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STRESS CONCENTRATION DUE TO A PROLATE SPHEROIDAL INCLUSION.(U)
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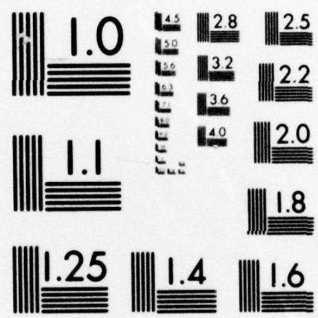

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9 Technical Report No. 6

to the

Office of Naval Research

Contract No. 15 N00014-75-C-0889

NR 031-781

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ONR Technical Report #6	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Stress Concentration Due to a Prolate Spheroidal Inclusion		5. TYPE OF REPORT & PERIOD COVERED Technical
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) M. Shibata and Kanji Ono		8. CONTRACT OR GRANT NUMBER(s) N00014-75-C-0889 NR 031-871
9. PERFORMING ORGANIZATION NAME AND ADDRESS Materials Department, School of Engineering 6531-Boelter Hall, University of California Los Angeles, California 90024		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Metallurgy Program Office of Naval Research, 800 N. Quincy Street Arlington, Virginia 22217		12. REPORT DATE January 1978
		13. NUMBER OF PAGES 12
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Unlimited DISTRIBUTION STATEMENT A Approved for public release; Distribution Unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Internal Stresses Inclusion Elasticity Theory Inhomogeneity		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) See Following Page		

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

STRESS CONCENTRATION DUE TO A PROLATE SPHEROIDAL INCLUSION

ABSTRACT

Internal stresses inside a prolate spheroidal inclusion parallel to a uniaxial applied stress are obtained by using Eshelby theory. Effects of the elastic modulus and the aspect ratio of the inclusion are evaluated. The present results are compared with others, in particular, with Argon's solution for a slender rod.

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1. INTRODUCTION

When external stress is applied to a composite material, internal stresses within the material are strongly affected by the shape, arrangement, and elastic stiffness of the reinforcing element. For optimum design and effective utilization of composite materials, thorough knowledge of the stress concentration, especially at the interface, is essential. It is pertinent to note that failure often initiates at such stress concentration. In fiber reinforced composite materials, the stress concentration at the ends of discontinuous fibers is of prime importance. This problem has been considered by several investigators [1~3]. Most recently, Argon obtained approximate solutions for stresses around slender elastic rods with moduli different from that of the infinite elastic matrix [1]. Earlier, Edwards [2] evaluated the stress concentration due to a prolate spheroidal inclusion which lies normal to the applied stress axis. Tanaka et al. [3] considered a prolate spheroidal inclusion parallel to the stress axis in the limit of an infinitely long inclusion. While a prolate spheroid is a good approximation for a fiber, no detailed study of the stress concentration has been made. In this study, we report on the stress concentration produced by a prolate spheroidal inclusion, which lies in an infinite elastic matrix parallel to the applied stress axis. Effects of the shape and the elastic moduli of the inclusion on the stress concentration are considered. Eshelby theory [4] of transformation induced stresses forms the basis of the present study. This approach is common to our previous studies on the stress concentration in and around an oblate spheroidal inclusion embedded in the elastic-plastic matrix [5,6].

2. INCLUSION AND APPLIED STRESS

In the Cartesian coordinates system, X_1, X_2 and X_3, a prolate spheroidal inclusion is located within

$$(X_1^2 + X_2^2)/a^2 + X_3^2/c^2 \leq 1 . \tag{1}$$

The aspect ratio k of the inclusion is defined as c/a, and is greater than unity for a prolate spheroid. The matrix and inclusion are assumed to be elastically isotropic. The elastic stiffnesses are given by

$$C_{ijk\ell} = \lambda\delta_{ij}\delta_{k\ell} + \mu(\delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk})$$

for the matrix, and

$$C^{*}_{ijk\ell} = \lambda^{*}\delta_{ij}\delta_{k\ell} + \mu^{*}(\delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk})$$

for the inclusion, respectively. λ and λ^* are the Lame constants, and μ and μ^* are the shear moduli of the matrix and inclusion, respectively. In order to simplify the presentation of numerical results, Poisson's ratios of the matrix and inclusion, ν and ν^* are assumed to be equal to 1/3, unless otherwise specified. In the following calculations, we employ the ratio of shear modulus $m = \mu^*/\mu$ under uniaxial applied stress σ^A along the X_3 axis. The elastic strain e^A_{ij} produced by σ^A can be written as,

$$e^A_{ij} = \begin{pmatrix} -\nu & 0 & 0 \\ 0 & -\nu & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{\sigma^A}{E} \qquad (2)$$

where E is the Young's modulus of the matrix.

3. THEORY

Eshelby [4] obtained general solution for the inhomogenity problem of an ellipsoidal inclusion using the so-called equivalent inclusion method. The internal stress inside the inclusion, which is uniform, is given by

$$\sigma^I_{ij} = C_{ijk\ell}(e^C_{k\ell} + e^A_{k\ell} - e^T_{k\ell}) = C^{*}_{ijk\ell}(e^C_{k\ell} + e^A_{k\ell}), \qquad (3)$$

where the constraint strain $e^C_{k\ell}$ is given in terms of Eshelby tensor, $S_{ijk\ell}$, as, $e^C_{k\ell} = S_{k\ell mn} e^T_{mn}$. Here, e^T_{mn} is the eigen strain of the equivalent inclusion. By solving (3) with respect to e^T_{ij}, we can obtain σ^I_{ij}. Because of the symmetry of Eqs. (1) and (2), e^T_{ij} takes the form of

$$e^T_{ij} = \begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & Z \end{pmatrix} \tag{4}$$

where X and Z are functions of the aspect ratio, k, and the ratio of shear modulus, m, as well as σ^A/E.

4. RESULTS

From Eq. (3), X and Z are found to be

$$X = \frac{(1-m)\,[(1-m)\,r(k) - 1]}{3[(1-m)^2 f(k) - (1-m) g(k) + 1]} \cdot \frac{\sigma^A}{E}$$

$$Z = \frac{-(1-m)\,[(1-m)\,s(k) - 3]}{3[(1-m)^2 f(k) - (1-m) g(k) + 1]} \cdot \frac{\sigma^A}{E} \tag{5}$$

where m is μ^*/μ, and

$$r(k) = S_{3333} + 3S_{1133}$$
$$s(k) = 3S_{1111} + 3S_{1122} + 2S_{3311}$$
$$f(k) = (S_{1111} + S_{1122})S_{3333} - 2S_{1133}S_{3311}$$
$$g(k) = S_{1111} + S_{1122} + S_{3333}\,.$$

The values of r, s, f and g have been evaluated for $k \geqq 1$ by using appropriate $S_{ijk\ell}$ [7]. The resultant internal stresses inside the inclusion can be written as,

$$\sigma^I_{11} = \sigma^I_{22} = 2\mu^* \cdot \frac{1}{1-m} \cdot (3X + Z)$$

$$\sigma_{33}^{I} = 4\mu^* \cdot \frac{1}{1 - m} \cdot (X + Z) \,. \tag{6}$$

The numerical results are shown in Figs. 1 and 2. The stresses are normalized by σ^A, and are plotted against k for selected values of m.

Results indicate that both σ_{11}^I and σ_{33}^I rapidly approach the asymptotic values with increasing k and significant deviations are only found when k is less than 10 to 30. For an infinitely long inclusion σ_{11}^I always vanishes, and σ_{33}^I becomes $m\sigma^A$. When no inhomogeneity effect exist (m = 1), we obtain $\sigma_{33}^I = \sigma^A$, and $\sigma_{11}^I = 0$ as expected. The shear modulus of the inclusion has a larger effect on σ_{33}^I for a more elongated inclusion.

5. COMPARISON WITH OTHER STUDIES

5-1. The Results for an Infinitely Long Inclusion (k = ∞)

Tanaka, Mori and Nakamura made a pioneering contribution to inclusion problems by using Eshelby theory. In one of their papers [3], the internal stress due to the inhomogeneity effect of an infinitely long fiber parallel to the stress axis was evaluated as,

$$\sigma_{33}^{I} = \left\{1 + \frac{(E^* - E)\,[(1 + \nu)\,(1 - 2\nu)E + (1 - \nu)E^*]}{(1 - \nu^2)\,[(1 - 2\nu)E + E^*]E}\right\} \sigma^A \tag{7}$$

Their result disagrees with the present calculation except when specific values of m are taken. Relevant non-zero components of Eshelby tensor for a prolate spheroidal inclusion parallel to X_3 for k = ∞ are

$$S_{1111} = S_{2222} = \frac{5 - 4\nu}{8(1 - \nu)}$$

$$S_{1122} = S_{2211} = \frac{4\nu - 1}{8(1 - \nu)}$$

$$S_{1133} = S_{2233} = \frac{\nu}{2(1 - \nu)} \;,$$

from which we have $e_{11}^{c} = e_{22}^{c} = \frac{X + \nu Z}{2(1 - \nu)}$ and $e_{33}^{c} = 0$. From Eq. (3), we have

$$\sigma_{33}^{I} = \frac{m}{(1 + \nu)} \cdot \frac{1}{(1 - 2\nu^* + m)} [1 - \nu^* + m(1 + \nu^*) - 2\nu\nu^*] \cdot \sigma^A \quad (8)$$

or

$$\sigma_{33}^{I} = \frac{E^*}{E} \{1 + \frac{2E(\nu^* - \nu)}{E(1 - 2\nu^*)(1 + \nu^*) + E^*(1 + \nu)}\} \sigma^A \quad (8)'$$

When the Poisson's ratios are equal, Eq. (8)' yields $\sigma_{33}^{I} = m\sigma^A$, but Eq. (7) does not. The present result is correct, since this case corresponds to the isostrain (or Voigt) condition.

5-2. Rod-Shaped Fiber

Recently, Argon [2] obtained an approximate solution for the stress concentration due to a rod-shaped inclusion in an infinite elastic medium when uniaxial external stress is applied along the rod-axis. He used Eshelby's concept of stress-free transformation in order to evaluate traction at interfaces. Then, he employed a solution of elasticity for two point forces in an infinite medium given by Timoshenko and Goodier [8]. The interface traction was represented by the two point forces concentrated at the end of the rod-shaped inclusion. A rod having a large length-to-diameter ratio is similar to a prolate spheroidal inclusion with a large aspect ratio. Therefore, it is interesting to examine differences between results obtained by Argon and by us.

When a rod-shaped inclusion with a diameter $2\ r_o$ and length 2ℓ, is parallel to the axis of external stress σ^A, Argon's solution for σ_{33}^{I} at the end of the rod is

$$\sigma_{33}^{I} = \frac{1 - ak'}{m - ak'} m\sigma^A \quad (9)$$

where ν is assumed to be 1/3, and a constant a is found to be 1.412. Here, the apsect ratio k' is defined as $k' = \ell/r_o$. In Fig. 3, values of σ_{33}^{I} obtained from Eq. (6) and Eq. (9) are plotted against k and k' for m = 6. (This value of m corresponds to a steel reinforced concrete.) As can be seen in Fig. 3, a good agreement was obtained. When k (or k') goes to infinity, both results agree exactly. Differences are less than 3% for k (or k') > 100 and become greater with decreasing k (or k'). However, it is still less than 20% at k = 1. As pointed out by Argon, his method based on two point forces in an infinite medium is applicable to the case of very large aspect ratios of the rod-shaped inclusion. A part of the difference should also be attributed to different geometries.

ACKNOWLEDGEMENT

This work was supported by the Metallurgy Program of the Office of Naval Research under contract N00014-75-C-0889.

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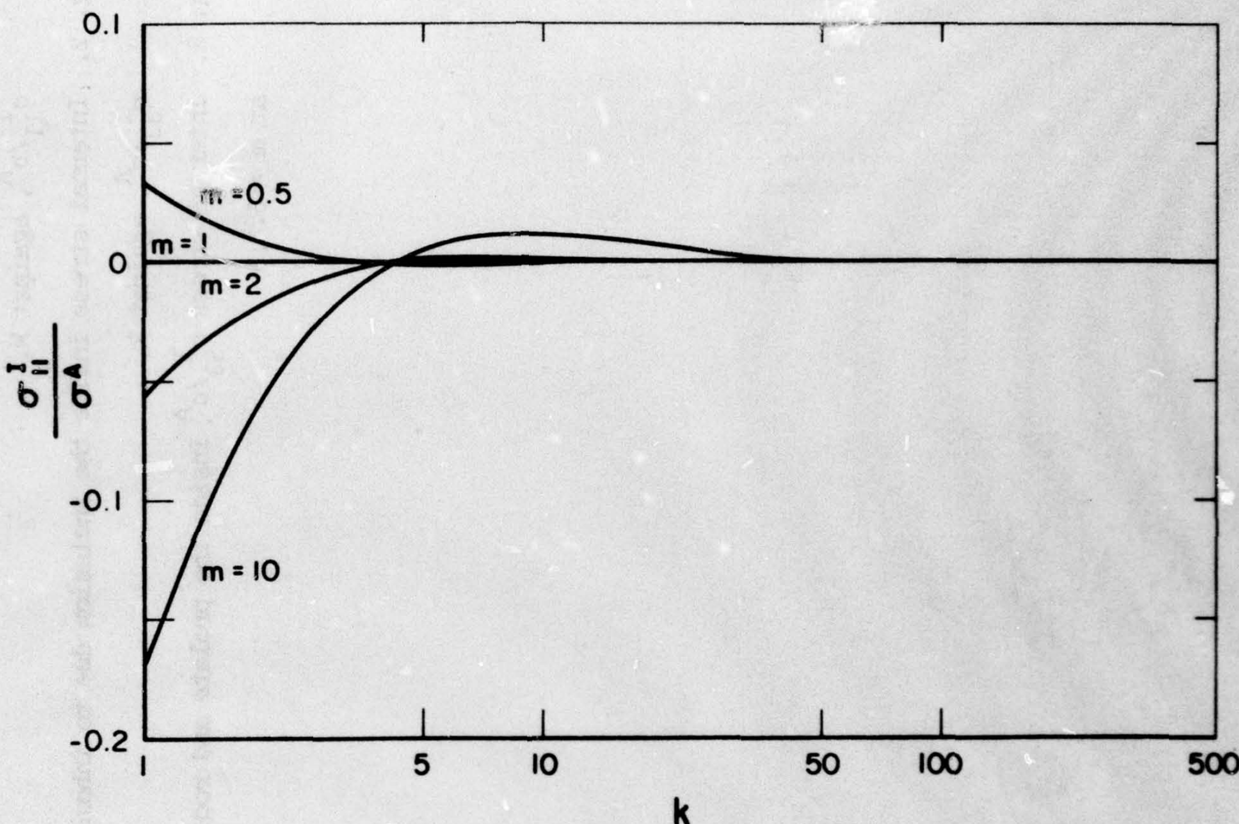
FIGURE CAPTIONS

Fig. 1. Internal stress inside the inclusion due to inhomogeneity effect, $\sigma_{11}^{I}/\sigma^{A}$, against k.

Fig. 2. Internal stress inside the inclusion due to inhomogeneity effect, $\sigma_{33}^{I}/\sigma^{A}$, against k.

Fig. 3. Internal stress $\sigma_{33}^{I}/\sigma^{A}$, inside the prolate and rod-shaped inclusions at m = 6.



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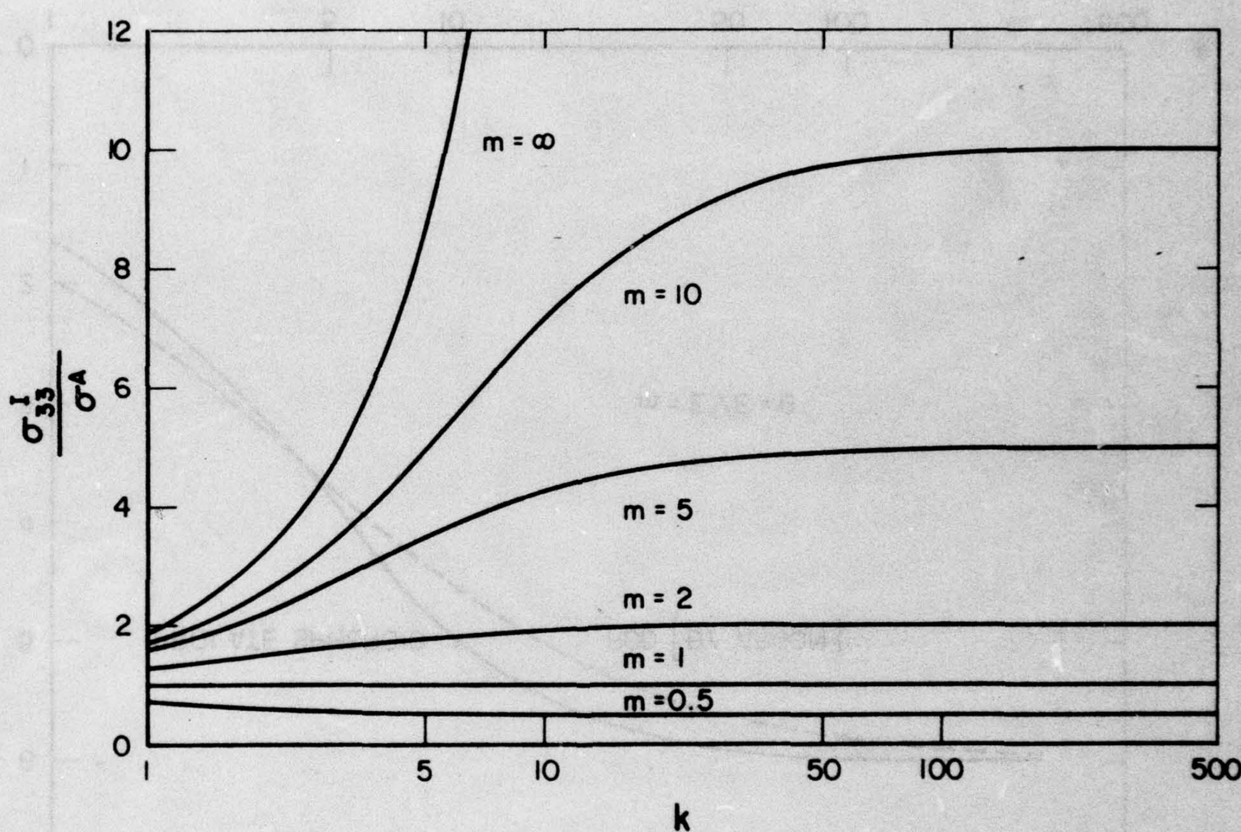

Fig. 2. Internal stress inside the inclusion due to inhomogeneity effect, $\sigma_{33}^{I}/\sigma^{A}$, against k.

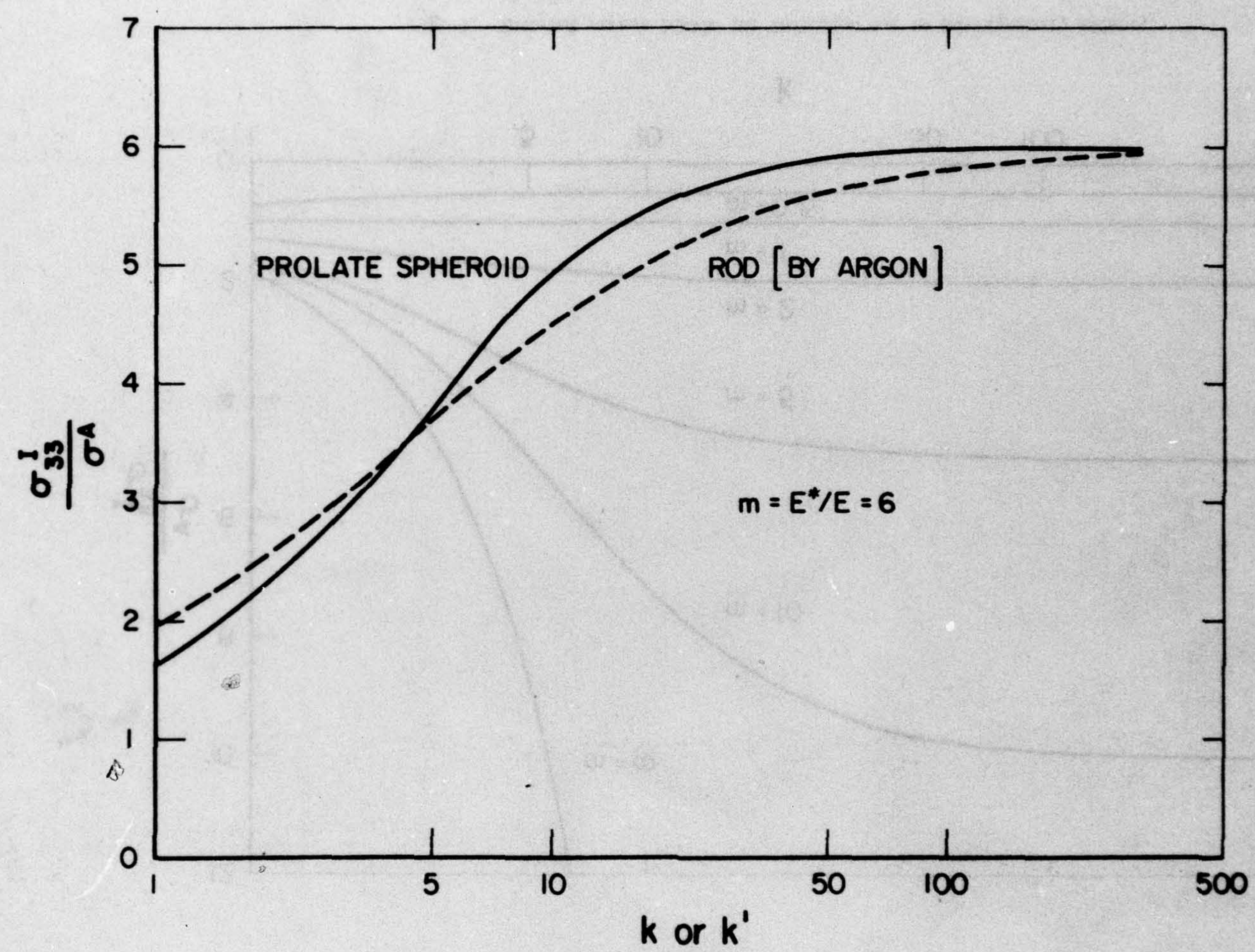

Fig. 3. Internal stress $\sigma_{33}^{I}/\sigma^{A}$ inside the prolate and rod-shaped inclusions at m = 6.