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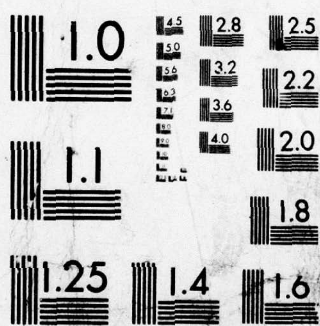

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FOREIGN TECHNOLOGY DIVISION

SOME LIMITING REGULAR LAWS FOR RANK

by

B. N. Gartshteyn

EDITED TRANSLATION

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SOME LIMITING REGULAR LAWS FOR RANK

By: B. N. Gartshteyn

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	А а	A, a	Р р	Р р	R, r
Б б	Б б	B, b	С с	С с	S, s
В в	В в	V, v	Т т	Т т	T, t
Г г	Г г	G, g	У у	У у	U, u
Д д	Д д	D, d	Ф ф	Ф ф	F, f
Е е	Е е	Ye, ye; E, e*	Х х	Х х	Kh, kh
Ж ж	Ж ж	Zh, zh	Ц ц	Ц ц	Ts, ts
З з	З з	Z, z	Ч ч	Ч ч	Ch, ch
И и	И и	I, i	Ш ш	Ш ш	Sh, sh
Й й	Й й	Y, y	Щ щ	Щ щ	Shch, shch
К к	К к	K, k	Ъ ъ	Ъ ъ	"
Л л	Л л	L, l	Ы ы	Ы ы	Y, y
М м	М м	M, m	Ь ь	Ь ь	'
Н н	Н н	N, n	Э э	Э э	E, e
О о	О о	O, o	Ю ю	Ю ю	Yu, yu
П п	П п	P, p	Я я	Я я	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

GREEK ALPHABET

Alpha	Α	α	α	Nu	Ν	ν	
Beta	Β	β		Xi	Ξ	ξ	
Gamma	Γ	γ		Omicron	Ο	ο	
Delta	Δ	δ		Pi	Π	π	
Epsilon	Ε	ε	ϵ	Rho	Ρ	ρ	ϱ
Zeta	Ζ	ζ		Sigma	Σ	σ	ς
Eta	Η	η		Tau	Τ	τ	
Theta	Θ	θ	ϑ	Upsilon	Υ	υ	
Iota	Ι	ι		Phi	Φ	φ	ϕ
Kappa	Κ	ϰ	κ ϰ	Chi	Χ	χ	
Lambda	Λ	λ		Psi	Ψ	ψ	
Mu	Μ	μ		Omega	Ω	ω	

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}

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rot	curl
lg	log

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SOME LIMITING REGULAR LAWS FOR RANK

B. N. Gartshteyn

(Presented by Acadamecian A. N. Kolmogorov 1 April 1948)

Let the random values of a finite sequence

$$\xi_1, \xi_2, \ldots, \xi_n$$

be independent and have the same distribution function F(x).

The random value $\rho_n = \zeta_n - \eta_n$ is called rank where

$$\zeta_n = \max(\xi_1, \xi_2, \ldots, \xi_n), \; \eta_n = \min(\xi_1, \xi_2, \ldots, \xi_n).$$

A considerable number of works have appeared recently which are devoted to the limiting distribution of rank in some special cases (see, for example, [1, 2] and for further published indications see [1]). The purpose of our work is obtaining several more general results in the same direction.

Let us designate the distribution function of the value ρ_n by $\Phi_n(r)$. Let us assume that with properly selected constants $a_n > 0$ and b_n and $n \to \infty$ functions $\Phi_n(a_n u + b_n)$ converge to some eigenfunction* of the distribution $\Phi(u)$. Our problem consists of

---

*I. e., which is different from the improper distribution function

$$e(u) = \begin{cases} 0 \text{ for } u \leq a, \\ 1 \text{ for } u > a, \end{cases}$$ where a - constant

determining the type of possible functions of $\Phi(u)$ which are limited in the indicated sense. We solve this problem with certain limitations which were imposed on the initial distribution function $F(x)$ of the values of ξ_k.

First of all, we assume that $F(x)$ is continuous. We require another limitation in the formulation of ~~serum~~ theorem 2.

From the obvious equality

$$P\{\zeta_n < z, \eta_n < y\} = P\{\zeta_n < z\} - P\{\zeta_n < z, \eta_n > y\} = \\ = F^n(z) - [F(z) - F(y)]^n .$$

we find that

$$\Phi_n(r) = \int_{-\infty}^{\infty} \int_{y}^{y+r} n(n-1)[F(z) - F(y)]^{n-2} dF(z)\, dF(y).$$

Hence, after integration

$$\Phi_n(r) = n \int_{-\infty}^{\infty} [F(z+r) - F(z)]^{n-1} dF(z).$$

By means of $\Phi_M^{(n)}(z)$ let us designate the distribution function of the value ζ_n, by means of $\Phi_{-m}^{(n)}(y)$ - the distribution function of the value η_n, and by means of $\Psi_n(r)$ - the composition of the function $\Phi_M^{(n)}(z)$ and $\Phi_m^{(n)}(y)$, i.e.,

$$\Psi_n(r) = \int_{-\infty}^{\infty} \Phi_M^{(n)}(r-y)\, d\Phi_m^{(n)}(y).$$

It is known that $\Phi_M^{(n)}(z) = F^n(z)$ and $\Phi_m^{(n)}(y) = [1 - F(-y)]^n$, whence

$$\Psi_n(r) = F^n(r) \cdot [1 - F(-r)]^n = \\ = n \int_{-\infty}^{\infty} F^n(r-y)[1 - F(-y)]^{n-1} dF(-y) = \\ = n \int_{-\infty}^{\infty} F^n(r+x)[1 - F(x)]^{n-1} dF(x).$$

Theorem 1. In order for the proper selection of the constants $a_n > 0$ and b_n of the function $\Phi_n(a_n u + b_n)$ to converge with $n \to \infty$ toward some eigenfunction of the distribution $\Phi(u)$, it is necessary and sufficient for $\Psi_n(a_n u + b_n)$ to converge with the same $a_n \to 0$

and b_n toward some eigenfunction of distribution. Here, both limiting functions coincide.

Let us now impose an additional limitation on the initial function F(x), and namely, let us assume that F(x) is such that the limiting distributions of the maximum ζ_n and the minimum term η_n exist.

In work [3] it is shown that the class of laws which are limiting for $\Phi_{M}^{(n)}(a_n u + b_n)$ (with $n \to \infty$), where $a_n > 0$ and b_n are selected constant in the proper manner consists of laws of the following three types:

$$1)\ \Phi_\alpha(u) = \begin{cases} 0 & \text{for } u \leqslant 0, \\ e^{-u^{-\alpha}} & \text{for } u > 0; \end{cases}$$

$$2)\ \Psi_\alpha(u) = \begin{cases} e^{-(-u)^\alpha} & \text{for } u \leqslant 0, \\ 1 & \text{for } u > 0; \end{cases}$$

$$3)\ \lambda(u) = e^{-e^{-u}}$$

α - a positive constant).

It is obvious that the class of laws which are limiting for the function $\Phi_{m}^{(n)}(a_n u + b_n)$ where $a_n > 0$ and b_n are constants selected in the proper manner, consists of laws of the same three types since if $\eta_n = \min(\xi_1, \xi_2, \ldots, \xi_n)$, then $-\eta_n = \max(-\xi_1, -\xi_2, \ldots, -\xi_n)$.

The following assumption which provides an estimate of the order of growth of values a_n is necessary to determine the class of possible limiting distribution of rank.

Lemma. If with some selection of constants $a_n > 0$ and b_n $\Phi_{M}^{(n)}(a_n u + b_n)$ with $n \to \infty$ are reduced to the proper limiting law, then with any $\varepsilon > 0$ and $n \to \infty$

1) в случае (1) $\Phi(u) = \Phi_\alpha(u)$ имеем (2) $a_n n^{-\frac{1}{\alpha}+\varepsilon} \to \infty$ (3) и $a_n n^{-\frac{1}{\alpha}-\varepsilon} \to 0$;

2) в случае (1) $\Phi(u) = \Psi_\alpha(u)$ имеем (2) $a_n n^{\frac{1}{\alpha}+\varepsilon} \to \infty$ (3) и $a_n n^{\frac{1}{\alpha}-\varepsilon} \to 0$;

3) в случае (1) $\Phi(u) = \lambda(u)$ имеем (2) $a_n n^{\varepsilon} \to \infty$ (3) и $a_n n^{-\varepsilon} \to 0$.

Key: (1) In the case; (2) we have; (3) and

On the basis of this lemma the following is proved

Theorem 2. The class of possible limiting laws for compositions of distribution functions of the maximum and minimum terms consists of laws of the following six types:

$$\Phi_\alpha(u), \quad \Psi_\alpha(u), \quad \lambda(u), \quad \Phi_\alpha(u)\cdot\Phi_\alpha(au), \quad \Psi_\alpha(u)\cdot\Psi_\alpha(au), \quad \lambda(u)\cdot\lambda(au),$$

a - constant.

The direct consequence of theorems 1 and 2 is confirmation that in our assumptions the maximum distribution of rank can belong only to one of the indicated 6 types.

Our results together with §6 [3] permits, in particular, obtaining Gumbel's result [1].

In conclusion, I consider it my duty to express my profound gratitude to B. V. Gnedenko for his formulation of the problem and constant assistance in its solution.

L'vov State University imeni
Ivan Franko

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[1] E. J. Gumbel, Ann. Math. Statistics, 18, No. 3, 384 (1947). [2] G. Elfving, Biometrika, 35, 111 (1947). [3] B. Gnedenko, Ann. Math., 44, No. 3, 423 (1943).

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