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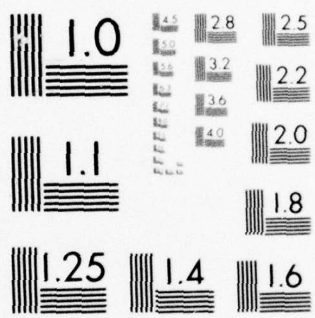

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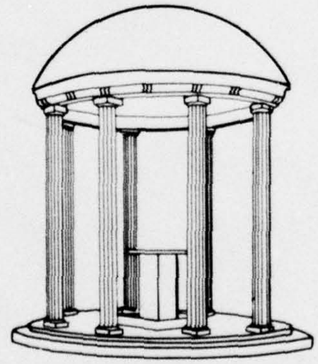

SCHOOL OF BUSINESS ADMINISTRATION AND CURRICULUM IN OPERATIONS RESEARCH AND SYSTEMS ANALYSIS

8

(s,S) INVENTORY POLICIES IN A NONSTATIONARY DEMAND ENVIRONMENT

Technical Report #11

Ronald Kaufman*

April 1977

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Harvey M. Wagner
Principal Investigator
University of North Carolina

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Scarf (1960) proves the optimality of (s,S) policies for a class of discrete review nonstationary inventory models. A considerable amount of inventory literature concerns computation of optimal and approximately optimal (s,S) policies under the Scarf hypotheses. Little research has dealt, however, with the case of nonstationary demands. This investigation examines the situation in which demand distributions are independent, but not identically distributed, and vary in a cyclic manner. Products that experience seasonal demands are a typical example of such a demand process.
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A detailed analysis of the nature of optimal policies in the nonstationary environment is presented. The behavior of selected operating characteristics such as period-end inventory, backlogged demand, frequency of stockout, replenishment frequency, and associated costs also are examined. The above performance measures for an individual inventoried good are aggregated over all goods to provide an analysis of the multi-item system behavior.

Approximately optimal (s,S) policies are derived for the nonstationary environment. The policies are based on the power approximation of Ehrhardt (1976), and require knowledge of only the mean and variance of demand. The operating characteristics of approximately optimal policies are compared with those of optimal policies.

The approximately optimal policy rule is examined in a statistical environment that generalizes MacCormick (1974). Policy parameters are revised periodically using a limited history of past demands to estimate the mean and variance of demand. Each time the policy parameters are revised, forecasts of system operating characteristics are calculated from a retrospective simulation employing the same sample of demands that was used to revise the policy.

The statistical phenomena are studied by means of a computer simulation program using time series analysis techniques.

FOREWORD

As part of the on-going program in "Decision and Control Models in Operations Research," Mr. Ronald Kaufman investigates the behavior of multi-item inventory control systems in which the means of the underlying demand distributions vary in a cyclic manner, corresponding to seasonal peaks for inventoried goods. Mr. Kaufman adapts the Ehrhardt power approximation (Technical Report No. 7), which was originally designed for stationary demand distributions, to the nonstationary environment. Mr. Kaufman tests the behavior of the power approximation in inventory systems where the demand distribution parameters are estimated from a limited amount of historical information. Several sections of this report parallel similar findings in earlier reports. Other related reports dealing with the research program are given below.

Harvey M. Wagner
Principal Investigator

MacCormick, A. (1975), Statistical Problems in Inventory Control, ONR and ARO Technical Report, December 1974, School of Organization and Management, Yale University, 244 pp.

Estey, A. S. and R. L. Kaufman (1975), Multi-Item Inventory System Policies Using Statistical Estimates: Negative Binomial Demands (Variance/Mean=9), ONR and ARO Technical Report 3, September 1975, School of Organization and Management, Yale University, 85 pp.

Ehrhardt, R. (1975), Variance Reduction Techniques for an Inventory Simulation, ONR and ARO Technical Report 4, September 1975, School of Organization and Management, Yale University, 24 pp.

Kaufman, R. (1976), Computer Programs for (s,S) Policies Under Independent or Filtered Demands, ONR and ARO Technical Report 5, School of Organization and Management, Yale University, 65 pp.

Kaufman, R. and J. Klincewicz (1976), Multi-Item Inventory System Policies Using Statistical Estimates: Sporadic Demands (Variance/Mean=9), ONR and ARO Technical Report 6, School of Organization and Management, Yale University, 58 pp.

Ehrhardt, R. (1976), The Power Approximation: Inventory Policies Based on Limited Demand Information, ONR and ARO Technical Report 7, School of Organization and Management, Yale University, 58 pp.

Klincewicz, J. G. (1976), Biased Variance Estimators for Statistical Inventory Policies, ONR and ARO Technical Report 8, School of Yale University, 24 pp.

Klincewicz, J. G. (1976), Inventory Control Using Statistical Estimates: The Power Approximation and Sporadic Demands (Variance/Mean=9), ONR and ARO Technical Report 9, School of Organization and Management, Yale University, 52 pp.

Klincewicz, J. G. (1976), The Power Approximation: Control of Multi-Item Inventory Systems with Constant Standard-Deviation-To-Mean Ratio for Demand, ONR and ARO Technical Report 10, School of Organization and Management, Yale University, 47 pp.

ABSTRACT

(s,S) INVENTORY POLICIES IN A
NONSTATIONARY DEMAND ENVIRONMENT

Ronald Louis Kaufman

University of North Carolina-1977

Scarf (1960) proves the optimality of (s,S) policies for a class of discrete review nonstationary inventory models. A considerable amount of inventory literature concerns computation of optimal and approximately optimal (s,S) policies under the Scarf hypotheses. Little research has dealt, however, with the case of nonstationary demands. This investigation examines the situation in which demand distributions are independent, but not identically distributed, and vary in a cyclic manner. Products that experience seasonal demands are a typical example of such a demand process.

A detailed analysis of the nature of optimal policies in the nonstationary environment is presented. The behavior of selected operating characteristics such as period-end inventory, backlogged demand, frequency of stockout, replenishment frequency, and associated costs also are examined. The above performance measures for an individual inventoried good are aggregated over all goods to provide an analysis of the multi-item system behavior.

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ACKNOWLEDGEMENTS

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I am indebted to my colleagues, Richard Ehrhardt, Arthur Estey, and Alastair MacCormick whose earlier work provided a framework for this study. Carl Schultz also provided valuable computer assistance.

For the typing of the manuscript, I am indebted to Helen Willard.

Finally, I gratefully acknowledge the support provided by the Army Research Office, the Office of Naval Research, the Yale University Graduate School, and the University of North Carolina.

CONTENTS

1. INVENTORY MANAGEMENT WITH INCOMPLETE DEMAND INFORMATION

This section describes the inventory model assumptions and the inventory operating characteristics that will be used to evaluate inventory policies. The Ehrhardt power approximation is also presented, and we will show how it can be successfully adapted to inventory systems with nonstationary demands.

1.1 The Model

1.1.1 Structure

We consider a single-item inventory model in which the inventory level is reviewed each period, and demand for the item is generated by a discrete time stochastic process. The demand process is nonstationary; Specifically, the demands are independent, but not identically distributed, non-negative integer-valued random variables. The distributions used to generate these demands are cyclic with period L. Let

$$\xi_1, \xi_2, \ldots, \xi_L, \xi_{L+1}, \ldots, \xi_{2L}, \xi_{2L+1}, \ldots, \xi_{3L}, \xi_{3L+1}, \ldots$$

represent the demand sequence: then the joint distributions of $\xi_{nL+1}, \ldots, \xi_{(n+1)L}$ and $\xi_{mL+1}, \ldots, \xi_{(m+1)L}$ are are assumed to be the same for any integers m and $n \geq 0$. Since there are at most L different demand distributions, Φ_i, for $i = 1,\ldots,L$, denotes the distribution function for the i^{th} demand within the cycle. We assume that demand is met as long as there is stock on hand, and when a stockout occurs, unfilled demand is backlogged until sufficient replenishment arrives.

Units of inventory are conserved, there being no losses by deterioration, obsolescence, or pilferage, and disposal is not allowed. Inventory on hand at the end of a current period is the inventory from the previous period plus any replenishment that arrives, less demand in the current period. If inventory on hand is negative, the amount represents backlogged demand. Replenishments are assumed to be delivered a fixed leadtime λ after being ordered. The time sequence of events within any period is taken to be order, delivery, and demand.

The cost functions for the model are simple. With no discounting over time and an unbounded planning horizon, a resonable criterion is minimization of long-run average expected total cost per period. The cost of a replenishment quantity is assumed to be linear with a fixed ordering cost K. The inventory holding cost is proportional to any stock on hand, at unit cost h, assessed at the end of each period; a unit penalty cost π is applied to any quantity on backorder at the end of each period.

The associated control problem is to make optimal replenishment decisions for this model. We assume that control over replenishment is exercised by an (s_i, S_i) policy, for $i = 1,\ldots,L$: Whenever inventory x on hand and on order at the start of the i^{th} period, $1 \leq i \leq L$, within a cycle of length L, drops below the value s_i, we place a replenishment order of size $S_i - x$. When the objective of control is to minimize long-run average expected total cost per period, and the distributions and parameters specifying the structure of the model are completely known, it follows from the theory of dynamic programming, [see Veinott (1966) and Karlin (1959)], that there is an (s_i, S_i) policy, for $i = 1,\ldots,L$, that is optimal among all possible policies.

When the distribution generating demand is not know, then even though this is the only assumption relaxed, it may no longer be true that a (s_i, S_i), for $i = 1,\ldots,L$, policy is optimal, using the criteria of long-run average expected total cost per period. Nevertheless, in this study we shall use such a policy, since it is in popular use in the applied situation of incomplete information. Information about the L demand distributions Φ_i, for $i = 1,\ldots,L$, must be gathered by inference from a limited sample of demands.

1.1.2 <u>Algorithms to Set (s,S) with Partial Knowledge</u>

When the means μ_i and variances σ_i^2, for $i = 1,\ldots,L$, of each demand distribution in a cycle are known, approximations to the optimal policy are available. The approximations are based on asymptotic theory that assume large unit penalty costs and large replenishment costs. The algorithm adopted here is the power approximation of Ehrhardt (1976), which is a generalization of earlier policies based on the work of Roberts (1962). Ehrhardt extended Roberts' work by generalizing the functional forms for s and S-s, and then fitting these functional forms to a larger set of known optimal policies using least-squares regression. The power approximation was designed by Ehrhardt for use in stationary environments; one of the purposes of this study is to investigate adaptations of the power approximation to nonstationary environments.

Let $\bar{\mu}_i^{(\lambda+1)}$ and $\bar{\sigma}_i^{2(\lambda+1)}$, for i = 1,...,L, denote the arithmetic average of the next $\lambda + 1$ demand means and variances, respectively, starting in the i^{th} period of the cycle. For example, if i = 5 and $\lambda = 2$, then $\bar{\mu}_5^{(3)}$ and $\bar{\sigma}_5^{2(3)}$ are the arithmetic averages of the means and variances, respectively, of the demands in periods 5, 6, and 7. For each pair of $\bar{\mu}_i^{(\lambda+1)}$ and $\bar{\sigma}_i^{2(\lambda+1)}$ the power approximation computes,

(1) $$D_p = (1.463)(\bar{\mu}_i^{(\lambda+1)})^{.364}\,(K/h)^{.498}\,[(\lambda+1)\bar{\sigma}_i^{2(\lambda+1)}]^{.0691}$$

and

$$s_1 = (\lambda+1)\,\bar{\mu}_i^{(\lambda+1)} + [(\lambda+1)\bar{\mu}_i^{(\lambda+1)}]^{.416}\,(\bar{\sigma}_i^{2(\lambda+1)}/\bar{\mu}_i^{(\lambda+1)})^{.603}\,U(z)$$

(2) $$S_1 = s_1 + D_p,$$

where U(z) is given by

$$U(z) = (.182/z) + 1.142 - 3.466z,$$

(3) $$z = \left[\frac{(\bar{\mu}_i^{(\lambda+1)})^{.364}\,(K/h)^{.498}}{(1+\frac{\pi}{h})[(\lambda+1)\,\bar{\sigma}_i^{2(\lambda+1)}]^{.431}}\right]^{1/2}$$

If $D_p/\bar{\mu}_i^{(\lambda+1)}$ is greater than 1.5, let $s_i = s_1$ and $S_i = S_1$.

Otherwise, compute

(4) $$S_2 = (\lambda+1)\bar{\mu}_i^{(\lambda+1)} + v\,[(\lambda+1)\,\bar{\sigma}_i^{2(\lambda+1)}]^{1/2} ,$$

where v is the solution to

(5) $$F(v) = \pi/(\pi+h) ,$$

and $F(\cdot)$ is the cumulative distribution function of the unit normal distribution. The policy parameters are then given by

(6) $$s_i = \text{minimum } \{s_1 , S_2\}$$
$$S_i = \text{minimum } \{S_1 , S_2\}$$

Since demands are integer-valued, we round s_1, D_p, and S_2 to the nearest integer.

The policy (s_i,S_i) is utilized at the beginning of the i^{th} period in the cycle. The policy (s_i,S_i), for $i = 1,\ldots,L$, computed using the power approximation, has been adopted in this study as the basic decision rule in the case of limited information about the demand distribution parameters.

1.2 Experimental Design

1.2.1 Nonstationary Demands

In this study, we assume that an item's demand distribution in any period is negative binomial. This distribution has parameters r and p, where $r>0$ and $0<p<1$, with probability mass function

$$f(x) = \frac{\Gamma(r+x)}{\Gamma(x+1)\,\Gamma(r)}\,(1-p)^r\,p^x \quad \text{for} \quad x = 0,1,\ldots ,$$

yielding mean $\frac{rp}{1-p}$ and variance $\frac{rp}{(1-p)^2}$.

Note that the variance-to-mean ratio is $\frac{1}{1-p}$, which is a function of p, and throughout this study the variance-to-mean ratio is fixed at 3; thus $p = 2/3$. We introduce nonstationarity by varying the parameter r

cyclically. We select cycle length $L = 12$, corresponding to a yearly demand cycle of 12 months' demands.

We select for study five general types of cyclic demand structures. Figure 1.1 illustrates the movement of the demand distributions' means in each cycle. Figure 1.1b displays a step model demand structure that henceforth we refer to as model I. The mean demand level for this model is α for the first nine periods of the cycle, and 2α for the last three periods of the cycle.

A more gradual increasing mean is shown for models II and III in Figures 1.1.c and 1.1.d, respectively. Both models maintain a constant mean α for the first seven periods of the cycle. The mean gradually increases to a maximum level in period 10 of 2α for model II and 5α for model III. After period 10 the mean demand slowly decreases.

Model IV, depicted in Figure 1.1.e shows mean demand varying in a sinusoidal manner where α is the lowest level of mean demand and 5α is the largest level.

Throughout this study models I, II, III, and IV are compared with the stationary model, depicted in Figure 1.1.a. The stationary model maintains a constant mean demand level α . The five models' values of α are selected to ensure that overall mean period demand for the entire cycle is equal to values specified in Section 1.2.2.

1.2.2 Design Parameters

The single-item model described in Section 1.1.1 is studied under the range of input parameters given in Table 1.1. The demand distribution is negative binomial with variance-to-mean ratio fixed at 3. The two mean values for demand are 8 and 16. Two values, 2 and 4, are assigned to leadtime. Since the cost function is linear in the parameters K, h, and π, the value of the unit holding cost is set at unity.

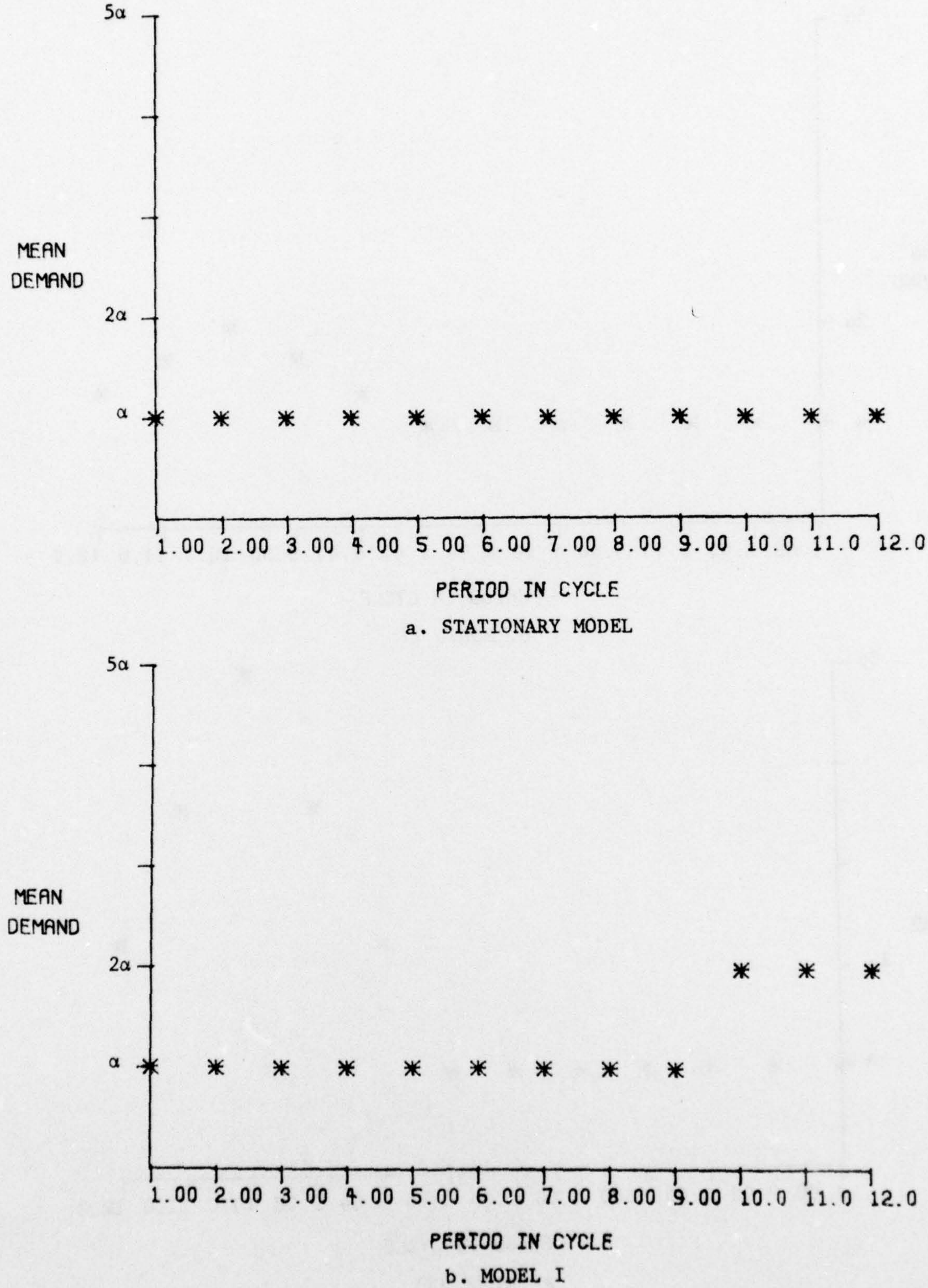
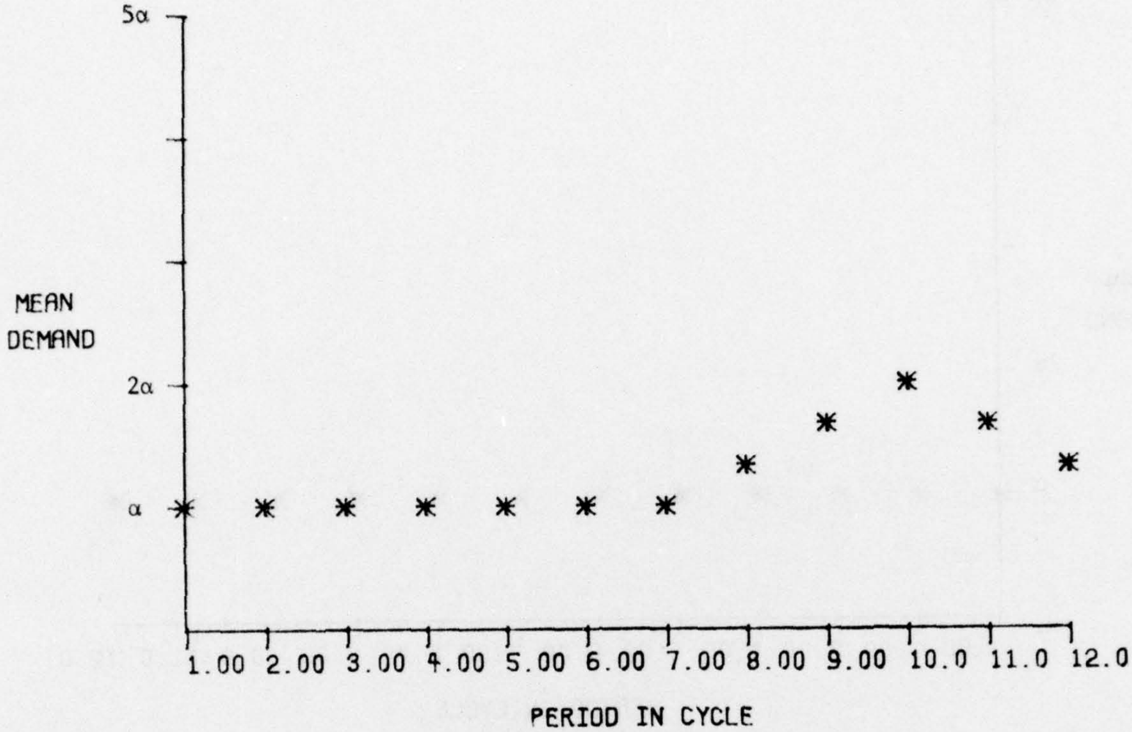
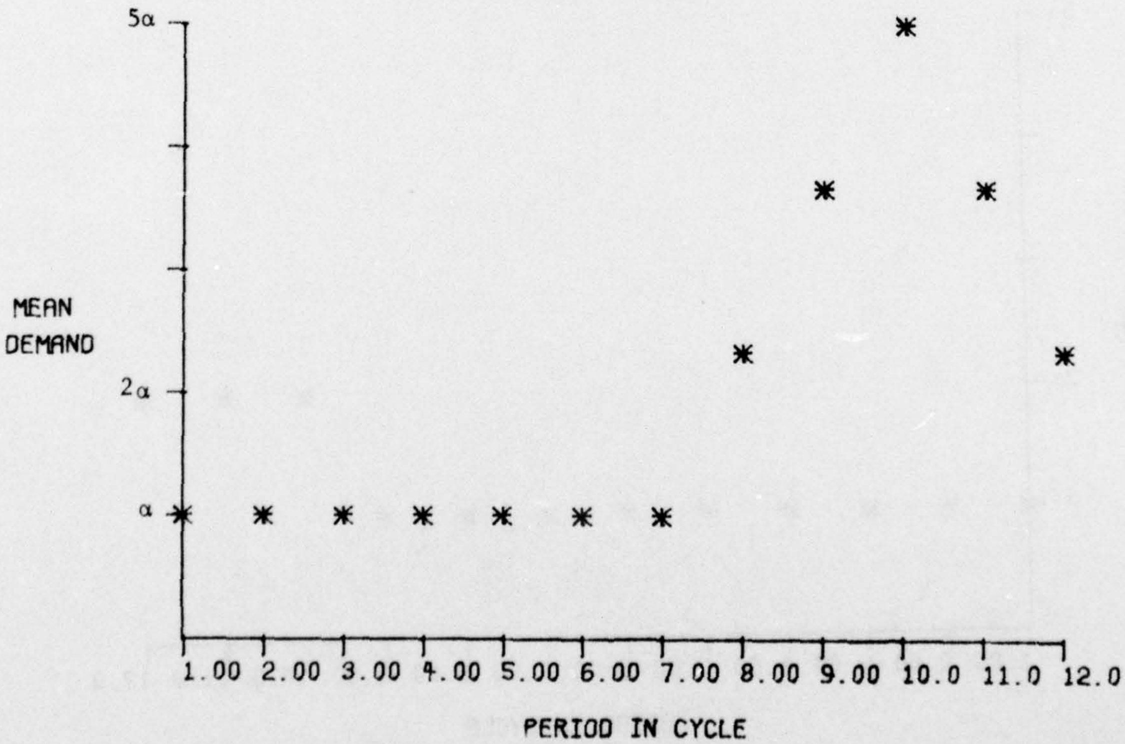


Figure 1.1 Demand Means



c. MODEL II



d. MODEL III

Figure 1.1 Demand Means

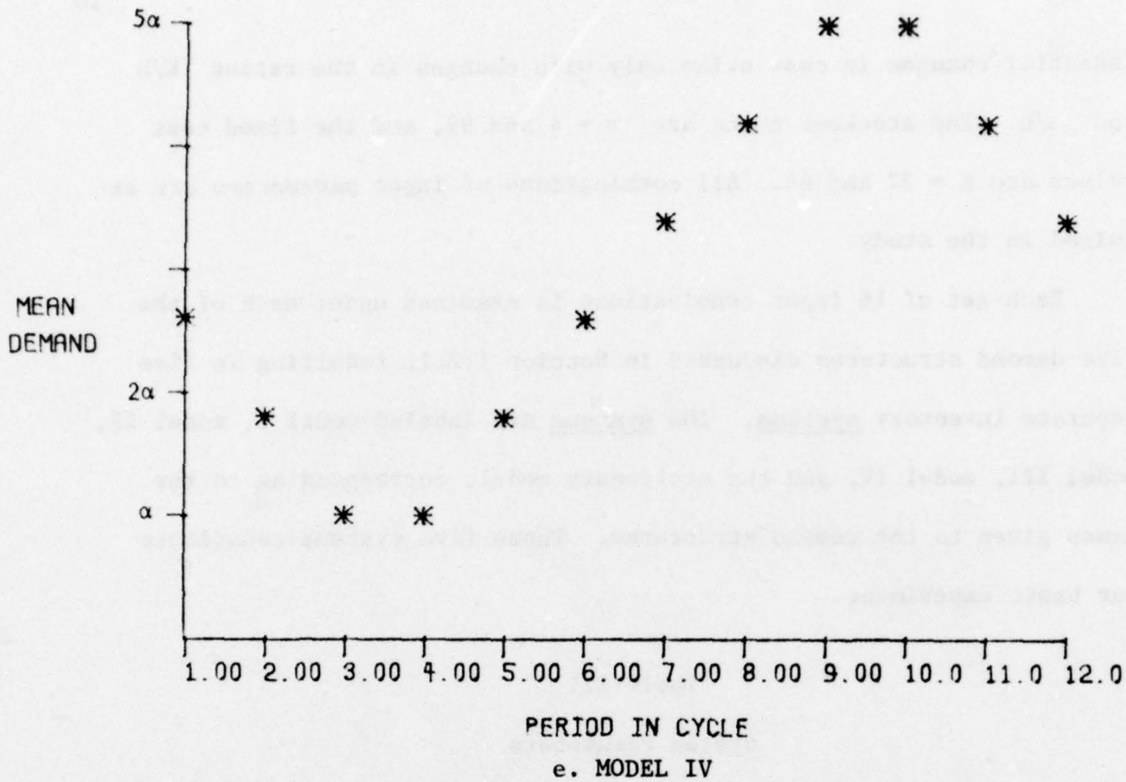


Figure 1.1 Demand Means

Essential changes in cost arise only with changes in the ratios K/h and π/h. The stockout costs are $\pi = 4$ and 99, and the fixed cost values are K = 32 and 64. All combinations of input parameters are examined in the study.

Each set of 16 input combinations is examined under each of the five demand structures discussed in Section 1.2.1, resulting in five separate inventory systems. The systems are labeled model I, model II, model III, model IV, and the stationary model, corresponding to the names given to the demand structures. These five systems constitute our basic experiment.

Table 1.1

System Parameters

Parameter	Parameter Settings	Number of Settings
Demand distribution	Negative Binomial ($V/\mu = 3$)	1
Mean demand	8, 16	2
Unit holding cost	1	1
Unit backlog penalty cost	4, 99	2
Replenishment setup cost	32, 64	2
Replenishment leadtime	2, 4	2

1.2.3 Decision Rules

This study examines three decision rules: optimal inventory rules, approximately optimal rules, and statistical approximation rules. The computation of optimal policies is explained in detail in Section 2.1. Approximately optimal policy rules are calculated using the power approximation, discussed previously in Section 1.1.2. A third method is the statistical power approximation, which is explained in the remainder of this section. Both the optimal and approximately optimal policies are analyzed analytically in Section 2. Section 2 also investigates the effects of demand structure misspecification on the power approximation. Computer simulation is used to investigate the statistical power approximation, and the findings are presented in Section 3.

For an arbitrary integer t, let $G<t$ be the number of observed demands during periods $t-G$ to $t-1$. In order to use the statistical power approximation of Ehrhardt (1976) in a stationary environment, the statistics required are the sample mean and variance of demand, $\bar{\xi}$ and $\bar{V}$, respectively. If t is the period in which we decide to compute a statistical policy, then

$$\bar{\xi} = G^{-1} \sum_{\tau=1}^{G} \xi_{t-\tau}$$

$$\bar{V} = (G-1)^{-1} \sum_{\tau=1}^{G} (\xi_{t-\tau} - \bar{\xi})^2 .$$

When using the statistical power approximation, we periodically obtain a new (s,S) policy by substituting $\bar{\xi}$ and $\bar{V}$ for $\bar{\mu}_i^{(\lambda+1)}$ and $\bar{\sigma}_i^{2(\lambda+1)}$ in equations (1) through (6).

In a regression context $\bar{\xi}$ and $\bar{V}$ are estimators of the regression coefficient α and the variance of the disturbance term for the regression model

$$y_{t-\tau} = \alpha x_{t-\tau} + \varepsilon_{t-\tau} \quad \text{for } \tau = 1,\ldots,G,$$

where, for the stationary environment

$$y_{t-\tau} = \text{demand in period } t-\tau$$

α = stationary mean demand level

$x_{t-\tau} = 1$

$E(\varepsilon_{t-\tau}) = 0$ for all $t>\tau$

$E(\varepsilon_{t-\tau}\varepsilon_{k-\tau}) = 0$ for $t \neq k$, $t>\tau$, $k>\tau$,

$E(\varepsilon^2_{t-\tau}) = \sigma^2$ for all $t>\tau$.

In our nonstationary environment, however, $x_{t-\tau}$ is no longer identically 1 for every demand period; now $x_{t-\tau}$ depends on the position of period $t-\tau$ within the demand cycle. We assume that we know the cycle length, and we let each period's mean demand in the cycle be represented as the product of the appropriate value of $x_{t-\tau}$ and α , which is the smallest demand mean of the demand structure. We also assume that the values of $x_{t-\tau}$ are known, but that α must be estimated. The appropriate regression model is

$$y_{t-\tau} = \alpha x_{t-\tau} + \varepsilon_{t-\tau} \qquad \text{for } \tau = 1,\ldots T(<t),$$

where

$y_{t-\tau}$ = demand in period $t-\tau$

α = smallest mean demand level in the demand structure

$x_{t-\tau}$ = appropriate constant such that $\alpha x_{t-\tau}$ is the mean demand level in period $t-\tau$

$E(\varepsilon_{t-\tau}) = 0$ for all $t>\tau$,

$E(\varepsilon_{t-\tau}\varepsilon_{k-\tau}) = 0$ for $t \neq k$, $t>\tau$, $k>\tau$,

$E(\varepsilon^2_{t-\tau}) = \sigma^2_{\gamma}(t-\tau)$ for all $t>\tau$,

$\dfrac{\sigma^2_{t-\tau}}{\alpha x_{t-\tau}} = c$ for all $t>\tau$,

$\gamma(t-\tau)$ = appropriate demand structure index used in period $t-\tau$.

$\gamma(t-\tau)$ identifies the demand distribution used in period $t-\tau$, relative to the cycle. For example, if $\gamma(t-\tau) = 5$, then period $t-\tau$ receives demand from the fifth period of the cycle, and the variance of demand in period $t-\tau$ is σ_5^2.

Table 1.2

Values for $x_{t-\tau}$

	Period in Cycle											
	1	2	3	4	5	6	7	8	9	10	11	12
Stationary Model	1	1	1	1	1	1	1	1	1	1	1	1
Model I	1	1	1	1	1	1	1	1	1	2	2	2
Model II	1	1	1	1	1	1	1	1.33	1.67	2	1.67	1.33
Model III	1	1	1	1	1	1	1	2.33	3.67	5	3.67	2.33
Model IV	2.60	1.80	1	1	1.80	2.60	3.40	4.20	5	5	4.20	3.40

The values of $x_{t-\tau}$ within a cycle for each of the demand structures studied are given in Table 1.2. Observe that $x_{t-\tau}$ is 1 when the mean demand level in period $t-\tau$ is α and 2 or 5 for the nonstationary systems when the mean demand level in period $t-\tau$ is at its largest value. Hence, in models I through IV, α represents a base level of mean demand, and $x_{t-\tau}$ determines how much above that level the mean demand in period $t-\tau$ will be.

The nonstationary regression model contains heteroscadastic disturbances. However, dividing both sides of the regression equation by $\sqrt{x_{t-\tau}}$ will eliminate the heteroscedasticity [see Johnston (1972)], and produce a transformed model with disturbance variance equal to the variance-to-mean ratio, which has been kept constant, times α. The resulting least squares estimator of α is

$$\hat{\alpha} = \frac{\sum_{\tau=1}^{G} y_{t-\tau}}{\sum_{\tau=1}^{G} x_{t-\tau}}$$

and a standard estimator of $c\alpha$ the variance-to-mean ratio times α , is

$$s^2 = \frac{\sum_{\tau=1}^{G} \frac{1}{x_{t-\tau}} (y_{t-\tau} - x_{t-\tau})^2}{G - 1} .$$

Once $\hat{\alpha}$ and s^2 are obtained, we can estimate $\bar{\mu}_i^{(\lambda+1)}$ and $\bar{\sigma}_i^{2(\lambda+1)}$, for $i = 1,\ldots,L$, by

$$\hat{\bar{\mu}}_i^{(\lambda+1)} = \frac{\sum_{j=R}^{R+\lambda} \hat{\alpha}\, x_j}{\lambda + 1} ,$$

and

$$\hat{\bar{\sigma}}_i^{2(\lambda+1)} = s^2 \hat{\bar{\mu}}_i^{(\lambda+1)} / \hat{\alpha}$$

respectively, where R is chosen such that $\gamma(R) = i$. These estimates are substituted for $\bar{\mu}_i^{\lambda+1}$ and $\bar{\sigma}_i^{2(\lambda+1)}$ in equations (1) through (6) of Section 1.1.2 to obtain the statistical policy (s_i,S_i).

1.2.4 Policy Revision

In implementing a statistical inventory policy, in a real-life environment, the policy parameters s_i and S_i, for $i = 1,\ldots,L$, would be revised at regular intervals. With such revisions taking place, there are system design choices to be made concerning the length of history kept and the amount of information stored. For the simulation experiments in this study, a history of fixed length is kept for each revision, and equal weight is given to each observation. This is not optimal if the demand process is known to be stationary, for then the entire history should be accumulated to give progressively better knowledge and performance. Even when demand is known to be nonstationary, but varying in a regular manner, such as by a trend or periodic cycle, or both, an optimal decision rule would generally utilize the entire history.

The decision-maker usually is <u>not</u> in a position to know, however, that conditions observed, even over the entire past history, will continue to prevail. This lack of complete information provides justification for making frequent revisions, placing greater weight on observations from the immediate past and less on distant history. For this study, an arbitrary choice has been made to keep a history of fixed length and give equal weight to all observations in the history. In this study, the revision interval and revision history length are both 24 periods, implying "monthly" demand data that are accumulated over the same time interval. For a detailed discussion of demand generation, system initialization, and output analysis, in the simulation experiment see MacCormick (1974, pp. 52-58).

1.2.5 Item Operating Characteristics

We examine detailed operating characteristics of each item in the five systems. These are the period-end inventory on hand, the period-end stockout quantity, the frequency of period-end stockouts, the replenishment quantity, the frequency of replenishment, and the total cost incurred. It is assumed that the decision-maker is interested only in the average performance of these characteristics between revisions, corresponding to the practice adopted by accountants of making reports to management at periodic intervals.

One objective of the study is to make inferences about the distributions of these operating characteristics for each of the three decisions rules (optimal policies under full information, approximately optimal policies, and statistical approximation policies), that were discussed in Section 1.2.3.

1.2.6 Forecasting

The method of MacCormick (1974) is used to forecast operating characteristics when the system is controlled by the statistical power approximation rule. Often termed "retrospective simulation," this method takes the recent demand history for the items in the sample and estimates the performance of the chosen policy for this history. Thus, the method uses the same data twice, once to fix the policy parameters, and once to forecast the performance. As a result, the forecasts are biased, and this study investigates the extent of the bias.

At each revision, after the (s_i, S_i) parameters, for $i = 1,\ldots,L$, have been set to new values, a forecast is made of the properties of the system. A history of the latest $G+\lambda$ demand values is kept to make the forecast. [We note that there are differences in information storage requirements of considerable importance in practice. To make a forecast of performance by retrospective simulation, the demands for the item in each of $G+\lambda$ periods must be stored, whereas the statistical decision rule may require only a handful of sufficient statistics to set the (s_i, S_i) policies. Also, for multi-item systems the cost of accumulating in storage the data needed to make forecasts by retrospective simulation is likely to be so high that histories will be kept for only a representative sample of items from the system.]

The forecasts are made every 24 periods by running the system, using the new policy values of s_i and S_i, for $i = 1,\ldots,L$, and observing the operating characteristics as if these policy values had been in force when the history occurred. Each time there is a policy revision, (every 24 periods), there is a forecast for each operating characteristic. The simulation initializes the stock on hand and on order at the actual value at the time of the forecast, that is, at the time of revision. As actual stock on hand is not recorded until the elapse of a leadtime after revision, the same interval is allowed to elapse before recording it for the forecast. The inventory on hand variable is therefore initialized at the initial value for inventory on hand and on order less the first λ demands in the history of $G+\lambda$ demands.

1.3 A Multi-Item System

Scientific techniques for inventory control are generally applied to systems of many items. This study combines the results from the five 16 single-item systems into five multi-item systems. Since management generally assesses the performance of control techniques for a multi-item inventory system by observing indices that are aggregate operating characteristics [Wagner (1962)], certain aggregate characteristics have been computed.

The operating characteristics of the multi-item system under statistical control have been measured by aggregating the sample values of the corresponding characteristics for each item in the system. When the system is operated under perfect information, these characteristics are computed analytically. The aggregate of average total cost per period is computed as the arithmetic sum of the corresponding costs for each item. The components of total cost for inventory storage, backlog penalty, and ordering replenishments are similarly computed. The aggregate backlog and replenishment frequencies are arithmetic averages of the corresponding frequencies observed for each item in the system. Since the unit inventory holding cost for all items is unity, the average number of units in inventory at period-end is numerically identical to the aggregate average holding cost per period. Finally, a weighted proportion of demand backlogged is computed as the ratio of a weighted sum of the average quantity backlogged per period to a weighted sum of the (exact) mean values of demand. The weights used in both the numerator and denominator of the ratio are the unit cost of backlogging demand for the respective item. Throughout this study the nonstationary systems will be compared to the system with stationary demands.

2. SYSTEM CONTROL WITH FULL INFORMATION ABOUT DEMAND

Section 2.1 contains a detailed explanation of the analytical models used to obtain optimal policies and evaluate the operating characteristics of arbitrary policies of the (s,S) type. Sections 2.2 and 2.3 record information descriptive of single-item and multi-item inventory systems controlled optimally and approximately optimally with full information about demand. The main intention of Section 2 is to establish benchmark values of the systems' operating characteristics for later comparison with systems controlled using only statistical information about demand. The data in this section also illustrates that the Ehrhardt power approximation can be adapted to control nonstationary inventory systems.

2.1 Analytical Models Used to Obtain and Evaluate Optimal Policies

In Section 1.1.1 we denoted the length of the demand distribution cycle by L. Hence if ϕ_i represents a demand density for the i^{th} period in a cycle of L periods, then demand densities in successive periods are

$$\phi_1, \phi_2, \ldots, \phi_L, \phi_1, \phi_2, \ldots, \phi_L, \ldots .$$

Thus, ϕ_1 is the demand density in period 1, ϕ_2 is the demand density in period 2, and ϕ_L is the demand density in period L. In period L+1, the demand density is again ϕ_1, and the cycle repeats. We denote the v+1- fold convolution of successive demand densities by ϕ_i^{v+1}; that is, ϕ_i^{v+1} is the convolution of i and the next v densities that follow i in the cycle, for i = 1,...,L.

When n periods remain in the finite horizon of a single-item inventory model, an optimal (s_n,S_n) policy is obtained by employing a recursive relationship based on the results of Scarf (1960) and Veinott (1966). We utilize the relationship for $n > \lambda$

$$f_n(x) = \min_{y \geq x} \{K\delta(y - x) + H(y,\gamma(n)) + \sum_{\xi=0}^{\infty} f_{n-1}(y - \xi)\phi_{\gamma(n)}(\xi)\}$$

where

x = inventory on hand and on order at the beginning of period n, prior to the ordering decision

y = inventory on hand and on order after the ordering decision in period n but prior to receiving demand in period n

n = number of periods that the item will be stocked

$\gamma(n)$ = the appropriate demand density index in the cycle when n periods remain

$$\delta(z) = \begin{cases} 0 & \text{for } z = 0 \\ 1 & \text{for } z \neq 0 \end{cases}$$

λ = delivery lag

$$H(y,\gamma(n)) = \begin{cases} h\sum_{\xi=0}^{y} (y-\xi)\,\phi^{\lambda+1}_{\gamma(n)}(\xi) + \pi\sum_{\xi>y} (\xi-y)\,\phi^{\lambda+1}_{\gamma(n)}(\xi) & \text{for } y \geq 0 \\ \pi\sum_{\xi>y} (\xi-y)\,\phi^{\lambda+1}_{\gamma(n)}(\xi) & \text{for } y<0 \end{cases}$$

$$f_\lambda(\cdot) \equiv 0.$$

Define

$$G_n(y) = H(y,\gamma(n)) + \sum_{\xi=0}^{\infty} f_{n-1}(y - \xi)\phi_{\gamma(n)}(\xi)$$

S_n = smallest integer that minimizes $G_n(\cdot)$.

s_n = smallest integer such that $G_n(s_n) < K + G_n(S_n)$.

Then

$$f_n(x) = \begin{cases} K + G_n(S_n) & \text{for } x < s_n \\ G_n(x) & \text{for } x \geq s_n, \end{cases}$$

and thus, (s_n, S_n) is an optimal policy when n periods remain.

Our algorithm calculates (s_n, S_n), for $n = \lambda + 1, \lambda + 2 \ldots$, recursively. Due to the cyclic nature of the demand structure, an optimal inventory policy consists of L, possibly distinct, (s,S) policies: that is, one for each period in the cycle. We terminate the algorithm when no further change in the policies occur. The justification for employing this approach is mainly heuristic, since we are not aware of any convergence theorem for this recursive process. In fact, the computer program used to implement the algorithm occasionally does not converge to a set of policies but cycles between sets of policies. An example is given in Table 2.1, which shows S_{12n} cycling between 94 and 95. Note, however, that the difference in total cost per period between these two inventory policies is negligible.

The operating characteristics of a specified inventory policy are calculated by extending the methods of Wagner (1969), who considered the stationary environment, to our cyclic environment. Wagner (1969) solves a Markov chain to determine the stationary distribution of inventory on hand and on order after ordering. These probabilities are denoted

$$r(y), \quad \text{for } y = s, \ldots, S,$$

where (s,S) is a stationary inventory policy in use.

When demand distributions vary periodically, another dimension must be added to the Markov state space, namely, the period in the cycle. Thus, our state space is (y,i) and represents the state of having y units of inventory on hand and on order after ordering at the beginning of the i^{th} period of the demand cycle. Since the policies used each period may vary, y can assume values from s_i to $\overline{S}$, where s_i is the reorder point used in the i^{th} period of the cycle and $\overline{S}$ is the maximum S_i of all L

Table 2.1

Optimal n-Stage Policies (s_n, S_n)

Model II

Mean = 16, π = 99, K = 64, λ = 2

n	s_n	S_n	n	s_n	S_n	n	s_n	S_n
1	56	73	25	61	96	49	61	96
2	72	93	26	71	105	50	71	105
3	88	115	27	87	121	51	87	121
4	92	131	28	92	132	52	92	132
5	85	138	29	85	137	53	85	137
6	68	112	30	68	112	54	68	112
7	59	94	31	59	94	55	59	94
8	54	100	32	54	100	56	54	100
9	55	90	33	55	90	57	55	90
10	56	90	34	56	90	58	56	90
11	55	93	35	55	93	59	55	93
12	55	<u>94</u>	36	55	<u>94</u>	60	55	<u>94</u>
13	61	96	37	61	96	61	61	96
14	71	105	38	71	105	62	71	105
15	87	121	39	87	121	63	87	121
16	92	132	40	92	132	64	92	132
17	85	137	41	85	137	65	85	137
18	68	112	42	68	112	66	68	112
19	59	94	43	59	94	67	59	94
20	54	100	44	54	100	68	54	100
21	55	90	45	55	90	69	55	90
22	56	90	46	56	90	70	56	90
23	55	93	47	55	93	71	55	93
24	55	<u>95</u>	48	55	<u>95</u>	72	55	<u>95</u>

TOTAL COST WITH S_{12n} = 94: 68.39679

TOTAL COST WITH S_{12n} = 95: 68.39684 periods in the cycle. In order to calculate replenishment probabilities, the states (S_i, i), for $i = 1,\ldots,L$, are decomposed into states $(S_i(\text{order}), i)$ and $S_i(\text{no order}), i)$, for $i = 1,\ldots,L$. The state $(S_i(\text{order}), i)$ represents entering period i with inventory on hand and on order before ordering less than s_i, so that an order occurs in period i. The state $(S_i(\text{no order}), i)$ represents already having S_i units of goods on hand and on order before ordering, so that no order occurs in period i.

Due to the cyclic nature of the model, the probability of entering state $(\cdot, j)$, for $1 \le j \le L$, given that the last state is $(\cdot, i)$, for $1 \le i \le L$, is zero for $j \neq i + 1$, unless $i = L$ and $j = 1$. Thus, the general structure of the transition matrix may be presented by examining only the transition probabilities from any period i to period $i + 1$, for $1 \le i \le L - 1$, within the cycle. Such a matrix is shown in Figure 2.1.

The structure depicted in Figure 2.1 assumes that $s_i < s_{i+1} < S_i < S_{i+1}$. Note in Figure 2.1 that the structure of this matrix is similar to an upper triangular structure, when the last $s_{i+1} - s_i$ rows are ignored. If $s_i \geq s_{i+1}$, then the upper triangular structure would appear only in the first $\overline{S} - s_i + 1$ rows of the matrix.

Let

$$r(S_i(\text{no order}), i) \quad \text{for } i = 1,\ldots,L$$

$$r(S_i(\text{order}), i) \quad \text{for } i = 1,\ldots,L$$

$$r(y, i) \quad \text{for } i = 1,\ldots,L \text{ and } y = s_i,\ldots,\overline{S},\ y \neq S_i,$$

denote the stationary probabilities for the Markov chain. For notational convenience, we also define

(y,i) : \ $(y,i+1)$:	$(\bar{S},i+1)$	$(\bar{S}-1,i+1)$	...	$(S_{i+1}$(no order)$,i+1)$	...	$(S_i,i+1)$	...	$(s_{i+1},i+1)$	$(S_{i+1}$(order)$,i+1)$
$(\bar{S},i)$	$\phi_i(0)$	$\phi_i(1)$	...	$\phi_i(\bar{S}-S_{i+1})$	...	$\phi_i(\bar{S}-S_i)$	...	$\phi_i(\bar{S}-s_{i+1})$	$1-\phi_i(\bar{S}-s_{i+1})$
$(\bar{S}-1,i)$		$\phi_i(0)$		$\phi_i(\bar{S}-1-S_{i+1})$	...	$\phi_i(\bar{S}-1-S_i)$	...	$\phi_i(\bar{S}-1-s_{i+1})$	$1-\phi_i(\bar{S}-1-s_{i+1})$
.									
.									
.									
$(S_i$(no order)$,i)$						$\phi_i(0)$	...	$\phi_i(S_i-s_{i+1})$	$1-\phi_i(S_i-s_{i+1})$
.									
.									
.									
(s_{i+1},i)								$\phi_i(0)$	$1-\phi_i(0)$
.									
.									
.									1
									⋮
(s_i,i)									1
$(S_i$(order)$,i)$						$\phi_i(0)$	...	$\phi_i(S_i-s_i+1)$	$1-\phi_i(S_i-s_i+1)$

Figure 2.1 Transition Probabilities

$$r(S_i,i) \equiv r(S_i(\text{order}),i) + r(S_i(\text{no order}),i) \quad \text{for } i = 1,\ldots,L.$$

After solving the Markov chain for $r(\cdot,\cdot)$, we can compute the operating characteristics:

$$\text{Expected Stockout Quantity} = \sum_{i=1}^{L} \sum_{y=s_i}^{\bar{S}} r(y,i)\left[\sum_{Q>y} (Q-y)\phi_i^{\lambda+1}(Q)\right]$$

$$\text{Expected Period-End Inventory} = \sum_{i=1}^{L} \sum_{y=s_i}^{\bar{S}} r(y,i)\left[\sum_{Q=0}^{y} (y-Q)\phi_i^{\lambda+1}(Q)\right]$$

$$\text{Replenishment Frequency} = \sum_{i=1}^{L} r(S_i(\text{order}),i)$$

$$\text{Stockout Probability} = 1 - \sum_{i=1}^{L} \sum_{y=s_i}^{\bar{S}} r(y,i)\left[\sum_{j=0}^{y} \phi_i^{\lambda+1}(y-j)\right]$$

Expected Replenishment Cost = K(Replenishment Frequency)

Expected Holding Cost = h(Expected Period-End Inventory)

Expected Backlog Cost = π(Expected Stockout Quantity)

Expected Total Cost = Expected Replenishment Cost +

Expected Stockout Cost +

Expected Holding Cost.

Note that each of the L periods in the cycle contributes to the expected value of the operating characteristic. Hence, when we refer to, for example, expected total cost, we are not referring to the expected cost incurred in a specific period of the cycle, but rather the average value per period for an arbitrary period in the cycle over the infinite horizon.

In the stationary environment Veinott and Wagner (1965) prove that if an (s,S) policy is optimal, the stockout probability is bounded above by $h/(h + \pi)$. This upper bound is valid also in the cyclic demand environment, as shown next.

Let $(s_i + C,\ S_i + C)$, for $i = 1,\ldots,L$, represent an inventory policy in a cyclic demand environment of cycle length L. The policy $(s_i + C,\ S_i + C)$ is used in the i^{th} period of the cycle. The total expected cost of such an inventory policy over an entire cycle can be expressed as

$$(1) \qquad K \sum_{i=1}^{L} r(S_i + C(\text{order}), i) + \sum_{i=1}^{L} \sum_{y=0}^{\overline{D}_i} H(\overline{S} + C - y, i)\, r(\overline{S} + C - y, i)$$

where

$$\overline{S} = \max_i \{S_i\}$$

$$\overline{D}_i = \overline{S} - s_i, \quad \text{for} \quad i = 1,\ldots,L.$$

We seek an integer C such that total cost with respect to the policy $(s_i + C,\ S_i + C)$, for $i = 1,\ldots,L$, is minimized. Since shifting every (s,S) policy in the cycle by C can not change replenishment frequency, minimizing (1) with respect to C is equivalent to minimizing

$$\sum_{i=1}^{L} \sum_{y=0}^{\overline{D}_i} H(\overline{S} + C - y, i)\, r(\overline{S} + C - y, i)$$

with respect to C.

It is easily verified that $H(\overline{S} + C - y, i)$ is convex in C. Thus we seek the smallest C such that

$$\sum_{i=1}^{L} \sum_{y=0}^{\overline{D}_i} H(\overline{S} + C + 1 - y, i)\, r_1(\overline{S} + C + 1 - y, i) - \sum_{i=1}^{L} \sum_{y=0}^{\overline{D}_i} H(\overline{S} + C - y, i)\, r_2(\overline{S} + C - y, i) \geq 0,$$

where $r_1(\cdot,\cdot)$ and $r_2(\cdot,\cdot)$ are the stationary distributions associated with policies $(s_i + C + 1,\ S_i + C + 1)$, for $i = 1,\ldots,L$, and $(s_i + C,\ S_i + C)$, for $i = 1,\ldots,L$, respectively. But since shifting all the (s,S) policies in the cycle by a constant can not affect the stationary distributions $r_1(z + 1, i)$ and $r_2(z,i)$ are equivalent, for $z = s_i + C,\ldots,\overline{S} + C$ and for all i. Hence, we can use $r'(z,\cdot)$ to denote $r_1(z + 1, \cdot)$ and $r_2(z,\cdot)$.

Veinott and Wagner (1965) express $H(\overline{S} + C + 1 - y, i)$

$- H(S + C - y, i)$ as

$$(h + \pi)\Phi_i^{\lambda+1}(\overline{S} + C - y, i) - \pi,$$

where

$$\Phi_i^{\lambda+1}(z) = \sum_{v=0}^{z} \phi_i^{\lambda+1}(v)$$

Therefore, we seek the smallest integer C such that

$$\sum_{i=1}^{L} \sum_{y=0}^{\overline{D}_i} \{(h + \pi)\Phi_i^{\lambda+1}(\overline{S} + C - y, i)r'(\overline{S} + C - y, i) - \pi r'(\overline{S} + C - y, i)\} \geq 0$$

or

$$\sum_{i=1}^{L} \sum_{y=0}^{\overline{D}_i} \Phi_i^{\lambda+1}(\overline{S} + C - y, i)r'(\overline{S} + C - y, i) \geq \pi/(h + \pi).$$

The left hand side of the above inequality is, for the minimizing value of C, the probability of having 0 or more units of inventory in stock. Hence, the stockout probability is bounded above by $1 - \pi/(h + \pi)$ or $h/(h + \pi)$.

Since this proof is valid for arbitrary (s,S) policies, it is valid for the class of policies $(s_i' + C, S_i' + C)$, for $i = 1,\ldots,L$, where (s_i', S_i'), for $i = 1,\ldots,L$, differ from the optimal policy by C. Hence, the proof remains valid for optimal policies.

2.2 Optimal Control with Full Information: Multi-Item Systems

We let each item in the 16-item systems described in Section 1.2.2 be controlled by using optimal replenishment policies. Table 2.2 lists the resulting expected values of average total cost per period and its components for the 16-item negative binomial systems with variance-to-mean ratio of 3. Each component's percent of total cost is shown in parentheses.

Table 2.2

Average Costs Per Period for a Multi-Item
Negative Binomial System Optimally Controlled

COST COMPONENT	STATIONARY MODEL	MODEL I	MODEL II	MODEL III	MODEL IV
INVENTORY	439 (57.8)	433 (58.0)	438 (58.2)	429 (59.7)	434 (58.6)
BACKLOG	109 (14.4)	106 (14.2)	106 (14.2)	102 (14.1)	104 (14.1)
REPLENISHMENT	211 (27.8)	208 (27.8)	208 (27.6)	188 (26.1)	202 (27.2)
	760 (100.0)	747 (100.0)	752 (100.0)	718 (100.0)	739 (100.0)

Observe that the stationary model's total cost serves as an upper bound for the nonstationary models. Appendices A to E reveal that the total cost of each individual case in the stationary system also provides an upper limit for the corresponding cases in the nonstationary systems. Further experimentation is required to determine if stationary costs can serve as useful upper bounds for nonstationary models. Models III and IV, which have the largest variation in demand means, have the lowest replenishment and total costs of all the models examined.

The apportionment of expected average costs for various classifications of the items in each multi-item system is shown in Table 2.3 which gives the distributions by percentage of the expected average total cost per period. Aggregate values of other operating characteristics for the various classifications are set out in Table 2.4. The classifications are one-way in the sense that items in the system are grouped according to the value taken by a single input parameter.

Table 2.3 reveals that the relative distributions of cost are virtually the same for the five models that we studied. This suggests that the cost parameters, the leadtime, and overall demand mean are the dominant factors in determining the distributions of system cost, whereas the nature of the nonstationarity has little influence on the cost distributions. With the exception of the replenishment frequencies of models III and IV,

Table 2.3a

Apportionment of Aggregate Average Costs Per Period for a Multi-Item System Under Optimal Control with Full Information, for Various Classifications

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	57.8	17.3	40.5	27.1	30.7	26.6	31.2	24.4	33.5
BACKLOG	14.4	8.0	6.3	7.0	7.4	6.6	7.8	6.0	8.4
REPLENISHMENT	27.8	13.2	14.6	11.1	16.7	14.2	13.6	11.4	16.3
TOTAL	100.0	38.6	61.4	45.2	54.8	47.4	52.6	41.8	58.2
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL I									
INVENTORY	58.0	17.4	40.5	27.3	30.7	26.6	31.4	24.6	33.4
BACKLOG	14.2	8.0	6.1	6.9	7.3	6.4	7.7	6.1	8.1
REPLENISHMENT	27.8	13.2	14.7	11.1	16.8	14.2	13.6	11.2	16.6
TOTAL	100.0	38.7	61.3	45.3	54.7	47.3	52.7	41.9	58.1

Table 2.3b

Apportionment of Aggregate Average Costs Per Period for a Multi-Item System Under Optimal Control with Full Information, for Various Classifications

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	57.8	17.3	40.5	27.1	30.7	26.6	31.2	24.4	33.5
BACKLOG	14.4	8.0	6.3	7.0	7.4	6.6	7.8	6.0	8.4
REPLENISHMENT	27.8	13.2	14.6	11.1	16.7	14.2	13.6	11.4	16.3
TOTAL	100.0	38.6	61.4	45.2	54.8	47.4	52.6	41.8	58.2
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL II									
INVENTORY	58.2	17.5	40.7	27.3	30.9	26.8	31.4	24.5	33.7
BACKLOG	14.2	8.0	6.1	6.9	7.3	6.4	7.7	6.0	8.1
REPLENISHMENT	27.6	13.1	14.5	11.1	16.6	14.1	13.6	11.3	16.4
TOTAL	100.0	38.6	61.3	45.2	54.7	47.3	52.7	41.8	58.2

Table 2.3c

Apportionment of Aggregate Average Costs Per Period for a Multi-Item System Under Optimal Control with Full Information, for Various Classifications

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	57.8	17.3	40.5	27.1	30.7	26.6	31.2	24.4	33.5
BACKLOG	14.4	8.0	6.3	7.0	7.4	6.6	7.8	6.0	8.4
REPLENISHMENT	27.8	13.2	14.6	11.1	16.7	14.2	13.6	11.4	16.3
TOTAL	100.0	38.6	61.4	45.2	54.8	47.4	52.6	41.8	58.2
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
INVENTORY	59.7	18.2	41.6	27.8	31.9	27.2	32.6	25.4	34.3
BACKLOG	14.1	8.0	6.2	7.0	7.2	6.4	7.8	6.1	8.0
REPLENISHMENT	26.1	12.6	13.6	10.8	15.4	13.6	12.5	10.5	15.6
TOTAL	100.0	38.7	61.3	45.5	54.5	47.2	52.8	42.0	57.9

Table 2.3d

Apportionment of Aggregate Average Costs Per Period for a Multi-Item System Under Optimal Control with Full Information, for Various Classifications

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	57.8	17.3	40.5	27.1	30.7	26.6	31.2	24.4	33.5
BACKLOG	14.4	8.0	6.3	7.0	7.4	6.6	7.8	6.0	8.4
REPLENISHMENT	27.8	13.2	14.6	11.1	16.7	14.2	13.6	11.4	16.3
TOTAL	100.0	38.6	61.4	45.2	54.8	47.4	52.6	41.8	58.2
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL IV									
INVENTORY	58.6	17.7	41.0	27.6	31.1	26.9	31.7	24.7	34.0
BACKLOG	14.1	8.0	6.1	6.8	7.2	6.4	7.7	6.1	8.0
REPLENISHMENT	27.2	13.0	14.2	10.9	16.4	13.9	13.3	11.2	16.0
TOTAL	100.0	38.7	61.3	45.3	54.7	47.2	52.8	42.0	58.0

Table 2.4a

Operating Characteristics of a Multi-Item System Under
Optimal Control with Full Information, for Various Classifications

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>STATIONARY MODEL</u>									
PERIOD-END INVENTORY	439	132	308	206	233	202	237	185	254
BACKLOG FREQUENCY	.100	.192	.009	.100	.101	.101	.100	.098	.103
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.011	.159	.005	.011	.011	.010	.012	.014	.010
REPLENISHMENT FREQUENCY	.288	.274	.302	.329	.247	.295	.282	.237	.340
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>MODEL I</u>									
PERIOD-END INVENTORY	433	130	302	204	229	199	234	183	249
BACKLOG FREQUENCY	.100	.192	.009	.100	.101	.100	.101	.100	.101
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.011	.156	.005	.010	.011	.010	.012	.014	.009
REPLENISHMENT FREQUENCY	.284	.268	.300	.324	.244	.290	.278	.231	.337

Table 2.4b

Operating Characteristics of a Multi-Item System Under
Optimal Control with Full Information, for Various Classifications

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	439	132	308	206	233	202	237	185	254
BACKLOG FREQUENCY	.100	.192	.009	.100	.101	.101	.100	.098	.103
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.011	.159	.005	.011	.011	.010	.012	.014	.010
REPLENISHMENT FREQUENCY	.288	.274	.302	.329	.247	.295	.282	.237	.340
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL II									
PERIOD-END INVENTORY	438	132	306	205	232	201	236	184	253
BACKLOG FREQUENCY	.100	.192	.009	.100	.100	.100	.101	.100	.101
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.011	.157	.005	.010	.011	.010	.012	.014	.009
REPLENISHMENT FREQUENCY	.284	.270	.299	.326	.243	.290	.279	.232	.337

Table 2.4c

Operating Characteristics of a Multi-Item System Under
Optimal Control with Full Information, for Various Classifications

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	439	132	308	206	233	202	237	185	254
BACKLOG FREQUENCY	.100	.192	.009	.100	.101	.101	.100	.098	.103
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.011	.159	.005	.011	.011	.010	.012	.014	.010
REPLENISHMENT FREQUENCY	.288	.274	.302	.329	.247	.295	.282	.237	.340
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
PERIOD-END INVENTORY	429	130	298	200	229	195	234	182	247
BACKLOG FREQUENCY	.100	.191	.009	.100	.101	.100	.101	.099	.101
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.010	.149	.005	.010	.010	.009	.011	.013	.009
REPLENISHMENT FREQUENCY	.259	.248	.269	.302	.216	.270	.247	.209	.309

Table 2.4d

Operating Characteristics of a Multi-Item System Under
Optimal Control with Full Information, for Various Classifications

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>STATIONARY MODEL</u>									
PERIOD-END INVENTORY	439	132	308	206	233	202	237	185	254
BACKLOG FREQUENCY	.100	.192	.009	.100	.101	.101	.100	.098	.103
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.011	.159	.005	.011	.011	.010	.012	.014	.010
REPLENISHMENT FREQUENCY	.288	.274	.302	.329	.247	.295	.282	.237	.340
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>MODEL IV</u>									
PERIOD-END INVENTORY	434	131	303	204	230	199	234	183	251
BACKLOG FREQUENCY	.100	.191	.009	.100	.100	.100	.101	.100	.101
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.010	.154	.005	.010	.011	.010	.012	.014	.009
REPLENISHMENT FREQUENCY	.275	.264	.287	.314	.236	.281	.269	.227	.323

the aggregate values of operating characteristics in Table 2.4 of the nonstationary systems are similar to those of the stationary model.

The sensitivity of operating characteristics and policy parameters is summarized in Table 2.5, which has been constructed by observing any monotonicity in the relation for a multi-item system. With the exception of the stationary model's decreasing relationship of backlog frequency with leadtime, the relationships of operating characteristics to input parameters are identical for the stationary system and all non-stationary systems, suggesting that the nature of the nonstationarity does not play a major role in the sensitivity of operating characteristics to input parameters.

Policy parameters s and S also follow identical relationships for those systems studied; however, the difference $S-s$, which is denoted by D, does not generally maintain the same relationships for each system. For example, while D increases with respect to leadtime in the stationary system, no such simple relationship exists in any of the nonstationary systems. The only monotonic relationship for D that all systems have in common is that with respect to replenishment cost.

Finally, we explore one other point, an approximation well-known to writers on inventory control, which is potentially of great practical use. As proved in Section 2.1, for an optimally-controlled multi-period inventory model with linear costs, the probability of a stockout occuring is no larger than $h/(h + \pi)$. Inspection of Tables 2.4 shows that the backlog frequencies are close to their upper bounds for every system. For example, the 8 items having $\pi = 4$ in the model I system have a combined relative backlog frequency of .192, compared to the upper bound of .200.

Table 2.5a

Sensitivity of Input Parameters of Operating Characteristics
of an Optimally Controlled Inventory System When
Demand Has a Negative Binomial Distribution

	INPUT PARAMETERS			
	C(OUT)/C(IN)	C(FIX)/C(IN)	LEADTIME	MEAN DEMAND
STATIONARY MODEL				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↓	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↓	↑
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↑	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↑	↓	↓	↑
POLICY PARAMETERS				
D	↓	↑	↑	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑
MODEL I				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↓	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↑	↑
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↑	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↑	↓	↓	↑
POLICY PARAMETERS				
D	↓	↑	?	?
s	↑	↓	↑	↑
S	↑	↑	↑	↑

↑ indicates relation is monotone increasing
↓ indicates relation is monotone decreasing
∿ indicates relation is approximately constant
? indicates no conclusion can be drawn from the data

Table 2.5b

Sensitivity of Input Parameters of Operating Characteristics of an Optimally Controlled Inventory System When Demand Has a Negative Binomial Distribution

	INPUT PARAMETERS			
	C(OUT)/C(IN)	C(FIX)/C(IN)	LEADTIME	MEAN DEMAND
STATIONARY MODEL				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↓	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↓	↑
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↑	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↑	↓	↓	↑
POLICY PARAMETERS				
D	↓	↑	↑	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑
MODEL II				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↓	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↑	↑
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↑	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↑	↓	↓	↑
POLICY PARAMETERS				
D	?	↑	?	?
s	↑	↓	↑	↑
S	↑	↑	↑	↑

↑ indicates relation is monotone increasing
↓ indicates relation is monotone decreasing
∿ indicates relation is approximately constant
? indicates no conclusion can be drawn from the data

Table 2.5c

Sensitivity of Input Parameters of Operating Characteristics of an Optimally Controlled Inventory System When Demand Has a Negative Binomial Distribution

	INPUT PARAMETERS			
	C(OUT)/C(IN)	C(FIX)/C(IN)	LEADTIME	MEAN DEMAND
STATIONARY MODEL				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↓	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↓	↑
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↑	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↑	↓	↓	↑
POLICY PARAMETERS				
D	↓	↑	↑	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑
MODEL III				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↓	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↑	↑
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↑	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↑	↓	↓	↑
POLICY PARAMETERS				
D	?	↑	?	?
s	↑	↓	↑	↑
S	↑	↑	↑	↑

↑ indicates relation is monotone increasing
↓ indicates relation is monotone decreasing
∿ indicates relation is approximately constant
? indicates no conclusion can be drawn from the data

Table 2.5d

Sensitivity of Input Parameters of Operating Characteristics
of an Optimally Controlled Inventory System When
Demand Has a Negative Binomial Distribution

	INPUT PARAMETERS			
	C(OUT)/C(IN)	C(FIX)/C(IN)	LEADTIME	MEAN DEMAND
STATIONARY MODEL				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↓	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↓	↑
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↑	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↑	↓	↓	↑
POLICY PARAMETERS				
D	↓	↑	↑	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑
MODEL IV				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↑	↑
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↑
E(REPLENISHMENT COST)	↓	↑	↑	↓
E(REPLENISHMENT FREQUENCY)	↑	↑	↓	↑
	↑	↓	↓	↑
POLICY PARAMETERS				
D	?	↑	?	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑

↑ indicates relation is monotone increasing
↓ indicates relation is monotone decreasing
∿ indicates relation is approximately constant
? indicates no conclusion can be drawn from the data

2.3 Optimal Control with Full Information: Single-Item Systems

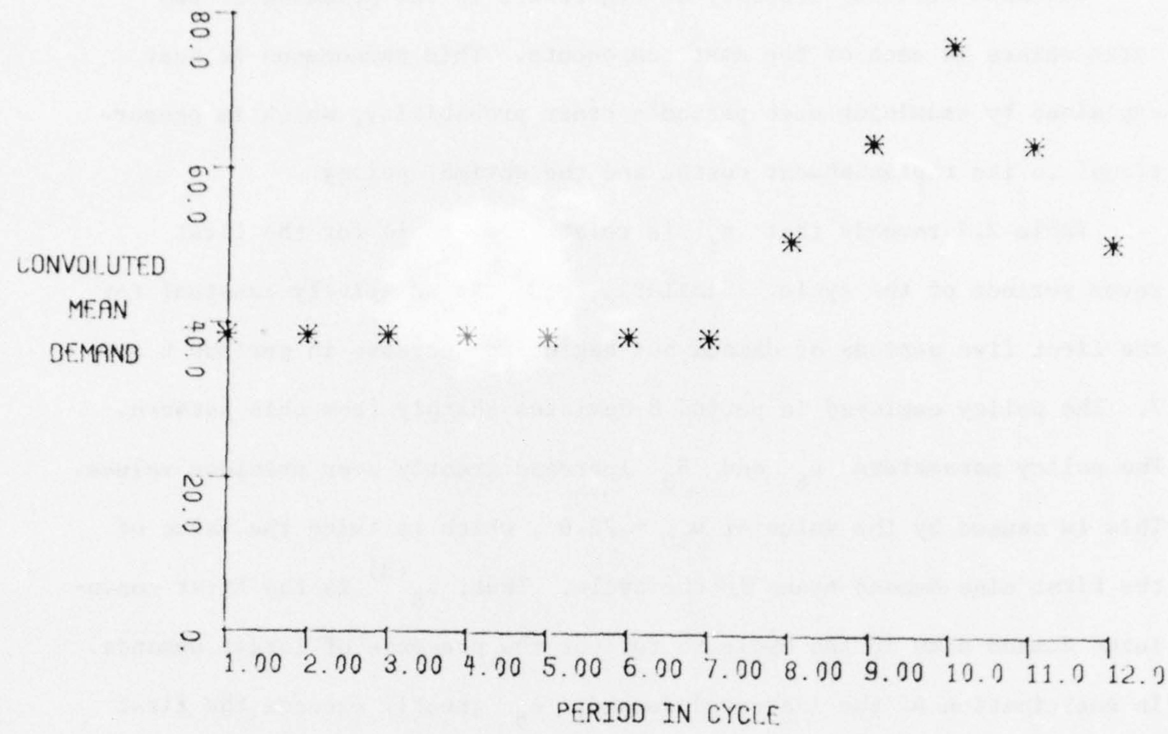
Summary information about the performance of single-item systems under optimal control with full information is tabulated in Appendices A to E. Two single-item cases are selected for the collection of more detailed information. Operating costs are examined for each period of the 12 month cycle. For convenient reference, the cases are labelled A and B, and the input parameters are set out in Table 2.6. These two cases are chosen because they best illustrate the effect of the nonstationary demand structure on the operating characteristics. To facilitate the analysis, Figure 2.2 displays the $\lambda + 1$ fold convoluted demand means for both cases. Note that the convoluted means differ from $\bar{\mu}_i^{(\lambda+1)}$ only by the factor $1/(\lambda + 1)$ and are denoted by $\mu_i^{(\lambda+1)}$.

Table 2.6

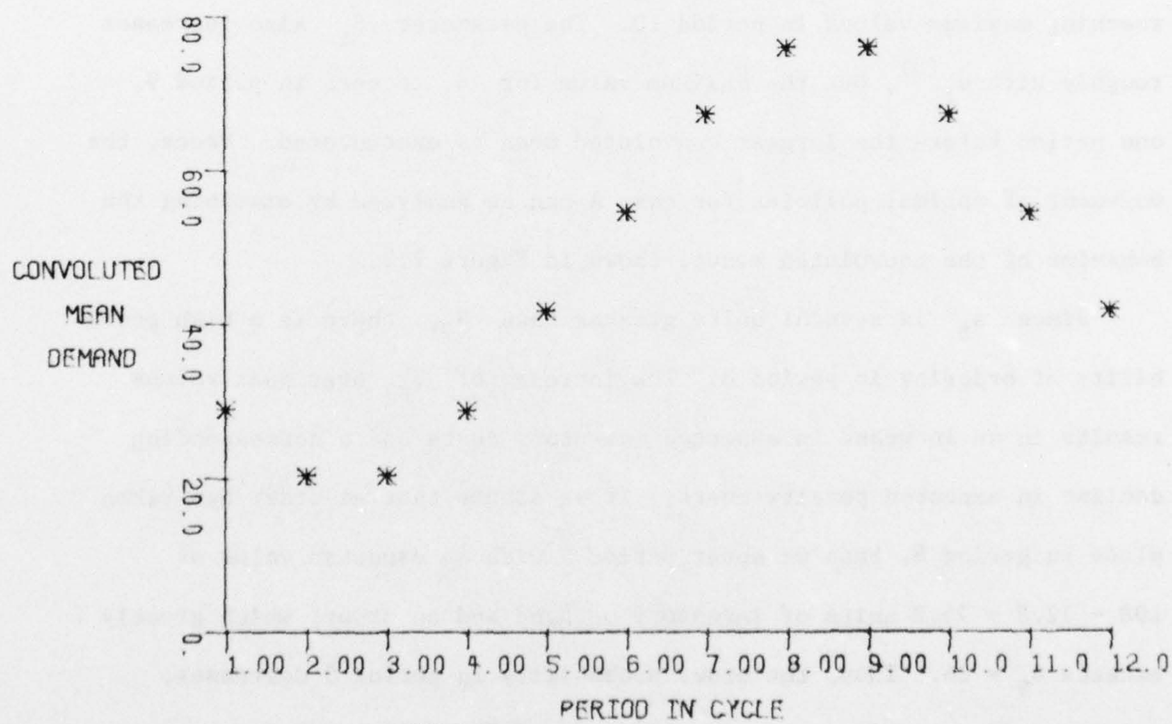
2 Single-Item Cases

Case	Demand Structure	MEAN	LEADTIME	C(FIX)/C(IN)	C(OUT)/C(IN)
A	Model I	16	2	32	99
B	Model IV	16	2	32	99

Figure 2.3 shows the monthly cost contributions to average total cost per period and its cost components for case A. Replenishment costs for month i are the result of orders made at the beginning of month i. Since an order decision at the beginning of month i can not affect the system until the order arrives λ periods later, we associate the backlog and holding costs incurred at the end of period $i + \lambda$ with period i. Thus, backlog and holding costs graphed in Figure 2.3 actually occur λ periods later. Table 2.7 lists case A's optimal (s_i, S_i) policy, for $i = 1,\ldots,12$.



a. Case A



b. Case B

Figure 2.2 Convoluted Mean Demands for Case A and Case B

The most striking property of Figure 2.3 is the presence of two large spikes in each of the cost components. This phenomenon is best explained by examining each period's order probability, which is proportional to its replenishment costs, and the optimal policy.

Table 2.7 reveals that s_i is relatively stable for the first seven periods of the cycle. Similarly, S_i is relatively constant for the first five periods of demand but begins to decrease in periods 6 and 7. The policy employed in period 8 deviates sharply from this pattern. The policy parameters s_8 and S_8 increase greatly over previous values. This is caused by the value of $\mu_{10} = 25.6$, which is twice the value of the first nine demand means in the cycle. Thus, $\mu_8^{(3)}$ is the first convoluted demand mean in the cycle to reflect the presence of larger demands. In anticipation of the increased demand, s_8 greatly exceeds the first seven values of s_i in the cycle, so that the probability of ordering in period 8 increases sharply. As $\mu_i^{(3)}$ increases, so does s_i, both reaching maximum values in period 10. The parameter S_i also increases roughly with $\mu_i^{(3)}$, but the maximum value for S_i occurs in period 9, one period before the largest convoluted mean is encountered. Hence, the movement of optimal policies for case A can be analyzed by examining the behavior of the convoluted means, shown in Figure 2.2.

Since s_8 is several units greater than S_7, there is a high probability of ordering in period 8. The increase of S_8 over past values results in an increase in expected inventory costs and a corresponding decline in expected penalty costs. If we assume that an order has taken place in period 8, then we enter period 9 with an expected value of $108 - 12.8 = 95.2$ units of inventory on hand and on order, which greatly exceeds $s_9 = 86$. Thus, the order probability in period 9 decreases, causing a decrease in expected holding cost and an increase in expected

Table 2.7

Optimal Policy for Case A

Period in Cycle: i	s_i	S_i
1	57	82
2	57	83
3	57	82
4	57	82
5	57	84
6	59	77
7	55	67
8	72	108
9	86	130
10	105	128
11	89	112
12	73	95

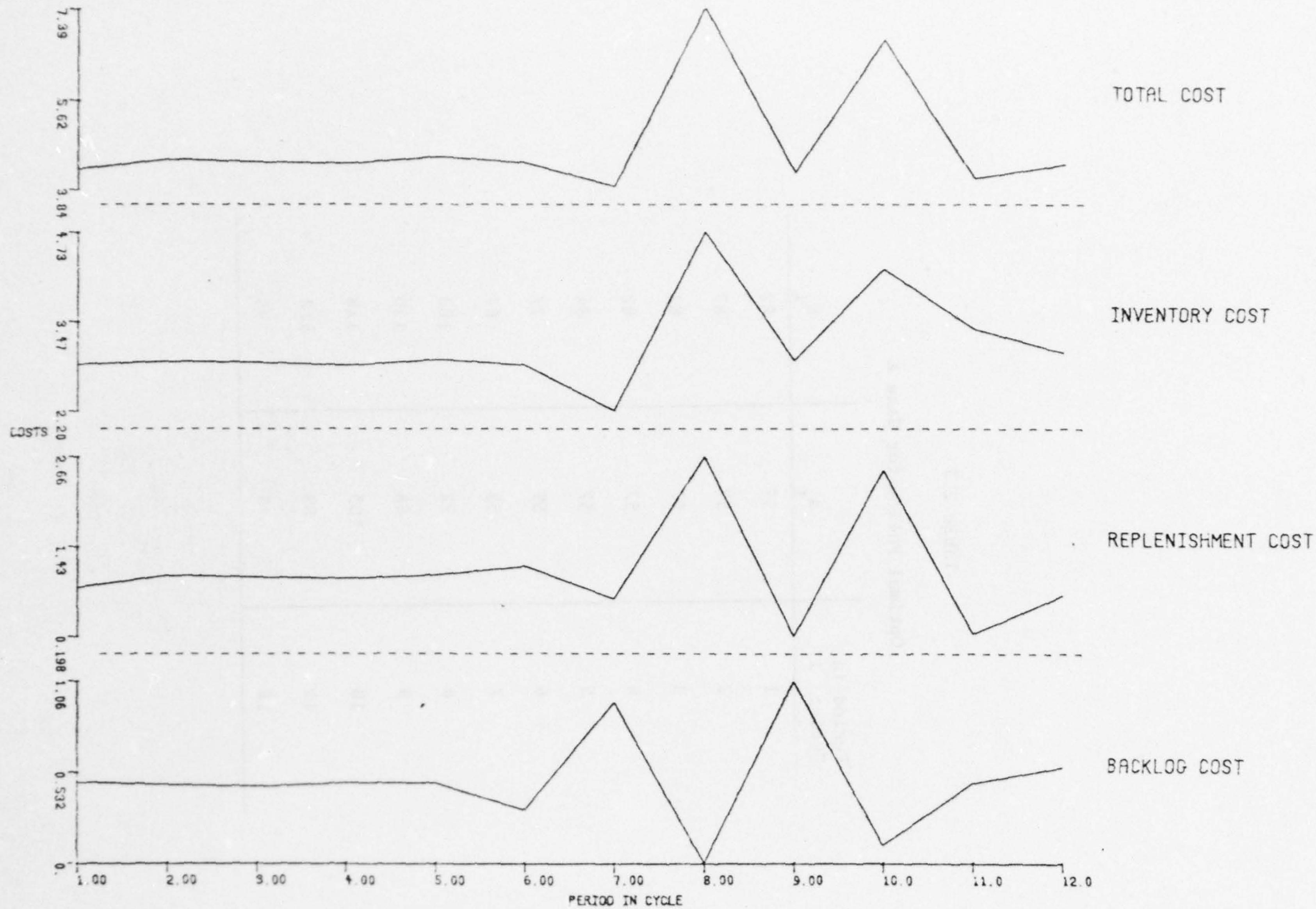


Figure 2.3 Cost Components for Case A Under Optimal Control With Full Information

penalty cost. Assuming that no order is placed in period 9, we then enter period 10 with an expected 95.2 - 12.8 = 82.4 units of inventory on hand and on order, which is significantly less than $s_{10} = 105$. Hence, we again see an increase in order probability that increases expected holding costs and reduces expected backlog costs. After period 10's demand the expected on hand and on order level is, assuming an order occurs in period 10, $S_{10} - 25.6 = 102.4$.

It is important to note that in explaining the structure of the cost components in the demand cycle, we have assumed a deterministic pattern in the order policy. Nevertheless, this deterministic approach approximates the actual probabilistic model well enough to explain the presence of spikes in the cost components. If we were to continue the analysis, we would observe a small probability of ordering in period 11 followed by a larger probability of ordering in period 12. After period 12 the (s,S) policies and mean period demands are stable. This stability is reflected in the cost components which are approximately constant in periods 1 through 6.

The underlying mean demand structure of case B, shown in Figure 1.1.e, exhibits considerably more gradual change than is observed in case A. This slower movement results in a gradual change in the optimal policies and their cost components. Table 2.8 displays the optimal policy for case B. Note that s_i and S_i attain their lowest values in periods 2 and 3 and highest values in periods 8 and 9. The convoluted mean, $\mu_i^{(3)}$, plotted in Figure 2.2.b, attains its minimum and maximum values in the same periods, respectively. The policy parameters s_i and S_i follow a sinusoidal pattern, similar to that of $\mu_i^{(3)}$.

The 12 period cost components, displayed in Figure 2.4, also follow the sinusoidal pattern of $\mu_i^{(3)}$. The cost components do not reflect the

Table 2.8

Optimal Policy for Case B

Period in Cycle: i	s_i	S_i
1	46	61
2	33	55
3	33	56
4	44	73
5	60	93
6	75	113
7	90	128
8	101	136
9	102	131
10	93	115
11	78	95
12	62	76

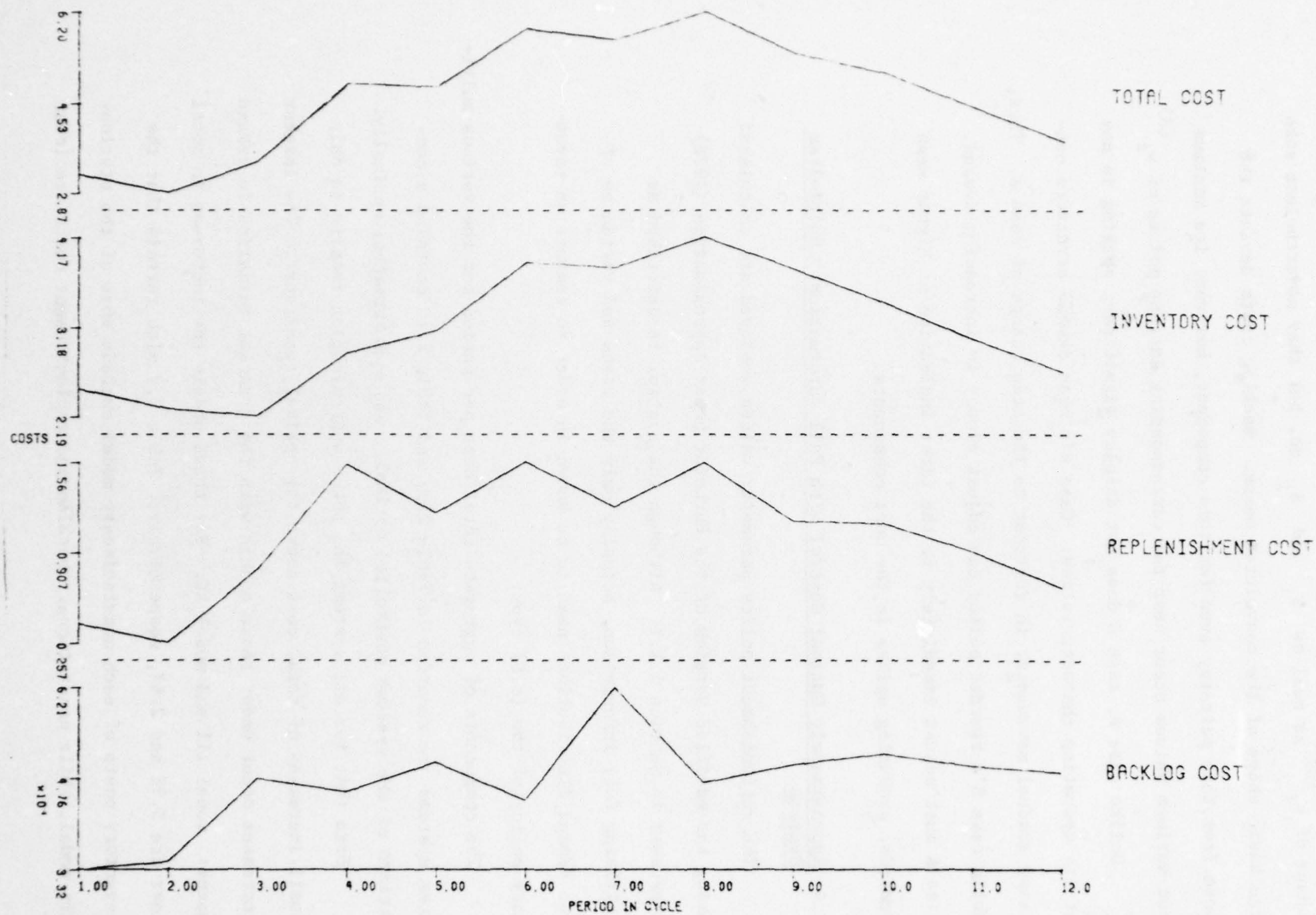


Figure 2.4 Cost Components for Case B Under Optimal Control With Full Information

shape of $\mu_i^{(3)}$ as well as s_i and S_i do, but they nevertheless echo the basic shape of the convoluted means. Backlog costs deviate the most from this pattern; even for this component, however, its minimum and maximum values occur near the corresponding extreme points of $\mu_i^{(3)}$.

Unlike case A, case B does not display significant spiking in any of its operating characteristics. Case B's mean demand structure contains gradual movements, in contrast to the step change of case A. Thus, while case B's reorder policy can adjust slowly to increasing demand, case A must adjust immediately to the first indication of higher mean demands, producing spikes in the cost components.

2.4 Approximately Optimal Control with Full Information: Multi-Item Systems

The replenishment policy parameter values employed are calculated using the modified version of the Ehrhardt power approximation (1976) discussed in Section 1.1.2. Although the control is described as utilizing full information, actually only the means and variances of the demand distributions need to be known in order to compute an inventory policy of the (s,S) type.

The components of aggregate total cost per period for the various multi-item systems are recorded in Table 2.9, and Table 2.10 contains a comparison of the systems controlled optimally and approximately optimally.

Note that for all systems the power approximation results in only small increases of total cost above the optimal total cost. The largest increases occur under those models with the greatest variation in demand means: model III and model IV. For these models the increases in total cost are 5.8% and 2.6%, respectively. Table 2.9 also reveals that the inventory costs of each nonstationary model exceeds that of the stationary model, while nonstationary backlog and replenishment costs are lower than those of the stationary model. These differences are most noticeable when contrasting models III and IV with the stationary model.

Table 2.9

Average Costs Per Period for a Multi-Item
Negative Binomial System Controlled Approximately Optimally

COST COMPONENT	STATIONARY MODEL	MODEL I	MODEL II	MODEL III	MODEL IV
INVENTORY	440 (57.8)	445 (58.8)	442 (58.4)	466 (61.3)	461 (60.7)
BACKLOG	109 (14.3)	105 (13.8)	106 (14.0)	95 (12.5)	97 (12.8)
REPLENISHMENT	212 (27.8)	207 (27.3)	209 (27.6)	199 (26.2)	201 (26.5)
TOTAL	761 (100.0)	757 (100.0)	758 (100.0)	760 (100.0)	759 (100.0)

Note: Numbers in parentheses are percent of total.

Table 2.10

Average Costs Per Period for a Multi-Item
Negative Binomial System
Comparison of Optimal and Approximately Optimal Control

COST COMPONENT	STATIONARY MODEL	MODEL I	MODEL II	MODEL III	MODEL IV
INVENTORY	1 (.1)	12 (2.9)	5 (1.1)	37 (8.7)	27 (6.3)
BACKLOG	0 (-.2)	-1 (-1.0)	0 (-.2)	-7 (-6.5)	-7 (-7.0)
REPLENISHMENT	1 (.3)	-1 (-.5)	1 (.5)	11 (6.0)	0 (-.2)
TOTAL	1 (.1)	10 (1.4)	6 (.8)	42 (5.8)	19 (2.6)

Note: Table 2.10 shows the absolute increase or decrease in the cost components of Table 2.9 over those in Table 2.2 with percent changes in parentheses.

Table 2.10 indicates that the nonstationary systems maintain higher levels of inventory than that found in the corresponding optimal systems which result in lower backlog costs under approximately optimal control. This is most apparent in models III and IV.

The apportionment of aggregate total cost per period for various classifications of the items in each system is shown in Table 2.11, which can be compared with Table 2.3. Observe that all systems follow approximately the same apportionment of costs. Model III's and model IV's

Table 2.11a

Apportionment of Aggregate Average Costs Per Period for a Multi-Item System Under Approximately Optimal Control with Full Information, for Various Classifications

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	57.8	16.3	41.5	27.6	30.2	26.7	31.1	24.0	33.8
BACKLOG	14.3	8.4	6.0	6.5	7.8	6.5	7.8	6.3	8.1
REPLENISHMENT	27.8	13.9	13.9	11.1	16.7	14.1	13.7	11.5	16.3
TOTAL	100.0	38.6	61.4	45.2	54.8	47.4	52.6	41.8	58.2
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL I									
INVENTORY	58.8	16.8	42.1	27.9	30.9	27.3	31.5	24.5	34.3
BACKLOG	13.8	8.1	5.8	6.5	7.4	6.3	7.6	6.1	7.8
REPLENISHMENT	27.3	13.8	13.6	10.8	16.5	13.8	13.5	11.4	15.9
TOTAL	100.0	38.6	61.9	45.2	54.8	47.4	52.6	41.9	58.0

Table 2.11b

Apportionment of Aggregate Average Costs Per Period for a Multi-Item System Under Approximately Optimal Control with Full Information, for Various Classifications

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	57.8	16.3	41.5	27.6	30.2	26.7	31.1	24.0	33.8
BACKLOG	14.3	8.4	6.0	6.5	7.8	6.5	7.8	6.3	8.1
REPLENISHMENT	27.8	13.9	13.9	11.[illegible]	16.7	14.1	13.7	11.5	16.3
TOTAL	100.0	38.6	61.4	45.2	54.8	47.4	52.6	41.8	58.2
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL II									
INVENTORY	58.4	16.6	41.8	27.7	30.7	26.9	31.5	24.4	34.0
BACKLOG	14.0	8.2	5.8	6.5	7.5	6.4	7.6	6.1	7.9
REPLENISHMENT	27.6	13.8	13.7	11.0	16.6	14.0	13.6	11.4	16.1
TOTAL	100.0	38.6	61.4	45.2	54.8	47.3	52.7	41.9	58.1

Table 2.11c

Apportionment of Aggregate Average Costs Per Period for a Multi-Item System Under Approximately Optimal Control with Full Information, for Various Classifications

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	57.8	16.3	41.5	27.6	30.2	26.7	31.1	24.0	33.8
BACKLOG	14.3	8.4	6.0	6.5	7.8	6.5	7.8	6.3	8.1
REPLENISHMENT	27.8	13.9	13.9	11.1	16.7	14.1	13.7	11.5	16.3
TOTAL	100.0	38.6	61.4	45.2	54.8	47.4	52.6	41.8	58.2
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
INVENTORY	61.3	17.5	43.8	28.7	32.6	27.6	33.7	26.6	34.7
BACKLOG	12.5	7.3	5.2	5.8	6.7	6.0	6.5	5.0	7.5
REPLENISHMENT	26.2	13.3	12.9	11.1	15.0	13.5	12.7	10.4	15.7
TOTAL	100.0	38.1	62.0	45.6	54.4	47.1	52.9	42.0	58.0

Table 2.11d

Apportionment of Aggregate Average Costs Per Period for a Multi-Item System Under Approximately Optimal Control with Full Information, for Various Classifications

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>STATIONARY MODEL</u>									
INVENTORY	57.8	16.3	41.5	27.6	30.2	26.7	31.1	24.0	33.8
BACKLOG	14.3	8.4	6.0	6.5	7.8	6.5	7.8	6.3	8.1
REPLENISHMENT	27.8	13.9	13.9	11.1	16.7	14.1	13.7	11.5	16.3
TOTAL	100.0	38.6	61.4	45.2	54.8	47.4	52.6	41.8	58.2
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>MODEL IV</u>									
INVENTORY	60.7	17.5	43.2	28.6	32.1	28.0	32.7	25.6	35.1
BACKLOG	12.8	7.5	5.3	6.0	6.7	5.8	7.0	5.5	7.3
REPLENISHMENT	26.5	13.4	13.0	10.4	16.1	13.3	13.2	11.0	15.5
TOTAL	100.0	38.4	61.6	45.1	54.9	47.1	52.9	42.1	57.9

Table 2.12a

Percentage Excess of Costs Per Period of a Multi-Item System Under Approximately Optimal Control Over the Costs of the Same System Under Optimal Control

COMPONENT	TOTAL	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	.1	-5.7	2.6	1.9	-1.4	.6	-.2	-1.3	1.2
BACKLOG	-.2	4.2	-5.7	-6.9	6.2	-1.0	.6	4.1	-3.2
REPLENISHMENT	.3	5.4	-4.2	.4	.3	-.1	.8	1.2	-.2
TOTAL	.1	.2	.1	.2	.1	.1	.2	.1	.1
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL I									
INVENTORY	2.9	-2.6	5.2	3.7	2.1	4.0	2.0	1.1	4.2
BACKLOG	-1.0	1.6	-4.4	-4.9	2.8	-1.2	-.7	1.2	-2.6
REPLENISHMENT	-.5	6.0	-6.4	-1.1	-.2	-1.7	.6	2.7	-2.7
TOTAL	1.4	1.2	1.5	1.2	1.5	1.6	1.2	1.5	1.3

Table 2.12b

Percentage Excess of Costs Per Period of a Multi-Item System Under Approximately Optimal Control Over the Costs of the Same System Under Optimal Control

COMPONENT	TOTAL	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	.1	-5.7	2.6	1.9	-1.4	.6	-.2	-1.3	1.2
BACKLOG	-.2	4.2	-5.7	-6.9	6.2	-1.0	.6	4.1	-3.2
REPLENISHMENT	.3	5.4	-4.2	.4	.3	-.1	.8	1.2	-.2
TOTAL	.1	.2	.1	.2	.1	.1	.2	.1	.1
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL II									
INVENTORY	1.1	-4.3	3.5	2.3	.1	1.2	1.0	.2	1.8
BACKLOG	-.2	2.3	-3.5	-4.8	4.1	.7	-1.0	1.3	-1.4
REPLENISHMENT	.5	6.4	-4.7	-.2	1.1	-.1	1.2	2.4	-.7
TOTAL	.8	.7	.8	.6	.9	.8	.8	1.0	.6

Table 2.12c

Percentage Excess of Costs Per Period of a Multi-Item System Under Approximately Optimal Control Over the Costs of the Same System Under Optimal Control

COMPONENT	TOTAL	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	.1	-5.7	2.6	1.9	-1.4	.6	-.2	-1.3	1.2
BACKLOG	-.2	4.2	-5.7	-6.9	6.2	-1.0	.6	4.1	-3.2
REPLENISHMENT	.3	5.4	-4.2	.4	.3	-.1	.8	1.2	-.2
TOTAL	.1	.2	.1	.2	.1	.1	.2	.1	.1
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
INVENTORY	8.7	2.1	11.6	9.1	8.3	7.7	9.5	11.0	7.0
BACKLOG	-6.5	-3.2	-10.9	-12.0	-1.2	-1.1	-11.0	-14.7	-.2
REPLENISHMENT	6.0	11.8	.7	9.6	3.5	4.9	7.3	4.8	6.8
TOTAL	5.8	4.2	6.9	6.0	5.7	5.7	6.0	5.7	6.0

Table 2.12d

Percentage Excess of Costs Per Period of a Multi-Item System Under Approximately Optimal Control Over the Costs of the Same System Under Optimal Control

COMPONENT	TOTAL	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	.1	-5.7	2.6	1.9	-1.4	.6	-.2	-1.3	1.2
BACKLOG	-.2	4.2	-5.7	-6.9	6.2	-1.0	.6	4.1	-3.2
REPLENISHMENT	.3	5.4	-4.2	.4	.3	-.1	.8	1.2	-.2
TOTAL	.1	.2	.1	.2	.1	.1	.2	.1	.1
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL IV									
INVENTORY	6.3	1.5	8.3	6.6	5.9	6.7	5.8	6.5	6.1
BACKLOG	-7.0	-3.9	-11.1	-9.3	-4.8	-7.0	-7.0	-7.5	-6.6
REPLENISHMENT	-.2	6.1	-5.9	-1.6	.7	-1.7	1.4	.8	-.9
TOTAL	2.6	1.9	3.1	2.2	3.0	2.4	2.8	2.9	2.4

apportionment of inventory costs, however, are slightly greater than the other systems. Detailed comparison of the systems under optimal and approximately optimal control, expanding on Table 2.10, is in Table 2.12, showing the percentage increases per period in each component of cost.

Tables 2.12a and 2.12b show that under low unit penalty costs, the stationary model, model I, and model II incur lower than optimal inventory costs at the expense of higher backlog costs. This strategy is reversed when the unit penalty cost is 99. Under the higher penalty cost, these systems carry excess inventory to lessen the incidence of backlogs. Models III and IV, however, carry excess inventory regardless of the value of unit penalty cost, although small penalty costs result in smaller excess inventory levels than do larger penalty costs.

Table 2.13 gives the values of other operating characteristics for the systems under approximately optimal control. The values of these operating characteristics for model III and model IV are different from those of the remaining systems. Model III and IV's high inventory levels are distributed differently from the other systems, and their backlog frequencies are less than those found in the stationary model, model I, and model II. These differences are also apparent in Table 2.14, which gives the percentage changes in the values of the operating characteristics in Table 2.13 from the corresponding ones in Table 2.4.

Table 2.15 illustrates the sensitivity of operating characteristics and policy parameters to input parameters. With the exception of backlog frequency with respect to leadtime, the operating characteristics of all four nonstationary systems follow identical relationships over their input parameters. The stationary model, model I and model

Table 2.13a

Operating Characteristics of a Multi-Item System Under Approximately Optimal Control with Full Information, for Various Classifications

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	440	124	316	210	230	203	237	183	257
BACKLOG FREQUENCY	.105	.200	.009	.101	.108	.103	.106	.106	.103
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.011	.166	.005	.010	.012	.010	.012	.014	.009
REPLENISHMENT FREQUENCY	.289	.289	.289	.331	.248	.293	.286	.240	.338
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL I									
PERIOD-END INVENTORY	445	127	318	211	234	206	239	185	260
BACKLOG FREQUENCY	.102	.195	.009	.100	.104	.101	.102	.105	.099
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.010	.159	.004	.010	.011	.010	.012	.014	.009
REPLENISHMENT FREQUENCY	.282	.284	.280	.320	.244	.285	.279	.236	.328

Table 2.13b

Operating Characteristics of a Multi-Item System Under Approximately Optimal Control with Full Information, for Various Classifications

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	440	124	316	210	230	203	237	183	257
BACKLOG FREQUENCY	.105	.200	.009	.101	.108	.103	.106	.106	.103
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.011	.166	.005	.010	.012	.010	.012	.014	.009
REPLENISHMENT FREQUENCY	.289	.289	.289	.331	.248	.293	.286	.240	.338
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL II									
PERIOD-END INVENTORY	442	126	317	210	233	204	239	185	158
BACKLOG FREQUENCY	.103	.197	.009	.100	.105	.103	.103	.105	.100
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.011	.161	.005	.010	.011	.010	.012	.014	.009
REPLENISHMENT FREQUENCY	.285	.287	.284	.325	.246	.289	.281	.237	.333

Table 2.13c

Operating Characteristics of a Multi-Item System Under Approximately Optimal Control with Full Information, for Various Classifications

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	440	124	316	210	230	203	237	183	257
BACKLOG FREQUENCY	.105	.200	.009	.101	.108	.103	.106	.106	.103
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.011	.166	.005	.010	.012	.010	.012	.014	.009
REPLENISHMENT FREQUENCY	.289	.289	.289	.331	.248	.293	.286	.240	.338
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
PERIOD-END INVENTORY	466	133	333	218	248	210	256	202	264
BACKLOG FREQUENCY	.093	.178	.008	.091	.094	.094	.091	.089	.096
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.010	.145	.004	.009	.010	.009	.010	.011	.009
REPLENISHMENT FREQUENCY	.277	.280	.274	.331	.223	.290	.264	.216	.338

Table 2.13d

Operating Characteristics of a Multi-Item System Under Approximately Optimal Control with Full Information, for Various Classifications

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	440	124	316	210	230	203	237	183	257
BACKLOG FREQUENCY	.105	.200	.009	.101	.108	.103	.106	.106	.103
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.011	.166	.005	.010	.012	.010	.012	.014	.009
REPLENISHMENT FREQUENCY	.289	.289	.289	.331	.248	.293	.286	.240	.338
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL IV									
PERIOD-END INVENTORY	461	133	328	217	243	212	248	194	266
BACKLOG FREQUENCY	.095	.181	.008	.093	.096	.094	.095	.095	.094
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.010	.148	.004	.009	.010	.009	.011	.012	.008
REPLENISHMENT FREQUENCY	.274	.278	.269	.309	.238	.276	.272	.229	.318

Table 2.14a

Percentage Excess of Operating Characteristics of a Multi-Item System Under Approximately Optimal Control Over the Same Characteristics for the System Under Optimal Control

OPERATING CHARACTERISTICS	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>STATIONARY MODEL</u>									
PERIOD-END INVENTORY	.1	-5.7	2.6	1.9	-1.4	.6	-.2	-1.3	1.2
BACKLOG FREQUENCY	4.2	4.6	-5.3	.9	7.4	1.8	6.5	7.7	.8
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	-.2	4.2	-5.7	-6.9	6.2	-1.0	.6	4.1	-3.2
REPLENISHMENT FREQUENCY	.4	5.5	-4.3	.4	.3	-.6	1.3	1.5	-.4
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>MODEL I</u>									
PERIOD-END INVENTORY	2.9	-2.6	5.2	3.7	2.1	4.0	2.0	1.1	4.2
BACKLOG FREQUENCY	1.2	1.6	-6.2	-.7	3.2	.7	1.7	4.7	-2.3
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	-1.0	1.6	-4.4	-4.9	2.8	-1.2	-.7	1.2	-2.6
REPLENISHMENT FREQUENCY	-.7	5.9	-6.6	-1.1	-.2	-1.7	.4	2.2	-2.6

Table 2.14b

Percentage Excess of Operating Characteristics of a Multi-Item System Under Approximately Optimal Control Over the Same Characteristics for the System Under Optimal Control

OPERATING CHARACTERISTICS	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>STATIONARY MODEL</u>									
PERIOD-END INVENTORY	.1	-5.7	2.6	1.9	-1.4	.6	-.2	-1.3	1.2
BACKLOG FREQUENCY	4.2	4.6	-5.3	.9	7.4	1.8	6.5	7.7	.8
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	-.2	4.2	-5.7	-6.9	6.2	-1.0	.6	4.1	-3.2
REPLENISHMENT FREQUENCY	.4	5.5	-4.3	.4	.3	-.6	1.3	1.5	-.4
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>MODEL II</u>									
PERIOD-END INVENTORY	1.1	-4.3	3.5	2.3	.1	1.2	1.0	.2	1.8
BACKLOG FREQUENCY	2.3	2.7	-4.8	0.0	4.6	2.6	2.0	5.5	-.8
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	-.2	2.3	-3.5	-4.8	4.1	.7	-1.0	1.3	-1.4
REPLENISHMENT FREQUENCY	.3	6.1	-4.9	-.2	1.1	-.3	.9	2.4	-1.1

Table 2.14c

Percentage Excess of Operating Characteristics of a Multi-Item System Under Approximately Optimal Control Over the Same Characteristics for the System Under Optimal Control

OPERATING CHARACTERISTICS	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	.1	-5.7	2.6	1.9	-1.4	.6	-.2	-1.3	1.2
BACKLOG FREQUENCY	4.2	4.6	-5.3	.9	7.4	1.8	6.5	7.7	.8
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	-.2	4.2	-5.7	-6.9	6.2	-1.0	.6	4.1	-3.2
REPLENISHMENT FREQUENCY	.4	5.5	-4.3	.4	.3	-.6	1.3	1.5	-.4
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
PERIOD-END INVENTORY	8.7	2.1	11.6	9.1	8.3	7.7	9.5	11.0	7.0
BACKLOG FREQUENCY	-7.3	-6.8	-17.7	-8.5	-6.2	-5.0	-9.6	-10.2	-4.5
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	-6.5	-3.2	-10.9	-12.0	-1.2	-1.1	-11.0	-14.7	-.2
REPLENISHMENT FREQUENCY	7.1	12.8	1.8	10.0	3.5	7.4	6.7	3.6	9.4

Table 2.14d

Percentage Excess of Operating Characteristics of a Multi-Item System Under Approximately Optimal Control Over the Same Characteristics for the System Under Optimal Control

OPERATING CHARACTERISTICS	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>STATIONARY MODEL</u>									
PERIOD-END INVENTORY	.1	-5.7	2.6	1.9	-1.4	.6	-.2	-1.3	1.2
BACKLOG FREQUENCY	4.2	4.6	-5.3	.9	7.4	1.8	6.5	7.7	.8
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	-.2	4.2	-5.7	-6.9	6.2	-1.0	.6	4.1	-3.2
REPLENISHMENT FREQUENCY	.4	5.5	-4.3	.4	.3	-.6	1.3	1.5	-.4
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>MODEL IV</u>									
PERIOD-END INVENTORY	6.3	1.5	8.3	6.6	5.9	6.7	5.8	6.5	6.1
BACKLOG FREQUENCY	-5.6	-5.2	-15.0	-6.8	-4.4	-5.4	-5.9	-4.8	-6.4
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	-7.0	-3.9	-11.1	-9.3	-4.8	-7.0	-7.0	-7.5	-6.6
REPLENISHMENT FREQUENCY	-.6	5.3	-6.0	-1.6	.8	-2.0	.9	.8	-1.6

IV have increasing backlog frequencies with respect to leadtime, whereas models II and III have decreasing backlog frequencies. Note also that unlike the stationary model, the nonstationary models do not follow constant relationships with respect to unit penalty costs for replenishment costs and replenishment frequency.

Policy parameters s and S follow identical relationships in all systems under approximately optimal control. These relationships also are in agreement with those observed under optimal control. Policy parameter D, under approximately optimal control, follows an increasing relationship with respect to mean demand in all systems, except in model III for those items with a replenishment cost of 32 and a leadtime of 2. Further examination of model III reveals that the largest values for $\bar{\mu}_i^{(\lambda+1)}$ in all the nonstationary systems occur in that model. The power approximation first computes D using equation (1) in Section 1.1.2; $D/\bar{\mu}_i^{(\lambda+1)}$ is a measure of order frequency, and if this quantity is less than or equal to 1.5, the algorithm attempts to find a smaller value for D that will increase order frequency [see Ehrhardt (1976)]. Under small leadtimes and unit replenishment costs, model III's demand structure occasionally resulted in small values for D that were not monotone with respect to mean demand for large $\bar{\mu}_i^{(\lambda+1)}$.

Finally, we examine the ratio $1/(1 + \pi)$ as an approximation to the probability of backlog in any period for each item (recall $h = 1$). The entries in Table 2.16 have been extracted from Table 2.13. The approximation is a reasonable one for the stationary model and models I and II. For models III and IV, however, the actual backlog probability is slightly overstated by $1/(1 + \pi)$.

Table 2.15a

Sensitivity of Input Parameters of Operating Characteristics of an Approximately Optimally Controlled Inventory System When Demand Has a Negative Binomial Distribution

	INPUT PARAMETERS			
	C(OUT)/C(IN)	C(FIX)/C(IN)	LEADTIME	MEAN DEMAND
STATIONARY MODEL				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↓	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↑	↓
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	∿	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	∿	↓	↓	↑
POLICY PARAMETERS				
D	∿	↑	↑	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑
MODEL I				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↓	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↑	↓
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↓	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↓	↓	↓	↑
POLICY PARAMETERS				
D	∿	↑	?	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑

↑ indicates relation is monotone increasing
↓ indicates relation is monotone decreasing
∿ indicates relation is approximately constant
? indicates no conclusion can be drawn from the data

Table 2.15b

Sensitivity of Input Parameters of Operating Characteristics of an Approximately Optimally Controlled Inventory System When Demand Has a Negative Binomial Distribution

	INPUT PARAMETERS			
	C(OUT)/C(IN)	C(FIX)/C(IN)	LEADTIME	MEAN DEMAND
STATIONARY MODEL				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↓	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↑	↓
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	∿	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	∿	↓	↓	↑
POLICY PARAMETERS				
D	∿	↑	↑	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑
MODEL II				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↓	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↓	↓
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↓	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↓	↓	↓	↑
POLICY PARAMETERS				
D	∿	↑	?	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑

↑ indicates relation is monotone increasing
↓ indicates relation is monotone decreasing
∿ indicates relation is approximately constant
? indicates no conclusion can be drawn from the data

Table 2.15c

Sensitivity of Input Parameters of Operating Characteristics of an Approximately Optimally Controlled Inventory System When Demand Has a Negative Binomial Distribution

	INPUT PARAMETERS			
	C(OUT)/C(IN)	C(FIX)/C(IN)	LEADTIME	MEAN DEMAND
STATIONARY MODEL				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↓	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↑	↓
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	∿	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	∿	↓	↓	↑
POLICY PARAMETERS				
D	∿	↑	↑	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑
MODEL III				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↓	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↓	↑
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↓	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↓	↓	↓	↑
POLICY PARAMETERS				
D	∿	↑	?	*
s	↑	↓	↑	↑
S	↑	↑	↑	↑

↑ indicates relation is monotone increasing
↓ indicates relation is monotone decreasing
∿ indicates relation is approximately constant
? indicates no conclusion can be drawn from the data
* indicates an explanation is given in the text

Table 2.15d

Sensitivity of Input Parameters of Operating Characteristics of
an Approximately Optimally Controlled Inventory System
When Demand Has a Negative Binomial Distirbution

	INPUT PARAMETERS			
	C(OUT)/C(IN)	C(FIX)/C(IN)	LEADTIME	MEAN DEMAND
STATIONARY MODEL				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↓	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↑	↓
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	∿	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	∿	↓	↓	↑
POLICY PARAMETERS				
D	∿	↑	↑	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑
MODEL IV				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↓	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↑	↓
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↓	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↓	↓	↓	↑
POLICY PARAMETERS				
D	∿	↑	?	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑

↑ indicates relation is monotone increasing
↓ indicates relation is monotone decreasing
∿ indicates relation is approximately constant
? indicates no conclusion can be drawn from the data

Table 2.16

Backlog Frequencies for Multi-Item Systems
With Negative Binomial Demands

	C(OUT)/C(IN)	$(=\pi)$
	4	99
$1/(1+\pi)$	.2	.01
SYSTEM:		
Stationary Model	.200	.009
Model I	.195	.009
Model II	.197	.009
Model III	.178	.008
Model IV	.181	.008

2.5 Approximately Optimal Control with Full Information: Single-Item Systems

The approximate policies of case A, displayed in Table 2.17, follow the same pattern observed in the optimal policy. Case A's power approximation policy maintains a constant policy for the first seven periods of demand. Both s_i and S_i increase in periods 8 through 10 and decrease in periods 11 and 12, following the similar pattern of the optimal policy. Comparing Table 2.17 with Table 2.7 reveals that although optimal and approximately optimal policies follow the same basic patterns, the two policies' values for S_i may differ a great deal in some periods of the cycle. The total percent excess cost for case A, however, is only 1.86%, suggesting that acceptable inventory policies may differ a good deal from optimal policies.

Figure 2.5 plots the cost components for Case A. Holding and replenishment costs govern the general shape of the total cost curve. Unlike

Table 2.17

Approximately Optimal Policy for Case A

Period in Cycle: i	s_i	S_i
1	57 (57)	85 (82)
2	57 (57)	85 (83)
3	57 (57)	85 (82)
4	57 (57)	85 (82)
5	57 (57)	85 (84)
6	57 (59)	85 (77)
7	57 (55)	85 (67)
8	72 (72)	104 (108)
9	87 (86)	122 (130)
10	102 (105)	140 (128)
11	87 (89)	122 (112)
12	72 (73)	104 (95)

Note: The optimal policy is given in parentheses.

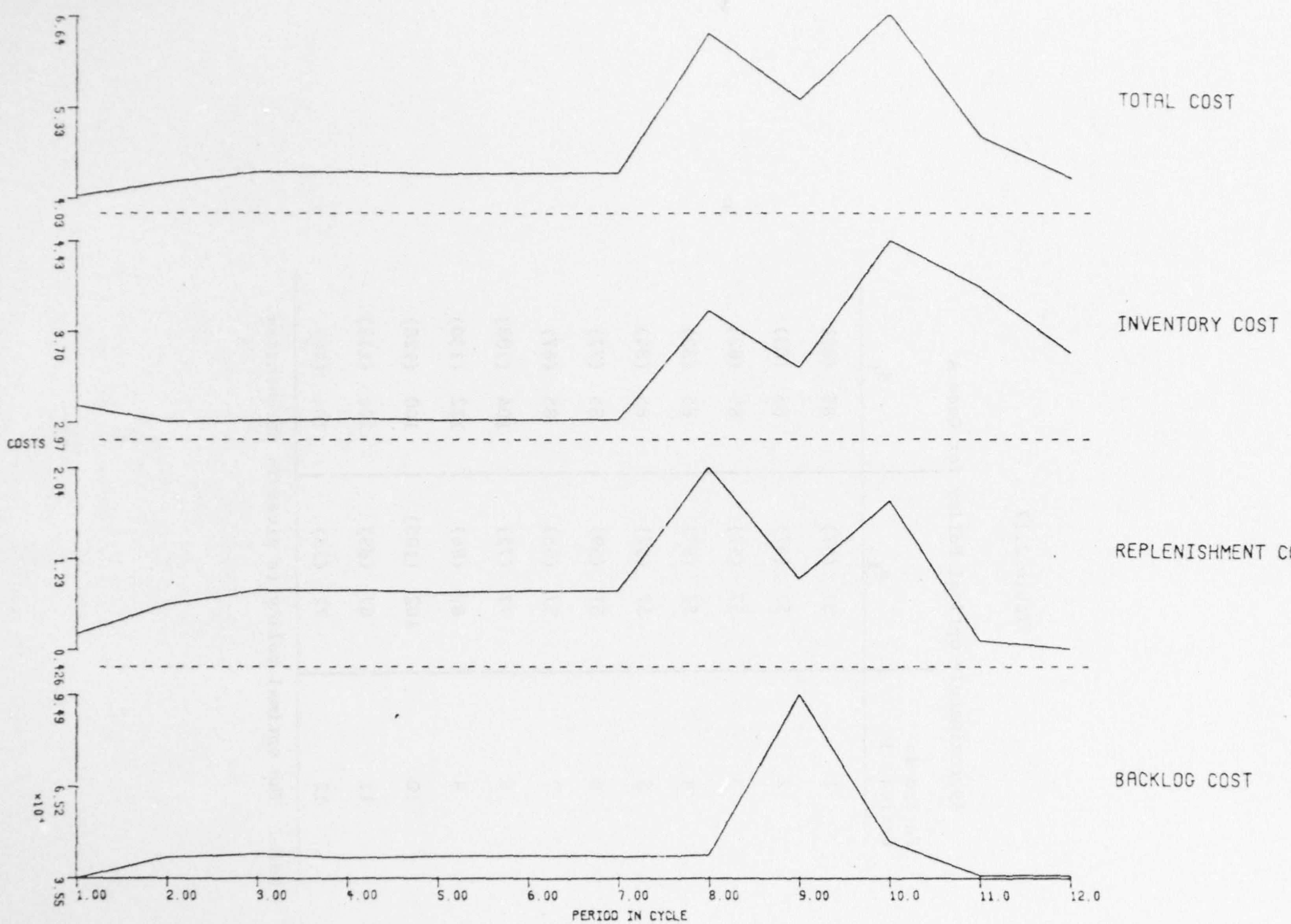


Figure 2.5 Cost Components for Case A Under Approximately Optimal Control With Full Information

Table 2.18

Approximately Optimal Policy for Case B

Period in Cycle: i	s_i	S_i
1	45 (46)	69 (61)
2	35 (33)	56 (55)
3	35 (33)	56 (56)
4	45 (44)	69 (73)
5	61 (60)	90 (93)
6	76 (75)	109 (113)
7	91 (90)	127 (128)
8	101 (101)	139 (136)
9	101 (102)	139 (131)
10	91 (93)	127 (115)
11	76 (78)	109 (95)
12	61 (62)	90 (76)

Note: The optimal policy is given in parentheses.

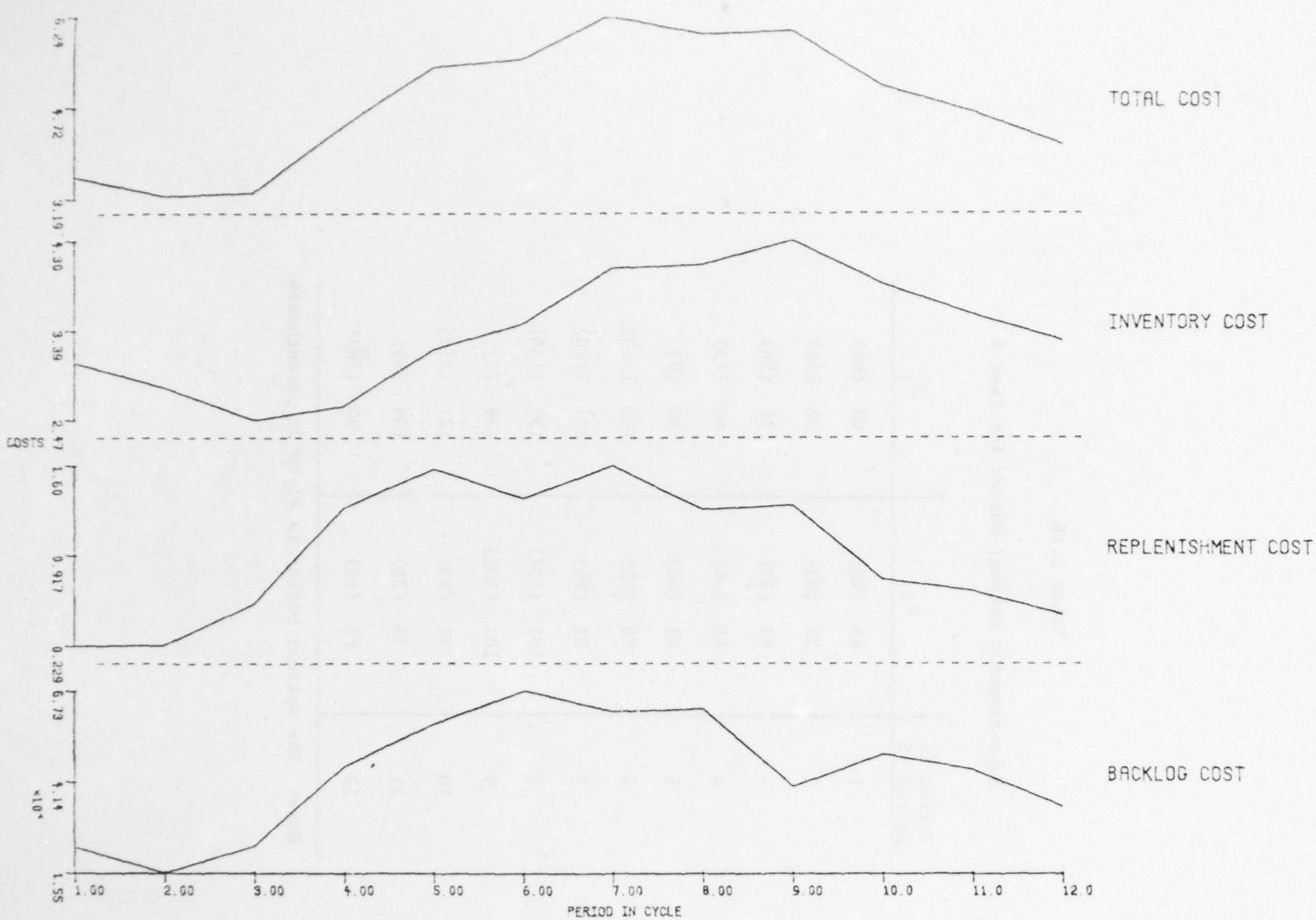


Figure 2.6 Cost Components for Case B Under Approximately Optimal Control With Full Information

case A under optimal control, peaks in the replenishment cost function do not always produce peaks in backlog cost. This is most noticeable in period 8 where a large increase in reorder probability has little effect on the corresponding backlog component.

Table 2.18 lists the approximate policy for case B. As under optimal control, the approximate policy closely follows the basic movement of the convoluted means. The differences between S_i in the optimal and power approximations policies are significant in some periods; however, the power approximation results in excess total costs of only 1.32%. The cost components of case B, illustrated in Figure 2.6 reflect the movement of $\mu_i^{(3)}$ in the same manner as the optimal cost components. For each period in the cycle, the total cost function under approximately optimal control does not deviate greatly from the total cost function under optimal control, shown in Figure 2.4.

2.6 The Effects of Demand Structure Misspecification on the Power Approximation

In Section 2.4 and 2.5 we assumed knowledge of the demand mean and variance for each period in the 12 month cycle. The decision maker, however, does not always have complete knowledge of the underlying demand structure. Section 2.6 investigates the effects of demand structure misspecification on the power approximation.

We first examine the situation in which the decision maker has knowledge of only the overall mean period demand and variance in the cycle. Thus, we apply the power approximation policies used in the stationary model to the nonstationary models. We will refer to these policies as stationary policies.

Table 2.19 contains the components of aggregate total cost per period for the nonstationary multi-item systems when controlled by stationary policies. Table 2.20 displays the absolute and percentage

Table 2.19

Average Costs Per Period for Multi-Item Negative Binomial Systems: Controlled With Stationary Policies

COST COMPONENT	STATIONARY MODEL	MODEL I	MODEL II	MODEL III	MODEL IV
INVENTORY	440 (57.8)	451 (45.7)	447 (48.9)	492 (22.1)	466 (35.6)
BACKLOG	109 (14.3)	326 (33.1)	258 (28.2)	1530 (68.8)	636 (48.6)
REPLENISHMENT	212 (27.8)	209 (21.2)	210 (23.0)	201 (9.0)	206 (15.8)
TOTAL	761 (100.0)	986 (100.0)	915 (100.0)	2222 (100.0)	1308 (100.0)

Note: Figures in parentheses are percent of total.

Table 2.20

Average Costs Per Period for Multi-Item Negative Binomial Systems: Comparison of Stationary Control with Optimal Control Given Full Information

COST COMPONENT	STATIONARY MODEL	MODEL I	MODEL II	MODEL III	MODEL IV
INVENTORY	1 (.1)	18 (4.1)	10 (2.2)	63 (14.6)	32 (7.4)
BACKLOG	0 (-.2)	221 (208.7)	151 (142.0)	1428 (1406.1)	532 (510.9)
REPLENISHMENT	1 (.3)	1 (.4)	2 (1.2)	13 (6.9)	5 (2.4)
TOTAL	1 (.1)	239 (32.0)	163 (21.7)	1504 (209.4)	569 (77.0)

Note: Table 2.20 shows the absolute increases in the cost components of Table 2.19 over those of Table 2.2 with percentage increases shown in parentheses.

incremental changes in costs when stationary policies are compared to optimal control given full information.

Table 2.20 clearly indicates that stationary policies used in nonstationary environments incur large percentage increases in expected total cost. Model I has the smallest percentage increase (32%) in expected total cost. Models I and II, however, have percentage increases in total cost that are much smaller than those of models III and IV. This suggests that larger variations in demand means contributes to the poor performance of the stationary policies. The increases in backlog cost component increases, varying from 142% to 1406%, are the primary source of excessive costs in all the nonstationary systems. With the possible exception of model III, replenishment and inventory costs do not differ greatly from those obtained under optimal control.

The distribution of expected average costs for the various parameter settings is shown in Table 2.21. The large penalty costs of the nonstationary systems result in different apportions for the components of total cost when contrasted with the stationary system. The total cost distribution of model I and model II, however, is similar to that of the stationary system, while that of model III and model IV bears little resemblance to the stationary system. (Recall in Section 2.4 that under approximately optimal control the distribution of total cost for all systems was similar to that of the stationary model.) This again suggests that greater variation in demand means results in further discrepancies in system characteristics from the stationary model under stationary control.

Table 2.22 records the percentage increases over optimal control for each component of cost for the various parameter settings. For all the nonstationary systems the backlog component is the primary source of excessive cost. This is especially apparent for those items with unit

Table 2.21a

Apportionment of Costs for a Multi-Item System Controlled with Stationary Policies

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	57.8	16.3	41.5	27.6	30.2	26.7	31.1	24.0	33.8
BACKLOG	14.3	8.4	6.0	6.5	7.8	6.5	7.8	6.3	8.1
REPLENISHMENT	27.8	13.9	13.9	11.1	16.7	14.1	13.7	11.5	16.3
TOTAL	100.0	38.6	61.4	45.2	54.8	47.4	52.6	41.8	58.2
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL I									
INVENTORY	45.7	13.5	32.2	21.9	23.8	21.1	24.6	18.8	26.9
BACKLOG	33.1	10.2	22.9	16.1	17.0	16.4	16.6	10.3	22.8
REPLENISHMENT	21.2	10.6	10.6	8.4	12.8	10.7	10.5	8.8	12.3
TOTAL	100.0	34.3	65.7	46.4	53.6	48.3	51.7	38.0	62.0

Table 2.21b

Apportionment of Costs for a Multi-Item System Controlled with Stationary Policies

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>STATIONARY MODEL</u>									
INVENTORY	57.8	16.3	41.5	27.6	30.2	26.7	31.1	24.0	33.8
BACKLOG	14.3	8.4	6.0	6.5	7.8	6.5	7.8	6.3	8.1
REPLENISHMENT	27.8	13.9	13.9	11.1	16.7	14.1	13.7	11.5	16.3
TOTAL	100.0	38.6	61.4	45.2	54.8	47.4	52.6	41.8	58.2
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>MODEL II</u>									
INVENTORY	48.9	14.3	34.6	23.4	25.5	22.6	26.3	20.1	28.8
BACKLOG	28.2	10.1	18.1	13.5	14.7	12.9	15.2	9.4	18.8
REPLENISHMENT	23.0	11.5	11.5	9.1	13.8	11.6	11.3	9.6	13.4
TOTAL	100.0	35.8	64.2	46.0	54.0	47.1	52.9	39.1	60.9

Table 2.21c

Apportionment of Costs for a Multi-Item System Controlled with Stationary Policies

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	57.8	16.3	41.5	27.6	30.2	26.7	31.1	24.0	33.8
BACKLOG	14.3	8.4	6.0	6.5	7.8	6.5	7.8	6.3	8.1
REPLENISHMENT	27.8	13.9	13.9	11.1	16.7	14.1	13.7	11.5	16.3
TOTAL	100.0	38.6	61.4	45.2	54.8	47.4	52.6	41.8	58.2
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
INVENTORY	22.1	7.3	14.8	10.7	11.4	10.2	12.0	8.8	13.3
BACKLOG	68.8	9.9	58.9	35.0	33.9	31.6	37.2	17.2	51.6
REPLENISHMENT	9.0	4.5	4.5	3.5	5.4	4.6	4.5	3.8	5.2
TOTAL	100.0	29.9	70.1	21.8	78.2	49.2	50.8	46.3	53.7

Table 2.21d

Apportionment of Costs for a Multi-Item System Controlled with Stationary Policies

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>STATIONARY MODEL</u>									
INVENTORY	57.8	16.3	41.5	27.6	30.2	26.7	31.1	24.0	33.8
BACKLOG	14.3	8.4	6.0	6.5	7.8	6.5	7.8	6.3	8.1
REPLENISHMENT	27.8	13.9	13.9	11.1	16.7	14.1	13.7	11.5	16.3
TOTAL	100.0	38.6	61.4	45.2	54.8	47.4	52.6	41.8	58.2
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>MODEL IV</u>									
INVENTORY	35.6	11.1	24.5	17.1	18.5	16.3	19.3	14.5	21.1
BACKLOG	48.6	11.4	37.2	24.0	24.6	19.1	29.5	13.6	35.0
REPLENISHMENT	15.8	7.9	7.9	6.2	9.5	8.0	7.8	6.6	9.2
TOTAL	100.0	30.5	69.5	47.3	52.7	43.4	56.6	34.7	65.3

Table 2.22a

Percentage Excess of Costs of a Multi-Item System Under Stationary Control Over the Costs of Controlling the Same System Optimally with Full Information

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>STATIONARY MODEL</u>									
INVENTORY	.1	-5.7	2.6	1.9	-1.4	.6	-.2	-1.3	1.2
BACKLOG	-.2	4.2	-5.7	-6.9	6.2	-1.0	.6	4.1	-3.2
REPLENISHMENT	.3	5.4	-4.2	.4	.3	-.1	.8	1.2	-.2
TOTAL	.1	.2	.1	.2	.1	.1	.2	.1	.1
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>MODEL I</u>									
INVENTORY	4.1	2.3	4.9	6.0	2.4	4.9	3.5	.9	6.5
BACKLOG	208.7	67.5	394.4	207.6	209.8	237.1	184.9	124.7	271.8
REPLENISHMENT	.4	6.2	-4.8	.1	.6	-.5	1.4	4.0	-2.0
TOTAL	32.0	17.2	41.4	35.2	29.4	34.9	29.5	19.7	41.0

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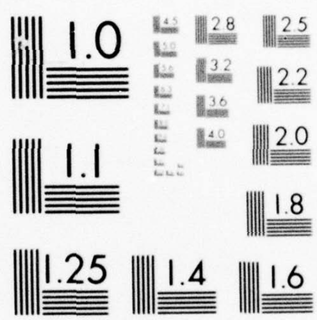

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Table 2.22b

Percentage Excess of Costs of a Multi-Item System Under Stationary Control Over the Costs of Controlling the Same System Optimally with Full Information

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	.1	-5.7	2.6	1.9	-1.4	.6	-.2	-1.3	1.2
BACKLOG	-.2	4.2	-5.7	-6.9	6.2	-1.0	.6	4.1	-3.2
REPLENISHMENT	.3	5.4	-4.2	.4	.3	-.1	.8	1.2	-.2
TOTAL	.1	.2	.1	.2	.1	.1	.2	.1	.1
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL II									
INVENTORY	2.2	-.6	3.4	4.4	.3	2.6	1.9	-.1	3.9
BACKLOG	142.0	52.2	260.2	137.8	146.1	143.5	140.8	89.0	181.7
REPLENISHMENT	1.2	6.6	-3.7	.4	1.7	.5	1.9	3.7	-.5
TOTAL	21.7	12.9	27.3	23.7	20.0	21.2	22.2	13.8	27.4

Table 2.22c

Percentage Excess of Costs of a Multi-Item System Under Stationary Control Over the Costs of Controlling the Same System Optimally with Full Information

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>STATIONARY MODEL</u>									
INVENTORY	.1	-5.7	2.6	1.9	-1.4	.6	-.2	-1.3	1.2
BACKLOG	-.2	4.2	-5.7	-6.9	6.2	-1.0	.6	4.1	-3.2
REPLENISHMENT	.3	5.4	-4.2	.4	.3	-.1	.8	1.2	-.2
TOTAL	.1	.2	.1	.2	.1	.1	.2	.1	.1
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>MODEL III</u>									
INVENTORY	14.6	25.2	10.1	18.9	11.0	15.7	13.8	7.6	19.9
BACKLOG	1406.2	283.4	2861.6	1456.4	1357.6	1431.5	1385.3	768.3	1896.4
REPLENISHMENT	6.9	11.0	3.1	3.2	9.6	3.7	10.5	11.4	3.9
TOTAL	209.4	119.8	274.5	73.8	295.2	234.7	188.3	203.7	214.6

Table 2.22d

Percentage Excess of Costs of a Multi-Item System Under Stationary Control Over the Costs of Controlling the Same System Optimally with Full Information

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	.1	-5.7	2.6	1.9	-1.4	.6	-.2	-1.3	1.2
BACKLOG	-.2	4.2	-5.7	-6.9	6.2	-1.0	.6	4.1	-3.2
REPLENISHMENT	.3	5.4	-4.2	.4	.3	-.1	.8	1.2	-.2
TOTAL	.1	.2	.1	.2	.1	.1	.2	.1	.1
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL IV									
INVENTORY	7.4	11.3	5.7	9.6	5.4	7.1	7.7	3.7	10.1
BACKLOG	510.9	153.4	980.9	519.7	502.6	428.1	579.8	298.1	671.8
REPLENISHMENT	2.4	7.2	-1.9	1.7	2.9	1.5	3.5	4.1	1.3
TOTAL	77.0	39.3	100.8	84.9	70.4	62.4	90.0	46.3	99.2

penalty cost of 99. Percentage increases in inventory and replenishment costs are small in comparison to the backlog component for all parameter settings in the nonstationary systems.

The values for selected operating characteristics are displayed in Table 2.23. Table 2.24 gives the percentage change in the value of these operating characteristics from the corresponding ones in Table 2.4. Backlog frequencies are extremely high for all nonstationary systems. For those items with unit costs of 99, the percentage increases range from 156.2% in model II to 1033.6% in model III.

In summary, stationary policies are unacceptable approximations in a nonstationary environment. The desirable simplicity of stationary policies does not adequately compensate for the excess values of backlog-related operating characteristics.

We now examine the situation where the decision maker has knowledge of the basic movement of demand means through the cycle, but incorrect information about the absolute values of the demand means. Model III is selected for a detailed examination under this type of misspecification.

We assume that we correctly identify the true values of the demand means for the first seven periods of the cycle. During the last five periods of the cycle, however, we assume that the mean demand gradually increases to only twice the level of the first seven periods, reaching a maximum value in period 10, and then gradually decreases. In other words, we expect mean demand to peak at a much lower mean demand level than is actually the case for model III. The variance-to-mean ratio is still assumed constant at 3, so that a misspecification in the mean demand results in a misspecification in the demand variance. We now investigate the consequences of such a misspecification on the system's operating characteristics for model III. The power approximation policy

Table 2.23a

Operating Characteristics of a Multi-Item System Controlled with Stationary Policies

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	440	124	316	210	230	203	237	183	257
BACKLOG FREQUENCY	.105	.200	.009	.101	.108	.103	.106	.106	.103
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.011	.166	.005	.010	.012	.010	.012	.014	.009
REPLENISHMENT FREQUENCY	.289	.289	.289	.331	.248	.293	.286	.240	.338
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL I									
PERIOD-END INVENTORY	451	133	317	216	235	208	242	185	266
BACKLOG FREQUENCY	.129	.230	.029	.129	.130	.124	.134	.123	.136
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.033	.262	.024	.032	.034	.033	.033	.031	.034
REPLENISHMENT	.285	.285	.285	.324	.246	.288	.282	.238	.337

Table 2.23b

Operating Characteristics of a Multi-Item System Controlled with Stationary Policies

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	440	124	316	210	230	203	237	183	257
BACKLOG FREQUENCY	.105	.200	.009	.101	.108	.103	.106	.106	.103
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.011	.166	.005	.010	.012	.010	.012	.014	.009
REPLENISHMENT FREQUENCY	.289	.289	.289	.331	.248	.293	.286	.240	.338
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL II									
PERIOD-INVENTORY	447	131	316	214	233	206	241	184	263
BACKLOG FREQUENCY	.124	.225	.024	.123	.126	.119	.130	.119	.129
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.026	.240	.017	.025	.027	.024	.028	.026	.026
REPLENISHMENT FREQUENCY	.287	.287	.287	.327	.247	.290	.284	.239	.334

Table 2.23c

Operating Characteristics of a Multi-Item System Controlled with Stationary Policies

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>STATIONARY MODEL</u>									
PERIOD-END INVENTORY	440	124	316	210	230	203	237	183	257
BACKLOG FREQUENCY	.105	.200	.009	.101	.108	.103	.106	.106	.103
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.011	.166	.005	.010	.012	.010	.012	.014	.009
REPLENISHMENT FREQUENCY	.289	.289	.289	.331	.248	.293	.286	.240	.338
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>MODEL III</u>									
PERIOD-END INVENTORY	492	196	296	163	328	237	254	226	266
BACKLOG FREQUENCY	.199	.176	.223	.294	.104	.203	.196	.183	.215
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.155	.572	.138	.157	.152	.142	.167	.116	.174
REPLENISHMENT FREQUENCY	.274	.274	.274	.311	.236	.277	.271	.230	.318

Table 2.23d

Operating Characteristics of a Multi-Item System Controlled with Stationary Policies

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	440	124	316	210	230	203	237	183	257
BACKLOG FREQUENCY	.105	.200	.009	.101	.108	.103	.106	.106	.103
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.011	.166	.005	.010	.012	.010	.012	.014	.009
REPLENISHMENT FREQUENCY	.289	.289	.289	.331	.248	.293	.286	.240	.338
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL IV									
PERIOD-END INVENTORY	466	146	320	223	242	213	253	189	276
BACKLOG FREQUENCY	.166	.278	.054	.169	.164	.153	.179	.149	.183
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.064	.390	.051	.064	.064	.050	.078	.054	.069
REPLENISHMENT FREQUENCY	.282	.282	.282	.320	.243	.284	.278	.236	.327

Table 2.24a

Percentage Excess of Operating Characteristics of a Multi-Item System Under Stationary Control Over Those for System Under Optimal Control with Full Information

OPERATING CHARACTERISTIC	VALUE	C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>STATIONARY MODEL</u>									
PERIOD-END INVENTORY	.1	-5.7	2.6	1.9	-1.4	.6	-.2	-1.3	1.2
BACKLOG FREQUENCY	4.2	4.6	-5.3	.9	7.4	1.8	6.5	7.7	.8
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	-.2	4.2	-5.7	-6.9	6.2	-1.0	.6	4.1	-3.2
REPLENISHMENT FREQUENCY	.4	5.5	-4.3	.4	.3	-.6	1.3	1.5	-.4
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>MODEL I</u>									
PERIOD-END INVENTORY	4.1	2.3	4.9	6.0	2.4	4.9	3.5	.9	6.5
BACKLOG FREQUENCY	28.7	19.6	217.5	28.3	29.2	24.1	33.4	22.8	34.6
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	208.7	67.5	394.4	207.6	209.8	237.1	184.9	124.7	271.8
REPLENISHMENT FREQUENCY	.3	6.2	-4.9	.1	.6	-.7	1.4	3.3	-1.7

Table 2.24b

Percentage Excess of Operating Characteristics of a Multi-Item System Under Stationary Control Over Those for System Under Optimal Control with Full Information

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>STATIONARY MODEL</u>									
PERIOD-END INVENTORY	.1	-5.7	2.6	1.9	-1.4	.6	-.2	-1.3	1.2
BACKLOG FREQUENCY	4.2	4.6	-5.3	.9	7.4	1.8	6.5	7.7	.8
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	-.2	4.2	-5.7	-6.9	6.2	-1.0	.6	4.1	-3.2
REPLENISHMENT FREQUENCY	.4	5.5	-4.3	.4	.3	-.6	1.3	1.5	-.4
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>MODEL II</u>									
PERIOD-END INVENTORY	2.2	-.6	3.4	4.4	.3	2.6	1.9	-.1	3.9
BACKLOG FREQUENCY	23.7	17.4	156.2	22.4	25.1	18.9	28.5	19.6	27.8
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	142.0	52.2	260.2	137.8	146.1	143.5	140.8	89.0	181.7
REPLENISHMENT FREQUENCY	1.0	6.3	-3.8	.4	1.7	.1	1.9	3.4	-.7

Table 2.24c

Percentage Excess of Operating Characteristics of a Multi-Item System Under Stationary Control Over Those for System Under Optimal Control with Full Information

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	.1	-5.7	2.6	1.9	-1.4	.6	-.2	-1.3	1.2
BACKLOG FREQUENCY	4.2	4.6	-5.3	.9	7.4	1.8	6.5	7.7	.8
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	-.2	4.2	-5.7	-6.9	6.2	-1.0	.6	4.1	-3.2
REPLENISHMENT FREQUENCY	.4	5.5	-4.3	.4	.3	-.6	1.3	1.5	-.4
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
PERIOD-END INVENTORY	14.6	25.2	10.1	18.9	11.0	15.7	13.8	7.6	19.9
BACKLOG FREQUENCY	98.7	53.7	1033.6	103.0	94.4	83.9	113.3	76.6	120.4
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	1406.2	283.4	2861.6	1456.4	1357.6	1431.5	1385.3	768.3	1896.4
REPLENISHMENT FREQUENCY	5.8	10.3	1.7	3.2	9.6	2.5	9.6	10.0	3.0

Table 2.24d

Percentage Excess of Operating Characteristics of a Multi-Item System Under Stationary Control Over Those for System Under Optimal Control with Full Information

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	.1	-5.7	2.6	1.9	-1.4	.6	-.2	-1.3	1.2
BACKLOG FREQUENCY	4.2	4.6	-5.3	.9	7.4	1.8	6.5	7.7	.8
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	-.2	4.2	-5.7	-6.9	6.2	-1.0	.6	4.1	-3.2
REPLENISHMENT FREQUENCY	.4	5.5	-4.3	.4	.3	-.6	1.3	1.5	-.4
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL IV									
PERIOD-END INVENTORY	7.4	11.3	5.7	9.6	5.4	7.1	7.7	3.7	10.1
BACKLOG FREQUENCY	65.8	45.3	494.8	68.8	62.9	53.9	77.6	49.5	82.0
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	510.9	153.4	980.9	519.7	502.6	428.1	579.8	298.1	671.8
REPLENISHMENT FREQUENCY	2.2	3.8	1.1	6.6	-1.8	1.7	2.9	1.2	3.4

resulting from such a misspecification will be referred to as a misspecified policy.

Table 2.25 contains the aggregate values of total cost per period and its components for model III when controlled by the misspecified policies. Backlog cost is clearly the dominant cost component, accounting for 89.2% of the total cost. Table 2.26 provides comparisons for the costs of misspecified policies with those of optimal control. Apparently the smaller misspecified peaks in demand result in insufficient levels of inventory. Holding costs are 54.9% below their optimal levels. As a result, backlog costs are over 3400% in excess of optimal control. This huge backlog cost is chiefly responsibile for a percentage increase in total cost of 462.2% for the multi-item system.

More detailed breakdowns of Tables 2.25 and 2.26 are provided in Tables 2.27 and 2.28. Although large percentages of excess backlog cost are found under every parameter setting, large unit penalty costs, leadtimes, or means result in the greatest increment in backlog costs. In contrast, the percentage decreases of inventory costs are much more uniform over all of the parameter settings.

The values of operating characteristics are contained in Table 2.29 with the percentage excess over optimal control recorded in Table 2.30. The higher values of backlog cost noted under large unit penalty costs, leadtimes, or means are also reflected in the corresponding values of backlog frequency and weighted proportion of demand backlogged in Table 2.29.

Hence, we conclude that the savings in inventory costs that result from the misspecified demand structure are not sufficient to compensate for the increase in backlog costs and backlog frequencies. Precise knowledge of the movement of demand means and variances throughout the cycle is crucial in obtaining approximately optimal policies.

Table 2.25

Average Costs Per Period for a
Multi-Item Negative Binomial System:
Controlled With Misspecified Policies

COST COMPONENT	MODEL III	
INVENTORY	193	(4.8)
BACKLOG	3606	(89.2)
REPLENISHMENT	243	(6.0)
TOTAL	4042	(100.0)

Note: Figures in parentheses are percent of total.

Table 2.26

Average Costs Per Period for a Multi-Item
Negative Binomial System: Comparison of
Misspecified Control with Optimal
Control Given Full Information

COST COMPONENT	MODEL III	
INVENTORY	-236	(-54.9)
BACKLOG	3505	(3450.8)
REPLENISHMENT	5555	(29.3)
TOTAL	3324	(462.9)

Note: Table 2.26 shows the absolute increases in the cost components of Table 2.25 over those of Table 2.2 with percentage increases shown in parentheses.

Table 2.27

Apportionment of Costs for a Multi-Item System Controlled with Misspecified Information About Demand

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
INVENTORY	4.8	1.1	3.6	2.2	2.6	2.5	2.3	2.2	2.6
BACKLOG	89.2	9.9	79.3	46.2	43.0	30.0	59.2	21.7	67.5
REPLENISHMENT	6.0	3.0	3.0	2.4	3.6	3.0	3.0	2.5	3.5
TOTAL	100.0	14.0	86.0	50.8	49.2	35.5	64.5	26.4	73.6

Table 2.28

Percentage Excess of Costs of a Multi-Item System Under Misspecified Control Over the Costs of Controlling the Same System Optimally with Full Information

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN) 4	C(OUT)/C(IN) 99	C(FIX)/C(IN) 32	C(FIX)/C(IN) 64	LEADTIME 2	LEADTIME 4	MEAN 8	MEAN 16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
INVENTORY	-54.9	-64.6	-50.7	-54.9	-54.9	-48.4	-60.3	-51.3	-57.6
BACKLOG	3450.8	597.1	7150.0	3639.3	3268.4	2548.5	4192.6	1887.0	4652.9
REPLENISHMENT	29.3	34.0	24.9	24.7	32.5	24.2	34.9	33.4	26.5
TOTAL	462.9	103.8	689.9	528.1	408.4	323.8	587.1	253.2	615.0

Table 2.29

Percentage Excess of Operating Characteristics of a Multi-Item System Under Misspecified Control Over Those for System Under Optimal Control with Full Information

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
PERIOD-END INVENTORY	-54.9	-64.6	-50.7	-54.9	-54.9	-48.4	-60.3	-51.3	-57.6
BACKLOG FREQUENCY	270.2	174.4	2261.6	279.8	260.6	219.0	320.6	228.7	310.9
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	3450.8	597.1	7150.0	3639.3	3268.4	2548.5	4192.6	1887.0	4652.9
REPLENISHMENT FREQUENCY	27.9	33.1	23.2	24.7	32.5	22.8	33.5	32.0	25.2

Table 2.30

Operating Characteristics of a Multi-Item System Controlled with Misspecified Information About Demand

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
PERIOD-END INVENTORY	193	46	147	90	103	100	93	89	104
BACKLOG FREQUENCY	.371	.525	.217	.379	.363	.318	.424	.327	.416
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.365	1.041	.337	.378	.352	.245	.484	.266	.414
REPLENISHMENT FREQUENCY	.331	.330	.332	.376	.286	.332	.330	.276	.386

3. SYSTEM CONTROL WITH STATISTICAL INFORMATION ABOUT DEMAND

We suppose that a manager responsible for the control of an inventory system as defined in this section knows the cycle length and the pattern of demand means through the cycle, that is, each mean relative to the smallest demand mean. It was shown in Section 1.2.3. that estimates of the mean and variance of the monthly demand distributions can be obtained from this information and an historical sample of realized demand. Such estimates are used below instead of the values for the true means and variances in control policies that are approximately optimal in the case of perfect information. Details of the control policies were given previously in Section 1.1.2.

The operating characteristics describing the performance of inventory systems under control using only statistical information about demand have not been computed analytically. The results in this section are statistical estimates of the performance of some selected inventory systems found by computer simulation. During each simulation run the control policy parameters are revised at regular intervals to allow for potential changes in the underlying demand structure. Such changes do not, in fact, occur during the simulation experiments, but in a real-life system, periodic revision is essential to detect and adapt to a possibly changing demand structure.

Section 3.1 discusses the results from the simulation of multi-item systems under statistical control. Revision intervals and history lengths are set to 24 periods, corresponding to two years of previous data. The sensitivity of the operating characteristics to values of the input parameters is investigated.

Under statistical control, the expected cost per period of the stationary system is estimated to be 7.1% higher than when the system is controlled optimally with full information. For the nonstationary systems, the estimated percentage increase in expected total cost per period depends on the nature of the nonstationarity, varying from 7.9% to 14.8%.

3.1 Performance under Control with Statistical Information about Demand

The input parameters for each item in the stationary and four nonstationary 16-item systems were specified in Section 1.2.2. Estimated average total cost per period and its components are shown in Table 3.1. Under statistical control, holding and backlog costs are significantly higher in each system than those found under optimal control, shown in Table 2.2. With the exception of model III, however, replenishment costs are almost identical.

Table 3.2 shows estimates of absolute and percentage incremental changes in expected costs per period when statistical control is compared with optimal control given full information (cf. Table 2.2). The stationary model has the smallest percent increase in expected total cost, 7.1%. Model I and model II have increases only slightly above that amount, but model III and model IV have percentage increases of 14.8% and 10.6%, respectively. Apparently larger variations in demand means produce greater percentage increases in total cost for model III and model IV. All systems studied incurred excess inventory and backlog costs.

When the items in each system are divided into subsystems according to values taken by input parameters, the resulting apportionment of costs between the subsystems is shown in Table 3.3. The distribution of costs are similar to those in Table 2.3, which shows the same systems controlled with full information.

Table 3.1

Average Costs Per Period for Multi-Item Negative Binomial Systems:
Controlled With Statistical Information About Demand
(Revision Interval 24 Periods, Revision History Length 24 Periods)

COST COMPONENT	STATIONARY MODEL		MODEL I		MODEL II		MODEL III		MODEL IV	
INVENTORY	463	(56.9)	473	(58.3)	469	(57.8)	496	(60.2)	492	(60.1)
BACKLOG	138	(16.9)	130	(16.1)	133	(16.4)	128	(15.6)	124	(15.2)
REPLENISHMENT	213	(26.2)	208	(25.6)	209	(25.8)	199	(24.2)	202	(24.7)
TOTAL	813	(100.0)	810	(100.0)	811	(100.0)	824	(100.0)	818	(100.0)

Note: Figures in parentheses are percent of total.

Table 3.2

Average Costs Per Period for Multi-Item Negative Binomial Systems:
Comparison of Statistical Control With Optimal
Control Given Full Information
(Revision Interval 24 Periods, Revision History Length 24 Periods)

COST COMPONENT	STATIONARY MODEL		MODEL I		MODEL II		MODEL III		MODEL IV	
INVENTORY	24	(5.4)	40	(9.2)	31	(7.2)	68	(15.8)	58	(13.4)
BACKLOG	28	(25.9)	24	(23.2)	26	(25.0)	27	(26.5)	20	(19.2)
REPLENISHMENT	2	(.9)	0	(-.2)	1	(.7)	12	(6.2)	1	(.3)
TOTAL	54	(7.1)	64	(8.6)	59	(7.9)	106	(14.8)	79	(10.6)

Note: Table 3.2 shows the absolute increases in the cost components of Table 3.1 over those of Table 2.2 with percentage increases shown in parentheses.

Table 3.3a

Apportionment of Costs for a Multi-Item System Controlled with Statistical Information About Demand
(Revision Interval 24 Periods, Revision History Length 24 Periods)

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	56.9	16.4	40.5	27.2	29.7	26.0	30.9	23.7	33.2
BACKLOG	16.9	7.8	9.1	8.0	8.9	7.4	9.5	7.3	9.6
REPLENISHMENT	26.2	13.1	13.0	10.4	15.7	13.3	12.9	10.9	15.3
TOTAL	100.0	37.4	62.6	45.6	54.4	46.8	53.2	42.0	58.0
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL I									
INVENTORY	58.3	16.8	41.5	27.7	30.6	26.8	31.6	24.5	33.8
BACKLOG	16.1	7.6	8.5	7.7	8.4	7.1	9.0	7.0	9.0
REPLENISHMENT	25.6	12.9	12.7	10.2	15.4	13.0	12.6	10.6	15.0
TOTAL	100.0	37.3	62.7	45.6	54.4	46.8	53.2	42.1	57.9

Table 3.3b

Apportionment of Costs for a Multi-Item System Controlled with Statistical Information About Demand
(Revision Interval 24 Period, Revision History Length 24 Periods)

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>STATIONARY MODEL</u>									
INVENTORY	56.9	16.4	40.5	27.2	29.7	26.0	30.9	23.7	33.2
BACKLOG	16.9	7.8	9.1	8.0	8.9	7.4	9.5	7.3	9.6
REPLENISHMENT	26.2	13.1	13.0	10.4	15.7	13.3	12.9	10.9	15.3
TOTAL	100.0	37.4	62.6	45.6	54.4	46.8	53.2	42.0	58.0
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>MODEL II</u>									
INVENTORY	57.8	16.7	41.1	27.6	30.2	26.5	31.3	24.3	33.5
BACKLOG	16.4	7.7	8.7	7.8	8.6	7.1	9.3	7.0	9.3
REPLENISHMENT	25.8	13.0	12.8	10.3	15.5	13.0	12.7	10.7	15.1
TOTAL	100.0	37.4	62.6	45.6	54.4	46.6	53.4	42.1	57.9

Table 3.3c

Apportionment of Costs for a Multi-Item System Controlled with Statistical Information About Demand
(Revision Interval 24 Periods, Revision History Length 24 Periods)

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	56.9	16.4	40.5	27.2	29.7	26.0	30.9	23.7	33.2
BACKLOG	16.9	7.8	9.1	8.0	8.9	7.4	9.5	7.3	9.6
REPLENISHMENT	26.2	13.1	13.0	10.4	15.7	13.3	12.9	10.9	15.3
TOTAL	100.0	37.4	62.6	45.6	54.4	46.8	53.2	42.0	58.0
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
INVENTORY	60.2	17.3	42.9	28.0	32.3	27.1	33.1	26.6	33.7
BACKLOG	15.6	7.1	8.5	7.5	8.0	7.0	8.6	6.2	9.3
REPLENISHMENT	24.2	12.3	11.9	10.3	13.8	12.4	11.8	9.6	14.6
TOTAL	100.0	36.7	63.3	45.8	54.2	46.5	53.5	42.4	57.6

Table 3.3d

Apportionment of Costs for a Multi-Item System Controlled with Statistical Information About Demand
(Revision Interval 24 Periods, Revision History Length 24 Periods)

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	56.9	16.4	40.5	27.2	29.7	26.0	30.9	23.7	33.2
BACKLOG	16.9	7.8	9.1	8.0	8.9	7.4	9.5	7.3	9.6
REPLENISHMENT	26.2	13.1	13.0	10.4	15.7	13.3	12.9	10.9	15.3
TOTAL	100.0	37.4	62.6	45.6	54.4	46.8	53.2	42.0	58.0
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL IV									
INVENTORY	60.1	17.5	42.6	28.3	31.8	27.4	32.7	25.7	34.4
BACKLOG	15.2	7.1	8.0	7.5	7.6	6.5	8.6	6.5	8.7
REPLENISHMENT	24.7	12.6	12.1	9.8	14.9	12.5	12.2	10.2	14.5
TOTAL	100.0	37.2	62.8	45.7	54.3	46.4	53.5	42.4	57.6

Table 3.4a

Percentage Excess of Costs of a Multi-Item System Under Statistical Control Over the Costs of Controlling the Same System Optimally with Full Information.
(Revision Interval 24 Periods, Revision History Length 24 Periods)

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>STATIONARY MODEL</u>									
INVENTORY	5.4	1.3	7.1	7.5	3.5	4.8	5.8	4.3	6.1
BACKLOG	25.9	4.2	53.5	-1.9	29.7	20.5	30.6	30.4	22.7
REPLENISHMENT	.9	6.3	-4.0	.9	1.0	.4	1.4	2.0	.2
TOTAL	7.1	3.6	9.2	8.1	6.2	5.7	8.3	7.4	6.8
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>MODEL I</u>									
INVENTORY	9.2	4.6	11.2	10.2	8.2	9.1	9.3	8.2	9.9
BACKLOG	23.2	2.5	50.5	21.5	24.8	19.4	26.4	25.3	21.6
REPLENISHMENT	-.2	6.4	-6.2	-.4	-.1	-.8	.4	2.6	-2.2
TOTAL	8.6	4.8	10.9	9.3	7.9	7.5	9.5	9.2	8.1

Table 3.4b

Percentage Excess of Costs of a Multi-Item System Under Statistical Control Over the Costs of Controlling the Same System Optimally with Full Information.
(Revision Interval 24 Periods, Revision History Length 24 Periods)

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	5.4	1.3	7.1	7.5	3.5	4.8	5.8	4.3	6.1
BACKLOG	25.9	4.2	53.5	-1.9	29.7	20.5	30.6	30.4	22.7
REPLENISHMENT	.9	6.3	-4.0	.9	1.0	.4	1.4	2.0	.2
TOTAL	7.1	3.6	9.2	8.1	6.2	5.7	8.3	7.4	6.8
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL II									
INVENTORY	7.2	3.0	9.0	9.1	5.5	6.7	7.6	7.1	7.2
BACKLOG	25.0	3.2	53.6	21.5	28.2	18.1	30.7	25.6	24.5
REPLENISHMENT	.7	6.6	-4.8	-.2	1.2	0.0	1.3	2.5	-.6
TOTAL	7.9	4.3	10.2	8.7	7.2	6.3	9.3	8.6	7.4

Table 3.4c

Percentage Excess of Costs of a Multi-Item System Under Statistical Control Over the Costs of Controlling the Same System Optimally with Full Information.
(Revision Interval 24 Periods, Revision History Length 24 Periods)

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	5.4	1.3	7.1	7.5	3.5	4.8	5.8	4.3	6.1
BACKLOG	25.9	4.2	53.5	-1.9	29.7	20.5	30.6	30.4	22.7
REPLENISHMENT	.9	6.3	-4.0	.9	1.0	.4	1.4	2.0	.2
TOTAL	7.1	3.6	9.2	8.1	6.2	5.7	8.3	7.4	6.8
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
INVENTORY	15.8	9.4	18.6	15.4	16.1	14.7	16.7	20.1	12.6
BACKLOG	26.5	2.2	58.0	24.6	28.3	25.4	27.4	16.5	34.2
REPLENISHMENT	6.2	12.1	.7	10.2	3.4	4.3	8.3	4.6	7.2
TOTAL	14.8	8.8	18.6	15.6	14.1	13.1	16.3	15.7	14.1

Table 3.4d

Percentage Excess of Costs of a Multi-Item System Under Statistical Control Over the Costs of Controlling the Same System Optimally with Full Information.
(Revision Intervals 24 Periods, Revision History Length 24 Periods)

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>STATIONARY MODEL</u>									
INVENTORY	5.4	1.3	7.1	7.5	3.5	4.8	5.8	4.3	6.1
BACKLOG	25.9	4.2	53.5	-1.9	29.7	20.5	30.6	30.4	22.7
REPLENISHMENT	.9	6.3	-4.0	.9	1.0	.4	1.4	2.0	.2
TOTAL	7.1	3.6	9.2	8.1	6.2	5.7	8.3	7.4	6.8
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>MODEL IV</u>									
INVENTORY	13.4	9.6	15.0	13.7	13.1	12.7	14.0	15.0	12.3
BACKLOG	19.2	-1.6	46.4	21.5	17.0	13.1	24.2	18.2	19.9
REPLENISHMENT	.3	6.8	-5.6	-.1	.6	-.8	1.4	.8	0.0
TOTAL	10.6	6.4	13.3	11.6	9.9	8.8	12.3	11.6	9.9

Table 3.4 shows the estimated percentage incremental changes in the expected average total cost and its components. For the various subsystems, this table elaborates on the changes summarized before in Table 3.2.

An examination of the percentage increases indicates that for all systems the excess backlog costs noted in Table 3.2 are due mainly to those items with high unit penalty costs. This is also true to a lesser extent for inventory costs. It should be noted, however, that the values in Table 3.4 are only estimates of the increase in costs of controlling the systems when the control is statistical instead of with full information, as rules for optimal control in the identical circumstances of statistical information and periodic revision have not been derived [cf. Hayes (1969)].

The physical operating characteristics of the multi-item system have their estimated values tabulated in Table 3.5, and Table 3.6 sets out the percentage increase of each over the corresponding values found in Table 2.4. As in the case of approximately optimal control, model III and model IV have higher inventory levels and lower backlog frequencies than the other systems.

The direction of the sensitivity of the policy variables and the operating characteristics to changes in the input parameter is indicated in Table 3.7. This table shows that the behavior of the stationary and nonstationary systems has the same pattern with one exception. Whereas the backlog frequency of model III is monotonically increasing with respect to mean demand and monotonically decreasing with respect to replenishment cost, the reverse is true for the other systems.

Some differences are apparent, however, when behavior is compared to the system under optimal control (Table 2.5). The monotonic patterns, for example, of expected replenishment cost, expected replenishment frequency,

Table 3.5a

Operating Characteristics of a Multi-Item System Controlled with Statistical Information About Demand
(Revision Interval 24 Periods, Revision History Length 24 Periods)

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	463	133	329	221	242	212	251	193	270
BACKLOG FREQUENCY	.104	.195	.013	.102	.107	.103	.105	.106	.102
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.014	.166	.008	.013	.015	.012	.016	.018	.012
REPLENISHMENT FREQUENCY	.291	.292	.290	.332	.250	.296	.286	.242	.340
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL I									
PERIOD-END INVENTORY	473	136	336	224	248	217	256	198	274
BACKLOG FREQUENCY	.101	.190	.012	.100	.103	.101	.102	.103	.100
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.013	.160	.007	.013	.014	.012	.015	.017	.011
REPLENISHMENT FREQUENCY	.283	.286	.281	.322	.244	.288	.278	.236	.331

Table 3.5b

Operating Characteristics of a Multi-Item System Controlled with Statistical Information About Demand (Revision Interval 24 Periods, Revision History Length 24 Periods)

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	463	133	329	221	242	212	251	193	270
BACKLOG FREQUENCY	.104	.195	.013	.102	.107	.103	.105	.106	.102
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.014	.166	.008	.013	.015	.012	.016	.018	.012
REPLENISHMENT FREQUENCY	.291	.292	.290	.332	.250	.296	.286	.242	.340
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL II									
PERIOD-END INVENTORY	469	136	333	224	245	215	254	197	272
BACKLOG FREQUENCY	.102	.192	.012	.099	.105	.100	.104	.103	.101
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.013	.162	.007	.013	.014	.012	.015	.017	.012
REPLENISHMENT FREQUENCY	.286	.288	.284	.325	.246	.289	.282	.238	.334

Table 3.5c

Operating Characteristics of a Multi-Item System Controlled with Statistical Information About Demand
(Revision Interval 24 Periods, Revision History Length 24 Periods)

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	463	133	329	221	242	212	251	193	270
BACKLOG FREQUENCY	.104	.195	.013	.102	.107	.103	.105	.106	.102
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.014	.166	.008	.013	.015	.012	.016	.018	.012
REPLENISHMENT FREQUENCY	.291	.292	.290	.332	.250	.296	.286	.242	.340
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
PERIOD-END INVENTORY	496	143	354	230	266	224	273	219	278
BACKLOG FREQUENCY	.095	.179	.012	.095	.095	.095	.095	.092	.098
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.013	.153	.007	.012	.013	.012	.014	.016	.012
REPLENISHMENT FREQUENCY	.278	.281	.275	.333	.223	.288	.268	.216	.340

Table 3.5d

Operating Characteristics of a Multi-Item System Controlled with Statistical Information About Demand
(Revision Interval 24 Periods, Revision History Length 24 Periods)

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	463	133	329	221	242	212	251	193	270
BACKLOG FREQUENCY	.104	.195	.013	.102	.107	.103	.105	.106	.102
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.014	.166	.008	.013	.015	.012	.016	.018	.012
REPLENISHMENT FREQUENCY	.291	.292	.290	.332	.250	.296	.286	.242	.340
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL IV									
PERIOD-END INVENTORY	492	143	348	232	260	224	267	210	282
BACKLOG FREQUENCY	.095	.178	.011	.095	.095	.094	.096	.095	.094
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	.012	.152	.007	.012	.013	.011	.014	.016	.011
REPLENISHMENT FREQUENCY	.276	.281	.271	.314	.238	.280	.272	.229	.322

Table 3.6a

Percentage Excess of Operating Characteristics of a Multi-Item System Under Statistical Control Over Those for System Under Optimal Control with Full Information. (Revision Interval 24 Periods, Revision History Length 24 Periods)

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	5.4	1.3	7.1	7.5	3.5	4.8	5.8	4.3	6.1
BACKLOG FREQUENCY	3.7	2.0	38.1	1.5	5.9	2.4	5.1	8.1	-.4
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	25.9	4.2	53.5	21.9	29.7	20.5	30.6	30.4	22.7
REPLENISHMENT FREQUENCY	.9	6.3	-4.0	.9	1.0	.2	1.6	2.3	-.1
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL I									
PERIOD-END INVENTORY	9.2	4.6	11.2	10.2	8.2	9.1	9.3	8.2	9.9
BACKLOG FREQUENCY	.7	-.8	32.9	-.7	2.1	.6	.8	3.0	-1.5
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	23.2	2.5	50.5	21.5	24.8	19.4	26.4	25.3	21.6
REPLENISHMENT FREQUENCY	-.3	6.5	-6.4	-.4	-.1	-.7	.2	2.1	-1.9

Table 3.6b

Percentage Excess of Operating Characteristics of a Multi-Item System Under Statistical Control Over Those for System Under Optimal Control with Full Information.
(Revision Interval 24 Periods, Revision History Length 24 Periods)

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>STATIONARY MODEL</u>									
PERIOD-END INVENTORY	5.4	1.3	7.1	7.5	3.5	4.8	5.8	4.3	6.1
BACKLOG FREQUENCY	3.7	2.0	38.1	1.5	5.9	2.4	5.1	8.1	-.4
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	25.9	4.2	53.5	21.9	29.7	20.5	30.6	30.4	22.7
REPLENISHMENT FREQUENCY	.9	6.3	-4.0	.9	1.0	.2	1.6	2.3	-.1
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: <u>MODEL II</u>									
PERIOD-END INVENTORY	7.2	3.0	9.0	9.1	5.5	6.7	7.6	7.1	7.2
BACKLOG FREQUENCY	1.5	-.1	35.7	-1.0	4.1	.1	2.9	3.0	0.0
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	25.0	3.2	53.6	21.5	28.2	18.1	30.7	25.6	24.5
REPLENISHMENT FREQUENCY	.4	6.4	-5.0	-.2	1.2	-.2	1.1	2.4	-.9

Table 3.6c

Percentage Excess of Operating Characteristics of a Multi-Item System Under Statistical Control Over Those for System Under Optimal Control with Full Information.
(Revision Interval 24 Periods, Revision History Length 24 Periods)

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	5.4	1.3	7.1	7.5	3.5	4.8	5.8	4.3	6.1
BACKLOG FREQUENCY	3.7	2.0	38.1	1.5	5.9	2.4	5.1	8.1	-.4
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	25.9	4.2	53.5	21.9	29.7	20.5	30.6	30.4	22.7
REPLENISHMENT FREQUENCY	.9	6.3	-4.0	.9	1.0	.2	1.6	2.3	-.1
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
PERIOD-END INVENTORY	15.8	9.4	18.6	15.4	16.1	14.7	16.7	20.1	12.6
BACKLOG FREQUENCY	-5.0	-6.5	25.4	-4.7	-5.3	-4.4	-5.7	-7.4	-2.7
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	26.5	2.2	58.0	24.6	28.3	25.4	27.4	16.5	34.2
REPLENISHMENT FREQUENCY	7.4	13.2	2.0	10.2	3.4	6.6	8.2	3.2	10.2

Table 3.6d

Percentage Excess of Operating Characteristics of a Multi-Item System Under Statistical Control Over Those for System Under Optimal Control with Full Information.
(Revision Interval 24 Periods, Revision History Length 24 Periods)

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	5.4	1.3	7.1	7.5	3.5	4.8	5.8	4.3	6.1
BACKLOG FREQUENCY	3.7	2.0	38.1	1.5	5.9	2.4	5.1	8.1	-.4
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	25.9	4.2	53.5	21.9	29.7	20.5	30.6	30.4	22.7
REPLENISHMENT FREQUENCY	.9	6.3	-4.0	.9	1.0	.2	1.6	2.3	-.1
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL IV									
PERIOD-END INVENTORY	13.4	9.6	15.0	13.7	13.1	12.7	14.0	15.0	12.3
BACKLOG FREQUENCY	-5.5	-6.9	24.0	-5.3	-5.7	-6.1	-4.9	-4.7	-6.3
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	19.2	-1.6	46.4	21.5	17.0	13.1	24.2	18.2	19.9
REPLENISHMENT FREQUENCY	.2	6.3	-5.4	-.1	.6	-.5	1.0	.8	-.2

Table 3.7a

Sensitivity of Input Parameters of Operating Characteristics of an Inventory System Controlled With Statistical Information About Demand When Demand Has a Negative Binomial Distribution

	INPUT PARAMETERS			
	C(OUT)/C(IN)	C(FIX)/C(IN)	LEADTIME	MEAN DEMAND
STATIONARY MODEL				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↑	↑	↑	↑
E(BACKLOG FREQUENCY	↓	↑	↑	↓
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↓	↑	↓	↑
E(REPLENISHMENT FREQUENCY	↓	↓	↓	↑
POLICY PARAMETERS				
D	∿	↑	↑	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑
MODEL I				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY	↑	↑	↑	↑
E(BACKLOG COST)	↑	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↑	↓
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↓	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↓	↓	↓	↑
POLICY PARAMETERS				
D	∿	↑	?	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑

↑ indicates relation is monotone increasing
↓ indicates relation is monotone decreasing
∿ indicates relation is approximately constant
? indicates no conclusion can be drawn from the data

Table 3.7b

Sensitivity of Input Parameters of Operating Characteristics of an Inventory System Controlled With Statistical Information About Demand When Demand Has a Negative Binomial Distribution

	INPUT PARAMETERS			
	C(OUT)/C(IN)	C(FIX)/C(IN)	LEADTIME	MEAN DEMAND
STATIONARY MODEL				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↑	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↑	↓
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↓	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↓	↓	↓	↑
POLICY PARAMETERS				
D	∼	↑	↑	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑
MODEL II				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↑	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↑	↓
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↓	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↓	↓	↓	↑
POLICY PARAMETERS				
D	∼	↑	?	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑

↑ indicates relation is monotone increasing
↓ indicates relation is monotone decreasing
∼ indicates relation is approximately constant
? indicates no conclusion can be drawn from the data

Table 3.7c

Sensitivity of Input Parameters of Operating Characteristics of an Inventory System Controlled With Statistical Information About Demand When Demand Has a Negative Binomial Distribution

	INPUT PARAMETERS			
	C(OUT)/C(IN)	C(FIX)/C(IN)	LEADTIME	MEAN DEMAND
STATIONARY MODEL				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↑	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↑	↓
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↓	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↓	↓	↓	↑
POLICY PARAMETERS				
D	∿	↑	↑	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑
MODEL III				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↑	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↓	↑	↓
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↓	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↓	↓	↓	↑
POLICY PARAMETERS				
D	∿	↑	?	*
s	↑	↓	↑	↑
S	↑	↑	↑	↑

↑ indicates relation is monotone increasing
↓ indicates relation is monotone decreasing
∿ indicates relation is approximately constant
? indicates no conclusion can be drawn from the data
* indicates an explanation is given in the text

Table 3.7d

Sensitivity of Input Parameters of Operating Characteristics of an Inventory System Controlled With Statistical Information About Demand When Demand Has a Negative Binomial Distribution

	INPUT PARAMETERS			
	C(OUT)/C(IN)	C(FIX)/C(IN)	LEADTIME	MEAN DEMAND
STATIONARY MODEL				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↑	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↑	↓
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↓	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↓	↓	↓	↑
POLICY PARAMETERS				
D	∿	↑	↑	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑
MODEL IV				
OPERATING CHARACTERISTIC				
E(COST)	↑	↑	↑	↑
E(PERIOD-END INVENTORY)	↑	↑	↑	↑
E(BACKLOG COST)	↑	↑	↑	↑
E(BACKLOG FREQUENCY)	↓	↑	↑	↓
PROPORTION DEMAND BACKLOGGED	↓	↑	↑	↓
E(REPLENISHMENT COST)	↓	↑	↓	↑
E(REPLENISHMENT FREQUENCY)	↓	↓	↓	↑
POLICY PARAMETERS				
D	∿	↑	?	↑
s	↑	↓	↑	↑
S	↑	↑	↑	↑

↑ indicates relation is monotone increasing
↓ indicates relation is monotone decreasing
∿ indicates relation is approximately constant
? indicates no conclusion can be drawn from the data

and expected backlog costs with respect to penalty cost are reversed from those observed for the same operating characteristics under optimal control.

Policy parameters s, S, and D appear to follow the same relationships as those found under approximately optimal control. The occasional departure from increasing monotonicity of D with respect to mean demand that was noted in model III under approximately optimal control is also apparent under statistical control in Appendix D.

We also consider the value taken by the ratio $1/(1 + \pi)$, where π is the unit backlog penalty cost, as an approximation to the frequency of backlog that can be observed for a typical item in a multi-item system. Table 3.8, which is derived from Table 3.5, shows that when $\pi = 4$, $1/(1 + \pi)$ is a good backlog frequency approximation for all systems except model III and model IV, where $1/(1 + \pi)$ is a loose upper bound; $1/(1 + \pi)$ is a good approximation of backlog frequency for all systems when $\pi = 99$, although it is slightly less than the backlog frequencies for each system.

Table 3.8

Backlog Frequencies for Multi-Item Systems
With Negative Binomial Demands

	C(OUT)/C(IN) ($=\pi$)	
	4	99
$1/(1+\pi)$	.2	.01
SYSTEM:		
Stationary Model	.195	.013
Model I	.190	.012
Model II	.192	.012
Model III	.179	.012
Model IV	.178	.011

3.2 Statistical Control: Single-Item Systems

The average (s_i, S_i) values, for $i = 1,\ldots,12$, of the statistical policy are recorded for case A and case B in Tables 3.9 and 3.10, respectively. The values for s_i do not differ greatly from these obtained under approximately optimal control. The statistical values for S_i are also close to those obtained under approximately optimal control with full information with one exception: when $\bar{\mu}_i^{(3)}$ is at its largest value, the corresponding S_i of the statistical policy is significantly less than the approximately optimal S_i. This occurs in period 10 in the cycle for case A and periods 8 and 9 for case B. The statistical estimates for $\bar{\mu}_i^{(3)}$ during these periods are occasionally much greater than the actual parameters. As mentioned previously in Section 2.4, large means can result in small values for $S_i - s_i$, that may cause the power approximation to produce small values for S_i. This explains the discrepancies in S_i under statistical and approximately optimal control. This difference, however, does not cause the general shape of the cost components, displayed in Figure 3.1 for case A and Figure 3.2 for case B, to differ from those obtained under approximately optimal control, shown in Figures 2.4 and 2.5.

Table 3.9

Statistical Policy for Case A

Period in Cycle: i	s_i			S_i		
1	58.4	(57)	[57]	86.5	(85)	[82]
2	58.4	(57)	[57]	86.5	(85)	[83]
3	58.4	(57)	[57]	86.5	(85)	[82]
4	58.4	(57)	[57]	86.5	(85)	[82]
5	58.4	(57)	[57]	86.5	(85)	[84]
6	58.4	(57)	[59]	86.5	(85)	[77]
7	58.4	(57)	[55]	86.5	(85)	[67]
8	73.9	(72)	[72]	105.8	(104)	[108]
9	89.1	(87)	[86]	123.6	(122)	[130]
10	104.0	(102)	[105]	131.2	(140)	[128]
11	89.1	(87)	[89]	123.6	(122)	[112]
12	73.9	(72)	[73]	105.8	(104)	[95]

Note: The approximately optimal policy is given in parentheses and the optimal policy is contained in brackets.

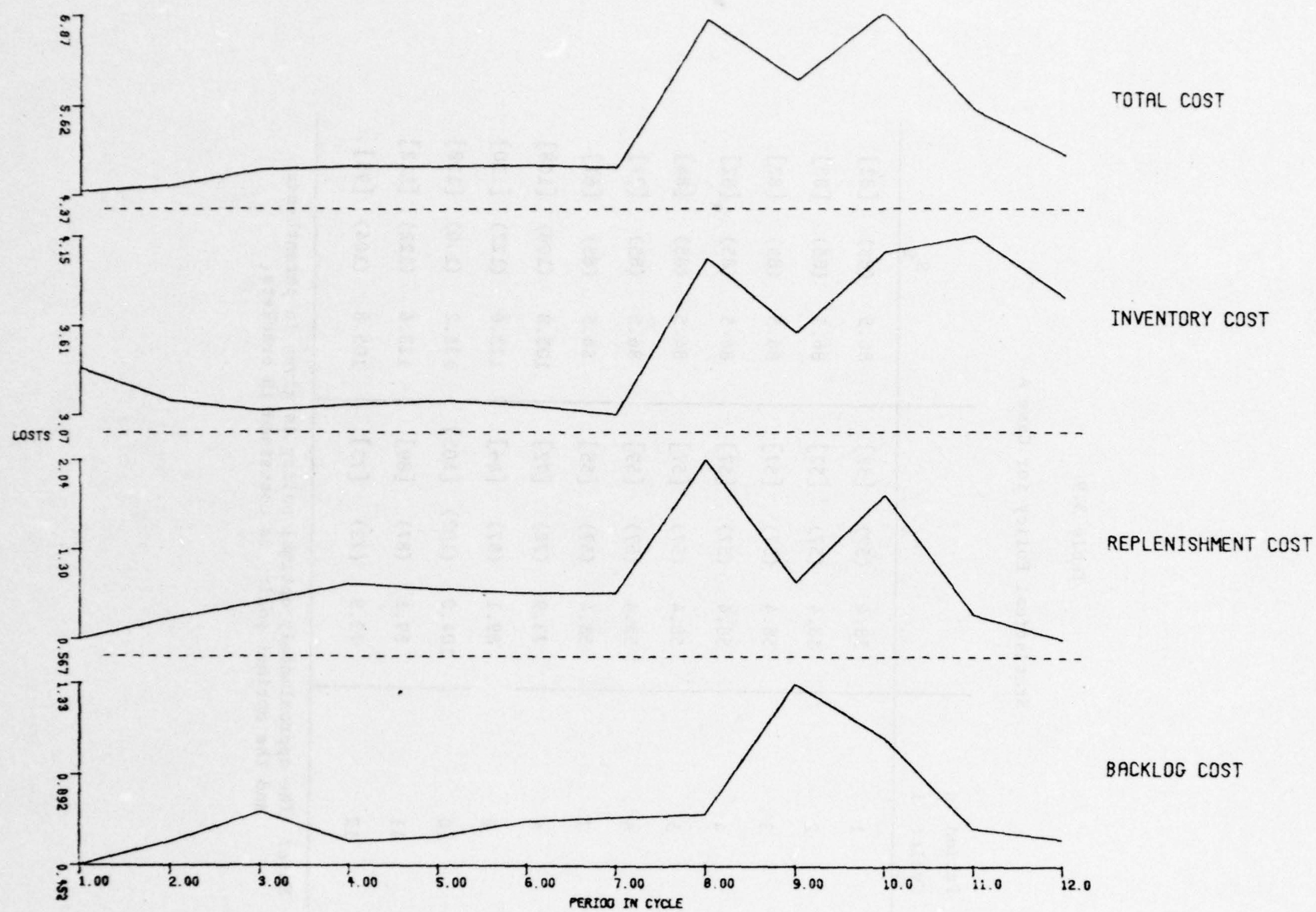

Figure 3.1 Cost Components for Case A Controlled With Statistical Information About Demand

Table 3.10

Statistical Policy for Case B

Period in Cycle: i	s_i			S_i		
1	46.4	(45)	[46]	71.7	(69)	[61]
2	35.4	(35)	[33]	56.5	(56)	[55]
3	35.4	(35)	[33]	56.5	(56)	[56]
4	46.4	(45)	[44]	71.1	(69)	[73]
5	62.2	(61)	[60]	91.3	(90)	[93]
6	76.6	(76)	[75]	110.4	(109)	[113]
7	92.7	(91)	[90]	127.3	(127)	[128]
8	102.6	(101)	[101]	132.0	(139)	[136]
9	102.6	(101)	[102]	132.0	(139)	[131]
10	92.7	(91)	[93]	127.3	(127)	[115]
11	76.6	(76)	[78]	110.4	(109)	[95]
12	62.2	(61)	[62]	91.3	(90)	[76]

Note: The approximately optimal policy is given in parentheses and the optimal policy is contained in brackets.

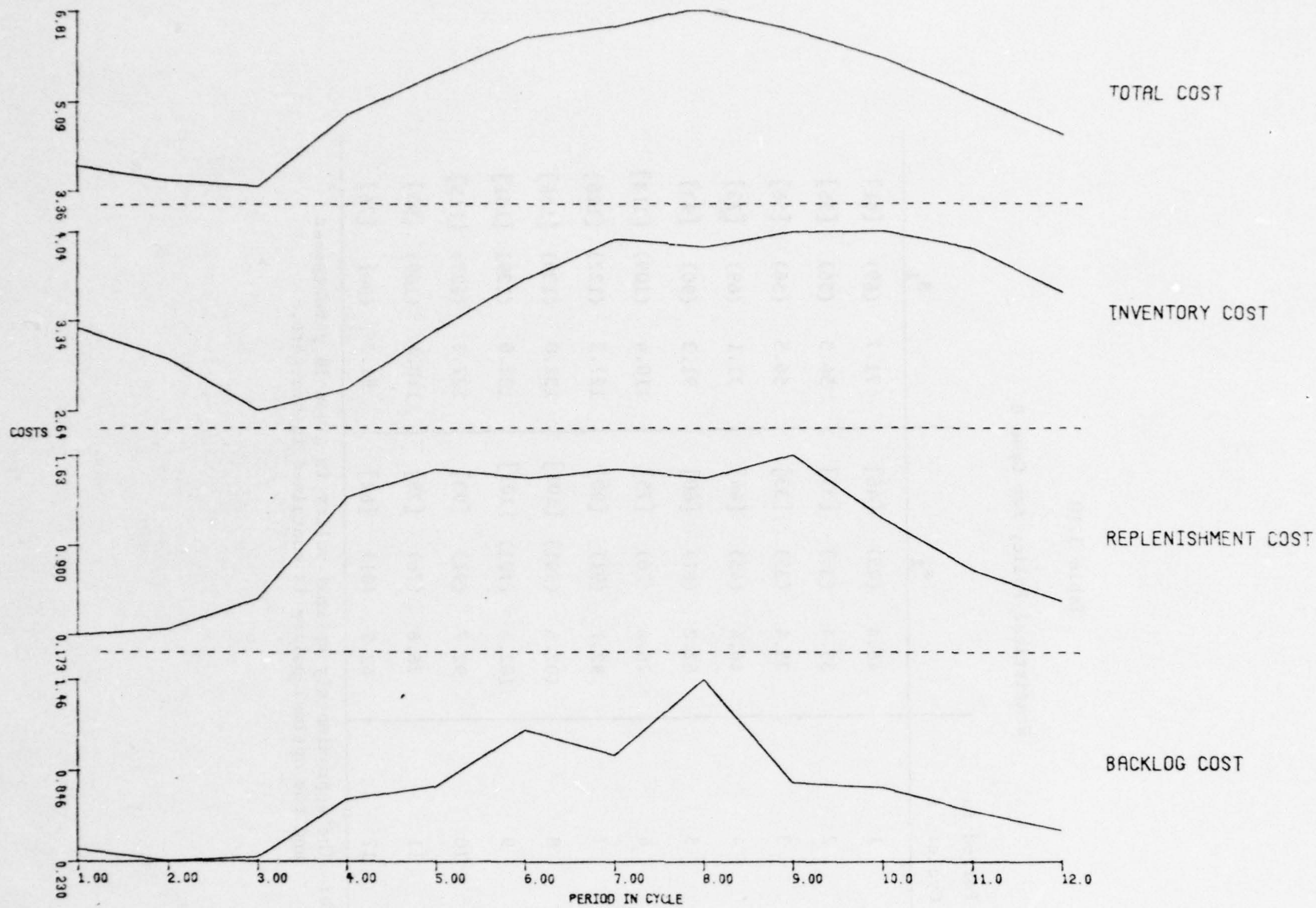

Figure 3.2 Cost Components for Case B Controlled With Statistical Information About Demand

4. FORECASTING THE OPERATING CHARACTERISTICS OF INVENTORY SYSTEMS UNDER STATISTICAL CONTROL

The behavior of an inventory system controlled with statistical information about demand was investigated in Section 3 using simulation techniques. In the absence of analytic expressions for the operating characteristics as functions of the input distribution functions and other system parameters, sample distributions for the operating characteristics were gathered from the simulations, particular attention being paid to the estimation of expected values.

The manager of an inventory system operating with incomplete information about demand will need to forecast the system's behavior, especially to justify the installation of scientific control. Forecasting also may be a routine requirement consequent upon the periodic revision of the parameters of the replenishment policies or even as part of a regular budgeting procedure.

Forecasting methods have been considered in MacCormick [(1974), Section 3.2]. In Section 4 information is gathered about forecasts using the same techniques as MacCormick which involve double use of the demand history, for control and forecasting. Sample distributions of the forecasts are compared with those of the operating characteristics, which were obtained in Section 3.

The major issues to be resolved here are the extent of bias and the level of dispersion of the forecasts, together with the variation of these with system settings, including the demand process, unit costs, and the replenishment leadtime. Little consideration is given to an important sampling problem, namely, that since it is usually prohibitively expensive to gather a history of demands for every item in a large multi-item system, a sample of the items must be selected to make a forecast. The accuracy of such a forecast is likely to be critically dependent on the number of items in the sample and its composition. Forecasts made for the multi-item system in this simulation experiment have used every item in the system.

4.1 Properties of Forecasts for Operating Characteristics of Multi-Item Systems

In the simulation experiments, point forecasts of the expected values of the system operating characteristics are made each time the control parameters (s,S) are revised. The demand history used to make a forecast has been selected to contain demands for the same number of periods as the revision interval. The actual values for the forecasts are obtained by retrospective simulation, which has been explained previously in Section 1.2.6.

Sample average values of the forecasts of average costs per period for a multi-item system are shown in Table 4.1, the sample standard deviations being shown in parentheses. The sample statistics for forecasts shown in Table 4.1 can be compared with those in Table 3.1 for the average cost per period. The comparison for expected average cost per period is explicit in Table 4.2, which contains the percentage difference between the forecasts and the estimated values.

On the average, every component of expected cost is underestimated by its forecast, behavior predicted by the analysis of simplified models in Section 2 of MacCormick (1974). For expected total cost per period, the underestimation is as high as 10.8% in the case of model III. For all systems the bias is greatest for the backlog component.

To check the sensitivity of the bias of the forecasts to other system parameters, Table 4.3 was constructed showing percentage values for

Table 4.1

Average Forecasted Values for Expected Costs Per Period for Multi-Item Negative Binomial Systems Controlled With Statistical Information About Demand (Revision Interval 24 Periods, Revision History Length 24 Periods)

COST COMPONENT	STATIONARY MODEL	MODEL I	MODEL II	MODEL III	MODEL IV
INVENTORY	450 (20) [60.8]	454 (21) [61.6]	452 (21) [61.3]	475 (28) [64.6]	470 (24) [63.6]
BACKLOG	78 (28) [10.6]	75 (26) [10.1]	76 (30) [10.3]	62 (27) [8.4]	67 (26) [9.0]
REPLENISHMENT	212 (5) [28.7]	209 (6) [28.3]	209 (5) [28.4]	199 (6) [27.0]	202 (6) [27.3]
TOTAL	740 (36) [100.0]	738 (34) [100.0]	738 (38) [100.0]	735 (38) [100.0]	739 (35) [100.0]

Note: Estimates of the standard deviations for the forecasts are shown in parentheses, and each cost component's percent of the total is shown in brackets.

Table 4.2

Average Costs Per Period for Multi-Item Negative Binomial Systems Controlled With Statistical Information About Demand: Comparison of Forecasts With Corresponding Estimated Expected Values (Revision Interval 24 Periods, Revision History Length 24 Periods)

COST COMPONENT	STATIONARY MODEL	MODEL I	MODEL II	MODEL III	MODEL IV
INVENTORY	2.9	4.0	3.6	4.3	4.3
BACKLOG	43.1	42.7	42.7	52.1	46.1
REPLENISHMENT	.4	-.6	-0.0	.2	.1
TOTAL	9.0	9.0	9.0	10.8	9.6

Note: Table 4.2 shows the percentage value of the underestimate by the forecast of the realized value.

Table 4.3a

Forecasting for a Multi-Item System Under Statistical Control:
Estimated Percentage Difference Between Forecasts and Expected Average Costs Per Period
(Revision Interval 24 Periods, Revision History 24 Periods, Forecasting History 24 Periods)

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	2.9	5.6	1.8	3.1	2.7	2.1	3.6	3.0	2.8
BACKLOG	43.1	11.1	70.8	47.4	39.4	34.5	49.9	42.9	43.3
REPLENISHMENT	.4	.5	.3	.3	.5	.4	.4	.8	.1
TOTAL	9.0	5.0	11.4	10.2	8.1	6.8	11.0	9.4	8.8
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL I									
INVENTORY	4.0	6.0	3.1	4.3	3.7	3.2	4.6	4.0	3.9
BACKLOG	42.7	9.7	72.2	44.6	40.9	33.3	50.0	44.4	41.3
REPLENISHMENT	-.6	-.5	-.8	-.8	-.6	-.4	-.9	-.7	-.6
TOTAL	9.0	4.5	11.7	10.0	8.2	6.8	11.0	9.6	8.6

Table 4.3b

Forecasting for a Multi-Item System Under Statistical Control:
Estimated Percentage Difference Between Forecasts and Expected Average Costs Per Period
(Revision Interval 24 Periods, Revision History 24 Periods, Forecasting History 24 Periods)

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	2.9	5.6	1.8	3.1	2.7	2.1	3.6	3.0	2.8
BACKLOG	43.1	11.1	70.8	47.4	39.4	34.5	49.9	42.9	43.3
REPLENISHMENT	.4	.5	.3	.3	.5	.4	.4	.8	.1
TOTAL	9.0	5.0	11.4	10.2	8.1	6.8	11.0	9.4	8.8
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL II									
INVENTORY	3.6	6.1	2.5	3.9	3.2	2.2	4.7	3.9	3.3
BACKLOG	42.7	9.2	72.2	45.6	40.0	35.6	48.1	44.4	41.4
REPLENISHMENT	-0.0	-0.0	-.1	-.1	0.0	-0.2	0.0	-.2	.1
TOTAL	9.0	4.6	11.7	10.1	8.1	6.6	11.2	9.6	8.6

Table 4.3c

Forecasting for a Multi-Item System Under Statistical Control:
Estimated Percentage Difference Between Forecasts and Expected Average Costs Per Period
(Revision Interval 24 Periods, Revision History 24 Periods, Forecasting History 24 Periods)

COST COMPONENT	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	2.9	5.6	1.8	3.1	2.7	2.1	3.6	3.0	2.8
BACKLOG	43.1	11.1	70.8	47.4	39.4	34.5	49.9	42.9	43.3
REPLENISHMENT	.4	.5	.3	.3	.5	.4	.4	.8	.1
TOTAL	9.0	5.0	11.4	10.2	8.1	6.8	11.0	9.4	8.8
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
INVENTORY	4.3	6.6	3.4	4.2	4.5	2.1	6.2	5.0	3.8
BACKLOG	52.1	17.6	81.0	53.4	50.9	47.2	56.0	52.2	52.0
REPLENISHMENT	.2	0.0	.5	.5	0.0	.7	-.2	.5	.1
TOTAL	10.8	6.5	13.3	11.5	10.2	8.5	12.8	11.0	10.7

Table 4.3d

Forecasting for a Multi-Item System Under Statistical Control:
Estimated Percentage Difference Between Forecasts and Expected Average Costs Per Period
(Revision Interval 24 Periods, Revision History 24 Periods, Forecasting History 24 Periods)

COST COMPONENTS	TOTAL	DISTRIBUTIONS BY VALUES OF INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
INVENTORY	2.9	5.6	1.[illegible]	3.1	2.7	2.1	3.6	3.0	2.8
BACKLOG	43.1	11.1	70.8	47.4	39.4	34.5	49.9	42.9	43.3
REPLENISHMENT	.4	.5	.3	.3	.5	.4	.4	.8	.1
TOTAL	9.0	5.[illegible]	11.4	10.2	8.1	6.8	11.0	9.4	8.8
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL IV									
INVENTORY	4.3	7.0	3.2	4.7	4.0	3.0	5.4	4.5	4.2
BACKLOG	46.1	12.7	75.6	49.6	42.6	40.4	50.4	47.6	45.0
REPLENISHMENT	.1	.1	-0.0	-0.0	.1	.3	-.2	.1	0.0
TOTAL	9.6	5.8	11.9	11.1	8.4	7.5	11.4	10.1	9.3

Table 4.4a

Forecasting for a Multi-Item System Under Statistical Control:
Estimated Percentage Difference Between Forecasts and Expected Values of Operating Characteristics
(Revision Interval 24 Periods, Revision History 24 Periods, Forecasting History 24 Periods)

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	2.9	5.6	1.8	3.1	2.7	2.1	3.6	3.0	2.8
BACKLOG FREQUENCY	5.7	2.2	58.2	6.5	4.9	4.4	6.9	5.8	5.6
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	43.1	11.1	70.8	47.4	39.4	34.5	49.9	42.9	43.3
REPLENISHMENT FREQUENCY	.4	.5	.3	.3	.5	.4	.4	.8	0.0
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL 1									
PERIOD-END INVENTORY	4.0	6.0	3.1	4.3	3.7	3.2	4.6	4.0	3.9
BACKLOG FREQUENCY	5.1	1.7	58.2	5.6	4.6	2.8	7.4	5.8	4.3
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	42.7	9.7	72.2	44.6	40.9	33.3	50.0	44.4	41.3
REPLENISHMENT FREQUENCY	-.7	-.6	-.8	-.8	-.6	-.5	-.8	-.7	-.7

Table 4.4b

Forecasting for a Multi-Item System Under Statistical Control:
Estimated Percentage Difference Between Forecasts and Expected Values of Operating Characteristics
(Revision Interval 24 Periods, Revision History 24 Periods, Forecasting History 24 Periods)

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	2.9	5.6	1.8	3.1	2.7	2.1	3.6	3.0	2.8
BACKLOG FREQUENCY	5.7	2.2	58.2	6.5	4.9	4.4	6.9	5.8	5.6
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	43.1	11.1	70.8	47.4	39.4	34.5	49.9	42.9	43.3
REPLENISHMENT FREQUENCY	.4	.5	.3	.3	.5	.4	.4	.8	0.0
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL II									
PERIOD-END INVENTORY	3.6	6.1	2.1	3.9	3.2	2.2	4.7	3.9	3.3
BACKLOG FREQUENCY	4.5	1.0	58.0	4.9	4.2	2.4	6.6	4.5	4.6
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	42.7	9.2	72.2	45.6	40.0	35.6	48.1	44.4	41.4
REPLENISHMENT FREQUENCY	-.1	-0.0	-.1	-.1	0.0	-.2	0.0	-.2	0.0

Table 4.4c

Forecasting for a Multi-Item System Under Statistical Control:
Estimated Percentage Difference Between Forecasts and Expected Values of Operating Characteristics
(Revision Interval 24 Periods, Revision History 24 Periods, Forecasting History 24 Periods)

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	2.9	5.6	1.8	3.1	2.7	2.1	3.6	3.0	2.8
BACKLOG FREQUENCY	5.7	2.2	58.2	6.5	4.9	4.4	6.9	5.8	5.6
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	43.1	11.1	70.8	47.4	39.4	34.5	49.9	42.9	43.3
REPLENISHMENT FREQUENCY	.4	.5	.3	.3	.5	.4	.4	.8	0.0
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL III									
PERIOD-END INVENTORY	4.3	6.6	3.4	4.2	4.5	2.1	6.2	5.0	3.8
BACKLOG FREQUENCY	8.1	4.2	69.8	9.7	6.6	6.3	10.0	8.1	8.1
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	52.1	17.6	81.0	53.4	50.9	47.2	56.0	52.2	52.0
REPLENISHMENT FREQUENCY	.3	0.0	.6	.5	0.0	.7	-0.0	.5	.2

Table 4.4d

Forecasting for a Multi-Item System Under Statistical Control:
Estimated Percentage Difference Between Forecasts and Expected Values of Operating Characteristics
(Revision Interval 24 Periods, Revision History 24 Periods, Forecasting History 24 Periods)

OPERATING CHARACTERISTIC	VALUE	INPUT PARAMETERS							
		C(OUT)/C(IN)		C(FIX)/C(IN)		LEADTIME		MEAN	
		4	99	32	64	2	4	8	16
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: STATIONARY MODEL									
PERIOD-END INVENTORY	2.9	5.6	1.8	3.1	2.7	2.1	3.6	3.0	2.8
BACKLOG FREQUENCY	5.7	2.2	58.2	6.5	4.9	4.4	6.9	5.8	5.6
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	43.1	11.1	70.8	47.4	39.4	34.5	49.9	42.9	43.3
REPLENISHMENT FREQUENCY	.4	.5	.3	.3	.5	.4	.4	.8	0.0
16-ITEM SYSTEM WITH NEGATIVE BINOMIAL DEMANDS: MODEL IV									
PERIOD-END INVENTORY	4.3	7.0	3.2	4.7	4.0	3.0	5.4	4.5	4.2
BACKLOG FREQUENCY	5.9	2.2	63.3	7.7	4.0	4.1	7.6	6.5	5.3
WEIGHTED PROPORTION OF DEMAND BACKLOGGED	46.1	12.7	75.6	49.6	42.6	40.4	50.4	47.6	45.0
REPLENISHMENT FREQUENCY	0.0	-0.0	.1	-0.0	.1	.3	-.3	.2	-0.0

the biases. In every system examined, the bias in the forecast of expected average total cost tends to increase with an increase in the unit backlog cost or in the mean demand. A very large increase in bias is observed in backlog cost when unit penalty costs increases from 4 to 99. Table 4.4 demonstrates biases for forecasts of operating characteristics other than costs, expressed as percentages of the estimates of the actual expected values of the operating characteristics. In particular, the forecasts of the backlog frequency do not have a percentage bias as large as those for the weighted proportion of demand backlogged. For both characteristics, however, the percent bias is much greater for those items with unit backlog costs of 99 than those with unit backlog costs of 4. More detailed versions of Tables 4.3 and 4.4 can be found in Appendices M, Q, U, and Y.

It is important to note that each of the 200 sample values for an operating characteristic that are collected during the simulation of a single item is based on 24 demand periods of the simulation. Since 24 is a multiple of the demand cycle length, which is 12, the first demand encountered in collecting each of the 200 sample forecast values is always sampled from the same demand distribution in the demand cycle. Demand distribution $12 - 2\lambda + 1$, where λ is the leadtime, is arbitrarily selected by the simulation as the initial distribution. Thus, the question arises as to whether initializing with a different demand distribution could result in changes in the forecasts, or whether 24 demand periods is sufficient to diminish the effects that the specific initial distribution may have on the forecasted operating characteristic.

This issue is investigated by an additional simulation of model IV, under statistical control, using a different initializing distribution. For each item in the system a new initial distribution is chosen such that, of all the distributions within a cycle, the new distribution's mean differs the most, in absolute value, from the previously selected initial distribution. Hence, the demand distributions in periods 3 and 10 are used as initial distributions for those cases with leadtimes of 2 and 4, respectively. The resulting forecast values, shown in Table 4.5, are virtually the same as the original simulation of model IV. We, thus, conclude that 24 demand periods is sufficient to deter any potential effect of an initial distribution, and that this issue does not warrant any further investigation.

Table 4.5

Average Forecasted Values for Expected Costs Per Period for Multi-Item Negative Binomial Systems Controlled With Statistical Information About Demand (Revision Interval 24 Periods, Revision History Length 24 Periods)

COST COMPONENT	MODEL IV (From Table 4.1)			MODEL IV (Using a different initializing distribution)		
INVENTORY	470	(24)	[63.6]	472	(24)	[64.0]
BACKLOG	67	(26)	[9.0]	65	(26)	[8.7]
REPLENISHMENT	202	(6)	[27.3]	202	(6)	[27.3]
TOTAL	735	(35)	[100.0]	741	(36)	[100.0]

Note: Estimates of the standard deviations for the forecasts are shown in parentheses, and each cost component's percent of the total is shown in brackets.

4.2 Properties of Forecasts for Operating Characteristics of Single-Item Systems

The average forecasted values of cost components are displayed in Figures 4.1 and 4.2, corresponding to cases A and B. The underestimation of backlog costs, observed in Table 4.1, for the multi-item systems becomes apparent in cases A and B when Figures 4.1 and 4.2 are compared with Figures 3.1 and 3.2. Both single-item systems considerably underestimate

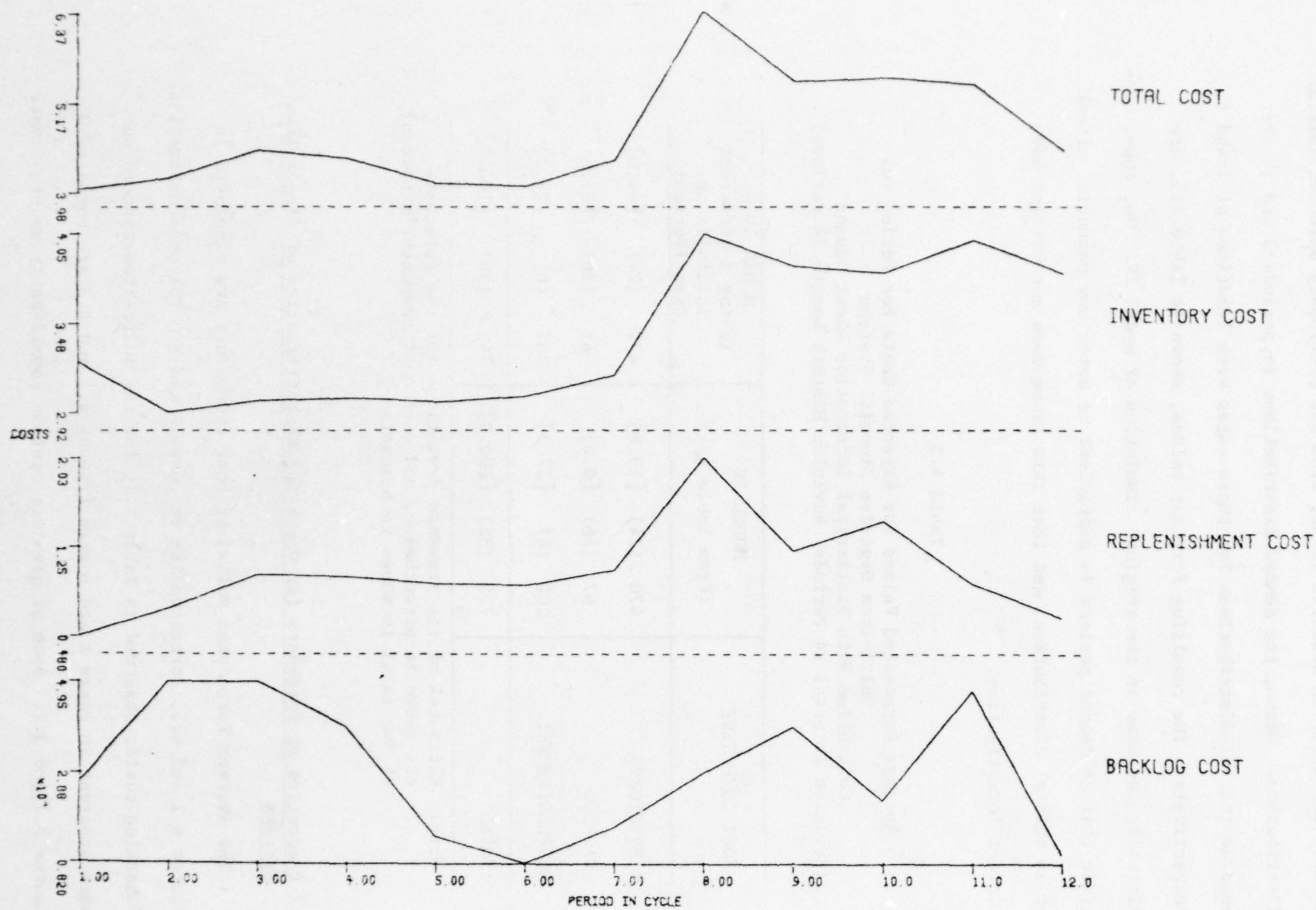

Figure 4.1 Cost Components of Forecasts for Case A

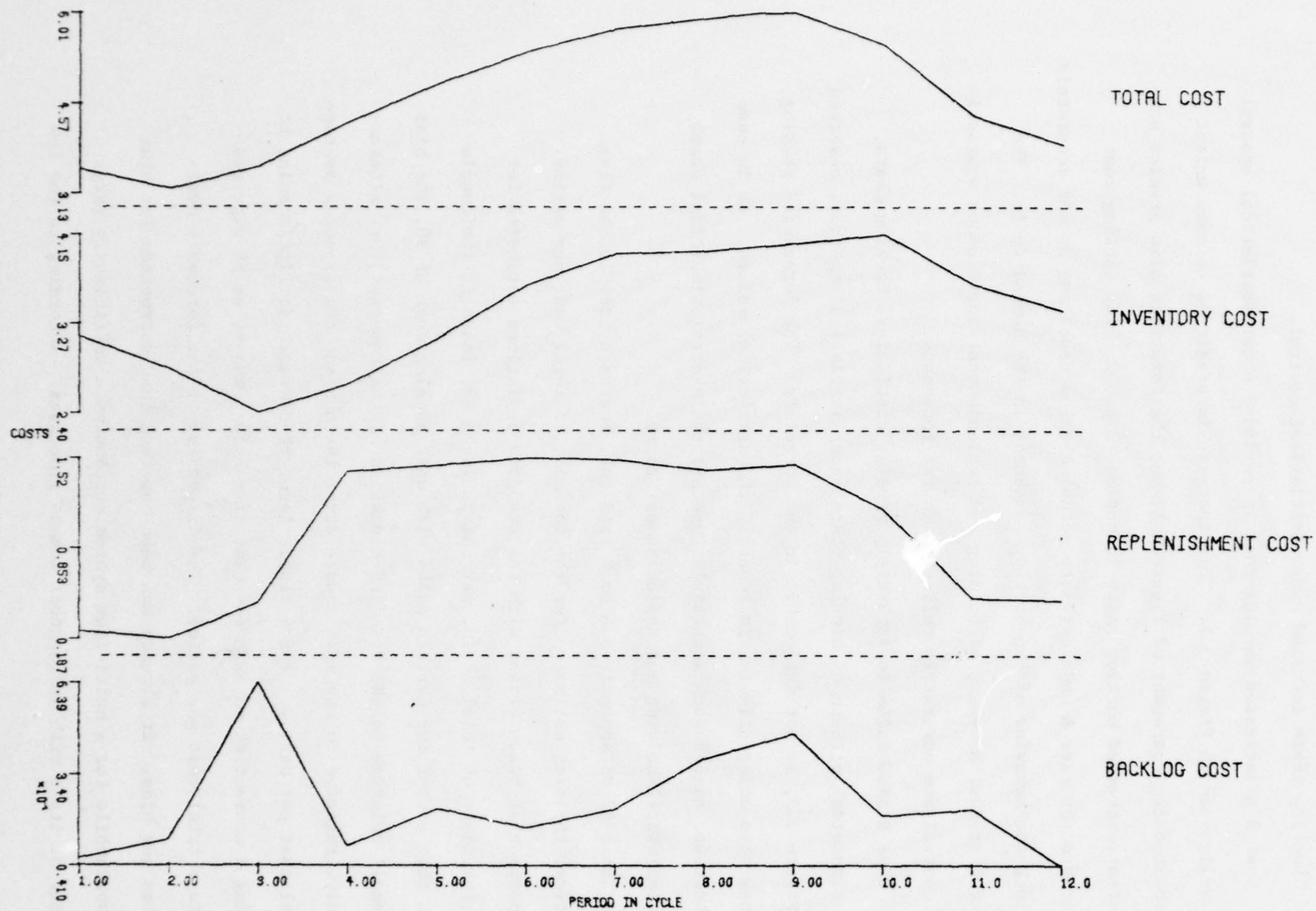

Figure 4.2 Cost Components of Forecasts for Case B

their backlog costs incurred under statistical control.

Case A's increased backlog cost in period 9 under statistical control is easily seen in Figure 3.1. This increase is predicted to some extent in the backlog component of Figure 4.1, but the forecasts also predict non-existent increased backlog costs in periods 2 and 3. The backlog cost forecasts for case A underestimate actual backlog costs and do not accurately reflect the behavior of the backlog component in the demand cycle. The general trends, however, of holding and replenishment costs under statistical control are accurately reflected in the forecasts.

Case B also fails to accurately predict statistical backlog costs. The sinusoidal pattern of backlog costs under statistical control, observed in Figure 3.2, is not detectable in the forecasts. The forecasted backlog values also underestimate, in general, the statistical values. As in case A, however, case B does accurately forecast the general sinusoidal shape of statistical holding and replenishment costs.

Table A7 of Appendices A to E sets out, for each item in the five systems, the bias estimated for the forecast of total cost per period. Comparison of these biases with the standard deviations estimated for the forecasts of total costs, set out in Table A8, leads to the conclusion that except for certain cases with unit backlog cost of 99, the bias is small relative to the standard deviation of the forecast, or, alternatively, relative to the mean square error taken about the expected average total cost per period. For a single item, then, there is little point to making a correction to each forecast of the long run value of expected average total cost per period. The dispersion of the forecasts overwhelms any bias. It already has been observed that correction for bias is worthwhile for a multi-item system containing a sufficiently large number of items with independent demand processes. According to the law of large numbers, asymptotically the dispersion becomes small relative to the bias in the forecast of an aggregate operating characteristic.

Table A7 also contains information useful to evaluate the forecasts for the short run. The table has been constructed by analysis of the time series of differences between the forecasts and the values subsequently realized for the operating characteristics. A convention has been adopted that the bias is positive if the realized value of an operating characteristic exceeds on the average the forecast value. To test whether bias observed in any operating characteristic is significantly positive or negative, a two-tail large sample test has been applied at the .05 level of significance, assuming that the sample mean differences for 200 observations are normally distributed. The standard deviation used to construct the unit normal test statistic is that estimated for the sequence of 200 differences. Results of the tests are shown in columns (6) to (11) of Table A7. Note that the average total cost per period has bias significantly positive for all the items in each of the five systems. A significantly positive bias also is present for average period-end inventory and backlog quantity in most cases. Stockout frequency is significantly positive when the unit penalty cost is large.

In summary, for single-item systems where the forecasts are to be used as predictions for long run expected values, the forecasts are estimated to have standard deviations large relative to the expected values. They are, therefore, not useful to predict well the long run performance of a single-item system. For a multi-item system, bias in the forecasts has been observed to be quite general. Both this bias and the dispersion of the forecasts are sensitive to the characteristics of the items in the system, however, and therefore the magnitudes depend on the overall composition of the multi-item system under consideration. For this reason, when forecasting is to be done using only a sample of items in the system, care must be taken to select a sample representative of the overall structure of the multi-item system.

5. TOPICS FOR FURTHER RESEARCH

In this section we summarize the main results of this report and suggest topics for further investigation.

In Section 2.3 we examined, in detail, the optimal policies and corresponding operating characteristics under optimal control for two typical single-item cases. We observed, in particular, that the movement of optimal policies through the demand cycle was similar to that of the means and variances of the $\lambda + 1$ - fold demand distributions, $\Phi_i^{\lambda+1}$, for $i = 1,\ldots, 12$. This suggested that an approximately optimal (s,S) inventory policy can be computed in the beginning of the i^{th} period within the cycle as a function of the cost parameters and the demand means and variances for only periods i to $\lambda + 1 + i$.

The 12 convoluted means and variances, $\mu_i^{(\lambda+1)}$ and $\sigma_i^{2(\lambda+1)}$, for $i = 1,\ldots, 12$, were used in the Ehrhardt power approximation to compute 12, possibly distinct, (s,S) policies, (see Section 1.1.2 for details). These policies resulted in operating characteristics that were very close to that of under optimal control.

When the 12 individual demand means and variances were not known, they were estimated using previous demand data. The technique adopted under statistical control assumed knowledge of the relative magnitudes of demand means within the cycle by representing mean demand in period i as αx_i, where α is the smallest demand mean encountered in the cycle and the x_i variables are assumed known. α is estimated by regression techniques, (see Section 1.2.3 for details). The results of Section 3 indicated that this statistical approach resulted in relatively small increases of total costs over that of optimal control.

We realize that in this study the amount of variation of mean demand for the various nonstationary systems has been arbitrarily selected. Models I and II vary mean demand from a base level α to 2α, while models III and IV vary mean demand from α to 5α. Sections 2 and 3 reveal a slight deterioration in the performance of the power approximation for models III and IV. Hence, it may be interesting to increase the mean demand levels for these models to much higher levels than 5α, to see if further degradation occurs. In the event of further degradation, the individual variances $\bar{\sigma}_i^{2(\lambda+1)}$, for $i = 1,\ldots,12$, might be slightly altered to account for the amount of variation of mean demand within a cycle. Further research with nonstationary models that have small variations in mean demand might also be considered to see if the stationary policies in Section 2.6 improve their performance.

The study has required that all demands are stocastically independent of each other. Hence, knowledge of simply the demand in one period provides no additional information about future demands. Correlated demand sequences can provide an interesting area for further investigation. The analysis of inventory policies when demands follow a Markov process remains an open area for research.

Under statistical control in Section 3 we assumed knowledge of the general shape of the demand means within a cycle via the x_i's, (see Section 1.2.3 for details). Complete knowledge of the x_i's indirectly gives us the cycle length. Relaxing these assumptions would require that new estimation schemes for the x_i's and the cycle length, based on the observed demands, be developed.

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Appendix A

Single-item Data

Stationary Model

Summary of Data for 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled with:

Optimal Policies (DP)

Power Approximation (PA)

Statistical Power Approximation

Note: For corresponding data in MacCormick (1974), see tables of the same number in Appendices A of those reports.

Appendix B

Single-item Data

Model I

Summary of Data for 16 Items with Negative Binomial Demand

Distributions (Variance/Mean = 3) Controlled with:

Optimal Policies (DP)

Power Approximation (PA)

Statistical Power Approximation

Note: For corresponding data in MacCormick (1974), see tables of the same number in Appendices A of those reports.

Appendix C

Single-item Data

Model II

Summary of Data for 16 Items with Negative Binomial Demand

Distributions (Variance/Mean = 3) Controlled with:

Optimal Policies (DP)

Power Approximation (PA)

Statistical Power Approximation

Note: For corresponding data in MacCormick (1974), see tables of the same number in Appendices A of those reports.

Appendix D

Single-item Data

Model III

Summary of Data for 16 Items with Negative Binomial Demand

Distributions (Variance/Mean = 3) Controlled with:

Optimal Policies (DP)

Power Approximation (PA)

Statistical Power Approximation

Note: For corresponding data in MacCormick (1974), see tables of the same number in Appendices A of those reports.

Appendix E

Single-item Data

Model IV

Summary of Data for 16 Items with Negative Binomial Demand

Distributions (Variance/Mean = 3) Controlled with:

Optimal Policies (DP)

Power Approximation (PA)

Statistical Power Approximation

Note: For corresponding data in MacCormick (1974), see tables of the same number in Appendices A of those reports.

Appendix F

Multi-item Data for the Power Approximation

Stationary Model

System of 16 Items with Negative Binomial Demand Distributions

(Variance/Mean = 3) Controlled Optimally with Full Information.

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendix C.

Appendix G

Multi-item Data for the Power Approximation

Stationary Model

System of 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled Approximately Optimally with Full Information.

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices E and F.

Appendix H

Multi-item Data for the (24,24) Statistical Power Approximation

Stationary Model

System of 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled with Statistical Information from a 24-Period Demand History, with Revision Every 24 Periods, Using Regression Estimates of Demand Means and Variances.

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices K and L.

Appendix I

Multi-item Forecasts for the (24,24) Statistical Power Approximation

Stationary Model

Forecasting Properties of Inventory System of 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled with Statistical Information, Revision Taking Place Every 24 Periods Using a 24-Period Demand History and Regression Estimates of Demand Means and Variances. Forecasts Made at Each Revision Using a 24-Period Demand History.

Note: The corresponding appendices in MacCormick (1974), are his Appendices T and U.

Appendix J

Multi-item Data for the Power Approximation

Model I

System of 16 Items with Negative Binomial Demand Distributions

(Variance/Mean = 3) Controlled Optimally with Full Information.

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendix C.

Appendix K

Multi-item Data for the Power Approximation

Model I

System of 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled Approximately Optimally with Full Information.

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices E and F.

Appendix L

Multi-item Data for the (24,24) Statistical Power Approximation

Model I

System of 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled with Statistical Information from a 24-Period Demand History, with Revision Every 24 Periods, Using Regression Estimates of Demand Means and Variances.

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices K and L.

Appendix M

Multi-item Forecasts for the (24,24) Statistical Power Approximation

Model I

Forecasting Properties of Inventory System of 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled with Statistical Information, Revision Taking Place Every 24 Periods Using a 24-Period Demand History and Regression Estimates of Demand Means and Variances. Forecasts Made at Each Revision Using a 24-Period Demand History.

Note: The corresponding appendices in MacCormick (1974), are his Appendices T and U.

Appendix N

Multi-item Data for the Power Approximation

Model II

System of 16 Items with Negative Binomial Demand Distributions

(Variance/Mean = 3) Controlled Optimally with Full Information.

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendix C.

Appendix O

Multi-item Data for the Power Approximation

Model II

System of 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled Approximately Optimally with Full Information.

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices E and F.

Appendix P

Multi-item Data for the (24,24) Statistical Power Approximation

Model II

System of 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled with Statistical Information from a 24-Period Demand History, with Revision Every 24 Periods, Using Regression Estimates of Demand Means and Variances.

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices K and L.

Appendix Q

Multi-item Forecasts for the (24,24) Statistical Power Approximation

Model II

Forecasting Properties of Inventory System of 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled with Statistical Information, Revision Taking Place Every 24 Periods Using a 24-Period Demand History and Regression Estimates of Demand Means and Variances. Forecasts Made at Each Revision Using a 24-Period Demand History.

Note: The corresponding appendices in MacCormick (1974), are his Appendices T and U.

Appendix R

Multi-item Data for the Power Approximation

Model III

System of 16 Items with Negative Binomial Demand Distributions

(Variance/Mean = 3) Controlled Optimally with Full Information.

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendix C.

Appendix S

Multi-item Data for the Power Approximation

Model III

System of 16 Items with Negative Binomial Demand Distributions

(Variance/Mean = 3) Controlled Approximately Optimally with

Full Information.

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices E and F.

Appendix T

Multi-item Data for the (24,24) Statistical Power Approximation

Model III

System of 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled with Statistical Information from a 24-Period Demand History, with Revision Every 24 Periods, Using Regression Estimates of Demand Means and Variances.

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices K and L.

Appendix U

Multi-item Forecasts for the (24,24) Statistical Power Approximation

Model III

Forecasting Properties of Inventory System of 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled with Statistical Information, Revision Taking Place Every 24 Periods Using a 24-Period Demand History and Regression Estimates of Demand Means and Variances. Forecasts Made at Each Revision Using a 24-Period Demand History.

Note: The corresponding appendices in MacCormick (1974), are his Appendices T and U.

Appendix V

Multi-item Data for the Power Approximation

Model IV

System of 16 Items with Negative Binomial Demand Distributions

(Variance/Mean = 3) Controlled Optimally with Full Information.

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendix C.

Appendix W

Multi-item Data for the Power Approximation

Model IV

System of 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled Approximately Optimally with Full Information.

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices E and F.

Appendix X

Multi-item Data for the (24,24) Statistical Power Approximation

Model IV

System of 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled with Statistical Information from a 24-Period Demand History, with Revision Every 24 Periods, Using Regression Estimates of Demand Means and Variances.

Note: For corresponding data in MacCormick (1974), see the table of the same number in his Appendices K and L.

Appendix Y

Multi-item Forecasts for the (24,24) Statistical Power Approximation

Model IV

Forecasting Properties of Inventory System of 16 Items with Negative Binomial Demand Distributions (Variance/Mean = 3) Controlled with Statistical Information, Revision Taking Place Every 24 Periods Using a 24-Period Demand History and Regression Estimates of Demand Means and Variances. Forecasts Made at Each Revision Using a 24-Period Demand History.

Note: The corresponding appendices in MacCormick (1974), are his appendices T and U.