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Effects of Structural Non-Linearities on Aircraft Vibration and Flutter

by

E.Breitbach

NORTH ATLANTIC TREATY ORGANIZATION

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EFFECTS OF STRUCTURAL NON-LINEARITIES ON AIRCRAFT VIBRATION AND FLUTTER,

by

(10) E. Breitbach

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PREFACE

Work on flutter prediction has always been based on the modal representation of the structure, which implies a linear model of the aircraft. A number of good results have been obtained, for many prototypes, using this method. Unfortunately, more and more difficulties have appeared during the last few years, both for aircraft carrying large stores and for light aircraft, where non-linear phenomena make it hazardous to use a linear approach.

This pilot paper, given by Dr E.Breitbach to the Sub-Committee on Aeroelasticity of the Structures and Materials Panel, presents a possibility for taking into account some non-linearities of the structure and their effect on flutter. It was the opinion of the Sub-Committee that it should be published by AGARD as an important milestone in the way of accurate flutter prediction in complicated conditions.

G.COUPRY
Chairman, Sub-Committee
on Aeroelasticity

EFFECTS OF STRUCTURAL NON-LINEARITIES ON AIRCRAFT VIBRATION AND FLUTTER

by

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SUMMARY

Experience has shown that aircraft structures are generally affected by structural non-linearities. The purpose of this paper is to find out the physical sources of the various types of non-linearities, to investigate their influence especially on the different parts of the flutter clearance process and to deal with those methods which permit quantitative solutions of non-linear aeroelastic problems.

NOTATION

t	time
P	structure point
f	frequency
f_r	normal frequency
h	normal deflection of the quarter-chord point of a wing section
α	rotation of a wing section
β	aileron rotation
$\beta_a, \beta_0, \beta_1$	characteristic angles of a non-linear force deflection diagram
β_k	critical angle of limited amplitude flutter
u	deflection
u_r	normal mode deflection
C	linear stiffness of an aileron hinge
C_1, C_2	characteristic stiffnesses of a non-linear force deflection diagram
F	force or moment, $\overline{F} = F/C$
M_r	generalized mass
K_r	generalized stiffness
D_{rs}	generalized damping
Q_r	generalized external force
γ_r	damping loss angle
δ_r	logarithmic decrement $= \gamma_r \pi$
q_r	generalized coordinate
ΔU_l	linear stiffness energy

ΔU_{nl} non-linear stiffness energy

Subscripts

r, s integers denoting the mode number

1. INTRODUCTION

As is well known, aeroelastic investigations are usually performed on the basis of the simplified assumption of structural linearity. However, it becomes more and more evident that many dynamics and flutter problems can only be solved satisfactorily by taking into account structural non-linearities.

Therefore, primarily a survey is made of the various types of non-linear effects which may arise on aircraft structures. Beyond that, it is attempted to understand their physical origin and to classify them as to their influence on ground and flight test methods as well as on the analytical flutter prediction. Emphasis is placed on methods which are suitable for treating non-linear vibration and flutter problems with special regard to systems with several degrees of freedom formulated in terms of calculated or measured modal characteristics.

The applicability of such methods is illustrated by a number of experimental and analytical results.

2. SURVEY OF RELEVANT STRUCTURAL NON-LINEARITIES ON AIRCRAFT

Non-linearities as arising in aircraft structures can generally be subdivided into distributed non-linearities which are continuously activated throughout the whole structure by elasto-dynamic deformations and into concentrated ones which act locally lumped especially in control mechanisms or in the connecting parts between wing and external stores. Some characteristic types of these two classes of non-linearities are described and discussed in the following sections.

2.1 CONCENTRATED NON-LINEARITIES

The control mechanisms of hand operated aircraft are affected by different types of strong concentrated non-linearities resulting from

- back-lash in the linkage elements of the control system
- solid friction in control cable and push rod ducts as well as in the hinge bearings
- kinematic limitation of the control surface deflection
- application of spring tab systems provided for relieving pilot operation.

These effects generally occurring in combined forms can best be demonstrated by means of results measured on real aircraft structures. Fig. 1 shows the aileron hinge moment of a glider versus hinge angle in case of static moments symmetrically acting in the sense opposite to the regular operation of the aileron system. The result represents the simplest form of a low damped back-lash. The antisymmetrical force deflection curve resulting from static loading in the regular operation sense of the aileron system is given in Fig. 2. It is a hysteresis type diagram with an elastic and nearly frictionless slope for forces below the critical slip-stick point where static friction changes to sliding friction. The latter causes an hysteresis increasing up to the maximum aileron deflection, beyond which the force deflection curve is characterized by nearly frictionless elastic deformation. Resulting from this non-linearity the resonance frequency of the antisymmetrical aileron vibration decreases from about 7 Hz at low amplitudes up to a minimum of 2.5 Hz at about 15 degrees, s. Fig. 3. The resonance frequency increases again along the dashed line for amplitudes exceeding the maximum aileron stroke. The force deflection diagram of a glider rudder in Fig. 4 is almost exclusively characterized by solid friction whereas elastic deformations are negligible.

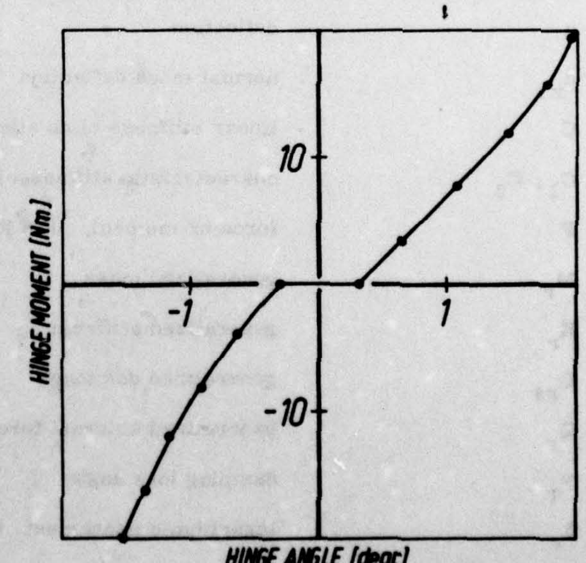

Fig. 1: Aileron hinge moment of a glider versus hinge angle, symmetrical loading

Quite similar non-linear effects due to back-lash and solid friction can be observed on aircraft

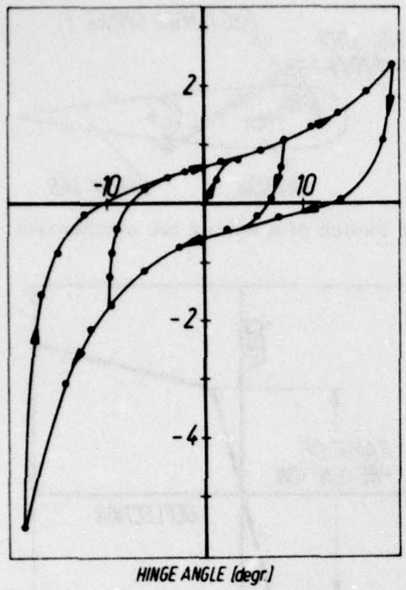

Fig. 2: Aileron hinge moment of a glider versus hinge angle, antisymmetrical loading

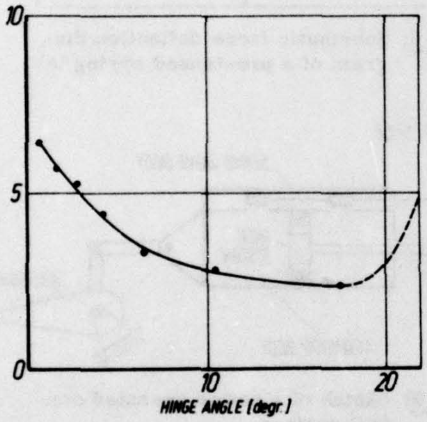

Fig. 3: Resonance frequency of the antisymmetrical aileron vibration versus hinge angle

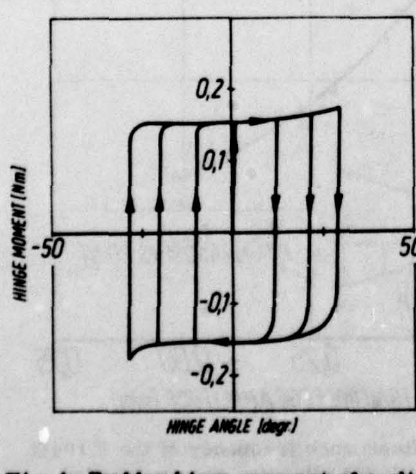

Fig. 4: Rudder hinge moment of a glider versus hinge angle

with external wing stores. In [1] and [2] special emphasis is placed on the investigation of the single point suspended under wing stores of a combat aircraft with sweepable wing. Main purpose of these studies is to find out the limits of linearisation and their possible consequences to test and analysis.

Another type of a concentrated non-linearity is occurring in spring tab systems deliberately introduced into the control mechanism in order to relieve pilot operation. Both the mechanism of a spring tab system and the appertaining force deflection curve are sketched in Fig. 5 and Fig. 6 respectively, assuming the hinge stiffness F_C of the main control surface to be zero. The stiffness of the spring tab system is relatively high in a rather small range around the origin of the diagram resulting from a special pre-tension of the tab spring F_T. At a certain tab deflection the pre-tension ceases and the stiffness suddenly drops to a much lower value. The force deflection diagram of the complete control mechanism is a combination of the tab spring F_T and the stiffness F_C between control stick and control surface which is generally characterized by back-lash and solid friction. As outlined in one of the following sections, such spring tab systems have already proved to be very susceptible to a dangerous kind of divergent flutter.

Power operated controls as schematically shown in Fig. 7 are also affected with non-linear effects. This might be illustrated for instance by the resonance frequency of the F 104G aileron system plotted in Fig. 8 as a function of the aileron deflection, [3]. Generally, it can be assumed that the dynamic behaviour of power control mechanisms is characterized by a combination of different sorts of non-linear effects such as outlined in the following:

- As pointed out in [3] and [4] the complex highly damped stiffness of the hydraulic actuator itself is a non-linear function of the vibration amplitude as well as of the actuator preload and of the position of the jack piston.
- The linkage mechanism between the actuator body and the aircraft structure is affected by considerable non-linear effects due to solid friction and back-lash especially in the hinge bearings.
- In case that vibratory motions of the aircraft structure result in relative motions between piston and cylinder of the servo valve, actuator forces are induced which also vary with the vibration amplitude. This kind of dynamic interaction can lead to considerable stability problems.

In addition to all that, the dynamic stiffness of hydraulic actuators is usually a frequency dependent function to be taken into account as a very important part of the flutter clearance process, see [5].

2.2 DISTRIBUTED NON-LINEARITIES

The vibration behaviour of aircraft structures is influenced not only by the concentrated non-linear effects described above but also by the so-called distributed ones which are induced by elastic deformations in riveted, screwed and bolted connections as well as within the structural components themselves. Because of the great number of rivets, screws and bolts it can be assumed that the resulting damping and stiffness non-linearities are more or less continuously distributed throughout the structure. In consequence of this special property, the effect of these distributed non-linearities on the normal mode shapes can mostly be regarded as negligible.

Experience has shown that the normal frequencies are weakly decreasing functions of the vibration amplitude. Quite contrary the overall damping values can undergo much higher variations against amplitude. Two typical results of a ground vibration test, carried out on the aircraft F 104G, [6], are plotted in Fig. 9.

As found out in special damping measurements on a fiber reinforced composite box spar, [7], structural damping of modern design components depends not only on the vibration amplitude but also on frequency. Fig. 10 shows the results of the fundamental bending normal mode, the normal frequency of which was modified from 235 up to 88 Hz by attaching additional masses. Thus, the stress distribution over the test structure could be kept invariant.

3. INFLUENCE OF STRUCTURAL NON-LINEARITIES ON TEST AND ANALYSIS

The survey above demonstrates that there are a lot of different types of structural non-linearities to be taken into account in ground vibration test as well as in flutter analysis and flight flutter test.

At first sight, the above described distributed non-linearities do not seem to be of great importance for aeroelastic investigations. However, many difficulties as occurring particularly during ground vibration and flight flutter tests can be attributed to these effects.

Thus, it has been shown in [8] that weak distributed stiffness non-linearities in concurrence with equally weak manufacture-conditioned structural asymmetries can be identified as physical sources of the well-known phenomenon of amplitude dependent normal mode asymmetries.

Another very interesting problem concerning the measurement of generalized masses is investigated in [9]. The study comes to the conclusion that measured generalized masses can be considerably affected by measuring errors due to small distributed stiffness non-linearities. Suitable means to overcome this problem are proposed.

As to the influence of distributed damping non-linearities, it is worth mentioning that certain changements of the critical flutter speed resulting from non-linear damping effects can be expected only in cases of mild flutter. However, damping becomes far more important in case of many dynamic response problems because the dynamic amplification of an externally excited structure is inversely proportional to the structural damping.

Regarding the so-called concentrated non-linearities it is self-evident that in most cases simplified linearized approaches must lead to inadmissible errors and misinterpretations. To settle these problems some propositions have been made especially in view of a more sophisticated preparation and execution of the ground vibration test.

According to [10] non-linear dry friction effects on control surfaces of hand controlled aircraft can be largely reduced by the application of an auxiliary periodic high frequency excitation. In this way an aircraft structure can be artificially linearized for frequencies much lower than that of the auxiliary excitation.

If the modal synthesis approach is utilized the detrimental influence of concentrated non-linearities on the accuracy of ground vibration tests can be

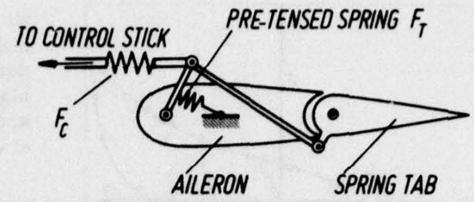

Fig. 5: Sketch of a spring tab mechanism

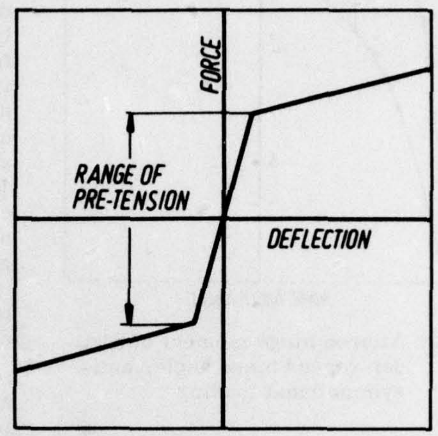

Fig. 6: Schematic force deflection diagram of a pre-tensed spring tab

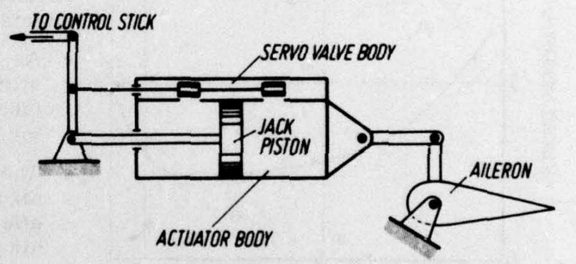

Fig. 7: Sketch of a power operated control surface

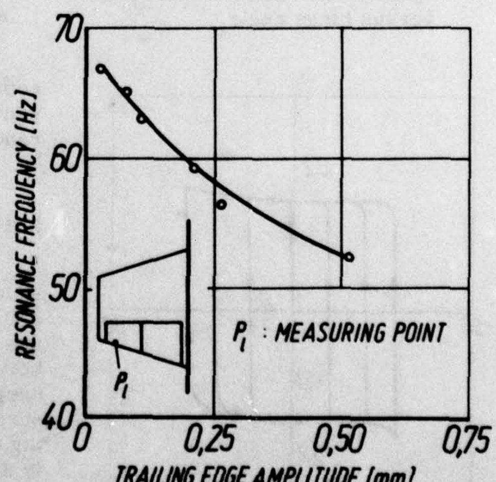

Fig. 8: Resonance frequency of the F 104G aileron vibration versus trailing edge amplitude

eliminated by means of three different procedures, which guarantee the measurement of a set of orthogonal normal modes:

- During the ground vibration test the mechanism of the control surfaces can be linearized by replacing the non-linear elements by linear artificial ones. As shown in [11], the real non-linear effects can be subsequently reintroduced into the flutter analysis by means of modal superposition.
- In accordance with [12] during the test the control surfaces can be fixedly clamped to the adjacent wing or tail structure in such a way that relative deflections are suppressed. This implies the elimination of the control surface degrees of freedom. The real vibration behaviour can be completely described in the subsequent analysis by introducing the control surface degrees of freedom in addition to the measured ones. The non-linear effects of the control mechanisme can be taken into account using an analytical approach quite similar to the one described in [11].
- As described in [13] the aircraft structure can be investigated in the ground vibration test without control surfaces. In this way non-linear effects cannot be activated. The normal modes of the control surfaces are separately measured. The mathematical model of the complete structure can be subsequently established by means of a modal coupling approach taking into account the non-linear properties of the coupling mechanism.

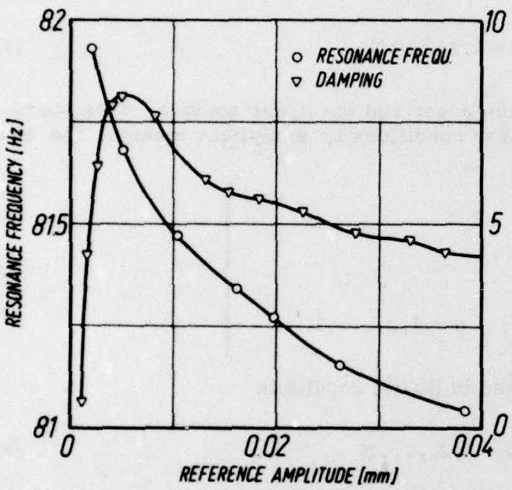

Fig. 9: Structural damping and resonance frequency of a typical F 104G normal mode versus reference amplitude

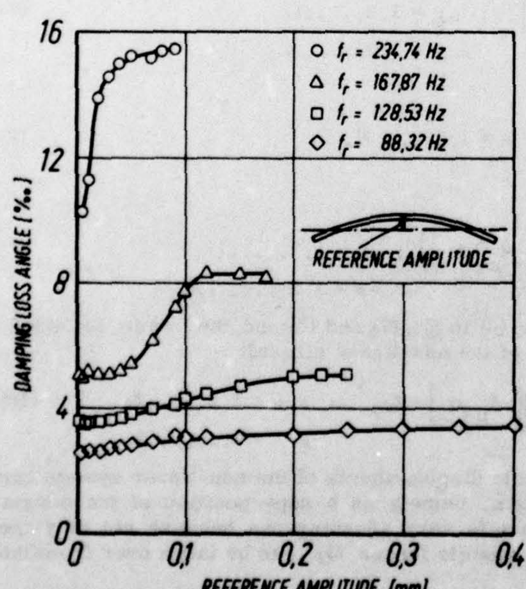

Fig. 10: Structural damping of a fiber reinforced composite box spar as function of vibration amplitude and frequency

Every one of the three procedures quoted above requires the force deflection diagrams of the control surface mechanisms. It has proved to be the most effective and accurate way to determine these properties experimentally.

As to flight flutter test, concentrated non-linearities must be taken into consideration as inherent properties of the test object. Manipulations as applied to ground vibration measurements in order to facilitate them are not possible here.

The methods to interprete and evaluate flight flutter test results reliably can be best improved by means of detailed analytical and wind tunnel investigations on systems with realistic non-linearities.

In the following the governing aeroelastic equations of an aircraft affected by concentrated non-linearities will be formulated in terms of modal characteristics. In order to give an impression of the influence of some typical non-linearities on the flutter stability the results of an analogue computer flutter simulation will be presented. Furthermore, the applicability of a slightly modified numerical approach based on the principle of "harmonic balance" will be checked up by means of the results of a wind tunnel flutter test on a non-linear wing aileron model.

3.1 FORMULATION OF THE NON-LINEAR AEROELASTIC EQUATIONS

Provided that the ground vibration test is carried out on an artificially linearized aircraft structure the governing equations can be written on the basis of the measured modal data as follows

$$M_r \ddot{q}_r + \sum_{(s)} D_{rs} \dot{q}_s + K_r q_r = Q_r \; ; \qquad r, s = 1, 2, \ldots, N \qquad (1)$$

with

M_r	generalized masses
K_r	generalized stiffnesses
D_{rs}	generalized damping coefficients
q_r	generalized coordinates, $\dot{q}_r(t)$, $\ddot{q}_r(t)$ denote the first and second derivation with respect to time

Q_r generalized unsteady aerodynamic forces depending on q_r, $\dot{q}_r$, $\ddot{q}_r$, Mach number and Reynolds number.

The generalized stiffness can be expressed by

$$K_r = 4\pi^2 f_r^2 M_r \tag{2}$$

with the normal frequency f_r of the r-th normal mode. The solution q_r of Eq. (1) can be retransformed into the corresponding geometrical displacements $u(P,t)$, which are defined as a function of time t and of the structure points P, by means of the series expansion of the measured normal mode functions $u_r(P)$

$$u(P,t) = \sum_{(s)} u_s(P)\, q_s(t) \ ; \quad s = 1,2,\ldots,N \ . \tag{3}$$

As mentioned before, in the ground vibration test the non-linear stiffness elements characterized by the force deflection functions $F_\mu(\beta_\mu)$ are replaced by artificial linear stiffnesses C_μ. Subscript μ indicates the control surfaces taken into consideration and β_μ denotes the hinge angle of the μ-th control surface.

The equations of motion of the non-linear system can be established by adding to Eq. (1) the Lagrange term

$$\frac{\partial}{\partial q_r}\left(\Delta U_{nl} - \Delta U_l\right) \ ; \quad r = 1,2,\ldots,N \tag{4}$$

with the stiffness energies ΔU_{nl} and ΔU_l stored in the non-linear and the linear springs. This operation is equivalent to the reestablishment of the real non-linear conditions by analytical means. The energy terms ΔU_l and ΔU_{nl} are defined as follows:

$$\left.\begin{aligned} \Delta U_l &= \frac{1}{2}\sum_{(\mu)} C_\mu \beta_\mu^2 \\ \Delta U_{nl} &= \sum_{(\mu)} \int_0^{\beta_\mu} F_\mu(\beta_\mu)\, d\beta_\mu \ ; \quad \mu = 1,2,\ldots,L \ . \end{aligned}\right\} \tag{5}$$

Applying to β_μ a series expansion similar to that formulated in Eq. (3) results in

$$\beta_\mu = \sum_{(s)} \beta_{\mu s} q_s \ ; \quad s = 1,2,\ldots,N \tag{6}$$

where $\beta_{\mu s}$ stands for the hinge angle of the μ-th control surface as measured in the s-th normal mode of the linearized system.

Insertion of Eq. (6) into Eq. (5) leads to

$$\Delta U_l = \frac{1}{2}\sum_{(\mu)} C_\mu \left[\sum_{(s)} \beta_{\mu s} q_s\right]^2 \ ; \quad \begin{aligned}\mu &= 1,2,\ldots,L \\ s &= 1,2,\ldots,N\end{aligned} \tag{7}$$

and with

$$d\beta_\mu = \sum_{(s)} \beta_{\mu s}\, dq_s \ ; \quad s = 1,2,\ldots,N \tag{8}$$

to

$$\Delta U_{nl} = \sum_{(\mu)} \int_0^{\beta_\mu} F_\mu(\beta_\mu) \left[\sum_{(s)} \beta_{\mu s}\, dq_s\right] ; \quad \begin{aligned}\mu &= 1,2,\ldots,L \\ s &= 1,2,\ldots,N \ .\end{aligned} \tag{9}$$

If the Lagrange operation is applied in accordance with Eq. (4) to Eq. (7) and (9) and the result is added to Eq. (1) this leads to the aeroelastic equations of motion of the non-linear aircraft

$$M_r \ddot{q}_r + \sum_{(s)} D_{rs}\dot{q}_s + K_r q_r + \sum_{(\mu)} \left[\beta_{\mu r} F_\mu(\beta_\mu) - C_\mu \sum_s \beta_{\mu s} q_s\right] = Q_r \ ; \quad \begin{aligned} r,s &= 1,2,\ldots,N \\ \mu &= 1,2,\ldots,L \ .\end{aligned} \tag{10}$$

Eq. (10) is based on the plausible hypothesis that the dynamic displacements of the non-linear system can be formulated in the same way as those of the linear system, namely as a superposition of the normal modes of the linearized test configuration. This formulation is very advantageous because not only the modal data M_r, D_{rs} and K_r but also the unsteady aerodynamic forces Q_r can be taken over from the linear system unchanged.

The equations can be solved

- in the time domain by means of an analogue computer or by numerical step by step integration,
- in the frequency domain by means of iteration procedures.

The derivation of the governing Eqs. (10), especially with regard to the unsteady aerodynamic forces, is dealt with more in detail in [11].

3.2 FLUTTER SIMULATION OF A SIMPLE NON-LINEAR WING AILERON SECTION

In order to investigate the influence of control surface non-linearities on the flutter stability, in [11] a real time flutter simulation by means of an analogue computer was carried out for a simple wing aileron section with a span width of 100 cm. The unsteady aerodynamic forces were determined by means of the incompressible strip theory including Wagner's function for non-uniform motions. As sketched in Fig. 11 the system is characterized by the following degrees of freedom

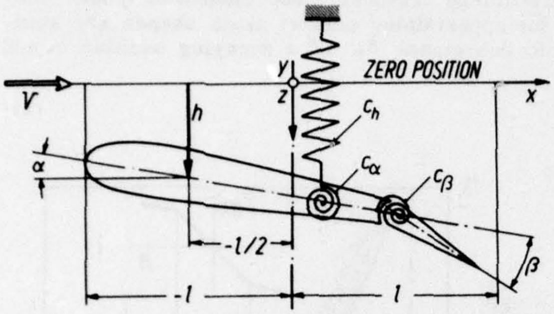

Fig. 11: Sketch of a wing aileron section with *three degrees of freedom*

wing normal deflection	h
wing torsion	α
aileron rotation	β .

The modal characteristics M_r, f_r and the normal mode shapes h_r, α_r, β_r of the "linearized" basic configuration are summarized in Table 1.

The aeroelastic stability curves resulting from three different types of non-linear force deflection characteristics are plottet in Fig. 12 to 14. The stability boundaries are presented in form of the ratio of the critical aileron hinge angle β_k to a reference angle β_a versus flight speed. By way of comparison, the flutter boundary of the linear basic system is demonstrated in form of a vertical line at V ~ 300 km/h.

Normal Mode	f_r [Hz]	M_r [kg cm^2]	h_r [cm]	α_r [rad]	β_r [rad]
r = 1	3.56	10.18	1	$0.66\ 10^{-3}$	$0.50\ 10^{-3}$
2	15.33	26.01	1	-0.039	-0.21
3	18.13	116.72	1	-0.045	1.17

Table 1: Modal characteristics of the "linearized" basic configuration of the wing aileron section

As shown in Fig. 12 non-linear flutter resulting from a back-lash type hinge stiffness leads to a limited amplitude flutter which is not explosive but may cause a long-term process of material fatigue.

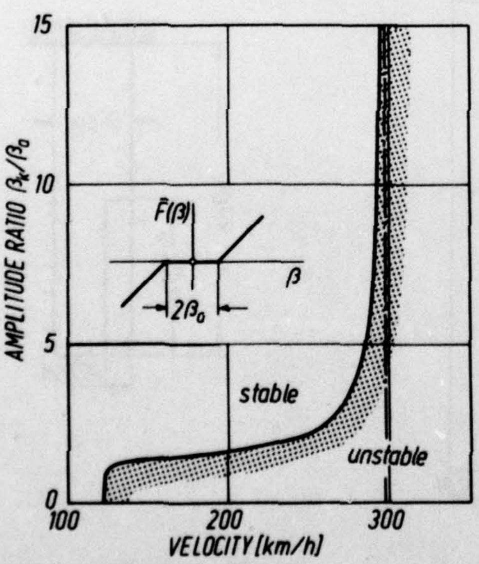

Fig. 12: Effect of back-lash on the flutter stability of a wing aileron section

In case that the force deflection curve is characterized by a combination of back-lash and solid friction the flutter boundary line is shifted to considerably higher speed values, see Fig. 13. This stabilizing effect is characteristic for all hysteresis type hinge stiffnesses.

In contrast to that, flutter resulting from decreasing or softening non-linear springs can be much more dangerous. It can be seen from Fig. 14, that this spring tab type hinge stiffness is very insidious, because flutter does not start until a certain critical amplitude limit is exceeded, for instance in consequence of a gust excitation. Above this critical limit the vibrations are steadily increasing until destruction of the aircraft.

As described in [14] in a qualitative manner for a great number of examples, the flutter behaviour of systems with non-linear control surface mechanisms is highly dependent on the manner the critical flutter speed of the appertaining "linearized" systems varies against changes of the linear hinge stiffness as shown in Fig. 15 for the above investigated wing aileron section. Functions of that kind can help towards a first qualitative asessment of the flutter stability of non-linear systems.

3.3 FLUTTER ANALYSIS AND WIND TUNNEL TESTS ON A NON-LINEAR WING AILERON MODEL

Recently, in the DFVLR/AVA low speed wind tunnel flutter tests were carried out on a half span wing aileron model with a non-linear aileron hinge stiffness. The main dimensions of the model can be seen from Fig. 16. In accordance with the above described procedure the ground vibration test was performed on a basic configuration with a "linearized" aileron hinge stiffness. The measured modal characteristics M_r, f_r, the damping loss angles γ_r and the appertaining normal mode shapes are summarized in Fig. 17. The relation between the logarithmic decrement δ_r of a decaying oscillation and the loss angle γ_r is

$$\gamma_r = \frac{\delta_r}{\pi} \quad . \tag{11}$$

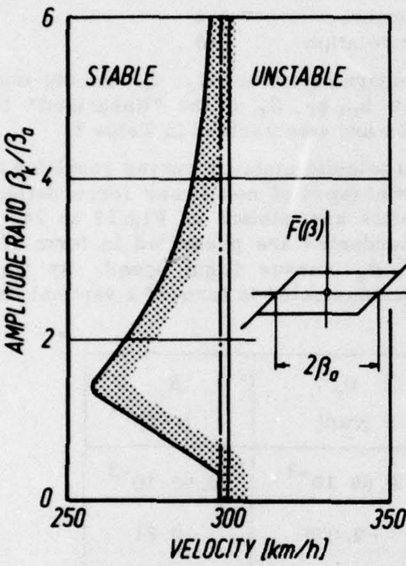

Fig. 13: Effect of hysteresis on the flutter stability of a wing aileron section

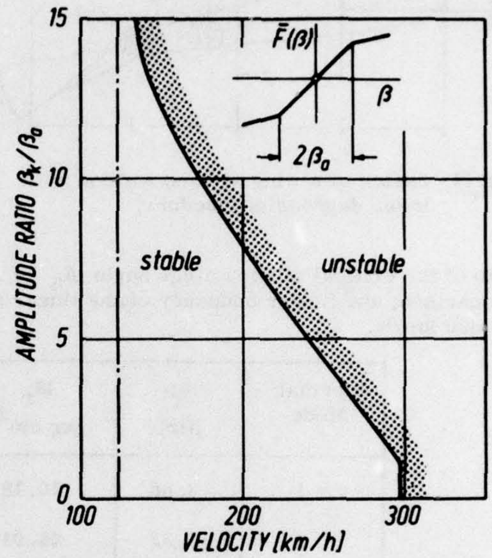

Fig. 14: Effect of a softening spring tab type stiffness on the flutter stability of a wing aileron section

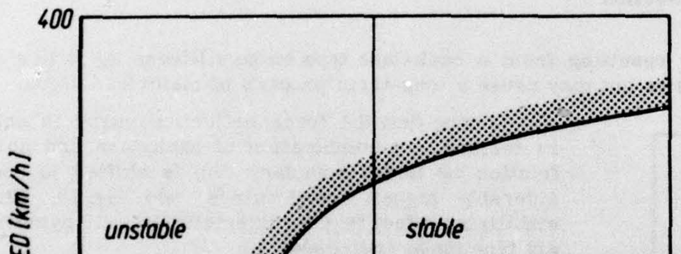

Fig. 15: Flutter speed of the "linearized" wing aileron section as function of changes of the linear hinge stiffness

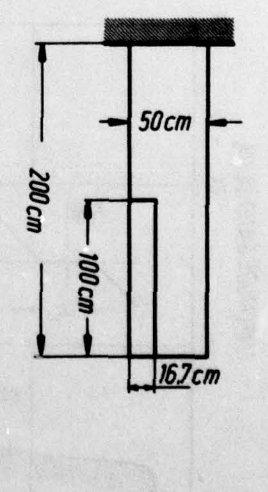

Fig. 16: Main dimensions of wind tunnel test model

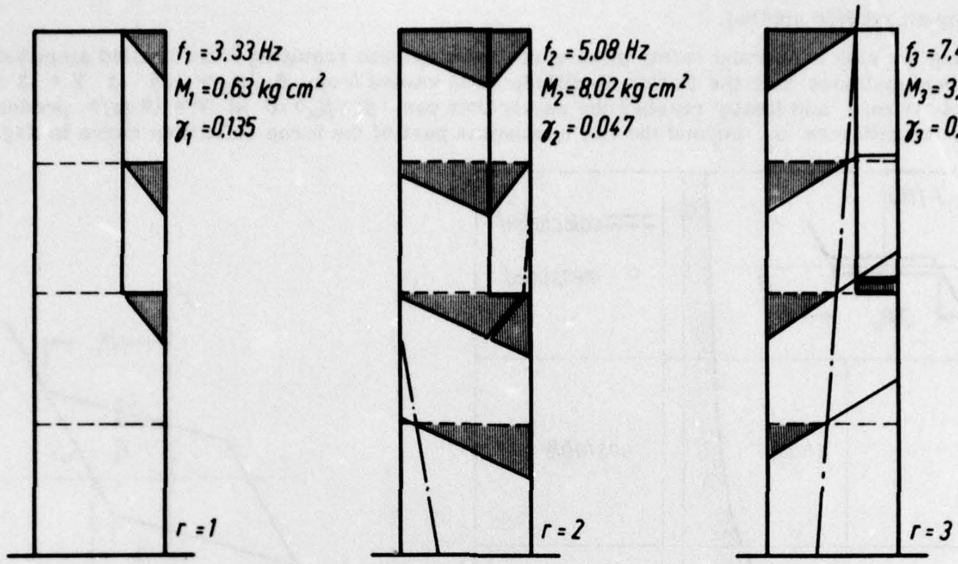

Fig. 17: Normal mode characteristics of a wing aileron wind tunnel test model

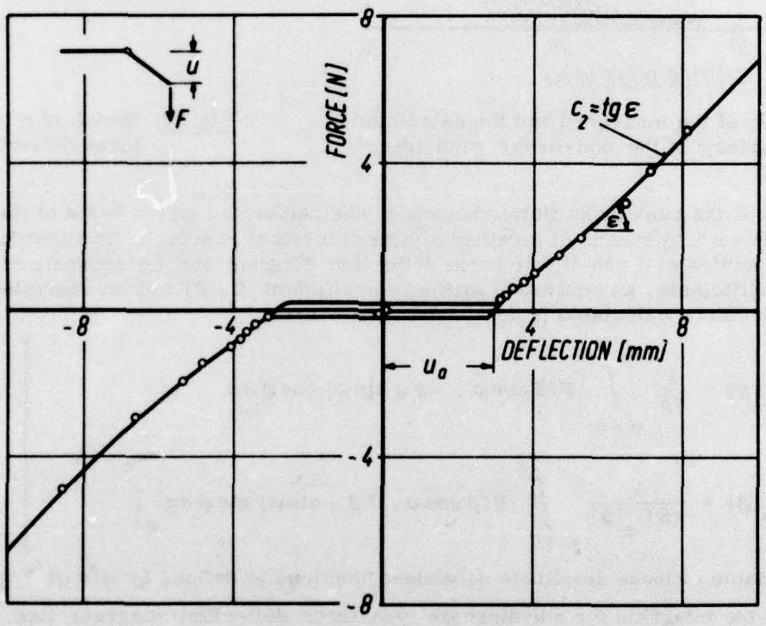

Fig. 18: Non-linear force deflection diagram of the aileron of the wind tunnel test model

The force deflection curve of the wind tunnel test configuration shown in Fig. 18 is characterized by back-lash of ± 3 mm in combination with a small hysteresis.

Fig. 19 shows the flutter boundary of the non-linear wind tunnel test model in comparison with calculated results in form of the critical amplitude ratio β_k / β_a of the aileron as a four valued function of velocity V. The two full lines indicate so-called limited amplitude flutter, whilst the dashed lines describe indifferent states only to be determined by calculation. Thus, along the upper dashed flutter boundary even extremely small external disturbances result in increasing or decreasing vibrations converging against one of the full lines.

According to the strength of external disturbances due to gust or manoevre loads the limited amplitude flutter turns up along the upper or the lower full line.

During the tests it turned out that the damping values were highly dependent on the momentary vibration amplitude so that a reliable damping measurement was impossible. This non-linear effect can be observed in flutter tests on wind tunnel models as well as in flight flutter tests, irrespective of the damp-

ing measurement method applied.

Regarding the risk of life and safety of an aeroelastic system resulting from limited amplitude flutter Fig. 19 demonstrates that the flutter amplitude ratio varies from $\beta_k/\beta_a \sim 2.5$ at $V = 12$ m/s to $\beta_k/\beta_a \sim 5$ at 15 m/s and finally reaches the asymptotic case $\beta_k/\beta_a \to \infty$ at $V = 19$ m/s predominantly defined by the stiffness c_2 beyond the flat hysteresis part of the force deflection curve in Fig. 18.

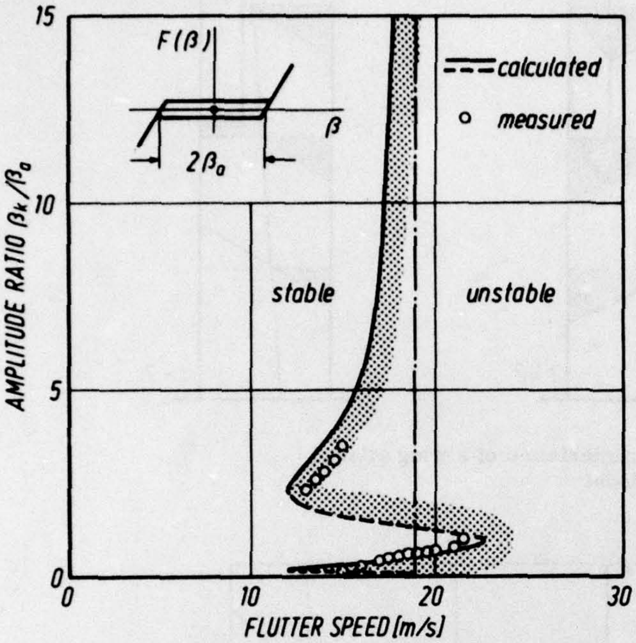

Fig. 19: Comparison of the measured and the calculated flutter boundary of the non-linear wind tunnel test model

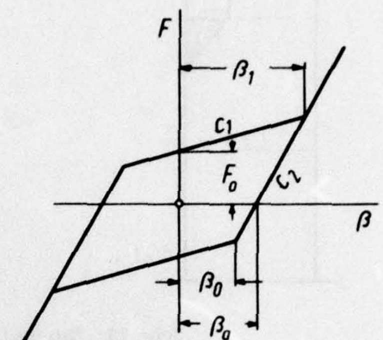

Fig. 20: Sketch of a hysteresis type force deflection diagram

The calculation of the non-linear flutter boundary was performed on the basis of the principle of the "harmonic balance" by means of a rather simple numerical approach. In accordance with [15] the elastomechanic properties of a non-linear force deflection diagram can be approximately described by equivalent linear coefficients, an equivalent stiffness coefficient $C_e(\beta)$ and an equivalent damping loss angle $\gamma_e(\beta)$, which can be calculated from

$$\left.\begin{aligned} C_e(\beta) &= \frac{1}{\pi\beta} \int_{\varphi=0}^{2\pi} F(\beta\cos\varphi\,,\ -\beta\omega\sin\varphi)\cos\varphi\, d\varphi \\ \gamma_e(\beta) &= \frac{1}{\pi\beta\, C_e(\beta)} \int_{\varphi=0}^{2\pi} F(\beta\cos\varphi\,,\ -\beta\omega\sin\varphi)\sin\varphi\, d\varphi\ . \end{aligned}\right\} \qquad (12)$$

The integration variable of these amplitude dependent functions is defined by $\varphi = \omega t$ with $\omega = 2\pi f$.

The solution of the integrals for a hysteresis type force deflection diagram like that sketched in Fig. 20 yields

$$\left.\begin{aligned} C_e(\beta) &= \frac{1}{\pi\beta}\Big[2F_0(\sin\varphi_0 - \cos\varphi_1) - 2\beta_a c_2(\sin\varphi_0 + \cos\varphi_1) + \\ &\quad + \frac{\beta}{2}(c_2 - c_1)(\sin 2\varphi_0 + \sin 2\varphi_1 + 2\varphi_0 - 2\varphi_1) + \frac{\beta}{2}\pi(c_2 + c_1)\Big] \\ \gamma_e(\beta) &= \frac{1}{\pi\beta\, C_e(\beta)}\Big[-2F_0(\sin\varphi_1 + \cos\varphi_0) + 2\beta_a c_2(\cos\varphi_0 - \sin\varphi_1) - \\ &\quad - \frac{\beta}{2}(c_2 - c_1)(\cos 2\varphi_0 + \cos 2\varphi_1)\Big] \end{aligned}\right\} \qquad (13)$$

with

$$\left.\begin{aligned} \varphi_0 &= \operatorname{arc\,cos}\frac{\beta_0}{\beta} \\ \varphi_1 &= \operatorname{arc\,sin}\frac{\beta_1}{\beta}\ . \end{aligned}\right\} \qquad (14)$$

The equivalent aileron hinge stiffness of the non-linear hinge mechanism of the wind tunnel model is plotted in Fig. 21 as a function of the amplitude ratio β / β_a. A further important result indispensable to the determination of the flutter boundary of the non-linear system is given in Fig. 22 illustrating the variation of the flutter speed against hinge stiffness, which is assumed to be linear for this calculation.

Finally, the flutter boundary of the non-linear wing aileron system as shown in Fig. 19 could be determined from a comparative evaluation of Fig. 21 and Fig. 22.

Calculated and measured results agree very well and thus demonstrate that the application of the principle of the "harmonic balance" is very efficient even in case of highly non-linear systems.

The "harmonic balance" approach can also be applied to systems with more than one concentrated non-linearity. Starting from the general equations of motion (10) this problem can be solved by numerical iteration.

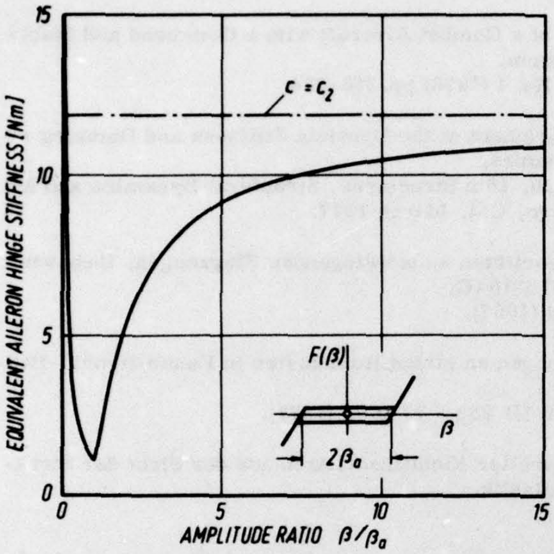

Fig. 21: Variation of the equivalent aileron hinge stiffness of the wind tunnel model versus hinge angle ratio β / β_a

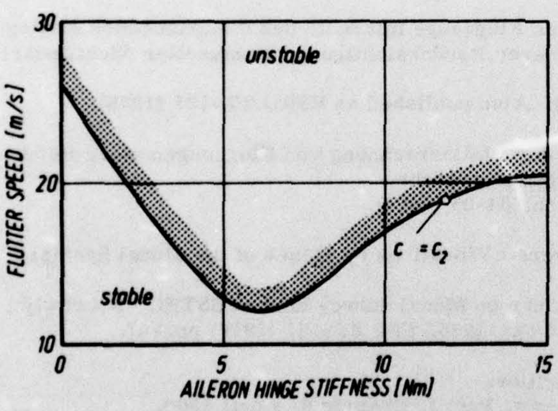

Fig. 22: Flutter speed of the "linearized" wing aileron wind tunnel model as function of changes of the linear hinge stiffness

4. CONCLUSION

Both a survey and a classification of structural non-linearities as occurring on aircraft have shown that a variety of non-linear effects have to be taken into account in the course of aeroelastic investigations. The physical properties of distributed and concentrated non-linearities have been exemplified by means of a number of typical results measured on real aircraft structures. Possibilities to overcome detrimental non-linear effects on ground and flight flutter tests have been discussed.

The influence of some different types of aileron hinge non-linearities on the flutter stability has been illustrated by the results of a flutter simulation carried out on a simple wing aileron section in the time domain. Furthermore the flutter behaviour of a wing aileron wind tunnel model with a non-linear aileron hinge mechanism has been investigated. The test results have been successfully compared with calculated ones obtained by a numerical procedure on the basis of the principle of the "harmonic balance".

As to future investigations emphasis should be placed on the following problems:

- Improvement of the ground vibration test concept for instance by artificial linearisation of the test structure and by subsequent introduction of the non-linearities into the aeroelastic equations.
- Investigation of the non-linear and frequency dependent dynamic properties of power operated controls; incorporation into the aeroelastic equations.
- Investigation of weak distributed non-linearities as to their effect on ground vibration test (with special regard to modern experimental-numerical methods) and flight flutter test.
- Amendment of flight flutter test methods aiming at a more reliable detection of the non-linear flutter boundaries.
- Further development of numerical methods to solve non-linear flutter problems with several intensively interacting concentrated non-linearities.
- Elaboration of suitable methods to predict unsteady aerodynamic forces due to arbitrary motions for real wings with finite aspect ratios and for all flight speed ranges.

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14. Abstract

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