-6M-1) - \frac{1}{30MN^2}(21M^4-30M^3+5M^2+4)\right.$$

$$\left. + \frac{1}{N^2}\left[N^2(N-1)-MN(M-1) + \frac{1}{3}(2M^3-3M^2+m) - \frac{1}{3}(2N^3-3N^2+N)\right]\right\} \qquad (27)$$

$$L((MN-1),M,N) = 1.0 \ . \qquad (28)$$

2. SOLUTION FOR ℓ

Our core problem assumes that only links in the network cost money. To optimize the design we must minimize the network GOS by distributing the channel miles of links we can purchase in an optimum way. The first thing we must discover is how many links we can buy; this is T in equation (1). Next we need to know the capacity, c, of the average link. To find this we must know ℓ, since the amount to be spent, D, is, from A4:

$$D = T\, k c \ell \ . \qquad (29)$$

+ This report used 10 as the log base throughout. Any other base will work equally well as long as consistency is exercised.

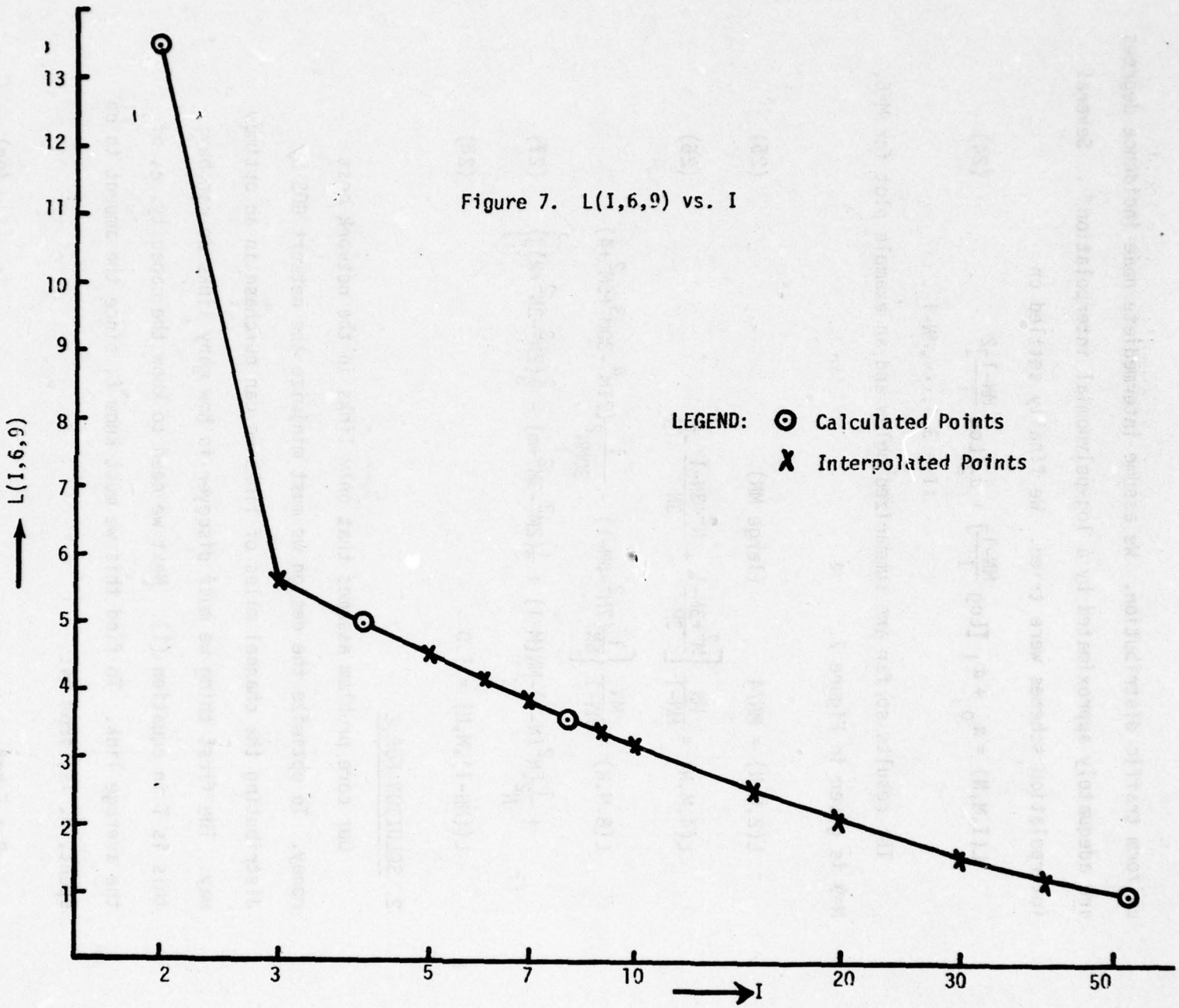


Figure 7. L(I,6,9) vs. I

Thus if we know ℓ, we can compute c.

As in the previous subsection, we must consider particular cases and the dependence of ℓ on I (as well as M and N). The cases considered are

$$I = \{2,4,8,(MN-1)\} \ . \tag{30}$$

After finding these we assume that $\ell(I,M,N)$ can be calculated by log-polynomial interpolation for the remaining values of I.

One of the factors involved in computing ℓ is the distance, in miles, between nodes at (i,j) and $(i+1,j)$; or equivalently (i,j) and $(i,j+1)$. This is our unit distance u. Since we are treating a square grid type of network with MN nodes covering an AB square mile area:

$$u = [AB/MN]^{1/2} \ . \tag{31}$$

a. Solutions for $\ell(2,M,N)$, $\ell(4,M,N)$, and $\ell(8,M,N$. The following equations can be written by inspection from Figures 3, 5, and 6 and equation (31)

$$\ell(2,M,N) = [AB/MN]^{1/2} \tag{32}$$

$$\ell(4,M,N) = [AB/MN]^{1/2} \tag{33}$$

$$\ell(8,M,N) = [AB/MN]^{1/2} \ \frac{1+\sqrt{2}}{2} \ . \tag{34}$$

b. Solution for $\ell((MN-1),MN)$. The solution for $\ell((MN-1),M,N)$ is not quite obvious. This situation corresponds to a completely connected network, however, and some thought reveals that it is mathematically analogous to the problem of finding the straight-line distance between calling pairs of nodes for the average call.

As a consequence of this analogy, we can write $\ell((MN-1),M,N)$ in terms of (11), (12) and (31), and the correction factor $\frac{MN}{MN-1}$ first used in (13). That is,

$$\ell((MN-1),M,N) = \frac{MN}{MN-1} \; (AB/MN)^{1/2} \; E\{(x^2+y^2)^{1/2}\} \tag{35}$$

or

$$\ell((MN-1),M,N) = \frac{MN}{MN-1} \; \left(AB/MN\right)^{1/2} \left[\frac{1}{M} \sum_{j=1}^{N-1} \frac{2(N-j)}{N^2}\right.$$

$$\left. + \frac{1}{N} \sum_{i=1}^{M-1} i \, \frac{2(M-i)}{M^2} + \sum_{i=1}^{M-1} \sum_{j=1}^{N-1} (i^2+j^2)^{1/2} \, \frac{4(M-i)(N-j)}{M^2N^2}\right] . \tag{36}$$

This reduces to

$$\ell((MN-1),M,N) = \frac{MN}{MN-1} \; (AB/MN)^{1/2} \left[\frac{1}{3MN}(M^2+N^2-2)\right.$$

$$\left. + \sum_{i=1}^{M-1} \sum_{j=1}^{N-1} (i^2+j^2)^{1/2} \, \frac{4(M-i)(N-j)}{M^2N^2}\right] . \tag{37}$$

c. Summary of Solutions for $\ell(I,M,N)$. In this subsection (II,2), we have obtained the average link length for an M by N

node grid network for various incidence degrees. We assume intermediate incidence degrees are adequately approximated by a log-polynomial interpolation. After trying several interpolation schemes, we finally settled on

$$\ell(I,M,N) = a_0 + a_1[\text{Log } \frac{I}{4}] + a_2[\text{Log } \frac{I}{4}]^2 \qquad (38)$$

$$: I = 4,5, \cdots MN-1.$$

The results are summarized below, and an example plot for M=6, N=9 is given in Figure 8.

$$\ell(2,M,N) = [AB/MN]^{1/2} \qquad (39)$$

$$\ell(4,M,N) = [AB/MN]^{1/2} \qquad (40)$$

$$\ell(8,M,N) = [AB/MN]^{1/2} \left[\frac{1+\sqrt{2}}{2}\right] \qquad (41)$$

or

$$\ell(MN-1),M,N) = \frac{MN}{MN-1}[AB/MN]^{1/2}\left[\frac{1}{3MN}(M^2+N^2-2) + \sum_{i=1}^{M-1}\sum_{j=1}^{N-1}(i^2+j^2)^{1/2}\frac{4(M-i)(N-j)}{M^2N^2}\right]. \qquad (42)$$

3. PUTTING IT TOGETHER

The reader has undoubtedly noticed by now that a key parameter in the subissues of our core problem is the node incidence degree, I. This observation suggests an algorithm which computes network GOS as a function of I. Suppose, then, that we are given:

A : width, in miles, of the rectangular area to be serviced

B : length, in miles, of the rectangular area to be serviced

D : the amount to be spent on the network

E : network offered traffic in erlangs

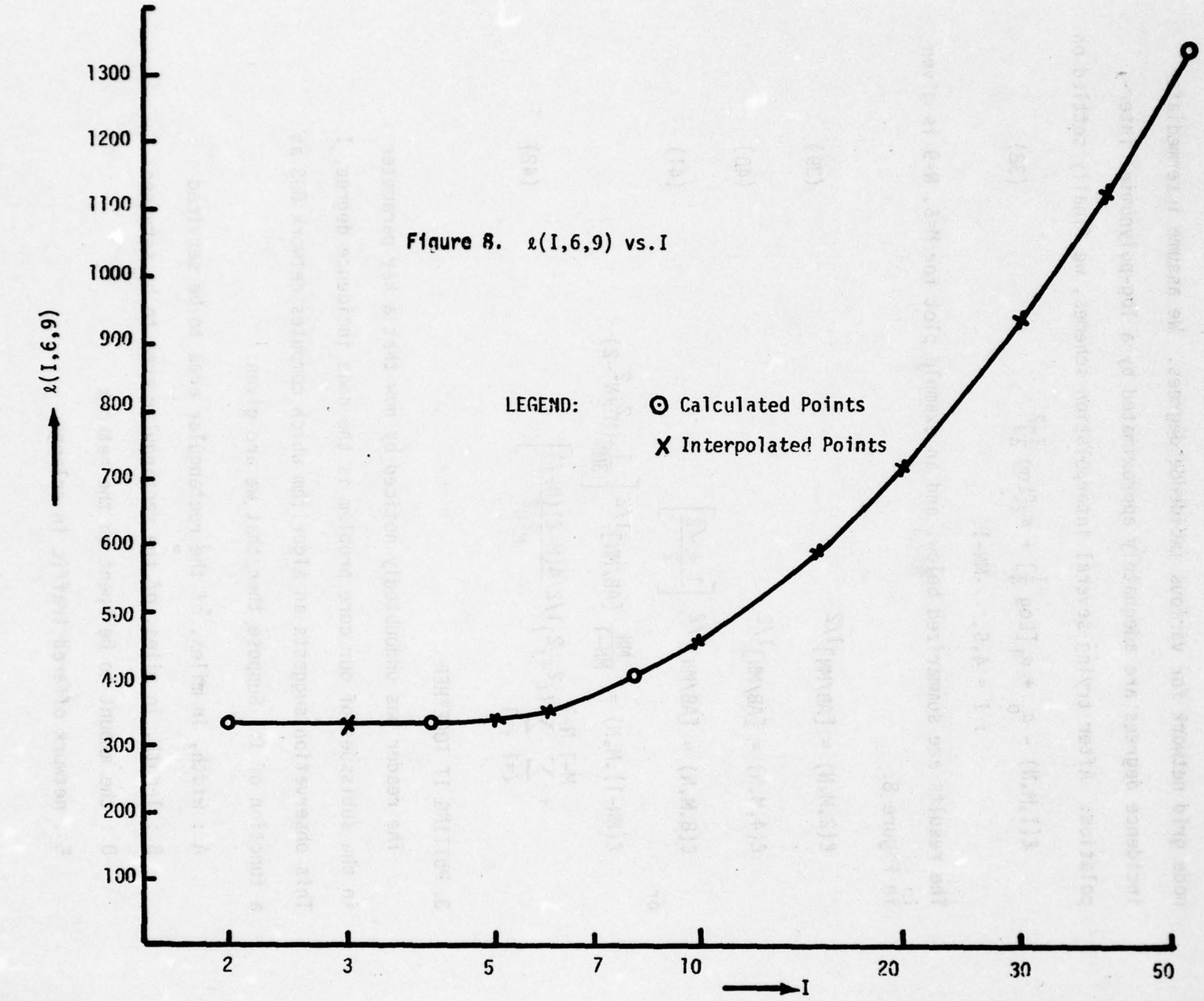


Figure 8. ℓ(I,6,9) vs.I

M : number of nodes along the A dimension of the rectangular area

N : number of nodes along the B dimension of the rectangular area

I : the average node incidence degree

k : cost per channel-mile for links.

Now we can compute, from (1)

$$T = MNI/2 \tag{43}$$

: total number of links in the network.

Next we can find, from (43), (29), and (38) through (42)

$$c = D/Tk\ell$$

$$= 2D/MNIk\ell \tag{44}$$

: the capacity of the average link.

Note that $\ell = \ell(I,M,N)$ is found by the procedure of section II, 2-3. By our assumptions of uniform traffic distribution and network symmetry we can focus our attention on a "typical" node in the network. This node is characterized by its average incidence degree, I, the average link capacity, c, and the originating traffic at that node, E_0; i.e.,

$$E_0 = E/MN \ . \tag{45}$$

Suppose, for the moment, that network GOS, G, is some function of c,I,E_0, and L(I,M,N); i.e.,

$$G = g(c,I,E_0,L) \tag{46}$$

where L = L(I,M,N) is given by the procedure of section II, 1,e. Then our core problem is solved by the algorithm of Figure 9.

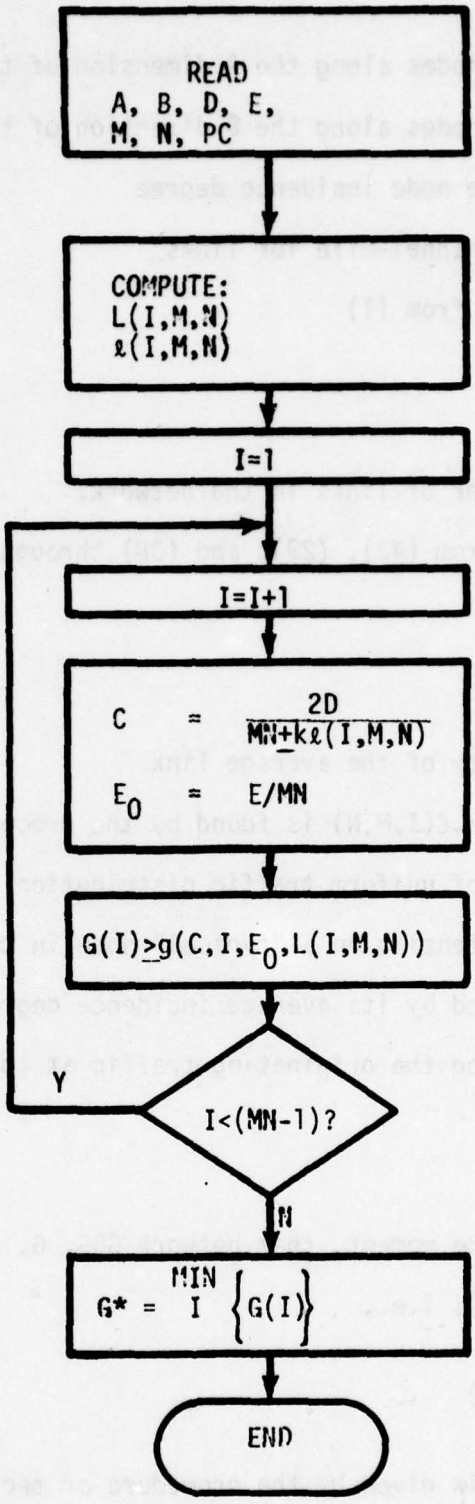


Figure 9. Algorithm for the Core Problem

a. Solving for G. By network GOS, G, we mean the probability that the average call is blocked by the network. Thus an equivalent measure is network call completion rate, R, which is the probability that the average call is completed:

$$R = 1.-G \ . \qquad (47)$$

A number of techniques are available for calculating these measures, in an approximate way, for the more traditional discrete network design problem. None are directly suitable for our purposes, for two reasons:

a. All depend on an assumption of statistical independence of traffic from one link to any other - which is certainly not a valid assumption.

b. All take a "microscopic" view of the discrete network structure; whereas our approach is "macroscopic" in nature and deals with average properties of the network.

Consequently we develop an approach which is felt to be similar in spirit to the Katz algorithm [5], but calculates R from first principles for our core problem.

To begin with we need to observe that a node operates on three identifiable categories of traffic:

E_0 : originating traffic, in erlangs.
This is calling traffic originating in the access area services by the node.

E_D : destination traffic, in erlangs.

This is traffic which originated elsewhere in the network and is destined for users in the access area serviced by the node.

E_T : tandem traffic, in erlangs.

This is traffic which originated elsewhere in the network and is destined for users not in the access area serviced by the node.

From these definitions and assumptions A1 and A2 it follows that

$$R = E_D/E_O \quad . \tag{48}$$

We need to find E_D.

E_D must comprise traffic entering our "typical" node over links incident on that node. However, not all of the traffic entering a node belongs to E_D. In fact we need a new parameter to describe all traffic entering a node via backbone links. Call this parameter E_T'. By the symmetry (average properties) of the network in question, as much traffic is flowing out of the node onto the links as is flowing into the node from the links.

E_T' : the sum of all traffic, in erlangs, entering a node from the links incident on the node. Conversely, it is also the sum of all traffic, in erlangs, leaving a node and entering links incident on the node.

Again, by symmetry, we see that this parcel of traffic, E_T' is divided equally amongst all links incident on the node. Thus each link carries E_{LC} erlangs where

$$E_{LC} = E_T'/I \quad . \tag{49}$$

How much traffic is offered to each link? Call this quantity E_{LO}. Then, by symmetry

$$E_{LO} = (E_O+E_T)/I \ . \tag{50}$$

Now, to a first approximation, E_{LO} and E_{LC} are related by the Erlang B equation

$$E_{LC} = E_{LO}(1-q) \tag{51}$$

$$q = E_B(E_{LO},c) \tag{52}$$

and $E_B(\cdot,\cdot)$ denotes the Erlang B equation.

We need one more observation; this concerns the origin of E_D. E_D, by the definitions, must be some fractional part of E_T'. The remaining fractional part of E_T' must be E_T. The preceding discussion is summarized in the block diagram of Figure 10.

We shall write an equation for solving Figure 10 presently; but first we need to determine the fractional relationship between E_T' and E_D. At this point L(I,M,N) becomes useful. Suppose, for the moment that L(I,M,N) represented the true average number of tandem links per call, rather than that average over the shortest route. Now if we randomly select a traffic parcel on a link, we need to determine the probability that that parcel exits the network (becomes part of E_D) at the node being fed by the link. This line of thinking shows that

$$E_D = E_T'/L(I,M,N) \ . \tag{53}$$

From (48) and (53) we have

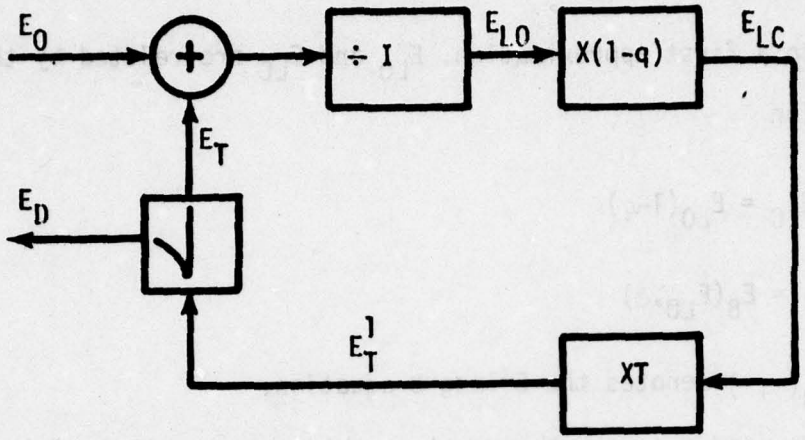


Figure 10. Block Diagram for Computing R

$$R = E_T'/E_0\ L(I,M,N) \tag{54}$$

or, by using (45) in (54)

$$R = E_T'\ MN/E\ L(I,M,N) \tag{55}$$

E_T' is found from Figure 10

$$E_T' = I\ E_C = I\ E_{LO}\ (1-q)\ . \tag{56}$$

However, from (50), we can rewrite (56) as

$$E_T' = (E_0+E_T)(1-q) \tag{57}$$

and, from Figure 10 and (53)

$$E_T = E_T'-E_D = E_T'\left(1- \frac{1}{L(I,M,N)}\right)\ . \tag{58}$$

Therefore

$$E_T' = \left[E_0+E_T'\left(1- \frac{1}{L(I,M,N)}\right)\right](1-q) \tag{59}$$

where, by using (52) and (50)

$$q = E_B\left(\frac{E_0+E_T}{I}\ ,\ c\right)\ . \tag{60}$$

Now using (45) and (58)

$$q = E_B\left(\frac{(E/MN) + E_T'\left(1- \frac{1}{L(I,M,N)}\right)}{}\ ,\ c\right) \tag{61}$$

and (59) becomes

$$E_T' = \left[\frac{E}{MN} + E_T'\left(1 - \frac{1}{L(I,M,N)}\right)\right](1-q)\ . \tag{62}$$

Note that (62) is transcendental due to folding in (61). We can solve (62) for E_T', and thereby find R and G via (55) and (47) by using a simple search algorithm which we need not go into here. Suffice it to say that this search algorithm performs the function

$$G(I) = g(c, I, E_0, L(I,M,N)) \qquad (63)$$

in Figure 9. One fine point needs to be mentioned. That is, due to the averaging properties used herein, c is not necessarily integer in (61). Therefore, we use a logarithmic interpolation technique suggested in [7], for finding $E_B(\rho,c)$ when c is not integer.

b. Some First Results on the Core Problem. We must still address our earlier assumption that L(I,M,N) represents the true average number of tandem links per call. This is done in the next subsection. First, however, we use that assumption to show the results of a problem solved with the algorithm of Figure 9.

The problem parameters are:

M=6 N=9

A = 2000 miles, B = 3000 miles

D = \$2,000,000 E = 4800 erlangs

k = \$0.50 per channel mile.

The results are shown in Figure 11.

With this graph one can pick out I for G*, the minimum GOS, and from this value of I, I*, L(I*,M,N), and ℓ(I*,M,N) can be determined. The average link capacity, c, can be computed through (1), (28), and Figure 8.

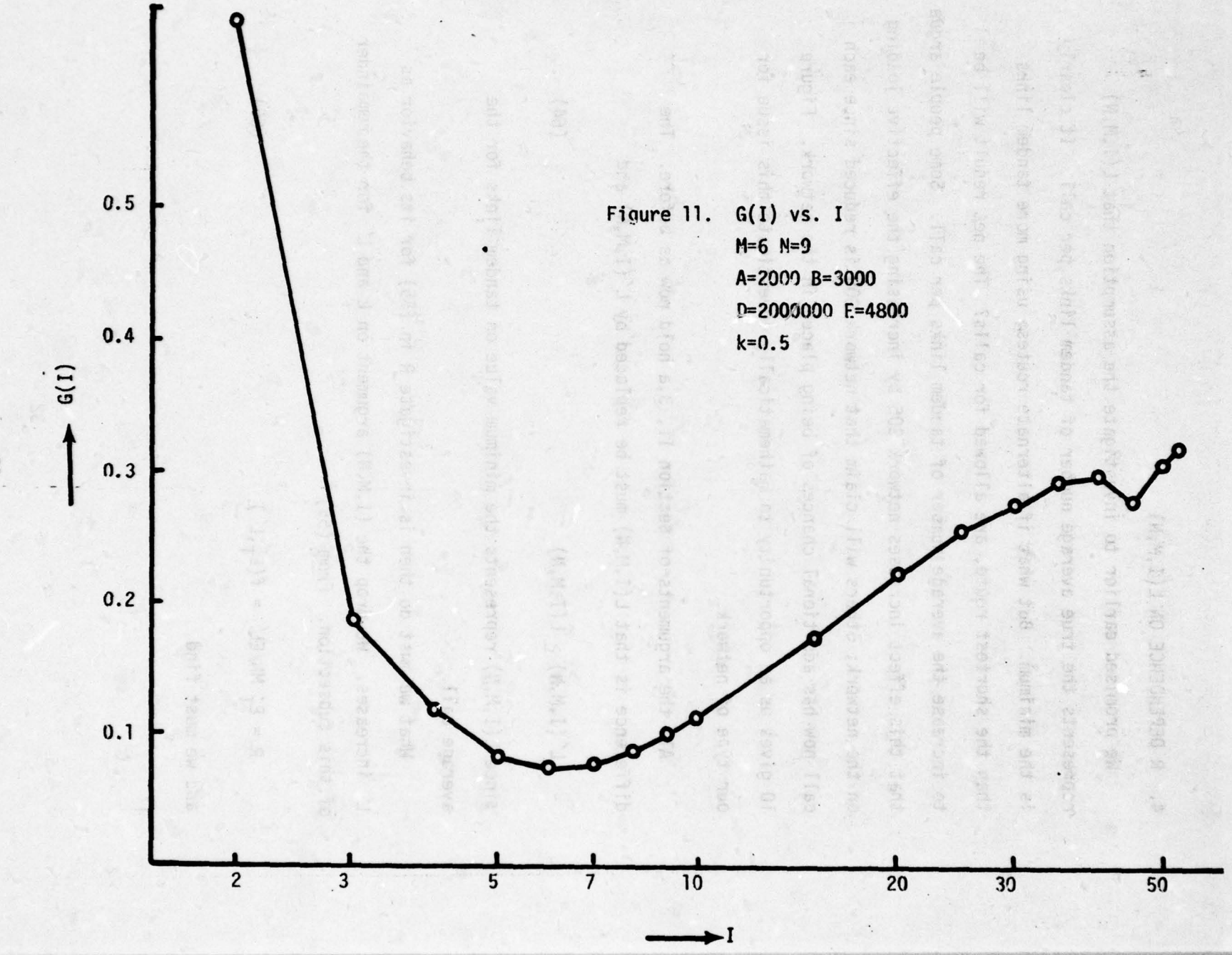


Figure 11. G(I) vs. I
M=6 N=9
A=2000 B=3000
D=2000000 E=4800
k=0.5

4. R DEPENDENCE ON L(I,M,N)

We promised earlier to investigate the assumption that L(I,M,N) represents the true average number of tandem links per call. It clearly is the minimum. But what if alternate routes, using more tandem links than the shortest route, are allowed for calls? The net result will be to increase the average number of tandem links per call. Some people argue that this effect increases network GOS by increasing the effective loading on the network; others will claim that network GOS is reduced since each call now has additional chances of being placed in the network. Figure 10 gives us an opportunity to mathematically investigate this issue for our type of network.

All the arguments of section II, 3,a hold now as before. The difference is that L(I,M,N) must be replaced by L′(I,M,N) and

$$L'(I,M,N) \geq L(I,M,N) \tag{64}$$

since L(I,M,N) represents the minimum value on tandem links for the average call.

What we must do then is investigate R in (55) for its behavior as L′ increases. We drop the (I,M,N) argument on L and L′ for the remainder of this subsection. From (55)

$$R = E_T' \, MN/EL' = f(E_T', L') \tag{65}$$

and we must find

$$\frac{dR}{dL'} = g(E_T', L'). \tag{66}$$

At this point we can observe that if $\frac{dR}{dL'}$ is everwhere negative for $L' > L$, it is established that the model of section II, 3,a is valid as is; and we can conclude that use of shortest path(s) only produces optimum network performance. If this is not the case, then we can find an L*, which is not the same as L, and which produces a maximum network call completion rate.

Equation (66) can be attacked by using the chain rule for differentiation of composite functions [8]

$$\frac{dR}{dL'} = \frac{\partial f(E_T', L')}{\partial E_T'} \frac{dE_T'}{dL'} + \frac{\partial f(E_T', L')}{\partial L'} \tag{67}$$

now

$$\frac{\partial f(E_T', L')}{\partial E_T'} = MN/EL' \tag{68}$$

and

$$\frac{\partial f(E_T', L')}{\partial L'} = -E_T' MN/EL'^2 . \tag{69}$$

Recalling (61) and (62) and using the notion of implicit functions [8]

$$\left[\frac{E}{MN} + E_T'\left(1-\frac{1}{L'}\right)\right]\left[1-E_B\left(\frac{\frac{E}{MN} + E_T'\left(1-\frac{1}{L'}\right)}{I}, c\right)\right] - E_T' = 0 = U = h(E_T', L'), \tag{70}$$

we see that

$$\frac{dE_T'}{dL'} = -\frac{\partial h/\partial L'}{\partial h/\partial E_T'} . \tag{71}$$

Now

$$\frac{\partial h}{\partial L'} = \left[\frac{E_T'}{L'} - E_T' - \frac{E}{MN}\right] \frac{\partial}{\partial L'} E_B\left(\frac{\frac{E}{MN} + E_T'\left(1-\frac{1}{L'}\right)}{I}, c\right)$$

$$+ \frac{E_T'}{L'^2}\left[1 - E_B\left(\frac{\frac{E}{MN} + E_T'\left(1-\frac{1}{L'}\right)}{I}, c\right)\right] \tag{72}$$

and

$$\frac{\partial h}{\partial E_T'} = \left[\frac{E_T'}{L'} - E_T' - \frac{E}{MN}\right] \frac{\partial}{\partial E_T'} E_B\left(\frac{\frac{E}{MN} + E_T'\left(1-\frac{1}{L'}\right)}{I}, c\right)$$

$$+\left(1-\frac{1}{L'}\right)\left[1-E_B\left(\frac{\frac{E}{MN} + E_T'\left(1-\frac{1}{L'}\right)}{I}, c\right)\right] . \tag{73}$$

If we make the following identifications

$$X \equiv \left[\frac{E_T'}{L'} - E_T' - \frac{E}{MN}\right] \tag{74}$$

$$S_1 \equiv \frac{\partial}{\partial L'} E_B(\cdot, \cdot) \tag{75}$$

$$S_2 \equiv \frac{\partial}{\partial E_T'} E_B(\cdot, \cdot) \tag{76}$$

$$p \equiv 1 - E_B(\cdot, \cdot) \tag{77}$$

Then (71) becomes

$$\frac{dE_T'}{dL'} = -\frac{XS_1 + \frac{E_T'}{L'^2} p}{XS_2 + \left(1-\frac{1}{L'}\right)p} . \tag{78}$$

Using the relation [4]

$$\frac{d}{dx}[f(r)] = \frac{d}{dr}[f(r)] \cdot \frac{dr}{dx} \tag{79}$$

and letting

$$r = \left[\frac{E}{MN} + E_T' \left(1-\frac{1}{L'}\right)\right] / I \tag{80}$$

we see that

$$S_1 = \left[\frac{\partial}{\partial r} E_B(r,c)\right] \cdot \left[E_T'/IL'^2\right] \tag{81}$$

and

$$S_2 = \left[\frac{\partial}{\partial r} E_B(r,c)\right] \cdot \left[\frac{1}{I}\left(1-\frac{1}{L'}\right)\right] . \tag{82}$$

Now if we identify

$$S_3 = \frac{\partial}{\partial r} E_B(r,c) \tag{83}$$

then (78) becomes

$$\frac{dE_T'}{dL'} = -\left[\frac{XS_3\left(\frac{E_T'}{IL'^2}\right) + \frac{E_T'}{L'^2} p}{XS_3\left[\frac{1}{I}\left(1-\frac{1}{L'}\right)\right] + \left(1-\frac{1}{L'}\right) p}\right]$$

or

$$\frac{dE_T'}{dL'} = -\left[\frac{\frac{E_T'}{L'^2}\left[XS_3/I + p\right]}{\left(1-\frac{1}{L'}\right)\left[XS_3/I + p\right]}\right]$$

which is just

$$\frac{dE_T'}{dL'} = - E_T'/L'^2\left(1-\frac{1}{L'}\right) . \tag{84}$$

Combining this with (68) and (69) into (67)

$$\frac{dR}{dL'} = - \frac{E_T' MN}{EL'^2} \left(\frac{L'}{L'-1} \right) \; ; \; L' > L \; . \tag{85}$$

Clearly

$$\frac{dR}{dL'} < 0 \text{ for } L' > L \geq 1 \; . \tag{86}$$

This result proves two things for our network problem:

a. Maximum network call completion rate, R, (or minimum network GOS, G) is obtained when shortest path(s) only are allowed in placing calls.

b. From this it follows that the model of Section II, 3,a for calculating R is valid, for computing optimum R, as is.

III. SWITCHES COST MONEY TOO

The purpose of this section is to augment the core problem of section II by folding in the node cost model given by assumption A5.

1. FOLDING IN THE NODE COST MODEL

By assumption, A5, the cost for a single node is

$$D_N = a + bt \ . \tag{87}$$

Now assume for the moment that M, N, and I are fixed. The link cost model given by A4 is

$$D_L = kc\ell \ . \tag{88}$$

Since M, N, and I are fixed, the total number of links is

$$T = MNI/2 \ . \tag{89}$$

From these we see that the amount spent on all nodes, D_{TN}, is

$$D_{TN} = [a + bIc]MN \tag{90}$$

and the amount spent on all links, D_{TL}, is

$$D_{TL} = Tkc\ell = MNIkc\ell/2 \ . \tag{91}$$

The total amount to be spent, D, is allocated between D_{TN} and D_{TL} such that

$$D = D_{TN} + D_{TL} \tag{92}$$

Substituting (90) and (91) into (92) and solving for c

$$c = [D - aMN]/[bMNI + kMNI\ell/2] \tag{93}$$

where $\ell = \ell(I,M,N)$. Using (93) instead of (44) in the algorithm of Figure 9 accounts for node costs.

2. ALLOWING M AND N TO VARY

Allowing M and N to vary in the modified (by (93)) algorithm of Figure 9 requires only the addition of an outer loop to step through various values for M and N. Two observations are worth making:

a. We required early on that the nodes be arranged in a square grid. In the spirit of this it is necessary to insure that

$$\frac{M}{A} \approx \frac{N}{B} \quad . \tag{94}$$

b. Changing the number of nodes in the complete A x B area does more than change the backbone network behavior. It also impacts: (1) the number of access areas (one per node), and (2) the cost of "wiring" the users in the access area to the node connecting them into the backbone. For this report we ignore the second issue. It can (and will) be incorporated into our model at a later date.

3. SOME RESULTS

Figure 12 shows the results of a run using the augmented algorithm of Figure 9. Two observations can be made from this run:

a. Network performance is steadily improved as the number of nodes

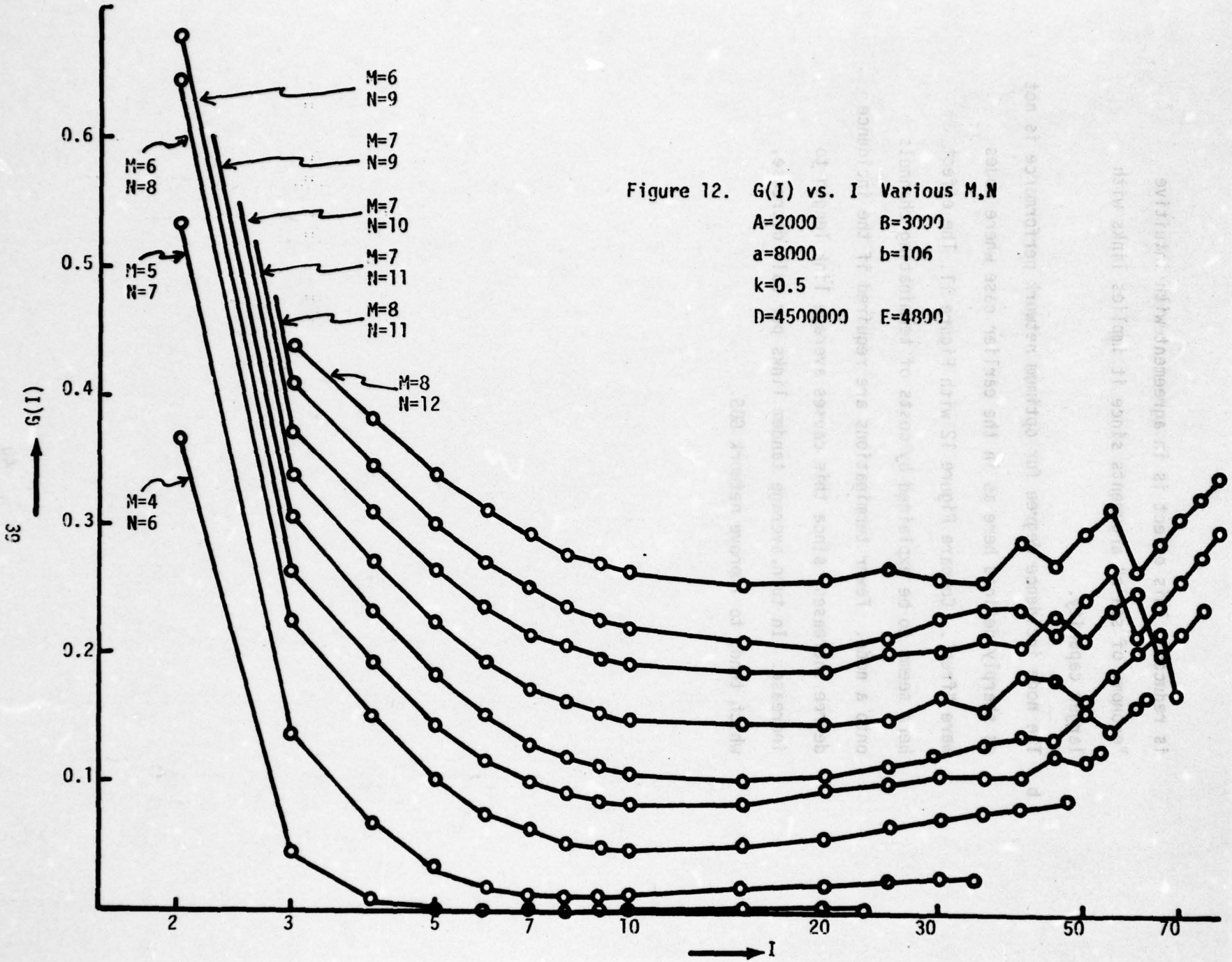


Figure 12. G(I) vs. I Various M,N

A=2000 B=3000

a=8000 b=106

k=0.5

D=4500000 E=4800

is reduced. This effect is in agreement with intuitive "economy of scale" arguments since it implies links with larger capacity.

b. The node incidence degree for optimum network performance is not as sharply defined here as in the earlier case where nodes were "free". Compare Figure 12 with Figure 11. The effect here seems to be explained by costs of terminating channels onto a node. Fewer terminations are required if the incidence degree increases, since this causes average link length to increase. In turn, average tandem links per call decrease, which tends to improve network GOS.

IV. SURVIVABLE TOPOLOGIES

The preceding sections have dealt with the problem of minimizing network GOS under constrained costs. It was assumed that no attacks occurred on the network. Thus the network(s) found were efficient under benign conditions. When designing the DCS it is also necessary to characterize the network(s) under attack conditions.

1. THE PERFORMANCE CHARACTERISTIC

Reference [1] develops the idea of "Performance Characteristic" in detail. Here we present only the essential features. Let a family of network designs be characterized by (M,N). Now let the most efficient design in that family, under benign conditions, be characterized by (I*,M,N); where I* is that node incidence degree which minimizes network GOS. The algorithm of Figure 9 can be used to find (I*,M,N) and produce G(I*,M,N) for that design. We define the utility under benign conditions, U_b, as

$$U_b = E(1-G(I^*,M,N)) \quad . \tag{95}$$

With that same design (I*,M,N), we associate a quantity called the utility after an attack, U_a. U_a is found by removing from that design a specified number of nodes, W, and calculating the "throughput" of the resulting network. That is

$$U_a = \frac{E}{MN}(M'N') \; (1-G(I^{*'},M',N')) \tag{96}$$

where

$$M'N' = MN-W \quad . \tag{97}$$

The following assumptions are implicit in (96):

A8. When a node is removed from the network, the traffic in the access area serviced by that node is lost.

A9. No cuts occur in the network.

A10. The post-attack network still has, on the average, a square grid structure characterized by M', N', and $I^{*'}$, where $M'N'$ is given by (97) and $I^{*'}$ is developed as shown below.

The pre-attack network has a total number of links, T, given by

$$T = MNI^*/2 \quad . \tag{98}$$

Since W nodes are removed to develop the post-attack networks, and each node has, on the average, I^* links incident on it; the total number of usable links, T', in the post-attack network is

$$T' = (MNI^*/2) - WI^* \quad . \tag{99}$$

There are $(MN-W) = M'N'$ nodes in the post-attack network. Thus its average incidence degree, $I^{*'}$, is

$$I^{*'} = (MNI^*-2WI^*)/(MN-W) \quad . \tag{100}$$

The implicit assumption is

A11. Nodes removed in the attack are not adjacent to each other.

The preceding conditions allows use of the model of section II, 3,a for

finding $G(I^{*\prime},M^{\prime},N^{\prime})$. The necessary relationships are

a. Redefining the post-attack network offered load, E , as

$$E = E(MN-W)/MN \qquad (101)$$

in accordance with assumption A8.

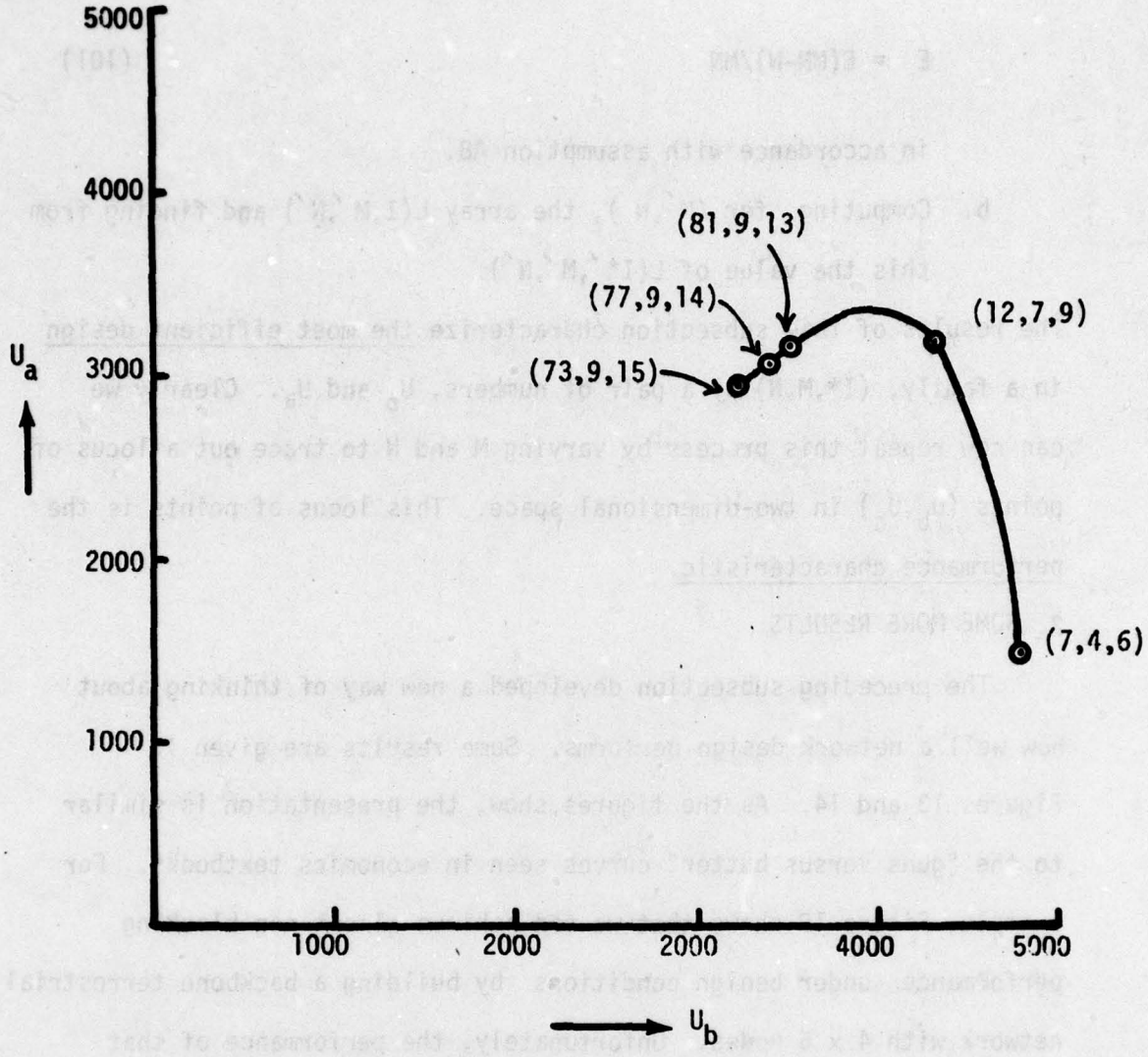
b. Computing, for $(M^{\prime},N^{\prime})$, the array $L(I,M^{\prime},N^{\prime})$ and finding from this the value of $L(I^{*\prime},M^{\prime},N^{\prime})$.

The results of this subsection characterize the most efficient design in a family, (I*,M,N) by a pair of numbers, U_b and U_a. Clearly we can now repeat this process by varying M and N to trace out a locus of points (U_b,U_a) in two-dimensional space. This locus of points is the performance characteristic.

2. SOME MORE RESULTS

The preceding subsection developed a new way of thinking about how well a network design performs. Some results are given in Figures 13 and 14. As the figures show, the presentation is similar to the "guns versus butter" curves seen in economics textbooks. For example, Figure 13 shows that we can achieve almost non-blocking performance under benign conditions by building a backbone terrestrial network with 4 x 6 nodes. Unfortunately, the performance of that network after an attack which removes nine nodes is very poor.

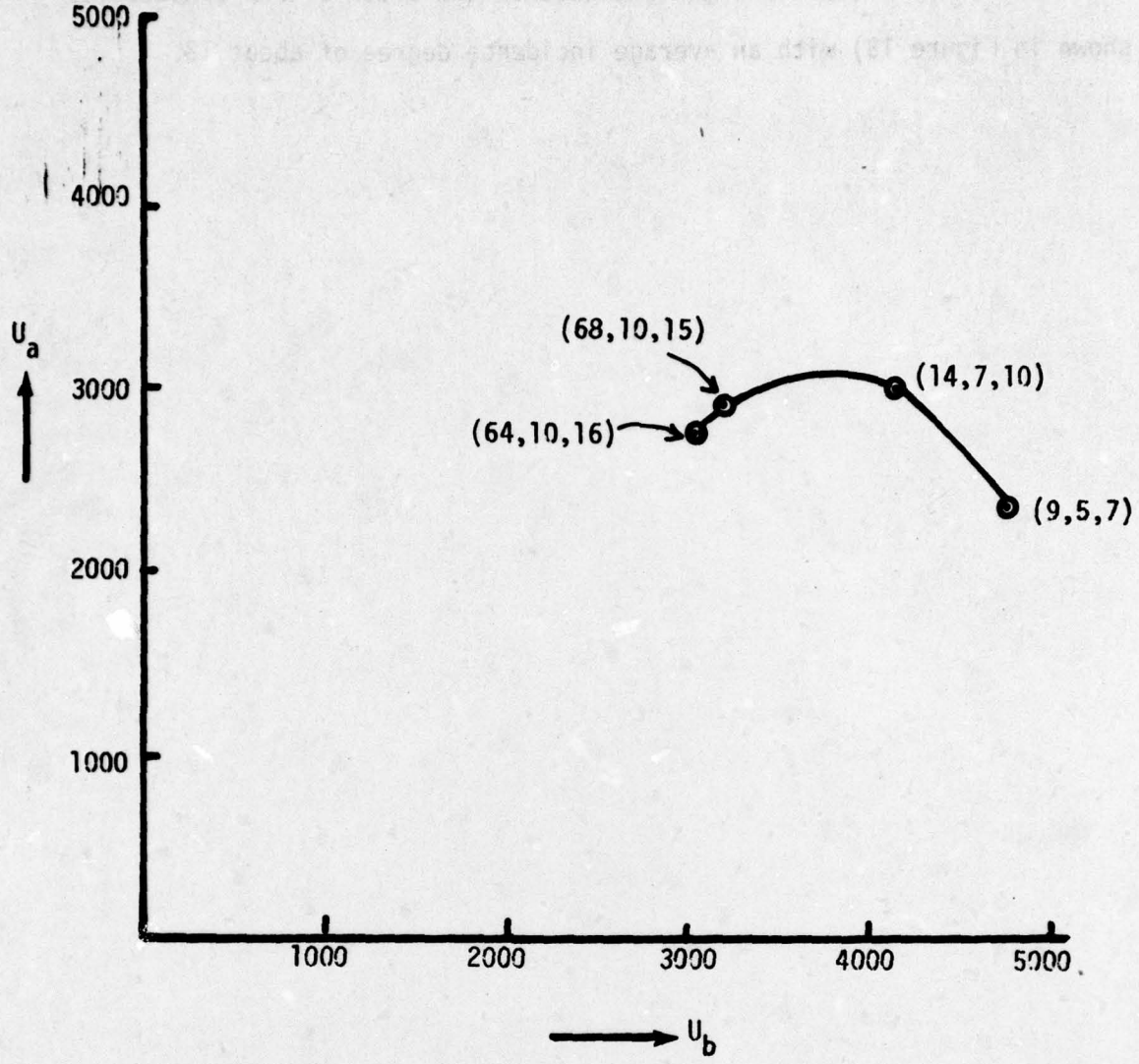
The presentation format allows an additional consideration - survivability - to be folded into network design from the first stages of design. That is, we can specify in advance a minimum acceptable post-attack throughput for the contemplated design. Thus, if the



LEGEND: (•,•,•) = (I*,M,N)

DATA: A=2000 B=3000 D=4500000 E=4800
a=8000 b=105 k=0.5

Figure 13. Performance Characteristic for a Grid Network; W=9



LEGEND: (•,•,•) = (I*,M,N)

DATA: A=2000 B=3000 D=4500000 E=4800
a=8000 b=106 k=0.5

Figure 14. Performance Characteristic for a Grid Network; W=10

requirement was, say, 3000 erlangs throughput after an attack removing nine nodes, the network design must have on the order of 7 x 9 nodes (as shown in Figure 13) with an average incidence degree of about 12.

V. CONCLUSIONS

Approximation techniques developed in this report permit analyzing average structural properties of terrestrial backbone networks. In addition, a new way of measuring the performance of a network design allows survivability-capacity tradeoffs to be made early in the design process where such decisions are most crucial.

The concepts introduced in this report can be developed in more detail for implementation in traditional discrete network design algorithms if desired. However, the analysis of average structural properties may be sufficient to guide future DCS planning. In this case serveral extensions to this report are desirable:

a. Access area costs need to be considered in the model.

b. Satellite/terrestrial tradeoffs must be considered.

c. Integration issues must be addressed.

d. Various switching possibilities (circuit, packet, SENET [9]), must be analyzed.

These issues represent not only a prospectus for future work, they also indicate the range of possible applications of models using the techniques developed herein.

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