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DUCTILE FRACTURE IN AXISYMMETRIC EXTRUSION AND DRAWING
Part 1 Deformation Mechanics of Extrusion and Drawing
Part 2 Workability in Extrusion and Drawing

MECHANICAL ENGINEERING
UNIVERSITY OF CALIFORNIA
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WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

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This investigation applies theories on flow and fracture in metalworking processes to prediction of workability of materials in axisymmetric bar extrusion and drawing, with special reference to central bursting. Part 1 deals with the determination of deformation mechanics in extrusion and drawing. The matrix method is used as the method of analysis and is shown to be an efficient numerical method which provides useful information on the detailed deformation characteristics for various process variables. (over)

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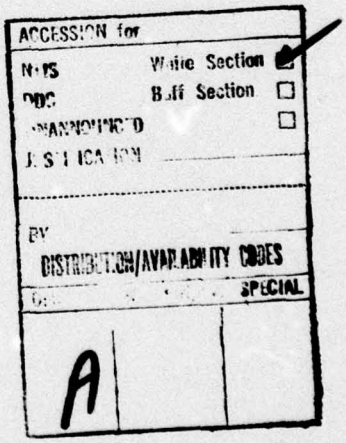
In Part 2 the validity of a theory on ductile fracture was examined by experimental data found in the literature. Then, combining the formulation of fracture criteria with the deformation mechanics found in Part 1, the workability of materials in extrusion and drawing was determined. Workability in extrusion was examined for aluminum alloy 2024-T351, using data found in the literature, and experiments of workability in drawing were attempted for SAE 1144 cold-drawn steel to test the prediction. The results of validation were inconclusive. Conclusive validation of the present workability theory in extrusion and drawing awaits more extensive and systematic experimental investigations, as well as theoretical calculations.

FOREWORD

This report was prepared by C. C. Chen, S. I. Oh, and Shiro Kobayashi, University of California, Berkeley, California, under U.S.A.F. Contract F33615-75-C-5151. The contract was initiated under Project No. 7351, "Metallic Materials," Task No. 735108, "Processing of Metals," and was administered under the direction of the Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio, with Mr. Vincent DePierre as Air Force Project Engineer.

This report discusses research conducted from March 1976 to March 1977. It was submitted by the authors in April 1977.



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Part 1

DEFORMATION MECHANICS OF EXTRUSION AND DRAWING

I. INTRODUCTION

In order to apply workability theory to metalworking processes, we must first establish flow and fracture theories. Then, by combining the detailed information on mechanics and the ductile fracture criterion, workability of materials in metalworking processes can be predicted. Based on this prediction, workability control for preventing fracturing can be achieved by selecting the proper set of process parameters.

In the previous report [1]*, theories on flow and fracture in metalworking processes were developed with an emphasis on applying the workability theory to metalworking processes where the occurrence of internal fracturing is a limiting factor. The present investigation is concerned with the application of these theories to the prediction of workability of materials in axisymmetric bar extrusion and drawing, with special reference to center bursting.

Workability of materials is the extent to which materials can deform without forming cracks during a mechanical working process. Workability, therefore, depends on the conditions imposed by the working process. Critical stress and strain conditions involved in the mechanical working processes must be known and should be specified in terms of process variables, such as height reduction, friction at the interface, and workpiece geometries and dimensions. Part I of the present investigation deals with the determination of deformation mechanics in extrusion and drawing.

The role of the detailed mechanics in the workability study is to provide the stress and strain paths at a critical site of a deforming

*Numbers in brackets refer to the references at the end of this report.

material. Therefore, the method of analysis should be capable of accurately determining not only overall quantities involved in metalworking processes, such as forming loads, but also stress and strain distributions during deformation. Furthermore, it is required to determine the stress and strain distributions under various process conditions. Therefore, to justify the approach, the computation involved in the method must be efficient. The matrix method developed by Lee and Kobayashi [2] comes close to fulfilling these requirements. In the previous report, this matrix method was refined and it was demonstrated that the method is effective for the analysis of steady-state processes as well as non-steady-state processes. In the present investigation, the matrix method, with further improvements, is used to determine steady-state deformation characteristics as functions of material property, die-workpiece interface friction, die angle, and reduction in bar extrusion and drawing.

II. METHOD OF ANALYSIS

The basic concepts of the matrix method for rigid-plastic deformation problems are the use of the Lagrange multiplier in a variational formulation and linearization of nonlinear stiffness equations. In the present formulation the dynamic effects, i.e., the inertia effects on the forces and the strain-rate effects on the material properties, are neglected.

For rigid-plastic materials the condition of incompressibility is imposed on the admissible velocity fields. This constraint can be removed by introducing a Lagrange multiplier. Consider a body V whose surface S, consists of S_U and S_T. The body is composed of a rigid-plastic material that obeys the von Mises yield criterion and its associated flow rule, under the boundary conditions, such that the entire body is deforming plastically. Body forces are assumed to be absent in the region V. It can be shown [3] that for the actual solution, the functional (1) becomes stationary with respect to the multiplier λ and the velocity fields that satisfy the velocity boundary conditions on S_U but not necessarily the incompressibility condition (kinematically complete):

$$\Phi = \int_V \bar{\sigma}\dot{\bar{\epsilon}}\, dV + \int_V \lambda \underset{\sim}{C}^T \dot{\underset{\sim}{\epsilon}}\, dV - \int_{S_T} \underset{\sim}{T}^T \underset{\sim}{U}\, dS, \tag{1}$$

where $\dot{\bar{\epsilon}}$ is the effective strain-rate; $\bar{\sigma}$, the effective stress; $\underset{\sim}{T}$, the traction vector specified on the boundary S_T; $\underset{\sim}{U}$, the velocity vector; C is the proper vector notation of the Kronecker delta such that $\underset{\sim}{C}^T \dot{\underset{\sim}{\epsilon}} = 0$ implies the incompressibility condition.

The formulation of the discrete variational problem follows the same procedure as that used in the finite-element method [4], [5]. The body V is divided into M elements interconnected at N nodal points. The approximation of the functional Φ by a function $\tilde{\phi}$ is performed on the elemental level by replacing U with a kinematically complete distribution, given by

$$\underset{\sim}{U} \simeq \underline{\underline{G}}\,\underset{\sim}{u}, \tag{2}$$

where $\underline{\underline{G}}$ is the interpolation function and $\underset{\sim}{u}$ is the vector whose components are velocities at nodal points associated with the element. The strain-rate vector is then derivable in the form

$$\dot{\underset{\sim}{\varepsilon}} = \underline{\underline{B}}\,\underset{\sim}{u}. \tag{3}$$

Assembling the function at the elemental level into an approximate finite-element model over all the elements and applying the stationary condition to the function $\tilde{\phi}$, we obtain the stiffness relations consisting of a large system of nonlinear equations. In order to solve the stiffness equations, we adopt the following procedure. The nonlinear stiffness equations were linearized by considering a small perturbation $\Delta\underset{\sim}{u}$ in the velocity vector $\underset{\sim}{u}$, such that

$$\underset{\sim}{u}_{(n)} = \underset{\sim}{u}_{(n-1)} + \Delta\underset{\sim}{u}_{(n)}, \tag{4}$$

for the n-th iteration process.† We then obtain the global perturbation equation for the n-th iteration as

---

†In the actual calculations, $\underset{\sim}{u}_{(n)} = \underset{\sim}{u}_{(n-1)} + \alpha\Delta\underset{\sim}{u}_{(n)}$ is used, where α is the deceleration coefficient.

$$\underset{\sim}{S}_{(n-1)}\left\{\begin{matrix}\Delta\underset{\sim}{u}\\ ---\\ \underset{\sim}{\lambda}\end{matrix}\right\}_{(n)} = \underset{\sim}{R}_{(n-1)}. \tag{5}$$

Specific formulations for the matrix $\underline{S}$ and the vector $\underset{\sim}{R}$, using a quadrilateral element with a bilinear velocity distribution, are presented elsewhere [1], [6]. It must be noted that, since the method is an iterative process, it is necessary to provide an initial guess for the velocity field $\underset{\sim}{u}_0$ (but no need for λ). The solution of linear equation (5) yields $\Delta\underset{\sim}{u}_{(n)}$ close to zero and a proper value of the mean stresses $\underset{\sim}{\lambda}_{(n)}$, if $\underset{\sim}{u}_{(n-1)}$ is close to the actual solution.

A. Convergence

In the previous studies the convergence of the solution has been measured by the quantity $\|\Delta\underset{\sim}{u}\|/\|\underset{\sim}{u}\|$, where the Euclidean vector norm is defined by

$$\|\underset{\sim}{u}\| = \sqrt{\sum_{i=1}^{N} |\underset{\sim}{u}_i|^2}, \tag{6}$$

where N = total number of nodal points.

The convergence criterion requires that the error norm at the n-th iteration ($\|\Delta\underset{\sim}{u}_{(n)}\|/\|\underset{\sim}{u}_{(n-1)}\|$) be less than that at the previous iteration This convergence criterion, with suitable selections of the deceleration coefficient α, has worked out well for the solutions of various metal-working problems. At the same time, however, it was realized that solution divergence has been indicated according to this criterion, although the solution was converging in its true sense. This can apparently be seen, particularly for a function which is not well behaved. In this case, the

criterion resulted in the use of unnecessarily small values of the deceleration coefficient, and thus in increased computing time. In the present program, instead of the error norm, the quantity f defined by

$$f = \sqrt{\sum_m \left\{ \left[\frac{\partial \tilde{\phi}^{(m)}}{\partial \underset{\sim}{u}^{(m)}} \right]^2 + \left[\frac{\partial \tilde{\phi}^{(m)}}{\partial \lambda^{(m)}} \right]^2 \right\}} \tag{7}$$

where

$$\Phi \cong \sum_m \tilde{\phi}^{(m)} \tag{8}$$

is utilized for solution convergence. In Eq. (7), the superscript (m) denotes the values of the m-th element and summation is made over all the elements. The criterion for convergence now is that the magnitude of f at the n-th iteration be less than the magnitude at the previous iteration. In this convergence scheme, the proper value of the deceleration coefficient at each iteration can be selected efficiently from the previous information on the function behavior and the convergence requirement at the current iteration.

B. Rigid-body treatment

The matrix method described in this section applies only if the entire body is deforming plastically and no rigid zone or unloading exists during the deformation process. In practical problems, however, situations do arise where the rigid zone as well as rigid unloading are involved. If these regions of no deformation are contained within the control volume V, the extremum principles do not apply to the problem of obtaining internal distributions. There are several approaches to handling this difficulty,

but a most effective technique is one which involves the approximate determination of the boundary of a nearly rigid zone.

A nearly rigid zone can be characterized by its very low value of effective strain rate in comparison to the deforming body. During the iteration process for an incremental solution over the entire body, the effective strain-rate in the possible rigid region approaches zero as the solution converges. Since the effective strain-rate appears in the denominator of the stiffness matrix S, the component of the normalized stiffness matrix will tend to become infinity if the nodal point associated with this component is contained in the rigid zone. Therefore, the elements for which the effective strain-rate is smaller than a certain value (say, 0.0001) are considered to be in the rigid zone. The effective strain-rates of the elements lying inside the rigid zone are then kept at this value in the perturbation relationship, and the iteration is continued for the solution in the plastically deforming region until a desired convergence is achieved. It should be noted, of course, that the converged solution gives the stress distribution only in the plastically deforming region.

The complete program of this improved version of the matrix method is listed in the appendix of this report.

III. ANALYSIS OF EXTRUSION

Several investigators have analyzed the extrusion process by using the finite-element method, mostly by elastic-plastic analysis [7], [8], [9]. However, almost all the analyses have been performed for the case of loading of the workpiece that fits the die and container and of extruding it a small amount, instead of extruding the workpiece until a steady state is reached. The exceptions are the work by Lee, Mallett, and Yang [10] for the plane-strain extrusion with frictionless curved dies using the elastic-plastic finite-element method and the rigid-plastic analysis of axisymmetric extrusion through frictionless conical dies by Shah and Kobayashi [6]. In the latter work, the authors investigated a possibility of applying the matrix method to steady-state metalworking processes and demonstrated that the analysis of a steady-state extrusion can be made by the matrix method. The analysis of extrusion in this section is an extension of this work.

A. Computational conditions and procedures

Boundary conditions and mesh system The boundary conditions and the mesh system used for the analysis of extrusion through conical dies are shown in Fig. 1. The material in the container moves axially with the uniform velocity of unit magnitude. The container is assumed to be frictionless, and along the conical die surfaces, the tangential traction, which is equal to the frictional stress at the die-workpiece interface, is prescribed. The extruded material moves axially with the uniform velocity of the magnitude determined from the area reduction and the incompressibility relationship. Also, no traction acts along the surfaces of the extruded part.

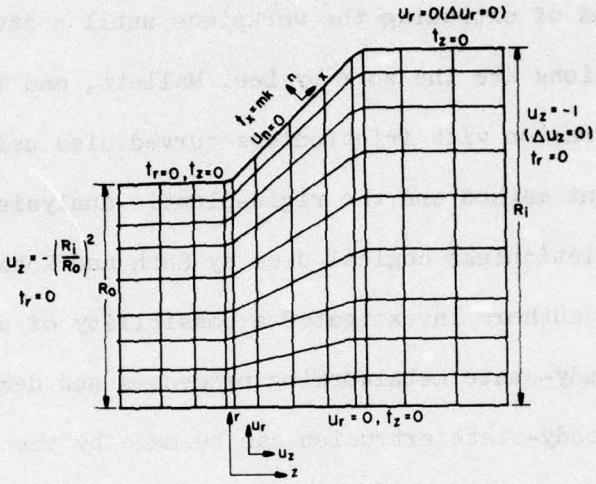


Fig. 1 Boundary conditions and mesh system for steady-state axisymmetric extrusion analysis.

Along the axis of symmetry, the conditions are that the shear traction and the radial velocity must vanish.

It must be noted here that the die corners were slightly modified by connecting the two material nodal points located closest to the die corners. This modification was made in order to avoid high singularity of the velocity components near the die corners.

B. Work-hardening materials

In the analysis of a non-steady-state process, the effect of work-hardening can be readily incorporated into the analysis by computing the incremental strains and modifying the flow stress at each deformation step according to the increase in the total effective strain. In the analysis of steady-state processes, however, the flow stress distribution must be consistent with the final effective strain distribution according to the material's work-hardening characteristics. This requirement can be achieved by using the following computational procedure. During the iteration process for a converging solution, the flow lines corresponding to the latest velocity field are constructed after each iteration. The network of grid distortions and the effective strain distributions are determined from these flow lines. The effective strains for all elements are then interpolated from these values, and using a given stress-strain relationship, corresponding flow stress distributions for elements are determined. Using this new flow stress distribution, the next iteration for the velocity field is carried out and the same procedure is repeated until the converged solution for the velocity field is obtained. Since the solution depends not only on $\underset{\sim}{u}_{(n-1)}$, but also on the flow stress distribution, when the

velocity solution has converged, the flow stress distribution and effective strain distribution also match each other according to the stress-strain behavior of the material. Thus the convergence of velocity solutions gives the correct solution for the work-hardening material.

C. Extrusion conditions

The extrusion process conditions under which the computation was carried out are summarized in Table 1. Since the final solution for the extrusion with nonhardening material was used as the initial guess for work-hardening materials, the results for non-work-hardening materials are also presented.

Table 1(a): Extrusion process conditions for computation.

α (die semi-cone angle) = 30°

$\frac{R_i}{R_0} = 2.366$, area reduction $1 - (\frac{R_0}{R_i})^2 = 0.82$

Material	Die-workpiece interface frictional stress f
Non-work-hardening $\bar{\sigma} = Y_0$	0, $0.2Y_0$, $0.4Y_0$
SAE 1112 steel $\bar{\sigma} = Y_0(1 + \frac{\bar{\varepsilon}}{0.3})^{0.25}$	0, $0.2Y_0$, $0.4Y_0$

Table 1(b): Extrusion process conditions for computation.

α (die semi-cone angle) = 45°

Work-hardening material: al alloy 2024-T351

$\bar{\sigma} = Y_0 \cdot 2.202(\bar{\varepsilon})^{0.1675}$ or $Y_0(1 + \frac{\bar{\varepsilon}}{0.01021})^{0.171}$

Reduction	Non-work-hardening friction f	Work-hardening friction f
$\frac{R_i}{R_0} = 1.25$	0	0 , $0.4Y_0$
1.6	0	0 , $0.4Y_0$
1.8	-	0 , $0.4Y_0$
2.0	0	0 , $0.4Y_0$
2.4	0	- , -

The stress-strain curves for SAE 1112 steel and for aluminum alloy 2024-T351 are shown in Fig. 2. The calculations for SAE 1112 steels are mainly to find the effect of friction on the deformation characteristics, while the cases for al alloy 2024-T351 emphasized the effect of area reduction, as well as the effect of friction, on the deformation in extrusion.

D. Results and discussion

The computation was performed for each solution until the accuracy of $f^2 = 10^{-5}$ was reached. This corresponds to the error norm of $\|\Delta \underset{\sim}{u}\|/\|\underset{\sim}{u}\| = 0.00008$. The number of iterations to reach the above convergence depends on the initial guess, but by using the best available initial guess for the velocity field, the average number of iterations required for the final solution was $25 \sim 30$ iterations.

The results were obtained for average extrusion pressure, normal pressure distribution on the die, grid distortions, velocity distributions, and stress, strain, strain-rate distributions.

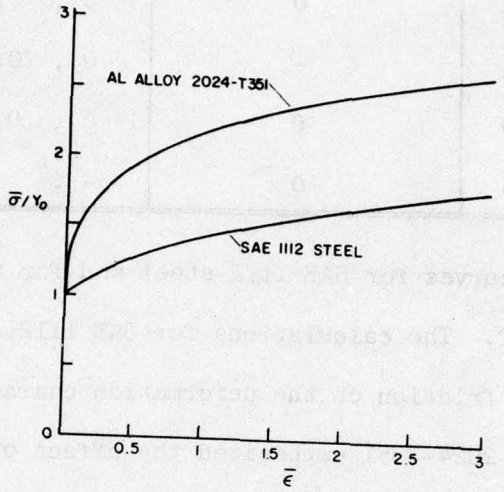


Fig. 2 Stress-strain curves for SAE 1112 steel (Y_0 = 79,800 psi; 550.21 MN/m^2) and for aluminum alloy 2024-T351 (Y_0 = 47,778 psi; 329.42 MN/m^2).

SAE 1112 steel The stress distributions computed differs from the actual distribution by a hydrostatic component. This hydrostatic component was determined by setting the net axial force along the exit boundary equal to zero, and the magnitude of stresses were corrected accordingly. The average extrusion pressures were then determined from the stresses along the entrance boundary. They are summarized in Table 2.

Table 2: Average extrusion pressure, p_{ave}/Y_0.

$\alpha = 30^\circ$

$\frac{R_i}{R_0} = 2.366$

Material	Friction 0	$0.2Y_0$	$0.4Y_0$
Non-work-hardening	1.755	2.292	2.784
SAE 1112 steel	2.425	3.148	3.665

The general trend of the velocity distributions is the same for all friction conditions. A typical example is given in Fig. 3. The two velocity components are plotted as functions of the radial coordinate at various locations in the axial direction within the deformation zone. These distributions are in general agreement with the measurements in the visio-plasticity study of axisymmetric extrusion of lead [11].

Detailed differences of deformation characteristics, due to material properties and friction at the die-workpiece interface, are more clearly indicated, for example, in grid distortions. The steady-state grid distortion patterns are compared for non-work-hardening and work-hardening cases for the two friction conditions in Fig. 4. With reference to Fig. 4(a),

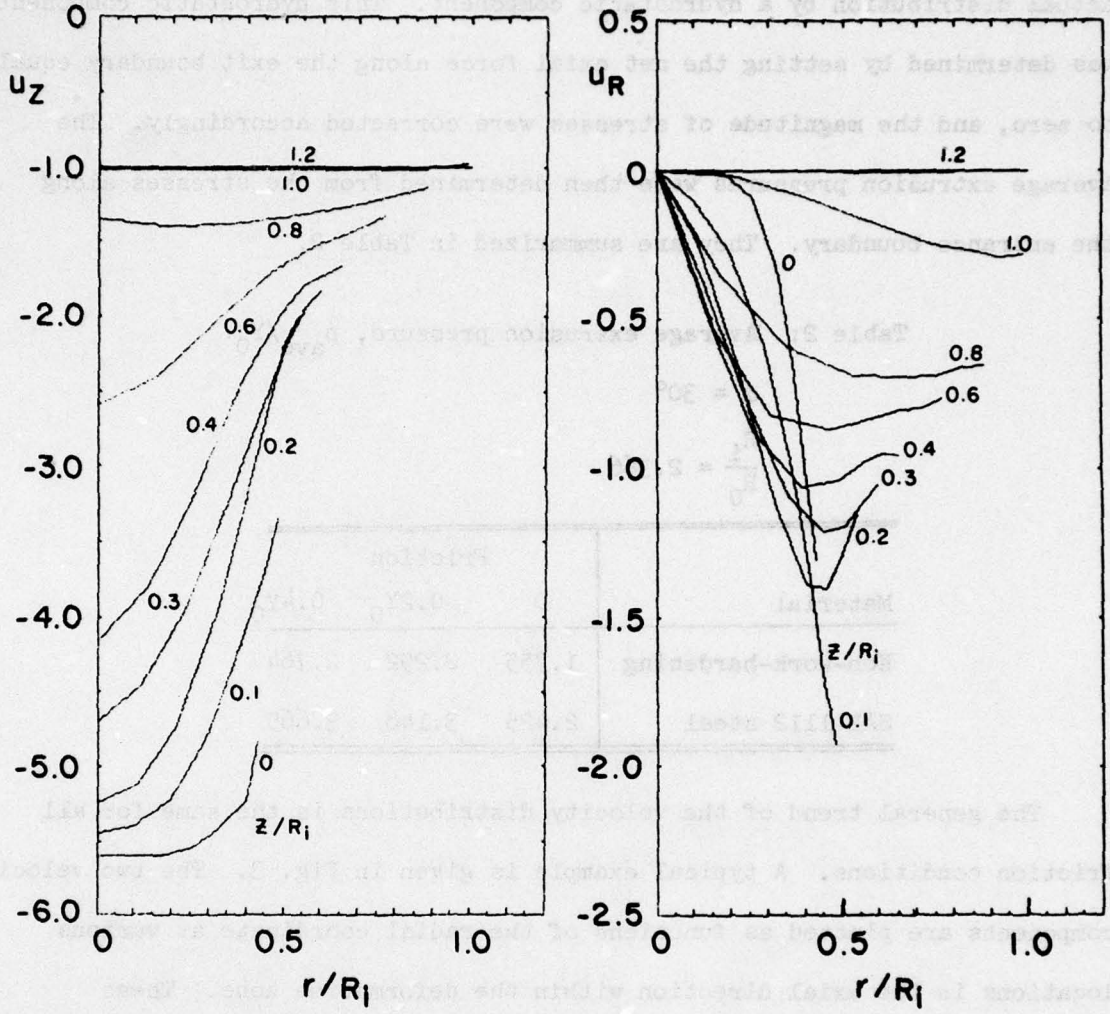


Fig. 3 Computed velocity distributions for SAE 1112 steel under the conditions $\alpha = 30°$, $R_i/R_0 = 2.366$, and $f = 0.2Y_0$.

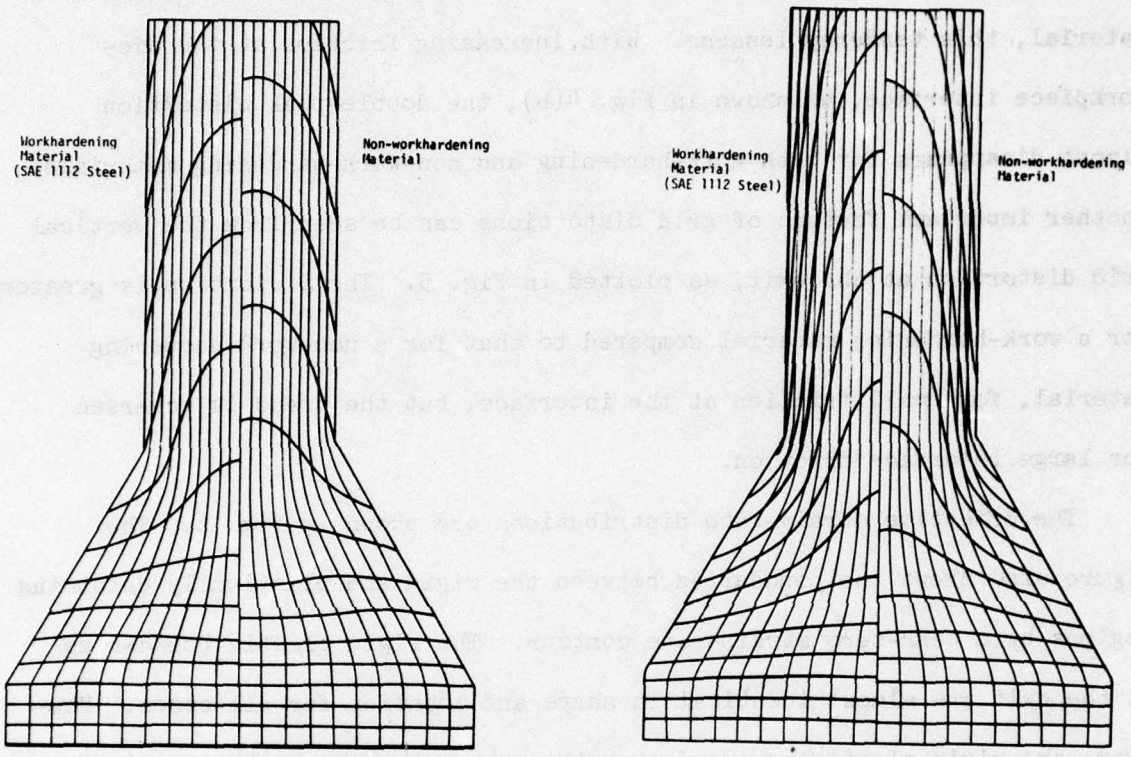


Fig. 4 Grid distortion patterns for non-workhardening and workhardening materials with (a) die frictional stress $f = 0$, and (b) $f = 0.4Y_0$.

it is clearly seen that the grid lines that are originally perpendicular to the axis of the workpiece distort and show the double peak in the extruded part for non-work-hardening material. However, for a work-hardening material, this tendency lessens. With increasing friction at the die-workpiece interface, as shown in Fig. 4(b), the double-peak distortion almost disappears for both work-hardening and non-work-hardening materials. Another important feature of grid distortions can be seen from the vertical grid distortion at the exit, as plotted in Fig. 5. The distortion is greater for a work-hardening material compared to that for a non-work-hardening material, for small friction at the interface, but the trend is reversed for large interface friction.

The effective strain-rate distributions are shown in Fig. 6. The figure also shows the boundaries between the rigid and plastically deforming regions by a near-zero strain-rate contour. The rigid plastic boundaries at the exit are almost identical in shape and location for all cases. However, the rigid-plastic boundaries at the entrance moves backwards, indicating larger deforming zones, with increasing interface friction. This observation applies to both work-hardening and non-work-hardening materials. Also, the results show that the deformation zone size becomes larger for work-hardening. The difference is more pronounced with increasing friction. Neglecting small details, the effective strain-rate distribution is identical for both materials. The strain-rate increases gradually from the entrance toward the exit and near the exit there is a sharp drop. As might be expected, there is some degree of strain-rate concentration near the die corner.

The effective strain-rates are integrated along the flow lines to yield

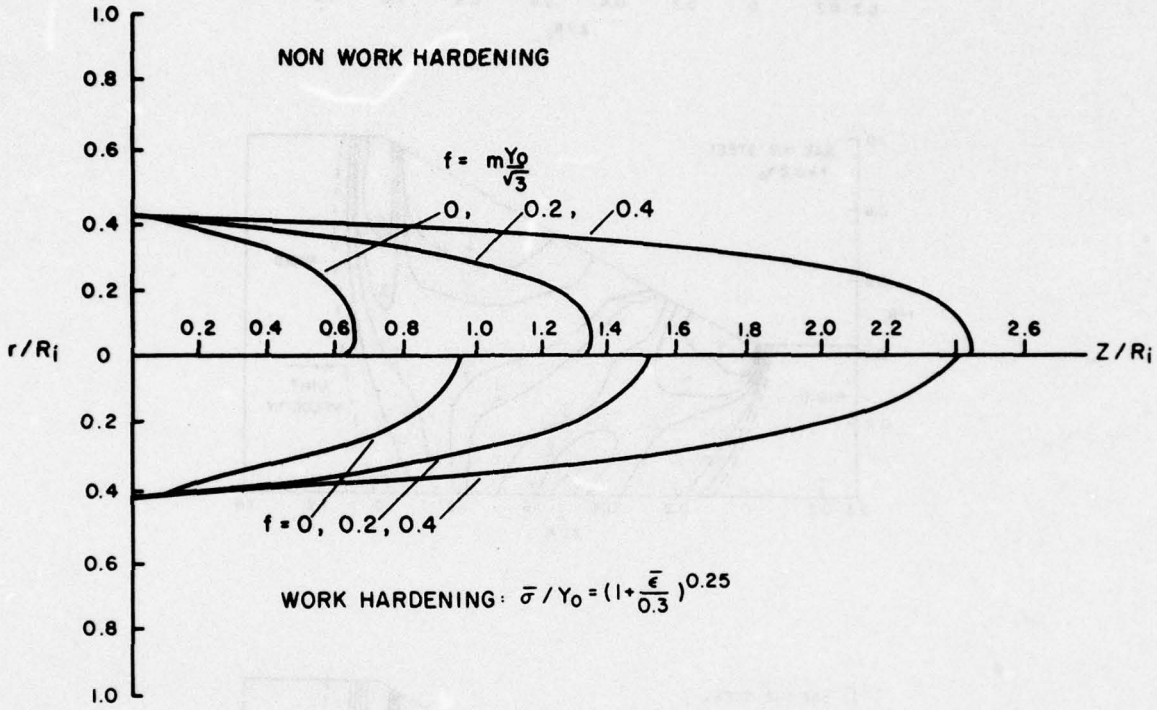


Fig. 5 Total grid distortions for various friction conditions; $\alpha = 30^{\circ}$, $R_i/R_0 = 2.366$.

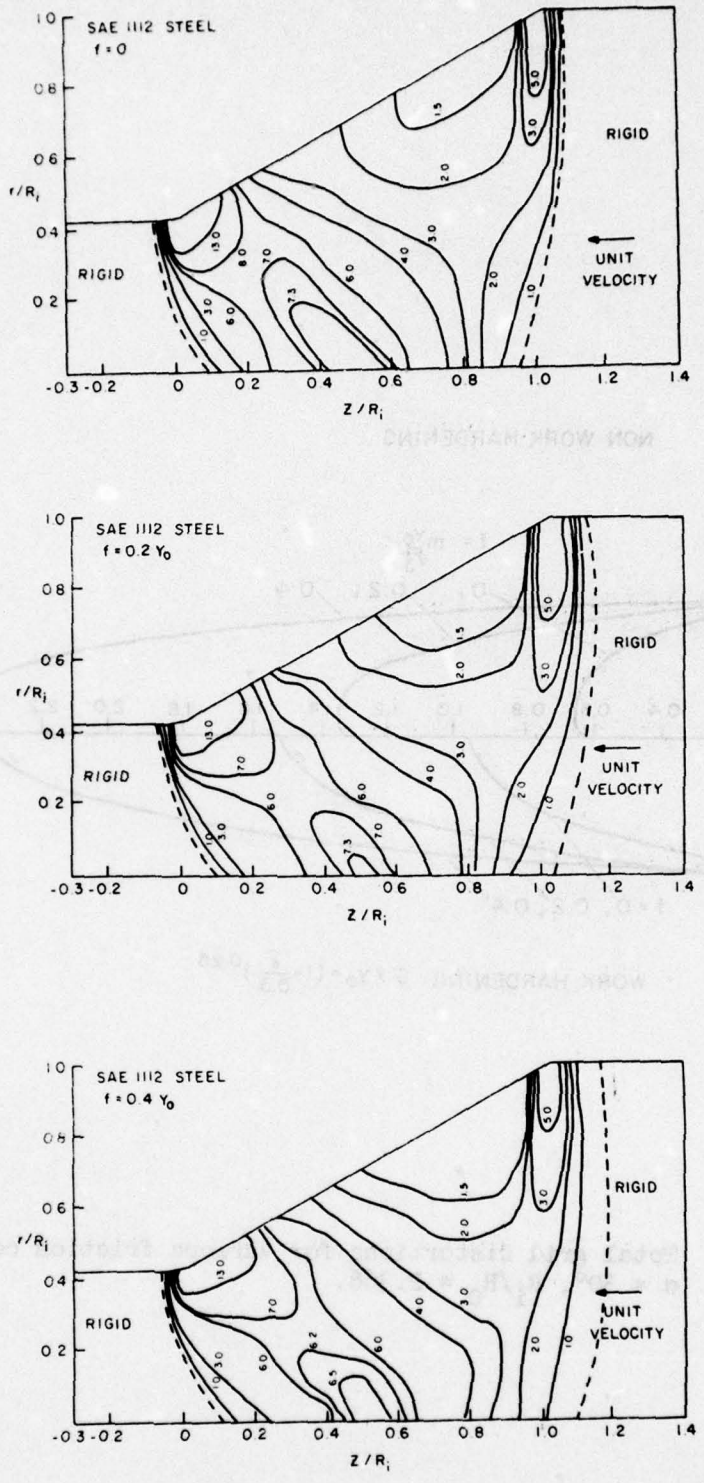


Fig. 6 Effective strain-rate distributions for SAE 1112 steel for various interface friction conditions.

the effective strain distributions. A typical effective strain distribution is given in Fig. 7. As for vertical sections, the strain is the highest near the die and the lowest near the extrusion axis throughout the deformation zone. The effective strain distribution at the exit section is also shown in the figure, which indicates that the strain is the lowest near the extrusion axis and increases toward the periphery. The strain distributions in the final product of the two materials are plotted for several friction values in Fig. 8. For both materials nonuniformity of deformation increases with increasing die-workpiece interface friction. The degree of nonuniformity is greater for small friction and less for large friction in work-hardening materials, while the opposite holds true for non-work-hardening materials. These results reflect exactly the findings in the vertical grid line distortion discussed with reference to Fig. 5.

The distributions of the hydrostatic pressure ($-\sigma_m/\bar{\sigma}$) are shown for the two friction conditions in Fig. 9. The distribution patterns are the same for both friction conditions and for both materials with and without work-hardening. Note that the hydrostatic pressure increases its magnitude throughout the deformation zone with increasing friction at the die-workpiece interface. It can be observed also that along the die-workpiece interface, the hydrostatic pressure is largest at the entrance and shows a minimum at some distance from the exit. This particular feature is seen in the normal pressure distributions along the die interface shown in Fig. 10. The pressure is the highest at the entrance and it decreases toward the exit, then increasing again near the exit. With increasing friction, the pressure increases but the distribution pattern remains the same. The trend of the curves for nonhardening and work-hardening materials is also the same with difference in magnitude of a constant amount.

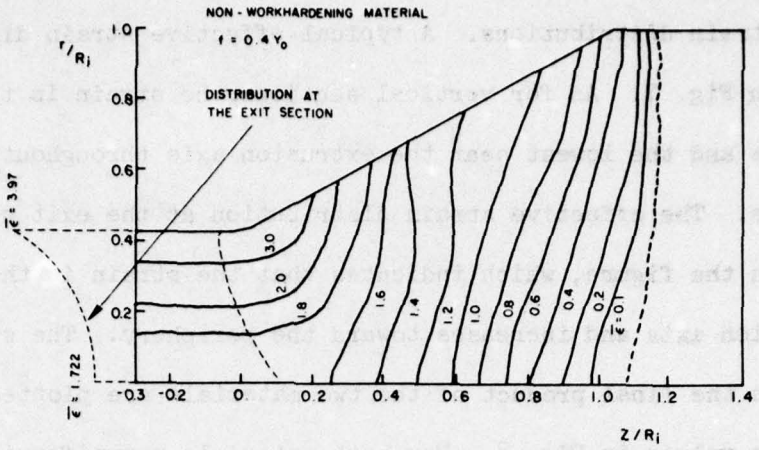


Fig. 7 A typical effective strain distribution.

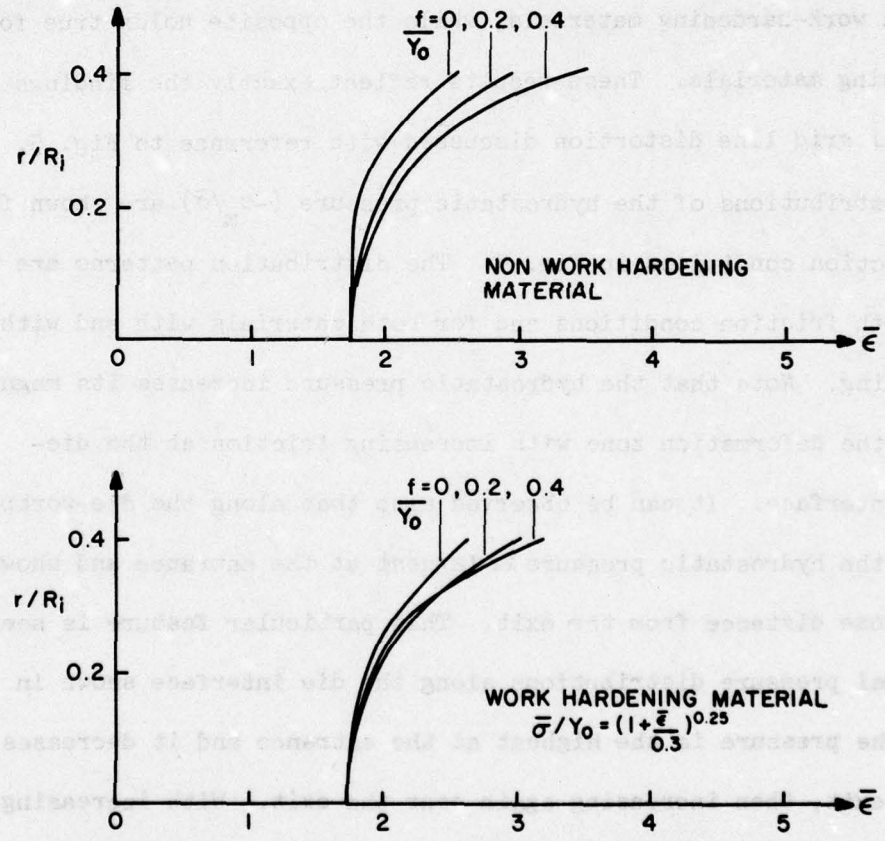


Fig. 8 Effective strain distributions at the exit sections for workhardening and non-workhardening materials under various interface friction conditions.

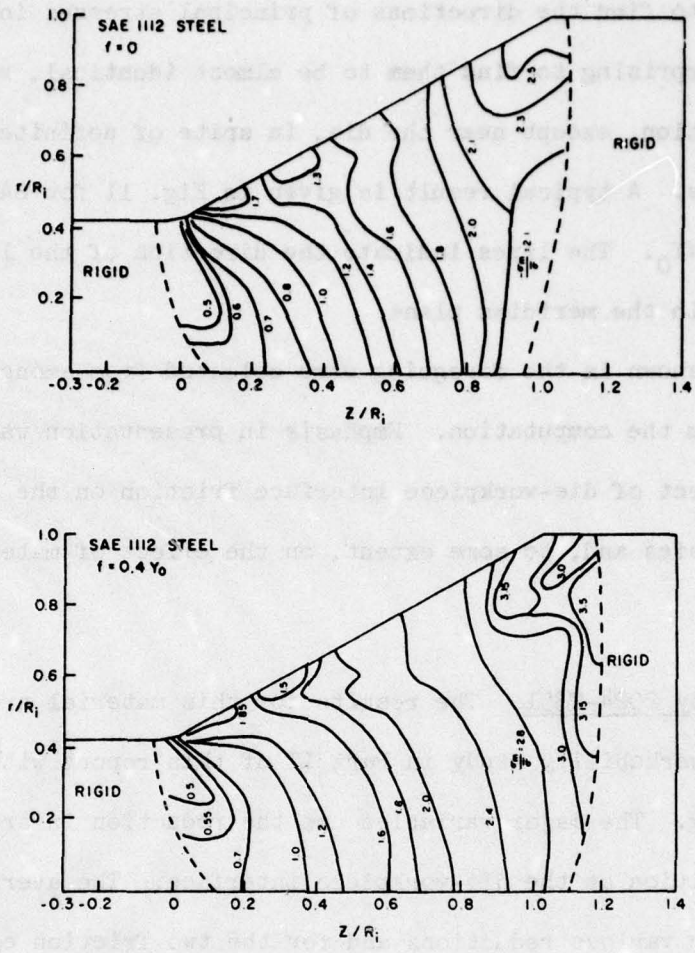


Fig. 9 Distributions of the hydrostatic pressure in SAE 1112 steel extrusion for the two friction conditions.

In other results obtained by the finite-element analysis, it is not only interesting to find the directions of principal stresses in the deformation zone, but surprising to find them to be almost identical, regardless of interface friction, except near the die, in spite of definite deviations of some quantities. A typical result is given in Fig. 11 for SAE 1112 steel with $f = 0.4Y_0$. The lines indicate the direction of the larger principal stress in the meridian plane.

The results shown in the foregoing were selected from among information obtained from the computation. Emphasis in presentation was placed mainly on the effect of die-workpiece interface friction on the detailed deformation mechanics and, to some extent, on the effect of materials properties.

Aluminum alloy 2024-T351 The results for this material are also utilized for the workability study in Part II of this report with regard to center bursting. The major variables are the reduction in area and the friction condition at the die-workpiece interface. The average extrusion pressures for various reductions and for the two friction conditions are summarized in Table 3.

Table 3: Average extrusion pressure, p_{ave}/Y_0.

$\alpha = 45°$

	R_i/R_0			
Friction	1.25	1.6	1.8	2.0
$f = 0$	2.102	2.913	3.289	3.567
$f = 0.4Y_0$	2.310	3.408	3.826	4.202

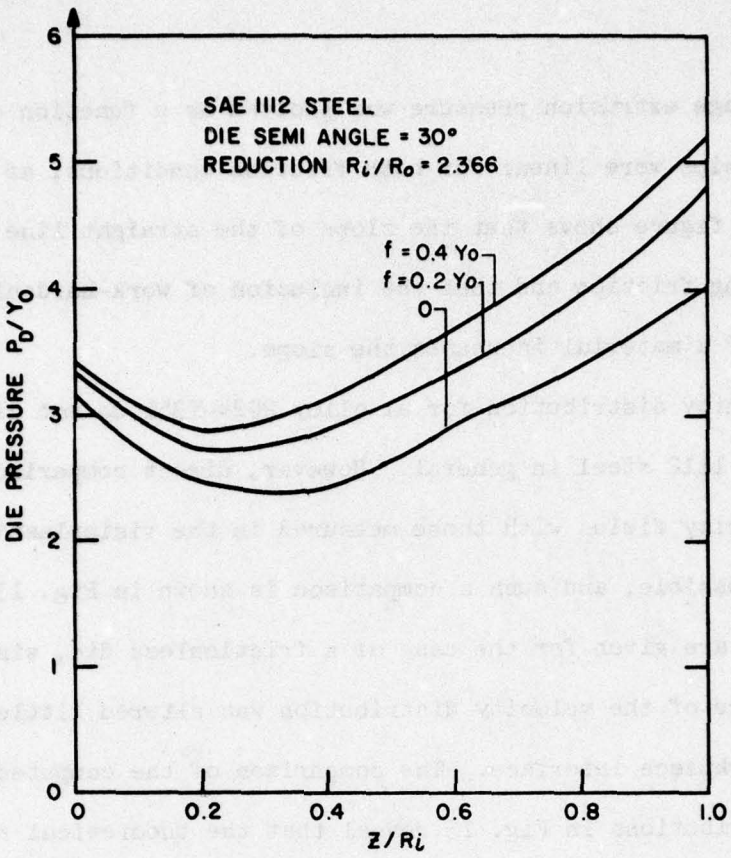


Fig. 10 The die pressure distributions under various friction conditions.

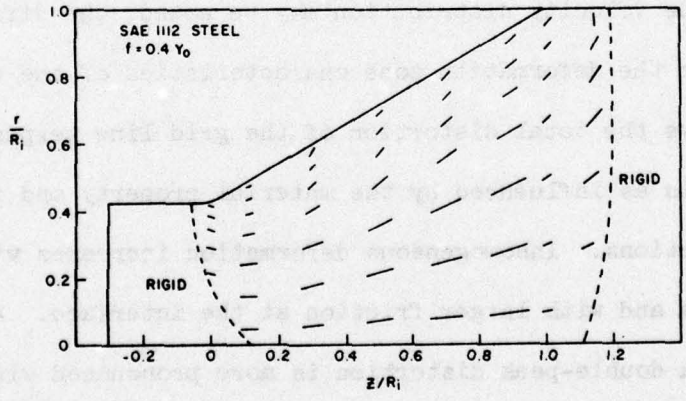


Fig. 11 Principal stress directions in extrusion.

When the average extrusion pressure was plotted as a function of $\ln(\frac{R_i}{R_o})$, the relationships were linear for both friction conditions, as shown in Fig. 12. The figure shows that the slope of the straight line increases with increasing friction and that the inclusion of work-hardening characteristics of a material increases the slope.

The velocity distribution for al alloy 2024-T351 do not differ from those for SAE 1112 steel in general. However, direct comparison of the computed velocity fields with those measured in the visioplasticity study [11] is now possible, and such a comparison is shown in Fig. 13. The computed results are given for the case of a frictionless die, since the general picture of the velocity distribution was altered little by friction at the die-workpiece interface. The comparison of the computed and measured velocity distributions in Fig. 13 reveal that the theoretical results in both velocity components are in agreement with the experimental results in every detail of distribution characteristics. This is one of the convincing evidences of the accuracy of the rigid-plastic finite-element analysis. Although in comparing Fig. 13 with Fig. 3 for SAE 1112 steel, some minor differences in the velocity distribution may be noted, the differences are mainly due to the deformation zone characteristics of the two materials.

Fig. 14 shows the total distortion of the grid line perpendicular to the extrusion axis as influenced by the material property and friction for several reductions. Inhomogeneous deformation increases with increasing reduction in area and with larger friction at the interface. An interesting feature is that a double-peak distortion is more pronounced with increasing reduction for a non-work-hardening material, while the contrary is true for a work-hardening material.

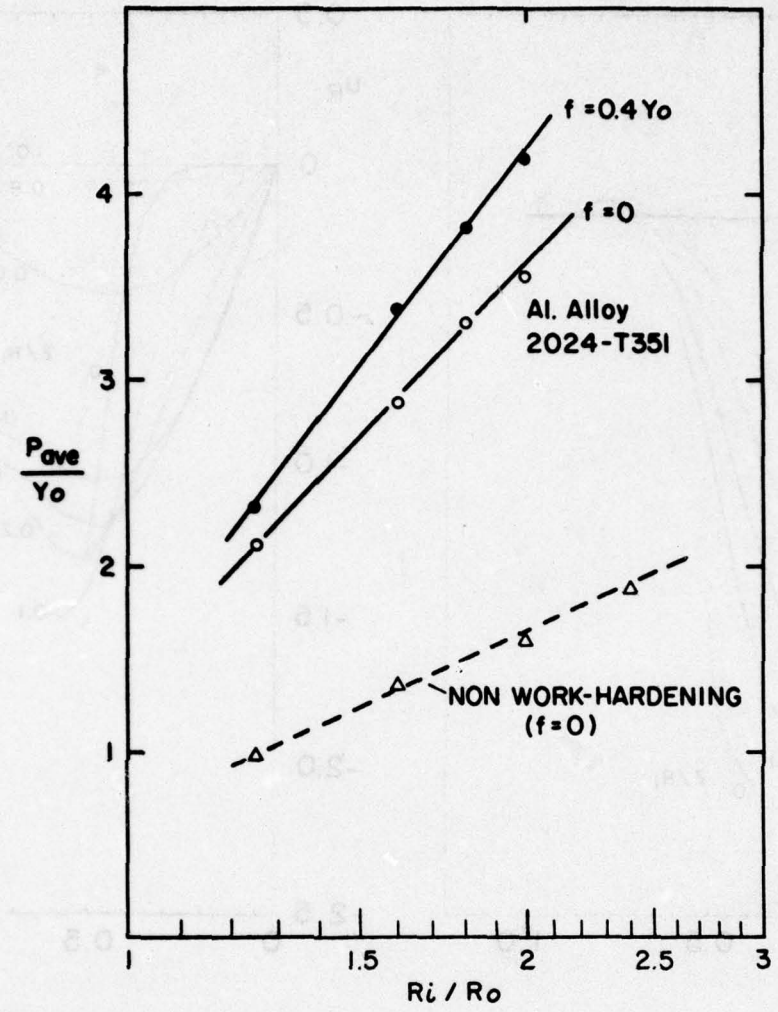
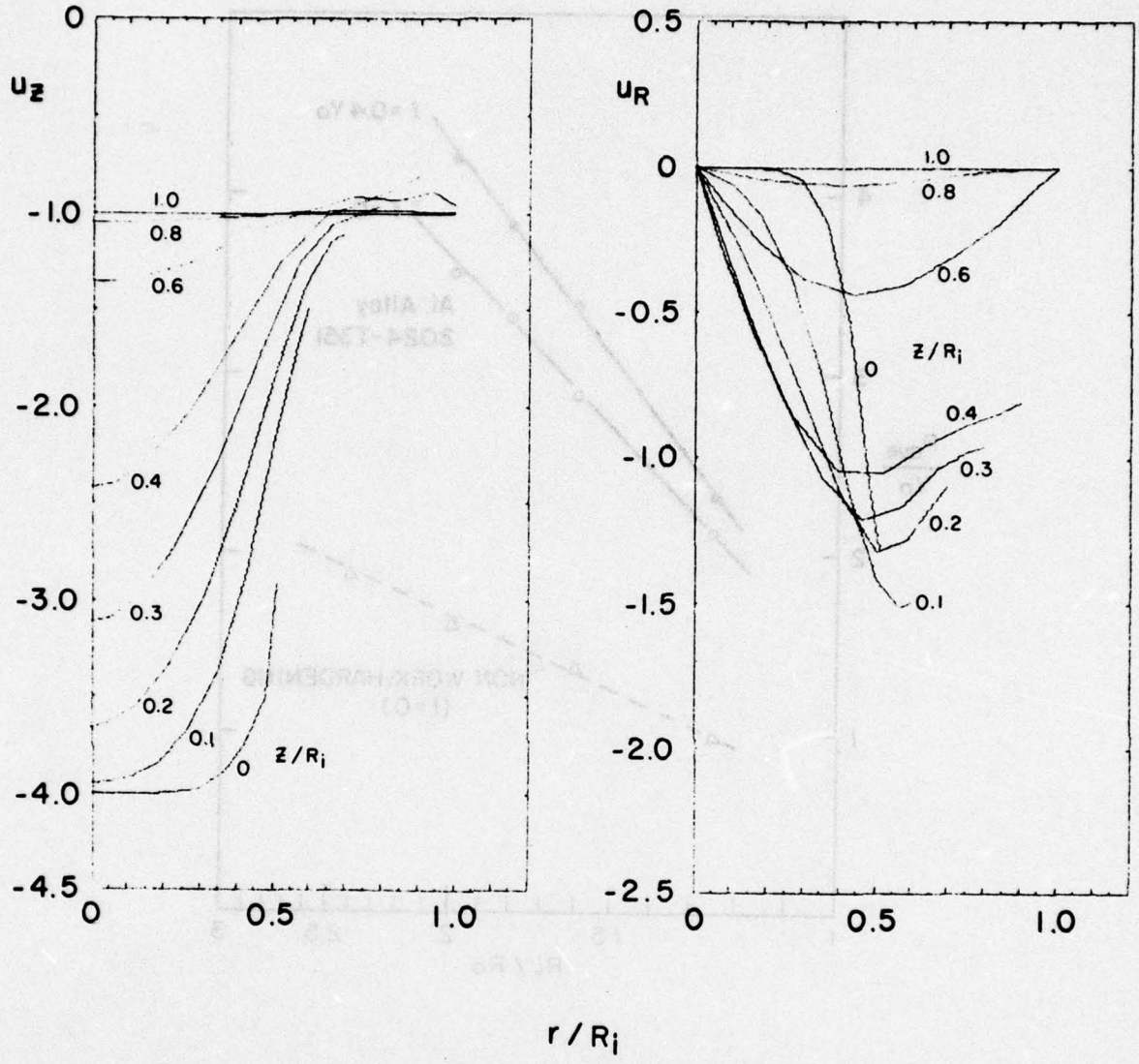
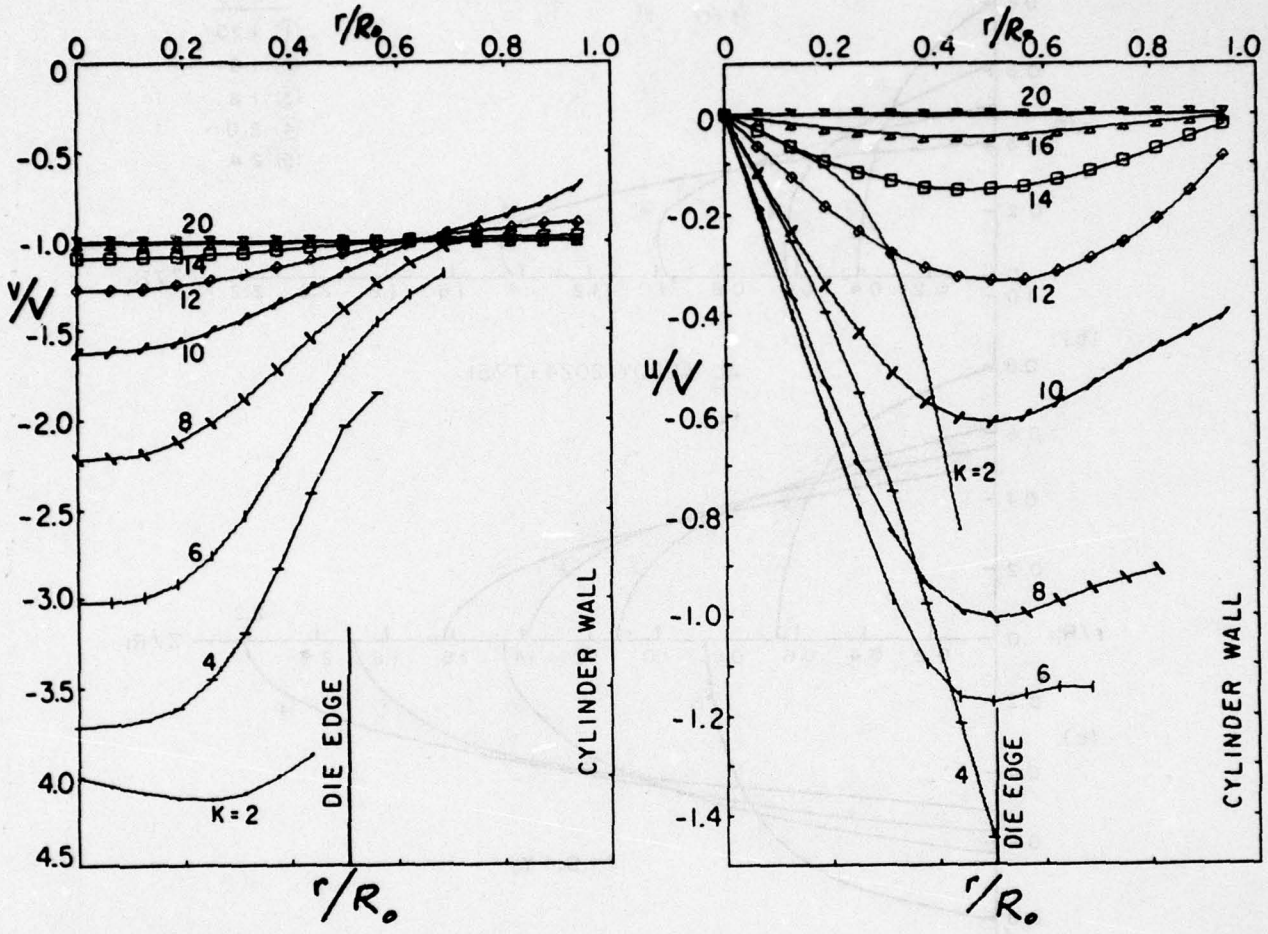


Fig. 12 Average extrusion pressures as functions of area reduction for aluminum alloy 2024-T351.



(a)

Fig. 13 Comparison of (a) computed and (b) experimental [11] velocity distributions; $R_i/R_0 = 2$, semi-die angle $\alpha = 45°$ (frictionless in computation).


r/R₀
0
0.2
0.4
0.6
0.8
1.0
v/V
u/V
20
16
14
12
10
8
6
4
K=2
DIE EDGE
CYLINDER WALL

(b)

Figure 13

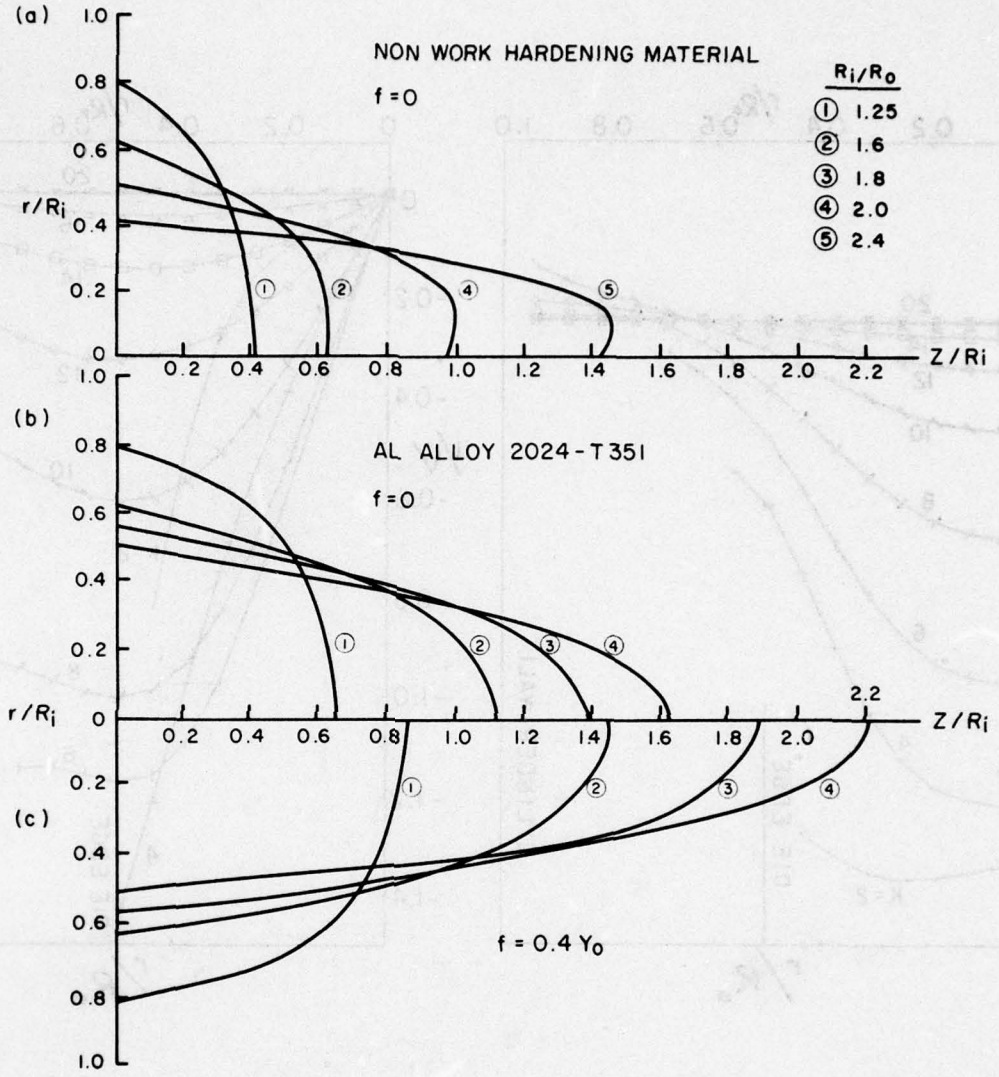


Fig. 14 Total grid distortions for various area reductions.

The effective strain-rate distributions for various area reductions, with two friction conditions, are shown in Fig. 15. With regard to the effect of friction on the deformation zone, the exit boundary remains about the same but the deformation zone expands as friction at the die-workpiece interface increases. This conclusion is the same as that drawn for SAE 1112 steel.

As the reduction in area increases, naturally the deformation zone becomes larger and the magnitude of strain-rate increases. Common features of the strain-rate distributions for all the reductions investigated are: (1) strain-rate concentrations occur near the corners at the entrance and at the exit, and (2) along the axis of symmetry the strain-rate peak appears in the middle of the deformation zone. The effective strain distributions across the extruded bar are plotted in Fig. 16. The strain is largest at the surface and smallest at the center. An interesting result is that the degree of nonuniformity in strain (difference between the largest and smallest strains) is greatest for the smallest area reduction. Also, the effect of friction on the final strain distribution is practically none, except that the magnitude of the surface strain increases slightly with increasing interface friction.

The distribution of the mean stress is similar to the pattern obtained for SAE 1112 steel. As the reduction decreases, the mean stress increases and becomes tensile in the zone near the center, as shown in Fig. 17. Another finding is seen, with respect to the die pressure, in Fig. 18. The die pressure is highest for the smallest reduction. This implies that the pressure distributions for smaller reductions are critical for die design. The distribution pattern is the same as that for SAE 1112 steel showing a minimum at some distance from the die exit.

Finally, the directions of the largest principal stresses are plotted for two reductions in Fig. 19. Again, the effect of die-workpiece interface friction is negligible and the pattern appears to be determined solely by the geometrical constraints.

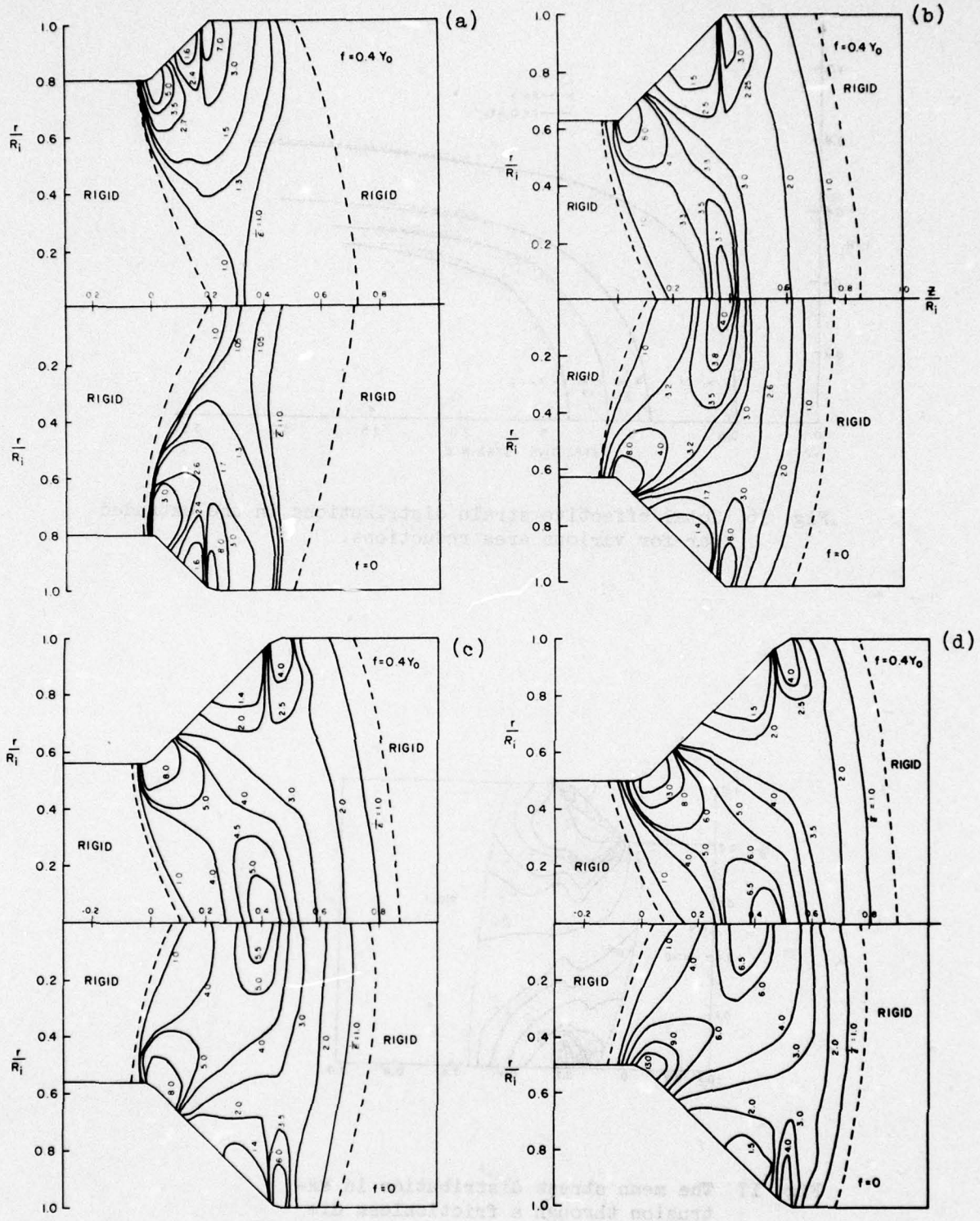


Fig. 15 Strain-rate distributions for two friction conditions at reductions of (a) $R_i/R_0 = 1.25$; (b) $R_i/R_0 = 1.6$; (c) $R_i/R_0 = 1.8$; (d) $R_i/R_0 = 2$.

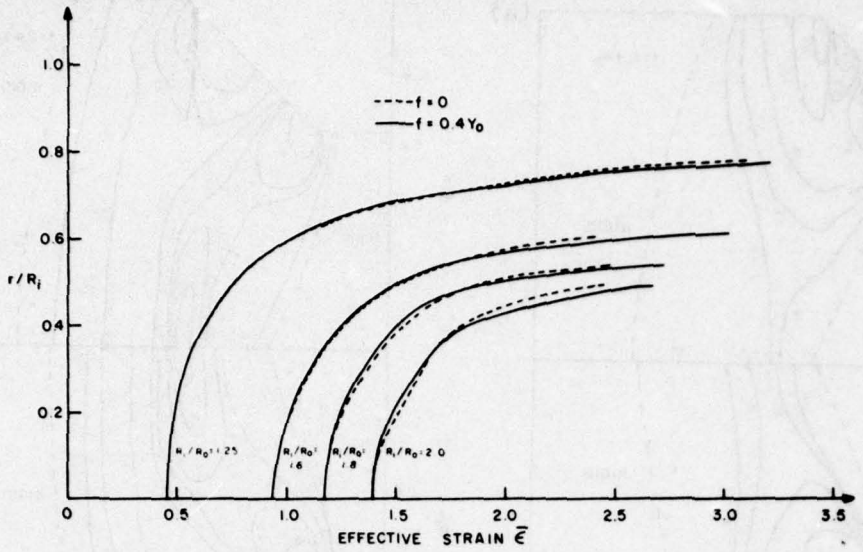


Fig. 16 Total effective strain distributions in the extruded bar for various area reductions.

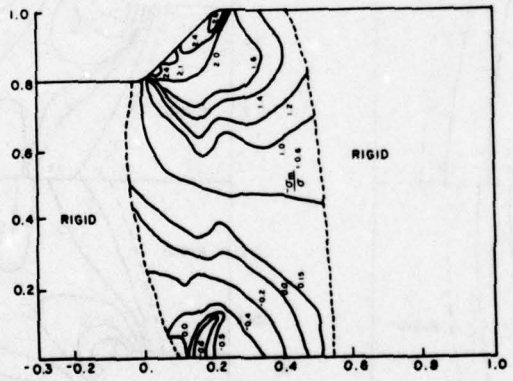


Fig. 17 The mean stress distribution in extrusion through a frictionless die at $R_i/R_0 = 1.25$.

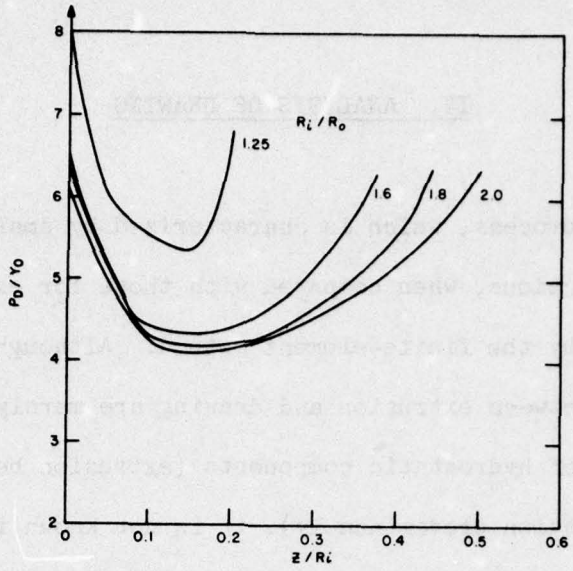


Fig. 18 Die pressure distributions for various area reductions.

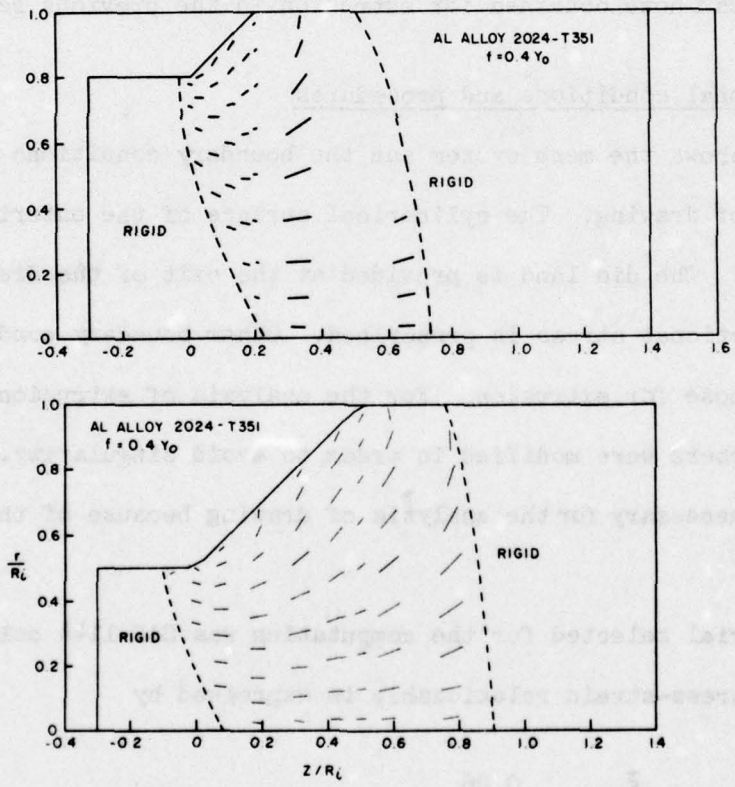


Fig. 19 Directions of the largest principal stresses.

IV. ANALYSIS OF DRAWING

The drawing process, which is characterized by smaller die angles and smaller area reductions, when compared with those for extrusion, has not yet been treated by the finite-element method. Although it appears that the differences between extrusion and drawing are merely geometrical and in the magnitude of hydrostatic components (extrusion being in compression and drawing in tension predominantly), it is not known if the results obtained in extrusion can be extrapolated to those in drawing according to the geometrical conditions and a simple concept of pushing and pulling. Therefore, we discuss the analytical results in drawing, emphasizing a comparison with those obtained for extrusion in the previous section.

A. Computational conditions and procedures

Fig. 20 shows the mesh system and the boundary conditions used for the analysis of drawing. The cylindrical surface of the entering bar is traction-free. The die land is provided at the exit of the die, along which the frictional stress is prescribed. Other boundary conditions are the same as those for extrusion. For the analysis of extrusion the shape of the die corners were modified in order to avoid singularity. However, this was not necessary for the analysis of drawing because of the small die angle.

The material selected for the computation was SAE 1144 cold-drawn steel whose stress-strain relationship is expressed by

$$\frac{\bar{\sigma}}{Y_0} = \left(1 + \frac{\bar{\varepsilon}}{0.262 \times 10^{-4}}\right)^{0.06}$$

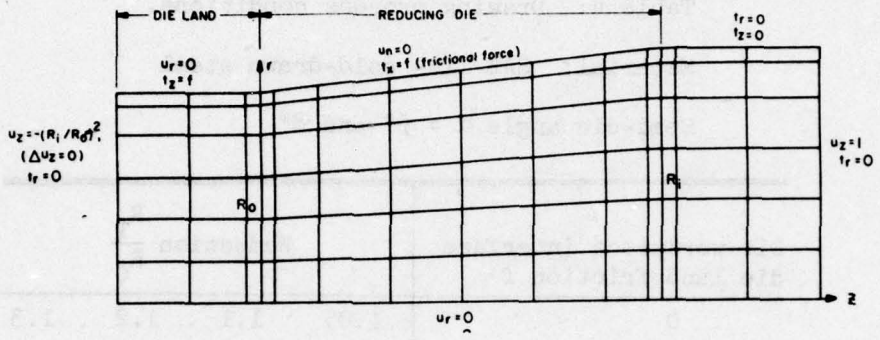


Fig. 20 The mesh system and boundary conditions for drawing.

with Y_0 = 84,800 psi

= 584.68 MN/m^2.

This material was selected mainly for the workability study in drawing given in Part 2 of this report.

The drawing process conditions for which the computation was performed are summarized in Table 4.

Table 4: Drawing process conditions.

Material: SAE 1144 cold-drawn steel

Semi-die angle α = 6° and 8°

Die-workpiece interface die land friction f	Reduction $\frac{R_i}{R_0}$			
0	1.05	1.1	1.2	1.3
0.25 Y_0	1.05	1.1	1.2	-

For computation purposes the solution of extrusion for α = 45° and R_i/R_0 = 1.25 was used as an initial guess for drawing with α = 6° and R_i/R_0 = 1.2 by modifying the solution according to geometrical proportions. Thirty-four (34) iterations were required to achieve the converged solution with

$$\sum_m \left\{ \left(\frac{\partial \tilde{\phi}^{(m)}}{\partial \underset{\sim}{u}^{(m)}} \right)^2 + \left(\frac{\partial \tilde{\phi}^{(m)}}{\partial \lambda^{(m)}} \right)^2 \right\} = 10^{-6}, \quad \left(\frac{\| \underset{\sim}{\Delta u} \|}{\| \underset{\sim}{u} \|} \sim 4 \times 10^{-6} \right).$$

This solution was then used as an initial guess for the computation of all the other cases. The convergence was excellent and only 6 to 10 iterations were necessary to obtain the solutions in most cases.

B. Results and discussion

The results for $\alpha = 6°$ are presented and discussed here. The results for $\alpha = 8°$ are presented and discussed in terms of workability in drawing in Part 2 of this report.

An example of the velocity distribution is shown for $R_i/R_0 = 1.2$ and $f = 0.25Y_0$ in Fig. 21. The distribution pattern looks somewhat different from those for extrusion. This is because of small reduction and small die angle and because the total distortion is small. Fig. 22 shows vertical grid line distortion in drawn bars. Greater friction produces more distortion. The difference in the total distortion due to die friction increases with increasing reductions.

The general pattern of the effective strain-rate distribution is the same as that for extrusion, as seen in Fig. 23. Concentrations of strain rate occur at the die corners. Along the axis of symmetry, strain-rate distribution shows a peak at the midpoint of the deformation zone. The deformation zone expands with larger die-workpiece interface friction as it did for extrusion.

The variation of effective strain distribution in drawn bars is comparatively small due to a small die angle and small reductions (Fig. 24). Fig. 24 shows also that the effect of the die friction on the strain distribution is negligible. By comparing Fig. 24 with the results of extrusion in Fig. 16, it can be seen that the die angle is a most important variable in controlling nonuniformity of the extruded or drawn bar properties.

The hydrostatic pressure distribution is shown in Fig. 25. It shows the combined characteristics of those for small reductions similar to

Fig. 17 and those for relatively extended deformation zone similar to Fig. 9. The magnitude of the hydrostatic component increases with larger die friction in drawing. This is contrary to the results with extrusion shown in Fig. 9. The die pressure distribution is plotted for various reductions and for two die friction conditions in Fig. 26. Higher die pressure is obtained for smaller reduction as it was for extrusion. However, contrary to the case of extrusion, the die pressure is higher with frictionless dies than those with friction. These findings on die pressure are in agreement with experimental results [12], although empirical values are average die pressures.

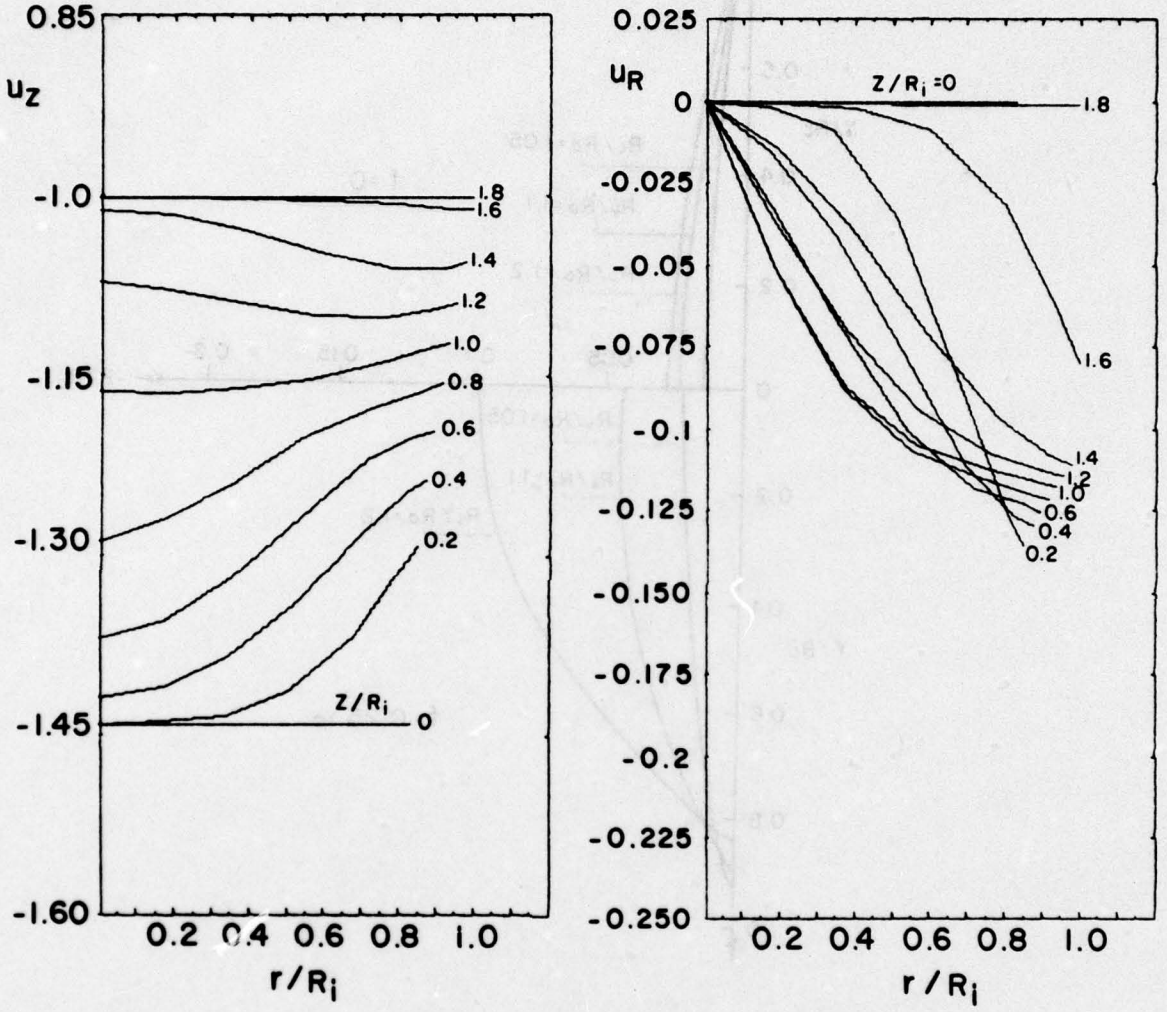


Fig. 21 Velocity distributions in drawing with $\alpha = 6°$, $R_i/R_0 = 1.2$, and $f = 0.25Y_0$.

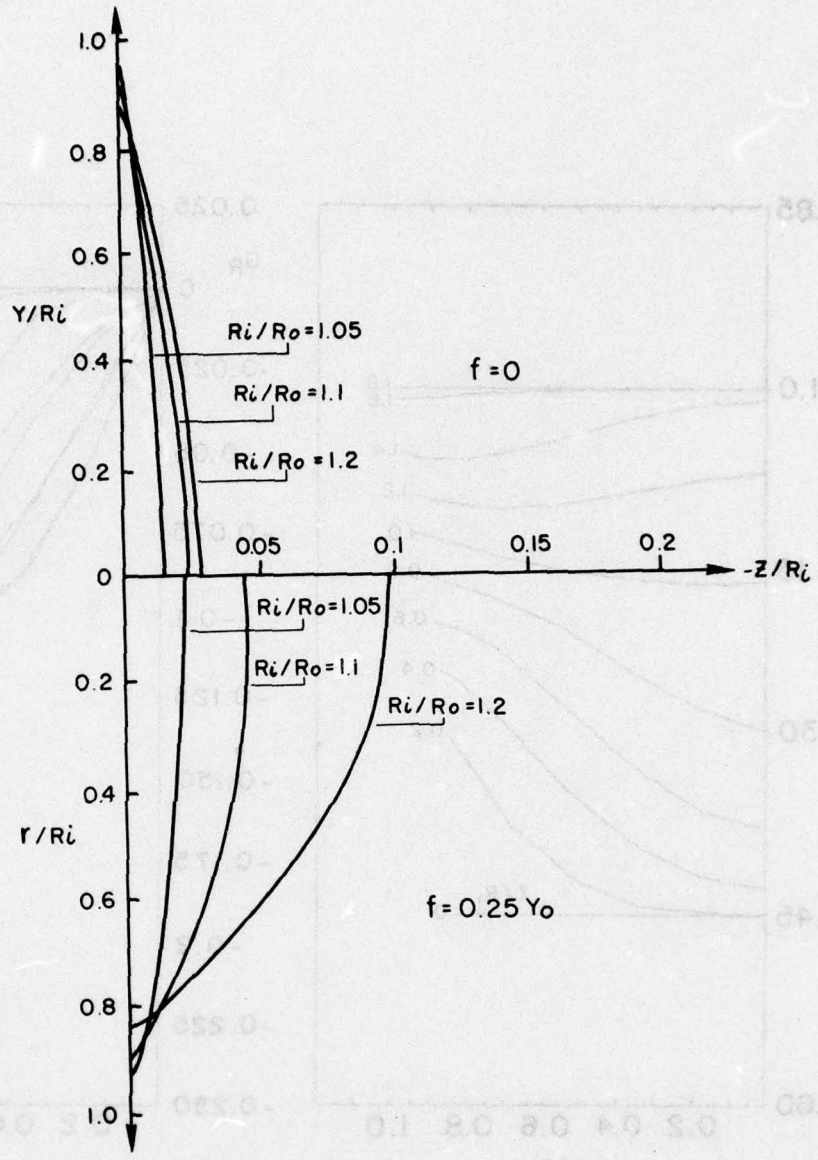
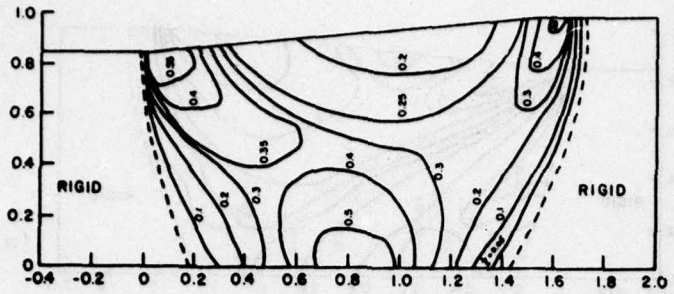
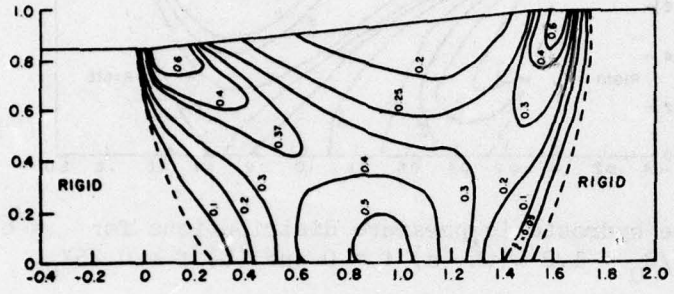


Fig. 22 Total distortion of a line originally perpendicular to the axis of drawing for various area reductions.



(a)



(b)

Fig. 23 Strain-rate distributions for $\alpha = 6°$ and $R_i/R_0 = 1.2$ with (a) $f = 0$ and (b) $f = 0.25Y_0$.

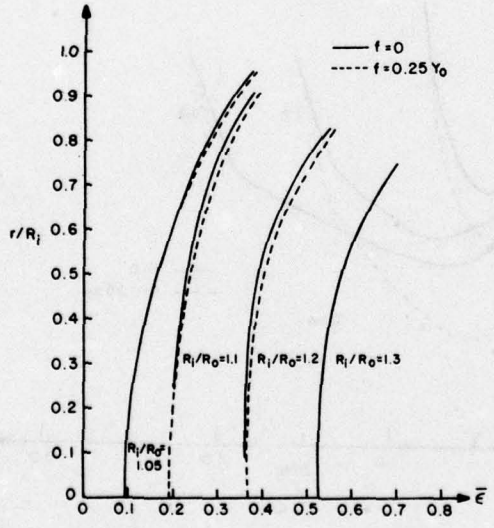


Fig. 24 Effective strain distributions in drawn bars.

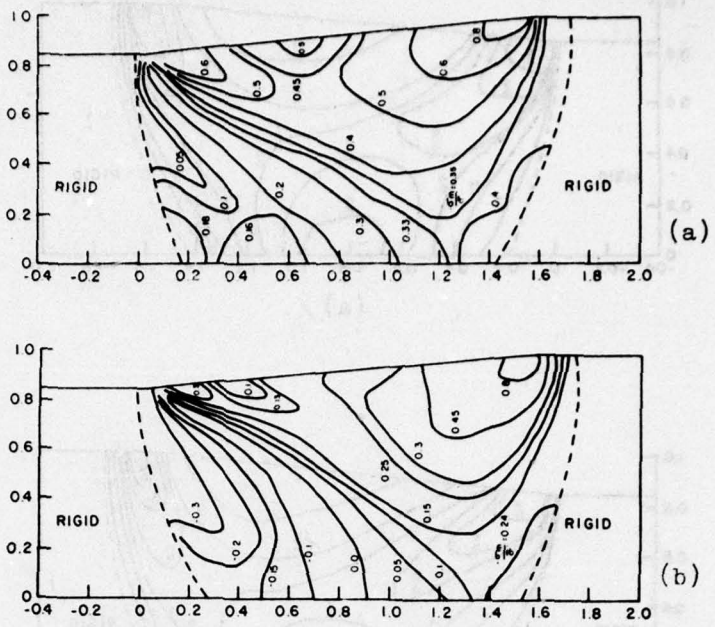


Fig. 25 The hydrostatic pressure distributions for = 6° and R_i/R_0 = 1.2 with (a) f = 0 and (b) f = $0.25Y_0$.

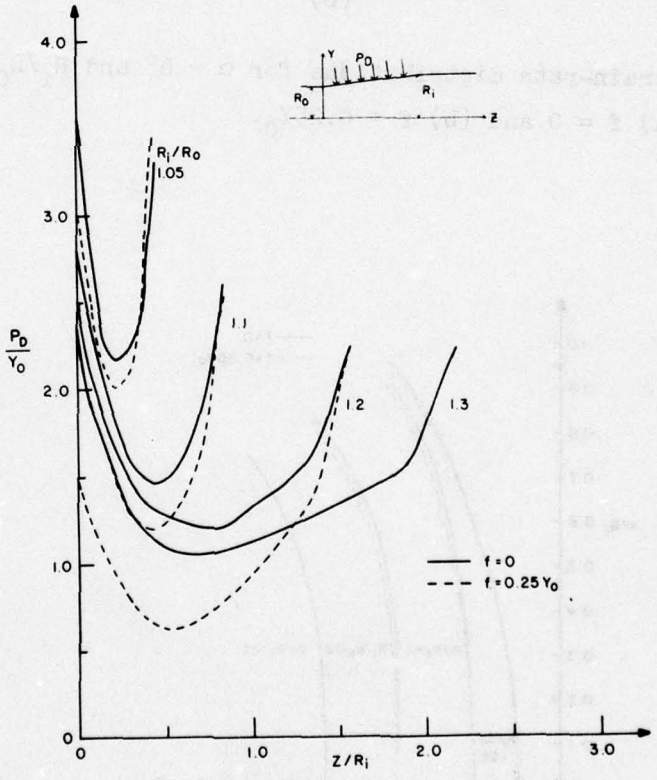


Fig. 26 The die pressure distributions for various reductions and for the two die-friction conditions.

V. SUMMARY AND CONCLUSIONS

In recent years the matrix method has been applied to the analysis of various metalworking processes, including the steady-state extrusion process. The present matrix method program incorporates the treatment of friction at the die-workpiece interface and an improvement of computational efficiency by adopting a new convergence criterion. Using this program, the detailed mechanics of steady-state extrusion and drawing were obtained under various process conditions. The stress-strain properties of SAE 1112 steel and aluminum alloy 2024-T351 were used for extrusion and the stress-strain curve of SAE 1144 cold-drawn steel was used for drawing. The variables included for the computation were the die angle, die-workpiece interface friction, and the area reduction. The computed results were presented in terms of velocity distribution, grid distortion, strain-rate distribution, and stress and strain distributions.

Although materials properties apparently influence metal flow, as seen in grid distortion patterns, their effects on the overall deformation characteristics appear to be insignificant in extrusion and drawing processes. Among other variables, friction at the die-workpiece interface plays an important role in determining the detailed mechanics in these processes. With increasing friction the degree of grid distortion becomes larger and the deformation zone size expands in both processes. But the effect of friction on nonuniformity of product properties is less significant, while the die angle is an important variable in controlling this nonuniformity. The contrast between the extrusion process and the drawing process can be

found in the die pressure distribution and the distribution of hydrostatic stress components. In both processes, the die pressure decreases with increasing reduction in area. However, the die pressure is greater for larger interface friction in extrusion, while the reverse is true in drawing. Similarly, the magnitude of the hydrostatic stress component is less with larger interface friction in extrusion, but increases with increasing friction in drawing.

It is concluded that the matrix method is indeed an efficient numerical method which provides useful information on the detailed deformation characteristics for various process variables. In order to ascertain the accuracy of the solutions presented here, however, an extensive theoretical and experimental investigation is still needed.

Part 2

WORKABILITY IN EXTRUSION AND DRAWING

I. INTRODUCTION

For occurrence of cracks at the free surface, such as in edge-cracking in sheet rolling and surface cracks in upsetting, the fracture criterion can be constructed experimentally. However, for predicting internal fracturing, formulations of fracture criteria under general deformation are required. In this part of the present investigation, the validity of the theory on ductile fracture developed in the previous report [1] was examined by the experimental data found in the literature. Then, combining the formulation of fracture criterion with the deformation mechanics found in Part 1, the workability of materials in extrusion and drawing was determined.

In an attempt to develop a general fracture criterion, a model for void growth was proposed to examine the stress and strain fields around voids. The solutions were obtained by the finite-element method. The results show that deformation is concentrated along the narrow band in the maximum shear stress direction. The fracture criterion assumed that voids of orders of magnitude smaller than the primary voids are distributed throughout and that fracturing occurred when the small voids grow and touch each other along the band connecting the large voids. The McClintock analysis was used for predicting the growth and coalescence of small voids. Based on this concept, the fracture strain formulation by McClintock was modified. Then the modified form, along with the formulation by Cockroft and Latham, was tested by the experiments on surface cracks and by the tension data by Bridgman. Its validity was further examined by applying the criterion to the determination of workability in bar extrusion and drawing.

II. FRACTURE CRITERIA

McClintock and his coworkers [13], [14] developed solutions for void growth under the transverse stress state. Their model consists of a single elliptic cylindrical void extending in one direction and imbedded in a rigid plastic media. The major and minor axes of the void coincide with the principal stress directions. They derived an approximate solution for the rigid work-hardening materials, $\bar{\sigma} = K\bar{\varepsilon}^n$, as

$$\log_e\left(\frac{R}{R_0}\right) = \frac{\sqrt{3}\,\bar{\varepsilon}}{2(1-n)} \sinh\left[\frac{\sqrt{3}(1-n)}{2}\,\frac{\sigma_a + \sigma_b}{\bar{\sigma}}\right] + \left(\frac{\varepsilon_a + \varepsilon_b}{2}\right) ; \tag{9}$$

$$m = \left(\frac{\sigma_a - \sigma_b}{\sigma_a + \sigma_b}\right) + \left[m_0 - \left(\frac{\sigma_a - \sigma_b}{\sigma_a + \sigma_b}\right)\right] \exp\left[-\frac{\sqrt{3}\,\bar{\varepsilon}}{(1-n)} \sinh\left\{\frac{\sqrt{3}(1-n)}{2}\left[\frac{\sigma_a + \sigma_b}{\bar{\sigma}}\right]\right\}\right] \tag{10}$$

where R and R_0 are the current and initial mean radii of the hole, respectively; σ_a and σ_b, principal stress components along the major and minor axes, respectively; $\bar{\sigma}$ is the effective stress and $\bar{\varepsilon}$ is the effective strain; m is the eccentricity of the ellipse defined in terms of the semi-major and semi-minor axes a and b, as

$$m = \frac{a - b}{a + b} , \tag{11}$$

and m_0 is the initial eccentricity. They assumed that the cylindrical hole is contained in the cell and that the material is composed of these cells. Fracturing was assumed to occur at the point where a growing void touches the cell boundary whose deformation is assumed to be the same as that of the boundary at infinity. The fracture strain, neglecting the void inter-

action, is then expressed (dropping a transient term) by

$$\bar{\varepsilon}_f = \frac{\log_e\left(\frac{\ell_a^0}{2a_0}\right)}{\left\{\frac{\sqrt{3}}{2(1-n)}\sinh\left[\frac{\sqrt{3}(1-n)}{2}\cdot\left(\frac{\sigma_a+\sigma_b}{\bar{\sigma}}\right)\right]+\frac{3}{4}\left(\frac{\sigma_b-\sigma_a}{\bar{\sigma}}\right)\right\}} \tag{12}$$

where ℓ_a^0 and a_0 are the initial values of hole spacing ℓ_a and a, respectively. The above derivation was given for generalized plane strain as an approximation for three-dimensional deformations. In three-dimensional configurations, there are six modes of fracture, two in each of the three perpendicular planes. Whichever one of these six modes gives the smallest fracture strain is the actual mode.

The model considers only the case where holes form at zero strain by complete separation of the particle-matrix interface. The review of the investigations on ductile fracture revealed that void nucleation is a complex process and that the nucleation pattern depends on particle size, particle composition, and possibly, particle distribution. The general observation is that void nucleation occurs at the large particles first. If a number of voids form and grow, then these voids may act as a stress raiser in the matrix and cause further void nucleation at other particle sites. Also, there are many indications that, during deformation, shear bands between large voids develop and that voids form at small particles in these bands, while the particles outside the bands remain relatively inactive. Thus, the formation of shear bands between large voids and growth and coalescence of small voids in the shear band may be a mechanism of eventual fracture.

Assuming this mechanism for fracture, a model for the growth of large voids has been proposed. Fig. 27 shows an assumed distribution of large voids, extending in arrays to infinity. The plane-strain condition is assumed in the direction perpendicular to the (x_a, x_b) plane. The minimum and maximum principal stresses act along the boundary at infinity in the directions of coordinate axes $(\sigma_a^\infty, \sigma_b^\infty)$. The solutions of this void growth model were obtained by the elastic-plastic finite-element model [1]. A result of the computation is shown in Fig. 28. As Fig. 28 indicates, deformation is concentrated along the narrow band in the maximum shear stress direction, and the concentration of strain along the band is about $2 \sim 3$ times the overall strain for most of the band and about $4 \sim 5$ greater near a large void. This implies that the band is the most favorable place for small voids to nucleate and grow.

The modified form of fracture criterion was obtained by assuming that voids of orders of magnitude smaller than the primary voids are distributed throughout and that fracturing occurred when the small voids grow and touch each other along the band connecting the large voids. When the McClintock analysis is applied locally under the condition prevailing within the shear band, we obtain

$$\bar{\varepsilon}^f = \frac{K}{\frac{2}{\sqrt{3}(1-n)} \sinh\left\{\frac{\sqrt{3}(1-n)}{2} \frac{\sigma_a + \sigma_b}{\bar{\sigma}}\right\} + \frac{\sigma_b - \sigma_a}{\bar{\sigma}}} \tag{13}$$

where $K = \frac{4}{3f} \log_e\left(\frac{\ell_a^0}{2a_0}\right)$ and f is a factor indicating the strain concentration along the shear band.

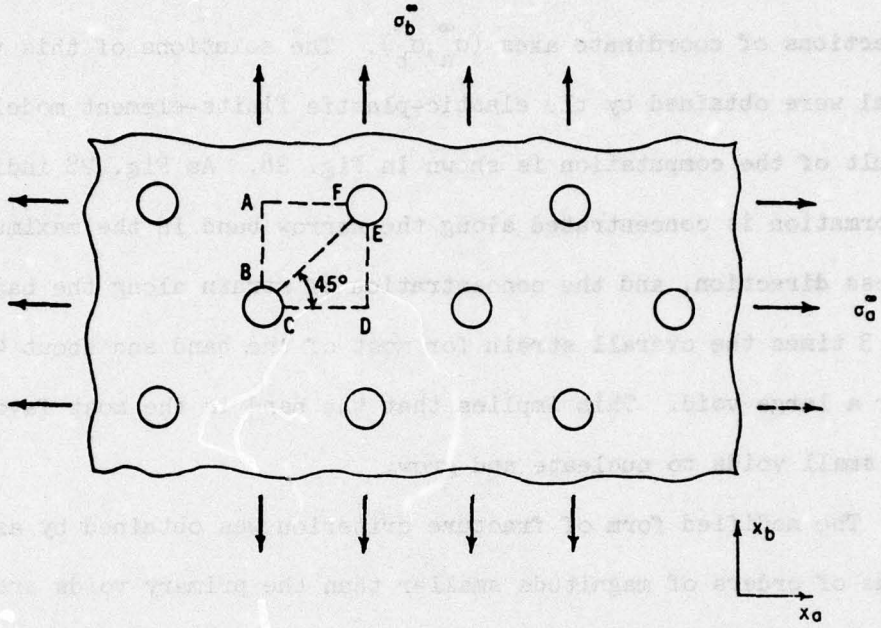


Fig. 27 Model for distribution of large voids.

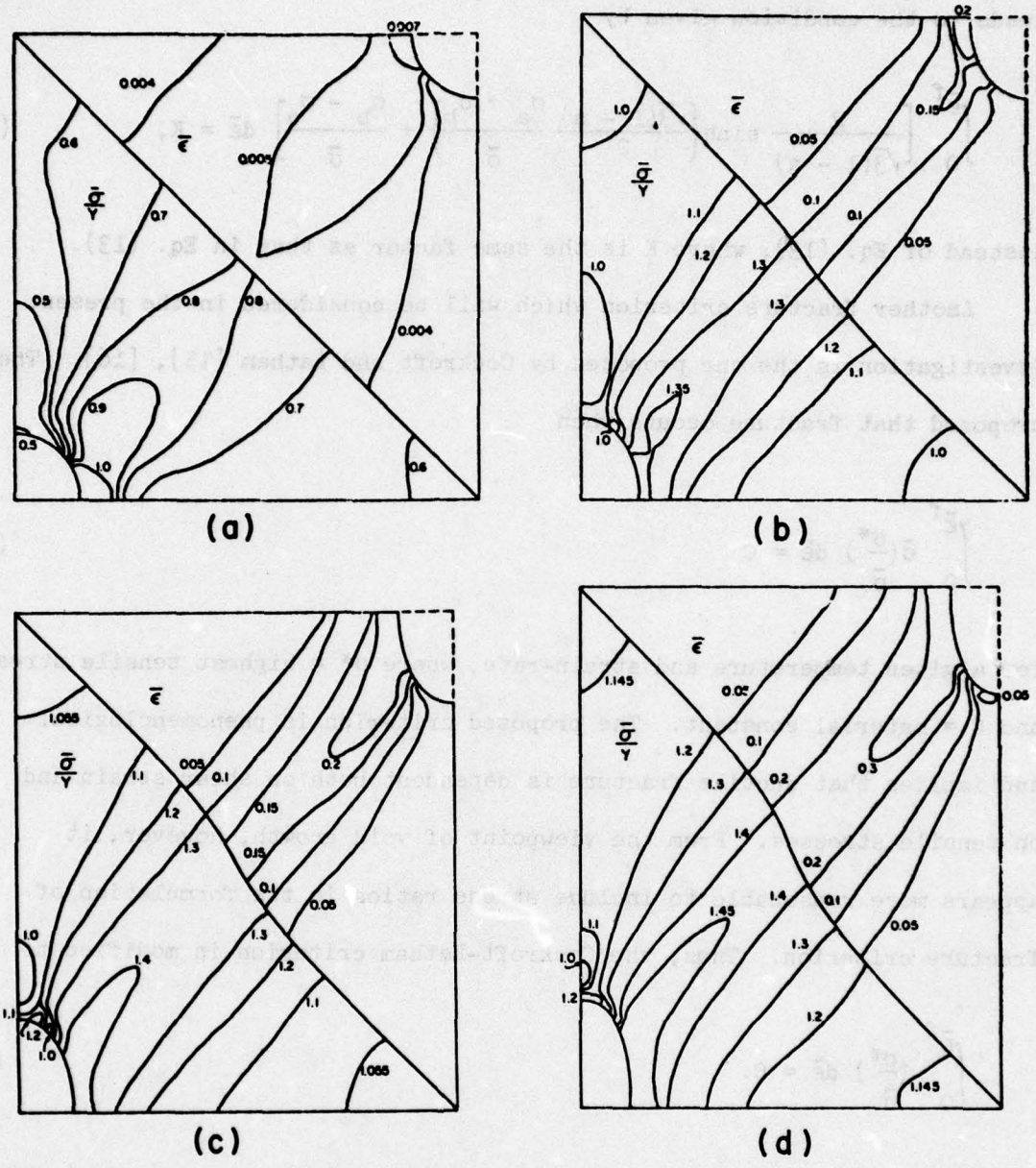


Fig. 28 Effective stress and effective strain distributions in the unit section. (a) $e_2 = 0.00525$; (b) $e_2 = 0.0423$; (c) $e_2 = 0.0699$; (d) $e_2 = 0.1163$.

When the stress ratios vary during deformation, McClintock's model leads to the condition given by

$$\int_0^{\bar{\varepsilon}^f} \left[\frac{2}{\sqrt{3}(1-n)} \sinh\left\{ \frac{\sqrt{3}(1-n)}{2} \frac{\sigma_a + \sigma_b}{\bar{\sigma}} \right\} + \frac{\sigma_b - \sigma_a}{\bar{\sigma}} \right] d\bar{\varepsilon} = K, \tag{14}$$

instead of Eq. (13), where K is the same factor as that in Eq. (13).

Another fracture criterion which will be considered in the present investigation is the one proposed by Cockroft and Latham [15], [16]. They proposed that fracture occurs when

$$\int_0^{\bar{\varepsilon}^f} \bar{\sigma}\left(\frac{\sigma^*}{\bar{\sigma}}\right) d\bar{\varepsilon} = C \tag{15}$$

for a given temperature and strain-rate, where σ^* = highest tensile stress and C = material constant. The proposed criterion is phenomenological and implies that ductile fracture is dependent both on shear strain and on tensile stresses. From the viewpoint of void growth, however, it appears more reasonable to include stress ratios in the formulation of fracture criterion. Thus, the Cockroft-Latham criterion is modified to

$$\int_0^{\bar{\varepsilon}^f} \left(\frac{\sigma^*}{\bar{\sigma}}\right) d\bar{\varepsilon} = C. \tag{16}$$

It may be of interest to note that Eq. (14) reduces to Eq. (16) with K = 2C, if the argument of sinh is small so that the hyperbolic sine function is approximated by a linear function. The magnitude of the argument of sinh depends on the stress state as well as on the work-hardening coefficient. When n = 1, Eqs. (14) and (16) are identical. For other values

of the work-hardening coefficient, the difference between the two criteria is small if $\frac{\sigma_a + \sigma_b}{\bar{\sigma}} < 1$, but increases with increasing values of $\frac{\sigma_a + \sigma_b}{\bar{\sigma}}$, as shown in Fig. 29.

We now examine in more detail the two criteria expressed by Eq. (14) and by Eq. (16).

A. Surface fracture

Since the fracture condition applicable to the free-surface cracks can be observed experimentally, several investigators [17]-[20] have used upsetting of cylindrical specimens for the study of ductile fractures. It was concluded from these investigations that the fracture criterion is expressed by

$$\varepsilon_1 = a - \frac{1}{2}\varepsilon_2, \tag{17}$$

where a is a material constant and ε_1 and ε_2 are the circumferential strain and the axial strain at the equatorial free surface, respectively. The two principal stresses in the surface, σ_1 and σ_2 ($\sigma_1 > \sigma_2$), during plastic deformation, are expressed by

$$\sigma_1 = \frac{\bar{\sigma}}{\sqrt{3}} \frac{2 + \alpha}{\sqrt{1 + \alpha + \alpha^2}} + \sigma_e , \qquad \sigma_2 = \frac{\bar{\sigma}}{\sqrt{3}} \frac{1 + 2\alpha}{\sqrt{1 + \alpha + \alpha^2}} + \sigma_e , \tag{18}$$

where $\alpha = \frac{d\varepsilon_2}{d\varepsilon_1}$ and σ_e is a hydrostatic stress acting on the surface. The incremental effective strain $d\bar{\varepsilon}$ is given by

$$d\bar{\varepsilon} = \frac{2}{\sqrt{3}} \sqrt{1 + \alpha + \alpha^2}\, d\varepsilon_1 , \tag{19}$$

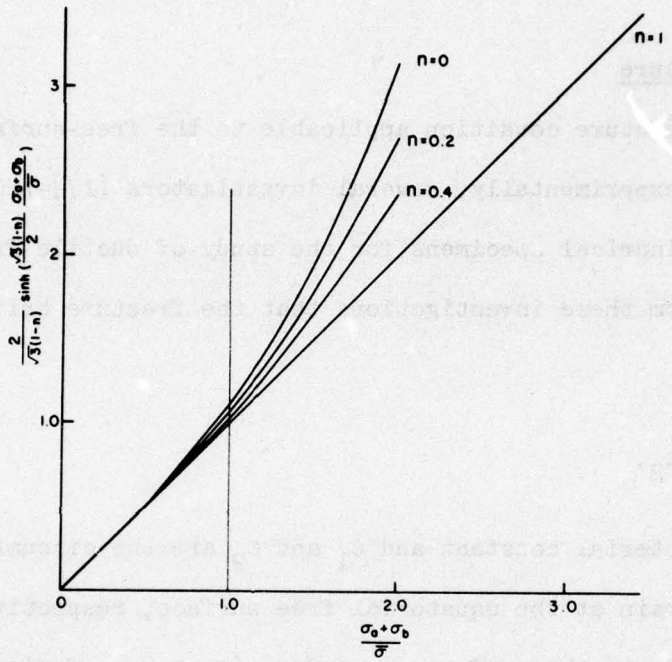


Fig. 29 Plot of hyperbolic sine function for various values of the work-hardening coefficient.

Under these conditions, the fracture criteria given by Eqs. (14) and (16) are both path-dependent, except when Eq. (16) becomes path-independent for $\sigma_e = 0$. Taking linear strain paths under $\sigma_e/\bar{\sigma}$ = const., and noting that σ_a and σ_b are the minimum and maximum stresses, respectively, Eq. (14) gives, for non-work-hardening materials (n = 0), the fracture strain as

$$\varepsilon_1^f = \frac{\sqrt{3}}{2\sqrt{1 + \alpha + \alpha^2}} \left[\frac{K}{\frac{2}{\sqrt{3}} \sinh\left(\frac{\sqrt{3}}{2} \frac{\sigma_1 + \sigma_2}{\bar{\sigma}}\right) + \frac{\sigma_1 - \sigma_2}{\bar{\sigma}}} \right] \tag{20}$$

for $\alpha \leq -\frac{1}{2}$, and

$$\varepsilon_1^f = \frac{\sqrt{3}}{2\sqrt{1 + \alpha + \alpha^2}} \left[\frac{K}{\frac{2}{\sqrt{3}} \sinh\left(\frac{\sqrt{3}}{2} \frac{\sigma_1 + \sigma_e}{\bar{\sigma}}\right) + \frac{\sigma_1 - \sigma_e}{\bar{\sigma}}} \right] \tag{21}$$

for $-\frac{1}{2} \leq \alpha \leq 0$. The criterion given by Eq. (16) becomes

$$\int_{\bar{\varepsilon}}^{\bar{\varepsilon}^f} \frac{\sigma^*}{\bar{\sigma}} \, d\varepsilon = \int_0^{\bar{\varepsilon}_f} \frac{\sigma_1}{\bar{\sigma}} \, d\bar{\varepsilon}$$

from which

$$\varepsilon_1^f = \frac{\sqrt{3}}{2\sqrt{1 + \alpha + \alpha^2}} \left(\frac{C}{\sigma_1/\bar{\sigma}}\right) = \frac{C}{\frac{2}{3}(2 + \alpha) + \frac{2}{\sqrt{3}}\left(\sqrt{1 + \alpha + \alpha^2}\left(\frac{\sigma_e}{\bar{\sigma}}\right)\right)}. \tag{22}$$

Eqs. (20), (21), and (22), with $\sigma_e/\bar{\sigma} = 0$ are plotted for K = 0.5 and C = 0.25 in Fig. 30.

It can be seen in Fig. 30 that the fracture line given by Eq. (22) has a slope of $-\frac{1}{2}$ for $\sigma_e/\bar{\sigma} = 0$ (free-surface cracks), and Eqs. (20) and

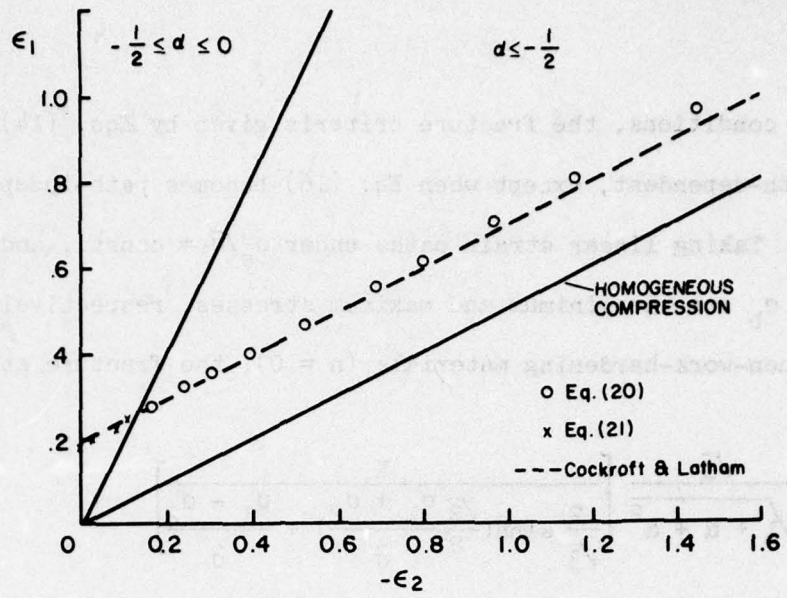


Fig. 30 Fracture conditions predicted from Eqs. (20) and (21) and by Cockroft and Latham [16].

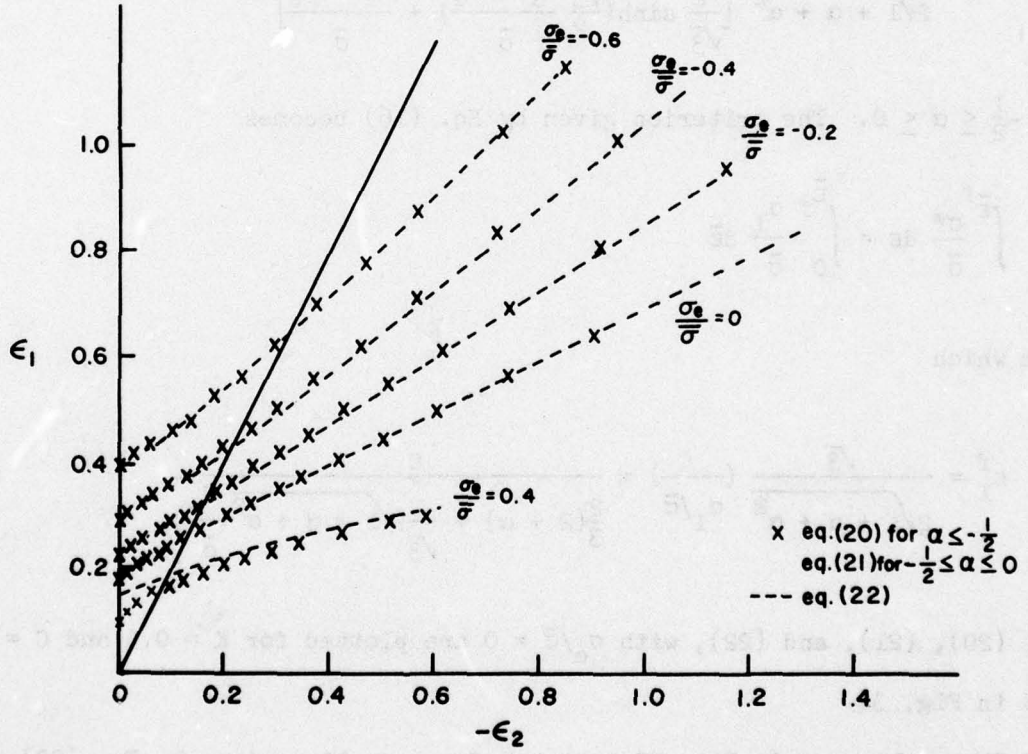


Fig. 31 Comparison of two fracture criteria for various values of $\sigma_e/\bar{\sigma}$.

(21) result in almost the same fracture strains as those according to Eq. (22) if the relationship $K = 2C$ is maintained. Similar plots of Eqs. (20), (21), and (22) for various values of $\sigma_e/\bar{\sigma}$ are given in Fig. 31.

B. Fracture in uniaxial tension

Bridgman [21] investigated plastic flow and fracture with special emphasis on the effects of hydrostatic pressure, using uniaxial tension. The two criteria given by Eqs. (14) and (16) are examined for internal fracturing, using the Bridgman experimental results. In uniaxial tension of a bar, the critical site for fracture is at the axis of symmetry in the neck section where $\varepsilon_1 = \varepsilon_z$ and $\alpha = -\frac{1}{2}$. Therefore, $\sigma_a = \sigma_r = \sigma_e$ and $\sigma_b = \sigma_z = \bar{\sigma} + \sigma_r$. Then, Eq. (14) is expressed by

$$\int_0^{\bar{\varepsilon}^f} (F_1 + 1)\, d\bar{\varepsilon} = K, \tag{23}$$

where $F_1 = h$ when $h > -1$,

$F_1 = -1$ when $h \leq -1$

and

$$h = \frac{2}{\sqrt{3}(1 - n)} \sinh\left\{\frac{\sqrt{3}(1 - n)}{2}\left(1 + 2\,\frac{\sigma_r}{\bar{\sigma}}\right)\right\},$$

assuming that the negative damage rate is not permitted.

The criterion given by Eq. (16) becomes

$$\int_0^{\bar{\varepsilon}^f} (F_2 + 1)\, d\bar{\varepsilon} = C, \tag{24}$$

where

$$F_2 = \frac{\sigma_r}{\bar{\sigma}} \quad \text{for} \quad \frac{\sigma_r}{\bar{\sigma}} > -1$$

and

$$F_2 = -1 \quad \text{for} \quad \frac{\sigma_r}{\bar{\sigma}} \leq -1.$$

Eqs. (23) and (24) are compared with experiments in Figs. 32 and 33, respectively. The experimental results by Bridgman used for comparison are summarized in Table 5.

In deducing experimental points in the figures from the data in Table 5, several assumptions and approximations are introduced: (1) the effective stress $\bar{\sigma}$ was assumed to be independent of the hydrostatic pressure, (2) the stress σ_r at the neck was estimated according to the Bridgman analysis, and (3) the effect of work-hardening coefficients was neglected.

The comparison shown in Figs. 32 and 33 reveals that the fracture criteria are both reasonably good for predicting the fracture strains.

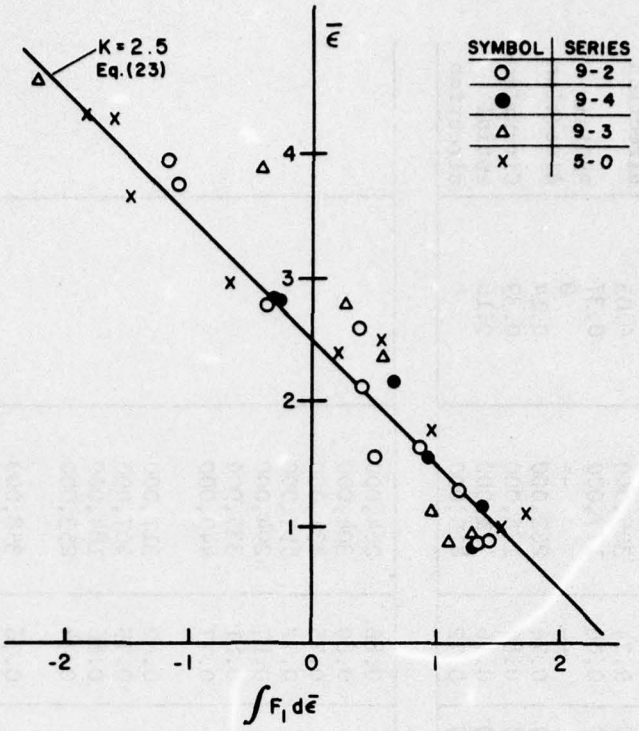


Fig. 32 Comparison between the fracture criterion given by Eq. (23) and Bridgman's fracture data.

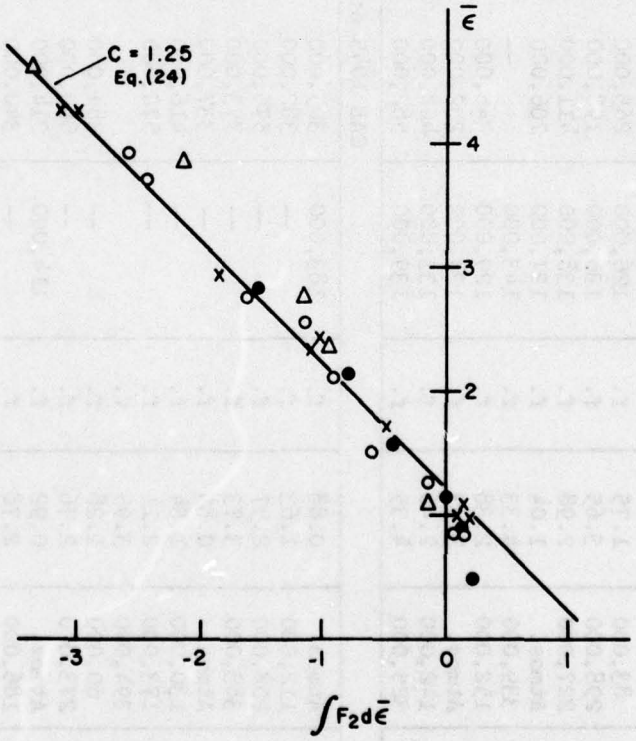


Fig. 33 Comparison between the fracture criterion given by Eq. (24) and Bridgman's fracture data.

Table 5 Bridgman experimental data.

Specimen	Pressure psi	Strain $\log_e A_i/A$	Fract. or not	Ave. stress at max.load psi	Ave.final stress,psi	a/R	Corr. factor	Final flow stress F,psi	Ratio of areas in fracture	Remarks
				Navy Gun Steel, as received						
5-0-1	Atmos.	1.12	f.	125,000	214,000	(0.74)	0.86	184,000	0.38	Longi-tudinal direction
5-0-2	83,000	1.75	f.	126,000	265,000	(1.19)	0.80	213,000	0.25	
5-0-3	298,000	3.65	f.	136,000	198,000	(1.96)	0.73	143,000	0	
5-0-5	227,000	2.98	f.	138,000	411,000	(1.78)	0.74	305,000	0.03	
5-0-6	Atmos.	1.04	f.	127,000	206,000	(0.70)	0.86	177,000	0.37	Radial direction
5-0-8	339,000	4.33	f.	143,000	--	--	--	--	0	
5-0-9	152,000	2.38	f.	129,000	345,000	(1.53)	0.76	262,000	0.14	
5-0-10	Atmos.	1.03	f.	124,000	202,000	(0.69)	0.87	175,000	0.39	Circumfer-ential direction
5-0-11	142,000	2.46	f.	133,000	408,000	(1.56)	0.76	311,000	0.16	
5-0-13	325,000	4.35	f.	139,000	582,000	(2.10)	0.72	415,000	0	
				SAE 1045 steel†						
9-2-1	Atmos.	0.88	f.	183,000	305,000	--	0.86	264,000		
9-2-6	112,000	1.63	f.	--	382,000	--	0.80	304,000		
9-2-7	208,000	2.57	f.	--	575,000	--	0.75	431,000		
9-2-8	383,000	3.73	f.	--	773,000	--	0.72	557,000		
9-2-18	Atmos.	0.85	f.	--	337,000	--	0.87	294,000		
9-2-20	130,000	1.54	f.	--	416,000	--	0.81	335,000		
9-2-22	173,000	2.11	f.	--	570,000	--	0.77	440,000		
9-2-23	394,000	3.95	f.							
9-2-24	60,000	1.28	f.	--	384,000	--	0.82	317,000		
9-2-26	273,000	2.76	f.	--	680,000	--	0.75	507,000		
9-3-1	Atmos.	0.92	f.	114,000	214,000	--	0.86	184,000		
9-3-5	186,000	2.78	f.	--	340,000	--	0.74	253,000		
9-3-6	269,000	3.87	f.							
9-3-7	145,000	2.37	f.	--	474,000	--	0.76	358,000		
9-3-9	34,000	1.10	f.	--	250,000	--	0.84	210,000		
9-3-10	387,000	4.61	f.							
9-3-11	Atmos.	0.81	f.	--	206,000	--	0.87	180,000		
9-4-1	Atmos.	0.89	f.	105,000	189,000	--	0.86	163,000		
9-4-7	18,000	1.13	f.	--	149,000	--	0.84	125,000		
9-4-9	116,000	2.14	f.	--	313,000	--	0.77	240,000		
9-4-12	Atmos.	0.90	f.	--	189,000	--	0.86	163,000		
9-4-13	74,000	1.56	f.	--	225,000	--	0.80	180,000		
9-4-14	182,000	2.83	f.	--	396,000	--	0.74	295,000		
9-4-15	389,000	4.80	f.							

Table 5 (continued)

†SAE 1045 steel
9-2 series: Quenched into water from 1575°F; drawn to 800°F
9-3 series: Quenched into salt at 800°F from 1575°F
9-4 series: Quenched into salt at 1100°F from 1575°F

III. WORKABILITY IN EXTRUSION

For workability in extrusion with reference to center bursting, experimental observations are limited and information about the stresses developed in metal being deformed by extrusion is insufficient. Thus, Latham and Cockroft [15] speculated the conditions in axisymmetric extrusion by assuming that the slip-line theory can be applied even though this is appropriate only to plane-strain conditions. Also, it was assumed that lubrication during cold extrusion is sufficiently good for the calculations to be based on zero friction. Based on the tensile plastic work density, they determined a damage factor for a range of extrusion conditions for several metals. Avitzur [22], [23] investigated theoretically center bursting in extrusion, using the upper-bound approach. Because the fracture condition was not incorporated in the approach, some objections may be raised regarding the validity of the results. However, the study led to defining the range of successful extrusions in terms of die angle, friction conditions, and the reduction.

Hoffmanner [24] also performed slip-line analysis for extrusion and compared with visioplasticity results for axisymmetric extrusion. He found that the calculated maximum center line principal stresses were in good agreement with visioplasticity results when the plane strain and axisymmetric results were compared on the basis of equal cross-sections. Combining the slip-line analysis for deformation and the fracture criterion proposed by Cockroft and Latham [16], Hoffmanner examined workability in extrusion. In Fig. 34, the three parabolic curves represent the process-

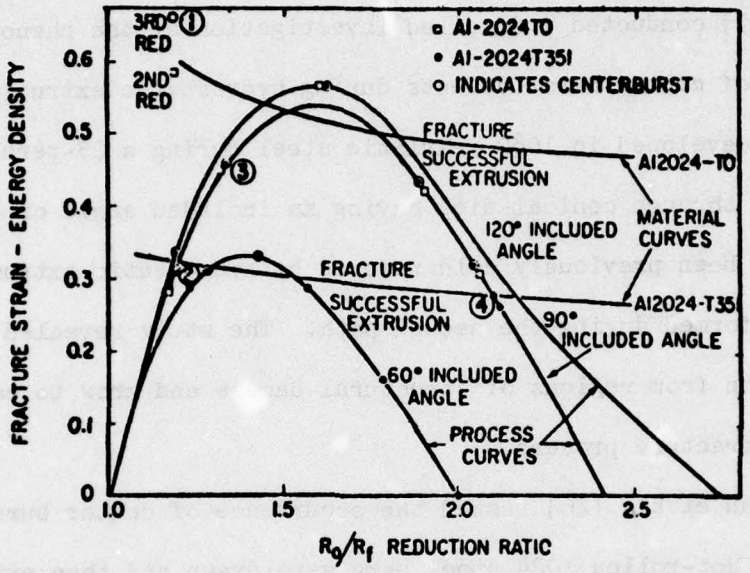


Fig. 34 Workability criteria for center burst based on a maximum tensile stress-strain energy criterion [23].

required strain energy as a function of the reduction ratio for three die angles. Whenever the process-required energy exceeds the energy available for successful deformation of the material, center burst should be observed. The extrusion observations are superimposed on these curves and demonstrate the near-perfect agreement between practice and prediction.

A. Experimental data

Pepe [25] conducted a detailed investigation of the phenomenological development of center burst defects during hydrostatic extrusion. Central bursts were developed in 1080 pearlitic steel during a 25-percent reduction in area pass through conical dies having an included angle of 45°. The material had been previously cold reduced by hydrostatic extrusion. Central bursts were formed during the second path. The study revealed that central bursts develop from regions of structural damage and grow to completion by a two-stage fracture process.

Zimmerman et al. [26] tested the occurrence of center bursting using 1024 steel. Hot-rolled 1024 steel bars were drawn and then extruded in three steps. The final extrusion step was utilized to examine, through a variety of die semi-angles, whether central bursting was produced or not. They found that among 4000 (1000 shafts from each of the four heats) shafts, center bursting was detected in about 45 shafts from one of the heats.

The above observations of center bursting in extrusion, as some other experimental studies show, were all made in material which has been pre-deformed. Although the present theoretical method is applicable to the determination of stress and strain distributions in extrusion for the material which has distributed strength and is capable of following the

previous deformation history, the experimental observation of center bursting during a single pass of deformation has been sought because the process condition in the computation can then be specified more faithfully according to the experimental procedure. In the present investigation, experiments reported by Hoffmanner [24] were used for examining the validity of the workability theory.

Hoffmanner observed center bursting in a single-step extrusion for aluminum alloy 2024-T351 at room temperature. The experimental conditions and the results were as follows:

1. Material: The specimens were obtained from a 3-inch diameter of aluminum alloy 2024 in as-received T351 condition.
2. Tensile test: Testing was performed at room temperature on an Instron testing machine at a nominal strain rate of 0.1 in/in/min. Measurements of extension were performed continuously until fracture with an extensometer exhibiting a strain sensitivity of at least 0.0005 in/in. Measurements of the minimum cross-sectional radius (a) and neck radius of curvature (R) were performed in two directions at 90° to each other either continuously by photographing the specimen or discontinuously by removing the specimen from the test fixtures and performing these measurements on an optical comparator.
3. Compression test: Compression testing was investigated by using simple upsetting of cylinders. For strain measurements, an orthogonal array of Vickers diamond pyramid impressions was accurately placed at 0.050-inch separations about the exact center of the specimens along subsequent directions of principal normal stress. These directions corresponded to the direction of loading and

the direction normal to it through the center of the specimen. Eight impressions were placed in each of the two directions.

4. Extrusion: Extrusion was performed at a ram speed of 12 in/min with dies of (60°), 90°, and (120°) included angles with anhydrous lanolin as a lubricant. Both the billets and the container were lubricated prior to extrusion and load-time curves were recorded on an oscillograph during deformation. The billets were machined with one end contoured to match the die and approximately 0.005 inch under the container diameter. A surface finish in the 20 to 30 microinch rms range was specified. Initial billet diameters of 1.780, 1.412, and 0.812 were used. The two larger diameters were used in extrusion through 90° dies to study single- and multiple-pass extrusion and the 0.812 diameter billets were used in single-pass reductions through all three die angles.

Results

1. Tension test results

Table 6

Specimen number	Gauge section diameter (inch)	Initial a/R ratio	Final a/R ratio	Fracture strain	Remarks
TNLH1	0.250	0	0.294	0.382	Longitudinal specimen
TNLH2	0.250	0	0.342	0.354	Longitudinal specimen
TNTH1	0.250	0	0	0.146	Transverse specimen
TNTH2	0.250	0	0	0.156	Transverse specimen

2. Compression test results

Table 7

Specimen no.	Initial	Final	Initial height/diameter ratio	Fracture strain	Remarks
UHN-1(upset)	0.50	1.61	1.145	0.265	
UHN-2(upset)	0.50	-	2.24	-	Teflon lub., sheared over
UHN-3(upset)	0.50	1.58	1.145	0.281	

3. Extrusion results

Table 8

Specimen no.	R_o/R_f	Included die angle	Remarks
HN-101A	1.245	90°	Unlubricated, billet stuck
HN-101B	1.326	90°	Centerburst
HN-101C	1.249	90°	Centerburst
HN-451	1.157	90°	Centerburst
HN-452	2.328	90°	
HN-990	1.988	90°	Centerburst
HN-999	1.089	90°	
HN-90B	2.132	90°	

B. Computation, results, and discussion

Using the matrix method, the extrusion process was analyzed for the material, al alloy 2024-T351, and the detailed mechanics were presented in Part 1 of this report. The process conditions were: semi-cone angle $\alpha = 45°$, area reduction $R_i/R_0 = 1.25$, 1.6, 1.8, and 2.0 for the two friction conditions, $f = 0$ and $0.4Y_0$. The critical site for occurrence of center bursting is along the axis of extrusion. The stress and strain states along the axis of extrusion resulted from the computation described in Part 1. Because the strain system along the axis of extrusion is identical

to that in uniaxial tension, the quantities necessary for workability are

$$\int_0^{\bar{\varepsilon}} (F_1 + 1)\, d\bar{\varepsilon} \quad \text{from Eq. (23) for one criterion}$$

and

$$\int_0^{\bar{\varepsilon}} (F_2 + 1)\, d\bar{\varepsilon} \quad \text{from Eq. (24) for another.}$$

These quantities are plotted as functions of area reduction in Figs. 35 and 36. From the figures, some conclusions immediately follow. The two quantities are almost identical, with a factor of 2. The magnitudes decreased greatly when work-hardening of materials was included. The magnitudes decreased further with friction at the die-workpiece interface. Note that the curve for non-work-hardening materials in Fig. 36 is in good agreement with the Hoffmanner result given in Fig. 34 for 90° included angle. This is somewhat surprising because the curve by Hoffmanner was obtained by approximations based on the slip-line analysis.

The critical values of these quantities (K in Eq. (23) and C in Eq. (24)) above which fracturing is predicted can be obtained from the fracture data in tension or compression tests. Hoffmanner estimated $C \simeq 0.3$ ($K = 2C$) from the tension test results. On the other hand, another estimate of $C \simeq 0.08$ was obtained from the compression test results. If we apply the critical value of $C = 0.3$ to the results in Fig. 36, fracturing should not occur in all the reductions. If we assume $C = 0.08$ to be a critical value, then center bursting would be observed over the range $1.08 < R_i/R_0 < 1.86$ with frictionless die and $1.16 < R_i/R_0 < 1.60$ with friction $f = 0.4Y_0$.

Although the prediction of workability with a critical value, $C = 0.08$, appears to be in good agreement with the experimental results in Table 8,

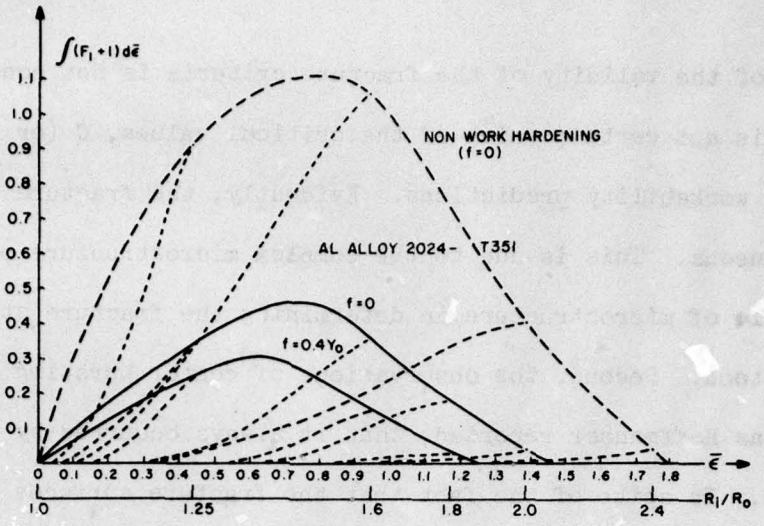


Fig. 35 Variations of $\int_0^{\bar{\epsilon}} (F_1 + 1)\, d\bar{\epsilon}$ as a function of area reduction in extrusion. Materials: non-work-hardening and al alloy 2024-T351.

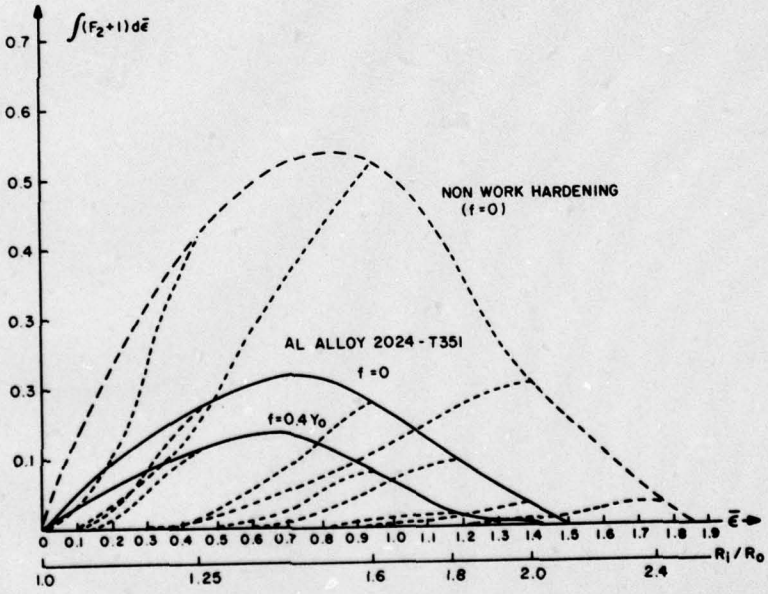


Fig. 36 Variations of $\int_0^{\bar{\epsilon}} (F_2 + 1)\, d\bar{\epsilon}$ as a function of area reduction in extrusion. Materials: non-work-hardening and al alloy 2024-T351.

the proof of the validity of the fracture criteria is not conclusive. First, it is not certain which of the critical values, C (or K), is proper to use for workability predictions. Evidently, the fracture behavior is not homogeneous. This is due to the complex microstructure of the material and the role of microstructure in determining the fracture strains is not yet understood. Second, the observations of center bursting in Table 8 revealed, as Hoffmanner reported, that it always occurred by complete separation. In spite of the fact that the fracture surfaces always possessed the centrally located conical cavity typical of the center burst, whether fracturing initiated at the center is in doubt.

IV. WORKABILITY IN DRAWING

To determine workability in drawing, the same approach as that for extrusion was applied. The process mechanics were analyzed by the matrix method and the stress and strain distributions along the axis of drawing were computed for various reductions of area. The tension and compression tests were performed to obtain the stress and strain property of the material and to determine the critical value for fracturing. Combining these data, the workability in drawing was predicted. The drawing experiments were conducted at room temperature to examine the predictions. The material was SAE 1144 cold-drawn steel. The stress-strain property of the material is given in Part 1.

A. Process mechanics

The process conditions were: semi-die angle $\alpha = 8°$; area reduction $R_i/R_0 = 1.05, 1.10, 1.2, 1.3$; and friction $f = 0, 0.25Y_0$. The stress and strain distributions were obtained and the significant quantities for fracturing were determined for each area reduction.

Because the two fracture criteria discussed in the previous sections resulted in almost identical predictions of fracture conditions, only the quantities (normalized energy density) defined by $\int_0^{\bar{\varepsilon}} (F_2 + 1)\, d\bar{\varepsilon}$ was shown as a function of the area reduction in Fig. 37. In calculating the energy density, the stress components were determined by correcting the hydrostatic part of the stress components from the total force balance. It is seen in Fig. 37 that the energy densities for the critical path along the axis of drawing increased monotonically with increasing area reduction.

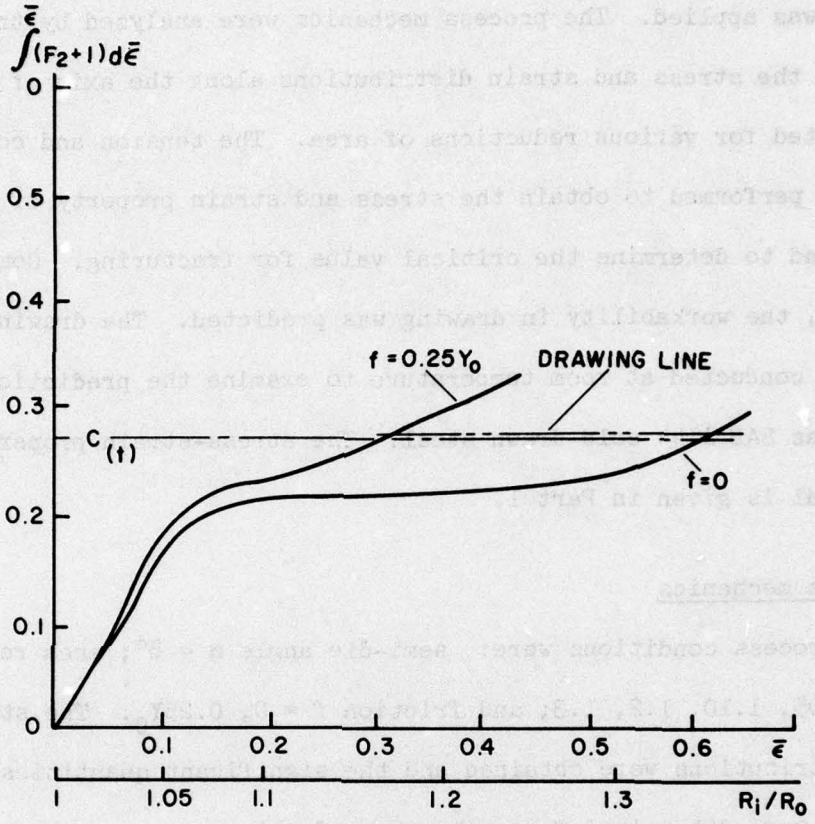


Fig. 37 Variations of $\int_0^{\bar{\varepsilon}} (F_2 + 1)\, d\bar{\varepsilon}$ as a function of area reduction in drawing. Material: SAE 1144 cold-drawn steel.

They were larger for larger die-workpiece interface friction. This is contrary to the extrusion results shown in Figs. 35 and 36. To determine workability, this energy density level must be compared with the critical value C for fracturing, estimated from the fracture data in tension and compression tests.

B. Tension and compression tests

Tension specimens were machined longitudinally from a 0.5-in. diameter bar. Testing was performed on an Instron machine with a plate speed of 0.02 in/min. The load-displacement curve was recorded and the instantaneous neck radii, as well as neck diameter, was measured until fracture. The critical energy density was estimated from the tension test as $C_{(t)} = 0.271$. The Bridgman analysis was used for an approximate stress analysis during neck formation.

Compression specimens were machined from the same bar of 0.5-in. diameter. The height and the diameter of the specimens were 0.50 in. and 0.49 in., respectively. The grid lines were printed on the cylindrical surface of the specimens, and the strains at fracture were determined from the grid distortion of the deformed specimens. The critical value C was estimated from the fracture strains according to Eq. (22) and was given by $C_{(c)} = 0.093$. It is to be noted that the fracture behavior is different in tension and in compression.

C. Workability

The energy density in drawing shown in Fig. 37 is compared with the critical value for fracturing in order to predict workability of SAE 1144

steel in drawing. It is seen again that the prediction is drastically different depending upon which critical value of tension and compression is applied. With the critical value of 0.09 which was estimated from the compression tests, center bursting would occur in all drawing with the area reduction larger than $R_i/R_0 = 1.03$, with very little effect of die-workpiece interface. On the other hand, if $C_{(t)}$ is taken as the critical value, the prediction is that for frictionless dies no center burst would occur up to $R_i/R_0 = 1.36$, and that this limiting value decreases with increasing die friction; for example, for friction $f = 0.25Y_0$, center burst would not be observed for R_i/R_o less than 1.16.

In order to examine the workability predictions, bar drawing experiments were performed. The bar specimen configuration is shown in Fig. 38. A threaded grip was used for drawing. This scheme, however, sets the drawing limit according to the maximum drawing force which can be sustained by the threaded part of the specimen. Commercially available carbide drawing dies with 8° semi-included die angle were used. The steady-state drawing force was measured, and the occurrence of center bursting was checked with drawn bars. The experimental conditions and results are summarized in Table 9.

Table 9 Experimental conditions and results.

Specimen no.	(Outlet diam.) $2R_0$	(Inlet diam.) $2R_i$	(Reduc. ratio) R_i/R_0	Lubrication	Ave. drawing pressure $(L/\pi R_0^2 Y_0)$	Center bursting
4	0.388"	0.485"	1.25	White lead in oil	0.907	
8	0.388"	0.465"	1.198	"	0.713	
5	0.4195"	0.485"	1.156	"	0.691	
16	0.4195"	0.465"	1.108	"	0.361	No
1	0.451"	0.485"	1.075	"	0.435	
9	0.451"	0.465"	1.031	"	0.428	
21	0.451"	0.5"	1.109	Dry	0.664	
7	0.465"	0.465"	1.198	Dry	0.997	

In Fig. 38 the experimental and theoretical average drawing stresses were plotted as functions of the reduction ratio R_i/R_0. Theoretical curves were for frictionless dies ($f = 0$) and for the frictional stress $f = 0.25Y_0$. The experimental values with and without lubrication fell between these two theoretical curves. The drawing limit due to the present experimental scheme is also shown in Fig. 38. Under the lubricated condition, drawing over the reduction range of $R_i/R_0 = 1.031 \sim 1.25$ produced no center bursting. This suggests that the use of the critical value $C_{(c)}$, estimated from the compression fracture data for workability prediction, was not appropriate. With regard to the critical value of $C_{(t)} = 0.271$, it was found that the drawing limit set by the present experimental scheme was very close to the critical value $C_{(t)}$, as shown in Fig. 37. In fact, drawing was not possible for three cases of drawing under the dry condition. As a result, it was not possible to determine the applicability of $C_{(t)}$ as a fracture criterion to the occurrence of center bursting in bar drawing.

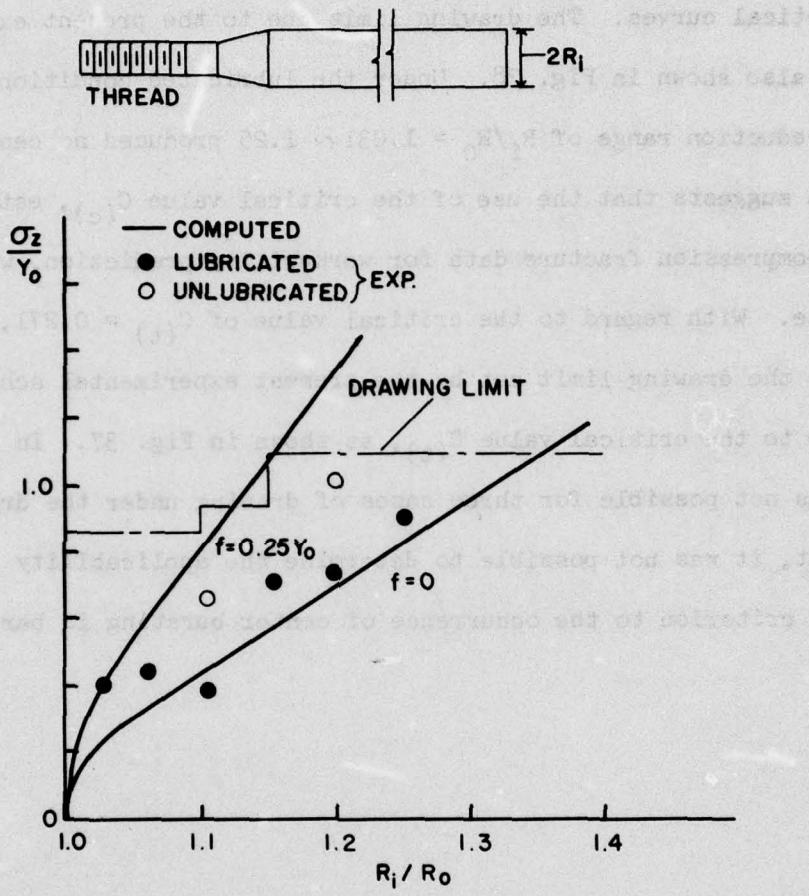


Fig. 38 Computed and experimental average drawing stress as functions of area reduction.

V. SUMMARY AND DISCUSSION

For workability in metalworking processes, formulation of a ductile fracture criterion must be simple (or in the form of easy applicability) and yet reflect physical mechanisms of fracture so that the predicted fracture behavior is close to the one of real materials within reasonable accuracy. With a two-size voids model a modified interpretation to the McClintock formulation of fracture strains was given. It was shown then that this formulation and the one by Cockroft and Latham resulted in almost identical fracture strains for occurrence of free-surface cracks and for fracturing in uniaxial tension. It was demonstrated further that these formulations predicted reasonably well the experimentally observed fracture strains. In examining the validity of the prediction by experiments, the stress-strain histories at a critical site were determined by using the continuous experimental observations on distortion in the case of surface cracks and by the Bridgman analysis combined with experimental measurements of load and neck geometry in the case of uniaxial tension. In applying the criteria to occurrence of center bursting in axisymmetric extrusion and drawing, the determination of stress and strain paths at a critical site was provided theoretically by the matrix method of analysis. The critical value of the material's capability against fracturing was obtained by tension and compression experiments. Workability in extrusion was examined for aluminum alloy 2024-T351 using the data found in the literature and the experiments of workability in drawing was attempted for SAE 1144 cold-drawn steel to test the predictions. The results of

validation were inconclusive. This is attributed to the fact that the fracture behavior has directionality as evidenced from the tension and compression fracture data. Furthermore, most of the previous investigations on center bursting in extrusion and drawing have reported observations of center bursting in pre-deformed materials. Although al alloy 2024-T351 was reported to have produced center bursting in a single extrusion path at room temperature, it occurred always by complete separation. With regard to bar drawing of SAE 1144 steel, the drawing limit set by the present experimental scheme was very close to the limiting energy density level, thereby making it extremely difficult to perform drawing near the critical boundary of the sound and defect zones.

In spite of inconclusive results of workability, the present investigation revealed several significant findings. In extrusion, the energy density level critical for center bursting was less for work-hardening materials when compared with that for a non-work-hardening material. This energy density level is reduced with increasing friction at the die-workpiece interface. It increases first with increasing area reduction and then decreases indicating a maximum as a function of area reduction. In drawing, the energy density level monotonically increases with the area reduction, and increases, as opposed to that in extrusion, with increasing interface friction.

Although conclusive validity of the present workability theory in extrusion and drawing awaits more extensive and systematic experimental investigations, as well as theoretical calculations, the method of computation is available and the approach has been cleared toward complete understanding of ductile fracture in metalworking processes.

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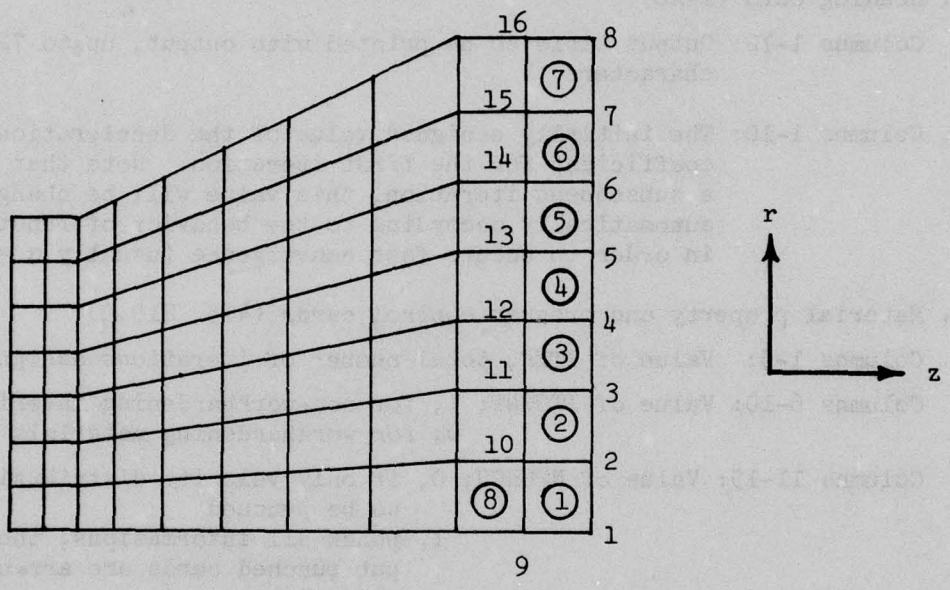
APPENDIX

COMPUTER PROGRAM OF THE MATRIX METHOD FOR THE ANALYSIS OF AXISYMMETRIC EXTRUSION AND DRAWING

I. Program Interpretation and Input Data Cards

The program was written according to the numbering system of nodal points and elements and the mesh system as shown in the figure.



The four surrounding nodal points (I, J, K, L) of an element are arranged in a clockwise direction with the lowest index assigned to I (e.g., for the first element in the figure (I, J, K, L) = (1, 9, 10, 2)).

The output values of stress and die force are all normalized by the initial yield stress and are positive in the sense of positive r or z direction.

In addition to preparing the input data cards, the following variables in program PURT need to be specified according to the mesh system and geometry considered:

THETA: Semi-included angle of conical die

NIPTS: Number of lines in mesh system parallel to the axis of symmetry

NJPTS: Number of lines in mesh system perpendicular to the axis of symmetry

NTIMES: Number of flow lines to be constructed

NMAX: Total number of increments for strain integration

RENTER: Radius of the billet at entrance (it is convenient to choose RENTER = 1.0)

YEXIT: z-coordinate of the billet at the die exit (YEXIT = 0.0)

A. Sequence of input data card preparation for the main program

1. Heading card (12A6)

   Columns 1-72: Output title to be printed with output, up to 72 characters

2. Columns 1-10: The initially assigned value of the deceleration coefficient for the first iteration. Note that in a subsequent iteration, this value will be changed automatically according to the behavior of functions in order to ensure fast convergence (usually $\alpha = 0.5$)

3. Material property and program control cards (4I5, F10.0)

   Columns 1-5: Value of ITER, total number of iterations assigned

   Columns 6-10: Value of ITCONT; 1, for non-workhardening materials
   0, for workhardening materials

   Columns 11-15: Value of NPUNCH; 0, if only velocity distributions to be punched
   1, punch all informations; the output punched cards are arranged as follows:

   a. Nodal point velocity distributions (u_r, u_z) (8F10.6)

   b. Total effective strain $(\bar{\varepsilon})$ distribution (8F10.6)

   c. Strain rate $(\dot{\varepsilon}_r, \dot{\varepsilon}_z, \dot{\varepsilon}_\theta, \dot{\varepsilon}_{rz}, \dot{\bar{\varepsilon}})$ distributions (8F10.6)

   d. Stress $(\sigma_r, \sigma_z, \sigma_\theta, \sigma_{rz}, \bar{\sigma}, \sigma_m)$ distributions (8F10.6)

   e. Boundary nodal point forces and velocities (F_r, F_z, u_r, u_z) (8F10.6)

   Columns 15-20: Value of NPRINT; 1, if nodal point and element data are to be printed
   0, otherwise

   Columns 21-30: Value of FLIMIT, assigned value of accuracy desired $(\|\Delta u\|/\|u\|)$, program will stop if this value is achieved (recommend value is 0.00008)

4. Geometry and traction boundary control card (4I5)

Columns 1-5: Value of NUMNP, total number of nodal points

Columns 6-10: Value of NUMEL, total number of elements

Columns 11-15: Value of NUMPC, number of traction boundary condition cards to be read (see Sequence 6, below)

Columns 16-20: Value of NBF, total number of nodal points along the die surface at which force calculation is required (see Sequence 6, below)

5. Nodal point data cards (I5, F5.0, 2F10.0)

The numbering system of nodal points for the boundary conditions

Columns 1-5: Nodal point number

Columns 6-10: Code for this nodal point
0.0 if R- and z-forces are specified
1.0 if R-velocity and z-force are specified
2.0 if R-force and z-velocity are specified
3.0 if R- and z-velocity are specified
5.0 for special boundary condition (nodal points on die surface)

Columns 11-20: R-coordinate of the nodal point

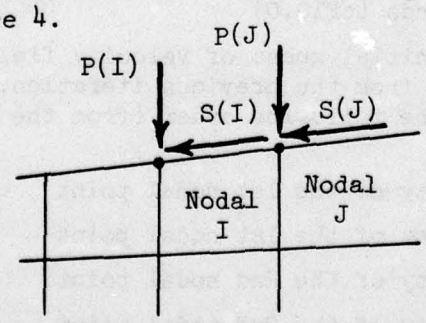
Columns 21-30: z-coordinate of the nodal point

6. Force calculation nodal points cards (16I5)

These cards provide the nodal point numbers at which the force calculations are desired. Each card provides 16 nodal points punched in every five columns. The total number of these nodal points should be the same as NBF specified in Sequence 4.

7. Traction boundary condition cards (2I5, 4F10.0)

These cards provide prescribed traction stresses along boundaries. If the prescribed stresses are all zero or no traction stresses are prescribed (e.g., for the case of extrusion (or drawing) with frictionless dies), these cards can be omitted and put NUMPC = 0 of Columns 11-15 of Sequence 4.



For the case with constant frictional stress (in terms of initial yield stress), these cards are needed. The total number of cards

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should be the same as the value of NUMPC. The shear stress and the normal stress are positive in the direction shown in the figure, which is opposite to the output positive stress and die force directions.

Columns 1-5: Nodal point number for point I

Columns 6-10: Nodal point number for point J

Columns 11-20: Prescribed normal stress in terms of initial yield stress at point I

Columns 21-30: Prescribed normal stress in terms of initial yield stress at point J (if normal stresses are not prescribed, put 0.0 into these two values)

Columns 31-40: Prescribed shear stress in terms of yield stress at point I

Columns 41-50: Prescribed shear stress in terms of yield stress at point J (e.g., for the case of constant friction stress with $f = 0.4Y_0$, put "-0.4" into these two values)

8. Flow line data cards (8F10.0)

Specify the R-coordinates of every flow line to be constructed before entering the die. The total number of these points should be the same as NTIMES specified in the program PURT. Each card is to be punched 8 entering points each in 10 columns.

9. Element data cards (5I5)

For each element these cards provide the four surrounding nodal point numbers (I,J,K,L) in a clockwise direction

Columns 1-5: Element number

Columns 6-10: Nodal point number of point I

Columns 11-15: Nodal point number of point J

Columns 16-20: Nodal point number of point K

Columns 21-25: Nodal point number of point L

10. Input velocity field cards (8F10.0)

These cards can be an initial guess of velocity field or a new velocity field obtained from the previous iteration. These input cards are arranged in the following order (from the nodal point to the last nodal point)

Columns 1-10: R-velocity of the 1st nodal point

Columns 11-20: z-velocity of the 1st nodal point

Columns 21-30: R-velocity of the 2nd nodal point

Columns 31-40: z-velocity of the 2nd nodal point

Each card provides four nodal point velocities

B. Sequence of input data cards for program EXDRSUM

The output stress distributions from the matrix iteration method are different from a uniform hydrostatic stress from the actual solution over all elements in extrusion or drawing.

This program calculates the average extrusion (or drawing) stress by either the energy balance method or the method of zero total axial force at the entrance boundary (for drawing) or at the exit boundary (for extrusion). After the converged solution is obtained from the main program, the output punched cards are then used in this program to calculate the correct stresses according to the following sequence of input data cards:

1. Problem control card (I5)
   Columns 1-5: Value of IEXDR; 1, for the extrusion problem
   0, for the drawing problem

2. Geometry control card (4I5, F10.0)
   Columns 1-5: Value of NUMNP, same as in the main program
   Columns 6-10: Value of NUMEL, same as in the main problem
   Columns 11-15: Value of NIPTS, same as in the main problem
   Columns 16-20: Value of NBF, same as in the main problem
   Columns 21-30: Value of THETA, semi-included cone die angle

3. Initial yield stress card (F10.0)
   Columns 1-10: Value of YIELD, initial yield stress of the material considered

4. Nodal point data cards (I5, F5.0, 2F10.0)
   Same as Sequence 5 of the main program

5. Element data cards (5I5
   Same as Sequence 9 of the main program

6. Input strain-rate distribution cards
   From output of the main program (Sequence 3-c)

7. Input stress distribution cards
   From output of the main program (Sequence 3-d)

8. Input boundary nodal point force and velocity cards
   From output of the main program (Sequence 3-e)

II. Computer program listing

```
EXDRSUM     2          PROGRAM EXDSM1(INPUT,OUTPUT,PUNCH)
EXDRSUM     3    C
EXDRSUM     4    C ***************************************************************************
EXDRSUM     5    C SINCE FOR EXTRUSION OR DRAWING, THE OUTPUT STRESS DISTRIBUTION FROM
EXDRSUM     6    C MATRIX ITERATION METHOD ARE IN DIFFERENCE OF A UNIFORM HYDROSTATIC
EXDRSUM     7    C PRESSURE FROM THE ACTURE SOLUTION OVER ALL ELEMENTS.   THIS PROGRAM
EXDRSUM     8    C CALCULATES THE AVERAGE EXTRUSION OR DRAWING PRESSURE AND THE CORREC-
EXDRSUM     9    C TION FACTOR OF THE MEAN STRESS.   THE OUTPUT STRESS AND DIE PRESSURE
EXDRSUM    10    C DISTRIBUTIONS OF THIS PROGRAM ARE THE ACTUAL SOLUTIONS.
EXDRSUM    11    C ***NOTE THAT ALL QUONTITIES INVOLVING VOLUMN OR SURFACE INTEGRAL ARE
EXDRSUM    12    C    MEASURED PER RADIAN(I.E. QUANTITIES DIVIDED BY (2*PAI))****
EXDRSUM    13    C ***NOTE ALSO ALL OUTPUT OF STRESSES AND DIE PRESSURE ARE IN TERMS
EXDRSUM    14    C    OF INITIAL YIELD STRESS  ***************
EXDRSUM    15    C ***************************************************************************
EXDRSUM    16    C
EXDRSUM    17          DIMENSION R(242),Z(242),UR(20),UZ(20),F(20),EFSTRN(210),IEL(210,4)
EXDRSUM    18         1  ,FZ(20),FR(20),STS(4,210),EFSTPS(210),ICAL(20),STSCR(4,210),
EXDRSUM    19         2 UDIE(20),NF(20),CODE(210)
EXDRSUM    20    C
EXDRSUM    21    C ***************************************************************************
EXDRSUM    22    C   READ  PROGRAM AND GEOMITRY CONTROL CARDS
EXDRSUM    23    C IEXDR=1...AXI-SYMMETRIC EXTRUSION PROBLEM
EXDRSUM    24    C       0...AXI-SYMMETRIC DRAWING PROBLEM
EXDRSUM    25    C NUMNP=TOTAL NODAL POINTS
EXDRSUM    26    C NUMEL=TOTAL ELEMENTS
EXDRSUM    27    C NIPTS=TOTAL LINES IN MESH SYSTEM PARALLEL TO AXIS
EXDRSUM    28    C NBF=TOTAL INPUT CARDS OF SURFACE NODAL POINT FORCE FROM OUTPUT OF MAIN
EXDRSUM    29    C     PROGRAM.
EXDRSUM    30    C THETA=SEMI-INCLUDED CONIC DIE ANGLE
EXDRSUM    31    C YIELD=INITIAL YIELD STRESS
EXDRSUM    32    C ***************************************************************************
EXDRSUM    33    C
EXDRSUM    34          READ 2300,IEXDR
EXDRSUM    35          READ 2311,NUMNP,NUMEL,NIPTS,NBF,THETA
EXDRSUM    36          READ 3312, YIELD
EXDRSUM    37          PAI=4.*ATAN(1.)
EXDRSUM    38          ANG=PAI*THETA/180.
EXDRSUM    39          RCOS=COS(ANG)
EXDRSUM    40          RSIN=SIN(ANG)
EXDRSUM    41    C
EXDRSUM    42    C ***************************************************************************
EXDRSUM    43    C READ NODAL POINT DATA AND ELEMENT DATA CARDS
EXDRSUM    44    C ***************************************************************************
EXDRSUM    45    C
EXDRSUM    46          DO 100 I=1,NUMNP
EXDRSUM    47          N=I
EXDRSUM    48      100 READ 1000,N,CODE(N),R(N),Z(N)
EXDRSUM    49          REXIT=R(NUMNP)
EXDRSUM    50          RENTER=R(NIPTS)
EXDRSUM    51          VEXIT=-(RENTER/REXIT)*(RENTER/REXIT)
EXDRSUM    52          N=0
EXDRSUM    53      130 READ 1001,M,(IEL(M,I),I=1,4)
EXDRSUM    54      140 N=N+1
EXDRSUM    55          IF(M-N) 170,170,150
EXDRSUM    56      150 DO 160 J=1,4
EXDRSUM    57      160 IEL(N,J)=IEL(N-1,J)+1
EXDRSUM    58      170 IF(M-N) 180,180,140
EXDRSUM    59      180 IF(NUMEL-N) 190,190,130
EXDRSUM    60      190 CONTINUE
EXDRSUM    61    C
EXDRSUM    62    C ***************************************************************************
EXDRSUM    63    C READ INPUT DATA CARDS
EXDRSUM    64    C ALL INPUT DATAS BELOW ARE FROM THE OUTPUTS OF THE MAIN PROGRAM
EXDRSUM    65    C EFSTRN=EFFECTIVE STRAIN RATE
EXDRSUM    66    C STS(1,N)=R-STRESS OF N-TH ELEMENT
EXDRSUM    67    C STS(2,N)=Z-STRESS OF N-TH ELEMENT
EXDRSUM    68    C STS(3,N)=TH-STRESS OF N-TH ELEMENT
EXDRSUM    69    C STS(4,N)=RZ-STRESS OF N-TH ELEMENT
EXDRSUM    70    C EFSTRS=EFFECTIVE STRESS
EXDRSUM    71    C AX, AY ,AZ,AM STAND FOR INPUT DATAS WHICH ARE NOT USED IN DOING THIS
EXDRSUM    72    C                 PROGRAM.
EXDRSUM    73    C UR,UZ,FR,FZ ARE THE VELOCITY AND FORCES AT NODAL POINT IN R AND Z-DIR.
EXDRSUM    74    C ***************************************************************************
EXDRSUM    75    C
EXDRSUM    76          READ 1003,((AX,AY,AZ,AM,EFSTRN(N)),N=1,NUMEL)
EXDRSUM    77          READ 1003,(((STS(I,N),I=1,4),EFSTPS(N),AX),N=1,NUMEL)
EXDRSUM    78          DO 220 I=1,NBF
EXDRSUM    79      220 READ 1002,NF(I),UR(I),UZ(I),FR(I),FZ(I)
EXDRSUM    80          IF(IEXDR.EQ.1) PRINT 3303
EXDRSUM    81          IF(IEXDR.EQ.0) PRINT 3304
EXDRSUM    82          PRINT 2201
```

```
EXDRSUM    83          PRINT 2202,(K,CODE(K),R(K),Z(K),K=1,NUMNP)
EXDRSUM    84          PRINT 2204
EXDRSUM    85          DO 250 I=1,NBF
EXDRSUM    86          FR(I)=FR(I)/YIELD
EXDRSUM    87          FZ(I)=FZ(I)/YIELD
EXDRSUM    88          J=NF(I)
EXDRSUM    89      250 PRINT 2203,NF(I),CODE(J),UR(I),UZ(I),FR(I),FZ(I)
EXDRSUM    90      C
EXDRSUM    91      C ***************************************************************
EXDRSUM    92      C FRTOTL=TOTAL DIE FORCE IN Z-DIRECTION BEFORE CORRECTION
EXDRSUM    93      C ***************************************************************
EXDRSUM    94      C
EXDRSUM    95          FRTOTL=0.
EXDRSUM    96          DO 300 I=1,NBF
EXDRSUM    97          J=NF(I)
EXDRSUM    98          IF(CODE(J).EQ.5.0.OR.CODE(J).EQ.15.0) GO TO 301
EXDRSUM    99          GO TO 300
EXDRSUM   100      301 FRTOTL=FRTOTL+FZ(I)
EXDRSUM   101      300 CONTINUE
EXDRSUM   102      C
EXDRSUM   103      C ***************************************************************
EXDRSUM   104      C METHOD=1...IF ENERGY BALANCE METHOD IS USED
EXDRSUM   105      C        2...IF ZERO TOTAL Z-FORCE AT RIGID BOUNDARY METHOD IS USED
EXDRSUM   106      C ***************************************************************
EXDRSUM   107      C
EXDRSUM   108          DO 900 METHOD=1,2
EXDRSUM   109          IF(METHOD.EQ.1) GO TO 901
EXDRSUM   110          IF(METHOD.EQ.2) GO TO 902
EXDRSUM   111      C
EXDRSUM   112      C ***************************************************************
EXDRSUM   113      C ***ENERGY BALANCE METHOD***********
EXDRSUM   114      C CALCULATE DISSIPATION ENERGY RATE OVER DIE SURFACE DUE TO FRICTION
EXDRSUM   115      C ***************************************************************
EXDRSUM   116      C
EXDRSUM   117      901 CONTINUE
EXDRSUM   118          FRICTN=0.
EXDRSUM   119          DO 103 I=1,NBF
EXDRSUM   120          J=NF(I)
EXDRSUM   121          IF(CODE(J).EQ.5.0.OR.CODE(J).EQ.15.0) GO TO 102
EXDRSUM   122          GO TO 103
EXDRSUM   123      102 FRICTN=FRICTN+(FR(I)*RSIN+FZ(I)*RCOS)*(UR(I)*RSIN+UZ(I)*RCOS)
EXDRSUM   124      103 CONTINUE
EXDRSUM   125      C
EXDRSUM   126      C ***************************************************************
EXDRSUM   127      C CALCULATE TOTAL STRAIN ENERGY RATE OVER TEH DEFORMED PORTION
EXDRSUM   128      C ***************************************************************
EXDRSUM   129      C
EXDRSUM   130          DSENGY=0.
EXDRSUM   131          DO 210 N=1,NUMEL
EXDRSUM   132          EFSS=EFSTRN(N)
EXDRSUM   133          IF(EFSTRN(N).LE.0.01) EFSS=0.0
EXDRSUM   134          I1=IEL(N,1)
EXDRSUM   135          I2=IEL(N,2)
EXDRSUM   136          I3=IEL(N,3)
EXDRSUM   137          I4=IEL(N,4)
EXDRSUM   138          VOL1=(Z(I2)-Z(I1))*(R(I1)*R(I1)+R(I2)*R(I2)+R(I1)*R(I2))
EXDRSUM   139          VOL2=(Z(I4)-Z(I3))*(R(I4)*R(I4)+R(I3)*R(I3)+R(I4)*R(I3))
EXDRSUM   140          VOL3=(Z(I3)-Z(I2))*(R(I3)*R(I3)+R(I2)*R(I2)+R(I3)*R(I2))
EXDRSUM   141          VOL4=(Z(I1)-Z(I4))*(R(I4)*R(I4)+R(I1)*R(I1)+R(I1)*R(I4))
EXDRSUM   142          VOLUMN=(VOL1+VOL2+VOL3+VOL4)/6.
EXDRSUM   143          DSENGY=DSENGY+EFSTRS(N)*EFSS*VOLUMN
EXDRSUM   144      210 CONTINUE
EXDRSUM   145      C
EXDRSUM   146      C ***************************************************************
EXDRSUM   147      C CALCULATE AVERAGE EXTRUSION PRESSURE OR AVERAGE DRAWING PRESSURE (EXT-
EXDRSUM   148      C   PSS) AND CORRECTION HYDROSTATIC PRESSURE(ALANDA) BY ENERGY BLANCE-
EXDRSUM   149      C   METHOD.
EXDRSUM   150      C ***************************************************************
EXDRSUM   151      C
EXDRSUM   152          PRINT 3300
EXDRSUM   153          IF(IEXDR.EQ.1) GO TO 502
EXDRSUM   154      501 EXTPSS=(DSENGY-FRICTN)*2./(-VEXIT*REXIT*REXIT)
EXDRSUM   155          ALANDA=(2.*FRTOTL-EXTPSS*REXIT*REXIT)/(RENTER*RENTER-REXIT*REXIT)
EXDRSUM   156          GO TO 503
EXDRSUM   157      502 EXTPSS=(DSENGY-FRICTN)*2./(RENTER*RENTER)
EXDRSUM   158          ALANDA=(2.*FRTOTL-EXTPSS)/(RENTER*RENTER-REXIT*REXIT)
EXDRSUM   159          GO TO 503
EXDRSUM   160      C
EXDRSUM   161      902 CONTINUE
EXDRSUM   162      C
EXDRSUM   163      C ***************************************************************
EXDRSUM   164      C ******METHOD OF ZERO TOTAL Z-FORCE OVER THE RIGID BOUNDARY**********
EXDRSUM   165      C ***************************************************************
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EXDRSUM  166   C
EXDRSUM  167         NI=NIPTS-1
EXDRSUM  168         IF(IEXDR.EQ.1) GO TO 705
EXDRSUM  169   C
EXDRSUM  170   C ***************************************************************
EXDRSUM  171   C CHECK ELEMENTS NEXT TO RIGID BOUNDARY AT ENTRANCE FOR DRAWING
EXDRSUM  172   C ***************************************************************
EXDRSUM  173   C
EXDRSUM  174         DO 708 I=1,NI
EXDRSUM  175         J=I
EXDRSUM  176     706 IF(EPSTRN(J).LE.0.01) GO TO 707
EXDRSUM  177         ICAL(I)=J
EXDRSUM  178         GO TO 709
EXDRSUM  179     707 J=J+NI
EXDRSUM  180         GO TO 706
EXDRSUM  181     708 CONTINUE
EXDRSUM  182         GO TO 800
EXDRSUM  183   C
EXDRSUM  184   C ***************************************************************
EXDRSUM  185   C CHECK ELEMENTS NEXT TO RIGID BOUNDARY AT EXIT FOR EXTRUSION
EXDRSUM  186   C ***************************************************************
EXDRSUM  187   C
EXDRSUM  188     705 DO 719 I=1,NI
EXDRSUM  189         J=NUMEL-NI+I
EXDRSUM  190     716 IF(EPSTRN(J).LE.0.01) GO TO 717
EXDRSUM  191         ICAL(I)=J
EXDRSUM  192         GO TO 718
EXDRSUM  193     717 J=J-NI
EXDRSUM  194         GO TO 716
EXDRSUM  195     718 CONTINUE
EXDRSUM  196     800 CONTINUE
EXDRSUM  197   C
EXDRSUM  198   C ***************************************************************
EXDRSUM  199   C CALCULATE TOTAL Z-FORCE(BEFORE CORRECTION) FOR ELEMENTS NEXT TO
EXDRSUM  200   C   RIGID BOUNDARY
EXDRSUM  201   C ***************************************************************
EXDRSUM  202   C
EXDRSUM  203         TFORCE=0.
EXDRSUM  204         I=1
EXDRSUM  205         INDEX=1
EXDRSUM  206     801 KK=ICAL(I)
EXDRSUM  207         I1=IEL(KK,1)
EXDRSUM  208         I2=IEL(KK,2)
EXDRSUM  209         I3=IEL(KK,3)
EXDRSUM  210         I4=IEL(KK,4)
EXDRSUM  211         Z1=0.
EXDRSUM  212         Z2=(Z(I1)-Z(I2))/2.
EXDRSUM  213         R2=(R(I3)+R(I4))/2.
EXDRSUM  214         IF(INDEX.EQ.1) GO TO 808
EXDRSUM  215         AREA1=(R2*R2-R1*R1)/2.
EXDRSUM  216         AREA2=R1*Z2
EXDRSUM  217         GO TO 809
EXDRSUM  218     808 R1=(R(I1)+R(I2))/2.
EXDRSUM  219         AREA1=(R2*R2-R1*R1)/2.
EXDRSUM  220         AREA2=0.
EXDRSUM  221     809 TFORCE=TFORCE+AREA1*STS(2,KK)+AREA2*STS(4,KK)
EXDRSUM  222         IF(I.EQ.NI) GO TO 810
EXDRSUM  223         R1=R2
EXDRSUM  224         KABOVE=KK+1
EXDRSUM  225         IF(KABOVE-ICAL(I+1)) 753,750,753
EXDRSUM  226     750 I=I+1
EXDRSUM  227         INDEX=1
EXDRSUM  228         GO TO 801
EXDRSUM  229     753 IF(KABOVE.LT.ICAL(I+1)) KNEXT=KABOVE+NI
EXDRSUM  230         IF(KABOVE.GT.ICAL(I+1)) KNEXT=KABOVE-NI
EXDRSUM  231         I1=IEL(KNEXT,1)
EXDRSUM  232         I2=IEL(KNEXT,2)
EXDRSUM  233         I3=IEL(KNEXT,3)
EXDRSUM  234         I4=IEL(KNEXT,4)
EXDRSUM  235         R2=(R(1)+R(2)+R(3)+R(4))/4.
EXDRSUM  236         AREA1=(R2*R2-R1*R1)/2.
EXDRSUM  237         AREA3=R1*Z1
EXDRSUM  238         AREA2=R2*Z2
EXDRSUM  239         TFORCE=TFORCE+STS(2,KABOVE)*(AREA1+AREA3)+STS(4,KABOVE)*AREA2
EXDRSUM  240         R1=R2
EXDRSUM  241         Z2=(Z(I1)-Z(I2))/2.
EXDRSUM  242         Z1=Z2
EXDRSUM  243         IF(KNEXT .EQ. ICAL(I+1)) GO TO 803
EXDRSUM  244         KABOVE=KNEXT
EXDRSUM  245         GO TO 753
EXDRSUM  246     803 I=I+1
EXDRSUM  247         INDEX=2
EXDRSUM  248         GO TO 801
EXDRSUM  249   C
EXDRSUM  250   C ***************************************************************
EXDRSUM  251   C CALCULATE CORRECTION HYDROSTATIC PRESSURE BY REQUIRING TOTAL Z-FORCE
EXDRSUM  252   C EQUAL TO ZERO AFTER CORRECTION (ALANDA).  CALCULATE AVERAGE EXTRUSION
EXDRSUM  253   C PRESSURE OR AVERAGE DRAWING PRESSURE (EXTPRS) BY FORCE BALANCING.
EXDRSUM  254   C ***************************************************************
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EXDRSUM   255    C
EXDRSUM   256      810 PRINT 3301
EXDRSUM   257          PRINT 2307,(ICAL(I),I=1,NI)
EXDRSUM   258          IF(IEXDR.EQ.1) GO TO 811
EXDRSUM   259          ALANDA=-(TFORCE/(RENTER*RENTER))*2.
EXDRSUM   260          EXTPS1=-ALANDA*(RENTER*RENTER-REXIT*REXIT)+2.*FRTOTL
EXDRSUM   261          EXTPSS=EXTPS1/(REXIT*REXIT)
EXDRSUM   262          GO TO 503
EXDRSUM   263      811 ALANDA=-(TFORCE/(REXIT*REXIT))*2.
EXDRSUM   264          EXTPSS=-ALANDA*(RENTER*RENTER-REXIT*REXIT)+2.*FRTOTL
EXDRSUM   265      503 CONTINUE
EXDRSUM   266          PRINT 2301,EXTPSS
EXDRSUM   267          PRINT 2302,ALANDA
EXDRSUM   268    C
EXDRSUM   269    C ***************************************************************
EXDRSUM   270    C CALCULATE THE ACTURE DIE PRESSURE
EXDRSUM   271    C ***************************************************************
EXDRSUM   272    C
EXDRSUM   273          PRINT 3310
EXDRSUM   274          DO 400 I=1,NBF
EXDRSUM   275          J=NF(I)
EXDRSUM   276          IF(CODE(J).EQ.5.0.OR.CODE(J).EQ.15.0) GO TO 409
EXDRSUM   277          GO TO 400
EXDRSUM   278      409 J2=J+NIPTS
EXDRSUM   279          J1=J-NIPTS
EXDRSUM   280          IF(J1.LT.NIPTS) J1=J
EXDRSUM   281          IF(J2.GT.NUMNP) J2=J
EXDRSUM   282          R1=(R(J1)+R(J2))/2.
EXDRSUM   283          R2=(R(J1)+R(J))/2.
EXDRSUM   284          H=(Z(J1)-Z(J2))/2.
EXDRSUM   285          AREA=(R1+R2)*SQRT((R2-R1)*(R2-R1)+H*H)/2.
EXDRSUM   286          P=((FR(I)*BCOS-FZ(I)*BSIN)/AREA)+ALANDA
EXDRSUM   287          PRINT 2303,J,CODE(J),R(J),Z(J),AREA,P
EXDRSUM   288      400 CONTINUE
EXDRSUM   289    C
EXDRSUM   290    C ***************************************************************
EXDRSUM   291    C CALCULATE THE ACTURAL STRESS DISTRIBUTION
EXDRSUM   292    C ***************************************************************
EXDRSUM   293    C
EXDRSUM   294          IF(METHOD.EQ.1) PRINT 3300
EXDRSUM   295          IF(METHOD.EQ.2) PRINT 3301
EXDRSUM   296          PRINT 2304
EXDRSUM   297          DO 405 N=1,NUMEL
EXDRSUM   298          STSCR(4,N)=STS(4,N)
EXDRSUM   299          DO 406 I=1,3
EXDRSUM   300      406 STSCR(I,N)=STS(I,N)+ALANDA
EXDRSUM   301          AVES=(STSCR(1,N)+STSCR(2,N)+STSCR(3,N))/3.
EXDRSUM   302      405 PRINT 2305,N,(STSCR(I,N),I=1,4),AVES,EFSTRS(N),EFSTRN(N)
EXDRSUM   303      900 CONTINUE
EXDRSUM   304    C
EXDRSUM   305     1000 FORMAT(I5,F5.0,5F10.0)
EXDRSUM   306     1001 FORMAT(16I5)
EXDRSUM   307     1002 FORMAT(I9,4F17.0)
EXDRSUM   308     1003 FORMAT(8F10.0)
EXDRSUM   309     2201 FORMAT(/,5X,*NODAL*,5X,*CODE*,5X,*R-COORD...*5X,*Z-COORD...*)
EXDRSUM   310     2202 FORMAT(5X,I5,5X,F5.2,5X,F10.6,5X,F10.6)
EXDRSUM   311     2203 FORMAT(2X,I5,5(2X,F10.6))
EXDRSUM   312     2204 FORMAT(///,5X,*INPUT DATA*/,2X,*NODAL* ,6X,*CODE*,8X,*R-VELOCITY*
EXDRSUM   313         1,2X,*Z-VELOCITY*,2X,*R-FORCE...*,2X,*Z-FORCE*)
EXDRSUM   314     2300 FORMAT(I5)
EXDRSUM   315     2301 FORMAT(//,5X,*AVERAGE EXTRUSION( OR DRAWING) PRESSURE=*,F12.6)
EXDRSUM   316     2302 FORMAT(//,5X,*THE CORRECTION HYDROSTATIC PRESSURE=*,F12.6)
EXDRSUM   317     2303 FORMAT(2X,*NODAL POINT=*,I4,2X,*CODE=*F5.2,2X,*R-COORD.=*F10.6,
EXDRSUM   318        1 2X,*Z-COORD.=*F10.6,2X,*APEA=*,F10.6,2X,*DIE PRESSURE=*,F15.8)
EXDRSUM   319     2304 FORMAT(/,10X,*ACTURAL STRESS DISTRIBUTION*//
EXDRSUM   320         1,5X,*ELE.NO.*,5X,*R-STRESS*,5X,*Z-STRESS*,5X,*TH-STRESS*,
EXDRSUM   321         2 5X,*RZ-STRESS*,*..MEAN STRESS..EF-STRESS*,3X,*EFF-STRAIN RATE*)
EXDRSUM   322     2305 FORMAT(7X,I6,7F13.6)
EXDRSUM   323     2307 FORMAT(//,5X,*ELEMENTS NEXT TO RIGID BOUNDARY=*,/,6X,16I5)
EXDRSUM   324     2311 FORMAT(4I5,F10.0)
EXDRSUM   325     3300 FORMAT(1H1,5X,*ENERGY BALANCE METHOD*)
EXDRSUM   326     3301 FORMAT(1H1,5X,*TOTAL ZERO Z-FORCE AT  RIGID BOUNDARY METHOD*)
EXDRSUM   327     3303 FORMAT(1H1,*....EXTRUSION PROBLEM....*)
EXDRSUM   328     3304 FORMAT(1H1,*....DRAWING PROBLEM....*)
EXDRSUM   329     3310 FORMAT(///,5X,*THE ACTUAL DIE PRESSURE DISTRIBUTION*)
EXDRSUM   330     3312 FORMAT(F10.0)
EXDRSUM   331          STOP
EXDRSUM   332          END
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EXTRUDE    2          PROGRAM PURT(TAPE1,INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,PUNCH)
EXTRUDE    3    C
EXTRUDE    4    C ***************************************************************************
EXTRUDE    5    C PROGRAM FOR STEADY STATE EXTRUSION OR DRAWING BY MATRIX METHOD.
EXTRUDE    6    C NOTE THAT THE FOLLOWUNG PROGRAM IS WRITING ACCORDING TO THE RIGHT
EXTRUDE    7    C ORDERING OF NODAL POINTS AND ELEMENT NUMBERS (SEE EXPLANATION)
EXTRUDE    8    C ***************************************************************************
EXTRUDE    9    C
EXTRUDE   10          COMMON/GENCON/NUMNP,NUMEL,HED(12),VOL,NEQ,NS,ITERNO,ISTOP,
EXTRUDE   11         1YIELD,MBAND,TEST,MDIAG,NBF,NUMPC,ITER,NBF2,NSCALE,NPUNCH,NPRINT,
EXTRUDE   12         2  NCHECK,ACFINI
EXTRUDE   13          COMMON /WALL/ THETA,FT,TANTH
EXTRUDE   14          COMMON /STRPATH/ STEP,YSTART,YDIE,YEXIT,YMIN,RENTER,REXIT,VEXIT
EXTRUDE   15          COMMON /DIMEN/ M1,M2,M3,M4,M5,M6,M7,M8,M9,M10,M11,M12,M13,M14
EXTRUDE   16    C
EXTRUDE   17    C ***************************************************************************
EXTRUDE   18    C  PROGRAM PURT IS FOR CONTROLLING THE DIMENSION OF THE COMPLETE PROGRAM
EXTRUDE   19    C  ITS PURPOSE IS TO PREVENT ASSIGNING A LARGER THAN NECESSARY DIMENSION
EXTRUDE   20    C  FOR ANY ARRAY THROUGH THE USE OF FOLLOWING STATEMENT...
EXTRUDE   21    C ***************************************************************************
EXTRUDE   22    C
EXTRUDE   23          COMMON A(24650)
EXTRUDE   24    C
EXTRUDE   25    C ***************************************************************************
EXTRUDE   26    C  NFIELD IS THE DIMENSION OF ARRAY A. ITS VALUE CAN BE DETERMINED
EXTRUDE   27    C  PRECISELY BY RUNNING THE PROGRAM ONCE.
EXTRUDE   28    C ***************************************************************************
EXTRUDE   29    C
EXTRUDE   30          NFIELD=24650
EXTRUDE   31          PAI=4.*ATAN(1.0)
EXTRUDE   32    C
EXTRUDE   33    C ***************************************************************************
EXTRUDE   34    C  THETA              SEMI-INCLUDED CONIC DIE ANGLE.
EXTRUDE   35    C  YSTART             Z-COORDINATE OF STARTING POINT TO CALCULATE TOTAL
EXTRUDE   36    C                     EFFECTIVE STRAIN AND IS SETTING SLIGHTLY LESS THAN
EXTRUDE   37    C                     THE CENTER OF FIRST ELEMENT.
EXTRUDE   38    C  STEP               THE INCREMENT SIZE FOR STRAIN-RATE INTEGRATION
EXTRUDE   39    C  YDIE               Z-COORDINATE AT ENTRANCE TO THE DIE
EXTRUDE   40    C  YEXIT              Z-COORDINATE AT EXIT FROM DIE
EXTRUDE   41    C  YMIN               MINIMUM Z-COORDINATE OF THE CONTROL VOLUME
EXTRUDE   42    C  RENTER             RADIUS OF BILLET BEFORE EXTRUSION
EXTRUDE   43    C  REXIT              RADIUS OF BILLET AFTER EXTRUSION
EXTRUDE   44    C  VEXIT              EXIT VELOCITY OF THE PRODUCT
EXTRUDE   45    C  NTIMES             NUMBER OF FLOW LINES TO BE CONSTRUCTED
EXTRUDE   46    C  NIPTS              NUMBER OF LINES IN MESH SYSTEM PARALLEL TO AXIS
EXTRUDE   47    C  NJPTS              NUMBER OF LINES IN MESH SYSTEM PERPENDICULAR TO AXIS
EXTRUDE   48    C  NMAX               TOTAL NUMBER OF INCREMENTS FOR INTEGRATION
EXTRUDE   49    C ***************************************************************************
EXTRUDE   50    C
EXTRUDE   51          THETA=6.
EXTRUDE   52          THETA=THETA*PAI/180.
EXTRUDE   53          TANTH=TAN(THETA)
EXTRUDE   54          YEXIT=0.
EXTRUDE   55          RENTER=1.0
EXTRUDE   56          NTIMES=11
EXTRUDE   57          NIPTS=7
EXTRUDE   58          NJPTS=15
EXTRUDE   59          NMAX=350
EXTRUDE   60    C
EXTRUDE   61    C ***************************************************************************
EXTRUDE   62    C       READ THE INPUT DATA CONTROL CARDS
EXTRUDE   63    C HED=OUTPUT TITLE UP TO 72 CHARACTERS.
EXTRUDE   64    C ACFINI=INITIAL DECCELARATION COEFFICIENTS, FOR FIRST ITERATION ONLY.
EXTRUDE   65    C ITER=TOTAL NUMBER OF ITERATIONS ASSIGNED.
EXTRUDE   66    C ITCONT=1, FOR NONWORKHARDENING MATERIALS
EXTRUDE   67    C          0, FOR WORK-HARDENING MATERIALS.
EXTRUDE   68    C NPUNCH=0, IF NOTHING OTHER THAN VELOCITIES TO BE PUNCHED.
EXTRUDE   69    C          1, OTHERWISE.
EXTRUDE   70    C NPRINT=1, IF THE NODAL POINT DATAS TO BE PRINTED.
EXTRUDE   71    C          0, OTHERWISE.
EXTRUDE   72    C FLIMIT=VALUE OF (ERROR NORM)/(SOLUTION NORM) REQUIRED FOR FINAL RESULT
EXTRUDE   73    C NUMNP=NUMBER OF NODAL POINTS(TOTAL).
EXTRUDE   74    C NUMEL=NUMBER OF ELEMENTS(TOTAL)
EXTRUDE   75    C NUMPC=NUMBER OF TRACTION BOUNDARY CONDITIONS CARDS TO BE READ.
EXTRUDE   76    C NBF=NUMBER OF NODAL POINTS AT WHICH FORCE CALCULATIONS ARE REQUIRED.
EXTRUDE   77    C ***************************************************************************
EXTRUDE   78    C
EXTRUDE   79          READ(5,1000) HED
EXTRUDE   80          READ(5,1004)ACFINI
EXTRUDE   81          READ(5,1003) ITER,ITCONT,NPUNCH,NPRINT,FLIMIT
EXTRUDE   82          READ(5,1005) NUMNP,NUMEL,NUMPC,NBF
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EXTRUDE   83          NIPTS=NIPTS+1
EXTRUDE   84          NBF2=NBF*2
EXTRUDE   85          NB=NBF
EXTRUDE   86          NB2=NBF2
EXTRUDE   87          NEL=NUMEL
EXTRUDE   88          NPC=NUMPC
EXTRUDE   89          NEQ=3*NUMNP
EXTRUDE   90          NQ=NEQ
EXTRUDE   91          NI=NIPTS
EXTRUDE   92          NJ=NJPTS
EXTRUDE   93    C
EXTRUDE   94    C *****************************************************************
EXTRUDE   95    C  DETERMINE THE LOCATION OF THE STARTING POINTS OF DIFFERENT ARRAYS ON
EXTRUDE   96    C  ARRAY A
EXTRUDE   97    C *****************************************************************
EXTRUDE   98    C
EXTRUDE   99          N1=1
EXTRUDE  100          N2=N1+NUMNP
EXTRUDE  101          N3=N2+NUMNP
EXTRUDE  102          N4=N3+NUMNP
EXTRUDE  103          N5=N4+NUMNP
EXTRUDE  104          N6=N5+NUMNP
EXTRUDE  105          N7=N6+4*NUMEL
EXTRUDE  106          N8=N7+NUMEL
EXTRUDE  107          N9=N8+6*NUMEL
EXTRUDE  108          N10=N9+5*NUMEL
EXTRUDE  109          N11=N10+5*NUMEL
EXTRUDE  110          N12=N11+NBF
EXTRUDE  111          N13=N12+NBF2
EXTRUDE  112          N14=N13+2*NUMPC
EXTRUDE  113          N15=N14+4*NUMPC
EXTRUDE  114          N16=N15+NBF
EXTRUDE  115          N17=N16+NBF
EXTRUDE  116          N18=N17+NBF
EXTRUDE  117          N19=N18+NTIMES
EXTRUDE  118    C
EXTRUDE  119    C *****************************************************************
EXTRUDE  120    C  READ THE INPUT DATA
EXTRUDE  121    C *****************************************************************
EXTRUDE  122    C
EXTRUDE  123          CALL PRELIM (A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N11),A(N13),
EXTRUDE  124         1  A(N14),A(N18),NEL,NPC,NTIMES,NIPTS,YSTART,YMIN,REXIT)
EXTRUDE  125          YDIF=(RENTER-REXIT)/TANTH
EXTRUDE  126          VEXIT=-(RENTER/REXIT)**2
EXTRUDE  127          STEP=(YSTART-YMIN)/(NMAX*1.15)
EXTRUDE  128          WRITE(6,1007) ACFINI,ITCONT,FLIMIT
EXTRUDE  129          WRITE(6,1008) STEP,YSTART,YDIF,VEXIT,YMIN,RENTER,REXIT,VEXIT,
EXTRUDE  130         1  NTIMES,NIPTS,NJPTS,NMAX
EXTRUDE  131    C
EXTRUDE  132          N20=N19+NEQ
EXTRUDE  133    C
EXTRUDE  134    C
EXTRUDE  135    C *****************************************************************
EXTRUDE  136    C  SINCE AFTER THE SOLUTION OF STIFFNESS EQUATIONS, THE STIFFNESS MATRIX
EXTRUDE  137    C  IS NOT NEDEED FOR THAT ITERATION, THE SPACE PROVIDED FOR STIFFNESS
EXTRUDE  138    C  MATRIX WILL BE USED FOR CONSTRUCTION OF FLOW LINES AND FOR STRAIN-
EXTRUDE  139    C  RATE INTEGRATION. THE NUMBERS M1 TO M14 ARE FOR DETERMINING THE
EXTRUDE  140    C  LOCATION OF PROPER ARRAY
EXTRUDE  141    C *****************************************************************
EXTRUDE  142    C
EXTRUDE  143          M1=1
EXTRUDE  144          M2=M1+NI*NJ
EXTRUDE  145          M3=M2+NI*NJ
EXTRUDE  146          M4=M3+NJ
EXTRUDE  147          M5=M4+NI
EXTRUDE  148          M6=M5+NI*NJ
EXTRUDE  149          M7=M6+NI*NJ
EXTRUDE  150          M8=M7+NTIMES*NMAX
EXTRUDE  151          M9=M8+NTIMES*NMAX
EXTRUDE  152          M10=M9+NUMEL
EXTRUDE  153          M11=M10+NUMEL
EXTRUDE  154          M12=M11+NI*NJ
EXTRUDE  155          M13=M12+NJ
EXTRUDE  156          M14=M13+NTIMES*NMAX
EXTRUDE  157    C
EXTRUDE  158    C *****************************************************************
EXTRUDE  159    C HOWEVER, IF THE SPACE OF STIFFNESS MATRIX IS NOT ENOUGH FOR STRAIN RATE
EXTRUDE  160    C INTEGRATION, CREATES MORE SPACE AS REQUIRED.
EXTRUDE  161    C *****************************************************************
EXTRUDE  162    C
EXTRUDE  163          NNN=MBAND*NEQ
EXTRUDE  164          N21=N20+NNN
EXTRUDE  165          IF(M14.GE.NNN) N21=N20+M14
EXTRUDE  166          N22=N21+NBF2*(2*MBAND-1)
EXTRUDE  167          N23=N22+NEQ
EXTRUDE  168          N24=N23+NEQ
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EXTRUDE  169    C
EXTRUDE  170    C ***********************************************************************
EXTRUDE  171    C N24 IS TOTAL SPACE REQUIRED, IF NFIELD , THE INITIAL ASSIGNED SPACE,
EXTRUDE  172    C IS NOT ENOUGH, STOP THE PROGRAM.
EXTRUDE  173    C ***********************************************************************
EXTRUDE  174    C
EXTRUDE  175          IF(N24.LE.NFIELD) GO TO 100
EXTRUDE  176          WRITE(6,1001) N24
EXTRUDE  177          STOP
EXTRUDE  178      100 CONTINUE
EXTRUDE  179          WRITE(6,1002) N24
EXTRUDE  180    C
EXTRUDE  181    C ***********************************************************************
EXTRUDE  182    C THE FOLLOWING SUBROUTINE PLAST DOING THE MAIN PART OF MATRIX ITERATION
EXTRUDE  183    C METHOD OF PRESENT PROBLEM.
EXTRUDE  184    C ***********************************************************************
EXTRUDE  185    C
EXTRUDE  186          CALL PLAST(A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9),
EXTRUDE  187         1A(N10),A(N11),A(N12),A(N13),A(N14),A(N15),A(N16),A(N17),A(N18),
EXTRUDE  188         2A(N19),A(N20),A(N21),A(N22),A(N23),NO,NEL,NPC,NB2,
EXTRUDE  189         3FLIMIT,ITCONT,NTIMES,NIPTS,NJPTS,NMAX)
EXTRUDE  190    C
EXTRUDE  191     1000 FORMAT(12A6)
EXTRUDE  192     1001 FORMAT(///* THE DIMENSION OF THE ARRAY (A) IS TOO SMALL*/
EXTRUDE  193         1* THE SIZE OF THE ARRAY (A) MUST BE *, I7)
EXTRUDE  194     1002 FORMAT(//* THE NECCESSARY SIZE OF THE ARRAY (A) IS*, I7)
EXTRUDE  195     1003 FORMAT(4I5,F10.0)
EXTRUDE  196     1004 FORMAT(8F10.0)
EXTRUDE  197     1005 FORMAT(4I5)
EXTRUDE  198     1007 FORMAT(//* ACOEF = *,F8.5,10X,* ITCONT = *,I2,
EXTRUDE  199         1  10X,* FLIMIT=*,F10.6,//)
EXTRUDE  200     1008 FORMAT(//* STEP SIZE = *,F8.4,10X,* YSTART = *,F8.4,10X,* YDIE = *
EXTRUDE  201         1  F8.4,10X,* YEXIT = *,F8.4,//* YMIN = *,F8.4,10X,* RENTER = *,
EXTRUDE  202         2  F8.4,10X,* REXIT = *,F8.4,10X,* VEXIT = *,F8.4,//* NTIMES = *,
EXTRUDE  203         3  I4,10X,* NIPTS = *,I3,10X,* NJPTS = *,I3,10X,* NMAX = *,I4,/)
EXTRUDE  204     1009 FORMAT(/ * STORAGE SPACE AVAILABLE FOR THE STRAIN CALCULATIONS IS
EXTRUDE  205         1NOT ENOUGH*/)
EXTRUDE  206    C
EXTRUDE  207          STOP
EXTRUDE  208          END
```

```
EXTRUDE  210          SUBROUTINE PRELIN (R,Z,UR,UZ,CODE,IEL,NF,IJBC,PSBC,RR,NEL,NPC,
EXTRUDE  211         1  NTIMES,NIPTS,YSTART,YMIN,REXIT)
EXTRUDE  212          COMMON/GENCON/NUMNP,NUMEL,HED(12),VOL,NEQ,NS,ITERNO,ISTOP,
EXTRUDE  213         1YIELD,MBAND,TEST,MDIAG,NBF,NJMPC,ITER,NBF2,NSCALE,NPUNCH,NPRINT,
EXTRUDE  214         2  NCHECK,ACFINI
EXTRUDE  215          DIMENSION R(1),Z(1),CODE(1),UR(1),UZ(1),IEL( NEL ,1),NF(1),
EXTRUDE  216         1  IJBC(NPC,1),PSBC(NPC,1),RR(1)
EXTRUDE  217    C
EXTRUDE  218    C ***********************************************************************
EXTRUDE  219    C THIS SUBROUTINE READS AND PRINTS ALL CONTROL INFORMATIONS AND
EXTRUDE  220    C NODAL POINT DATAS AND BOUNDARY CONDITIONS.
EXTRUDE  221    C ***********************************************************************
EXTRUDE  222    C
EXTRUDE  223          WRITE(6,1000)
EXTRUDE  224          WRITE (6,2000) HED,NUMNP,NUMEL
EXTRUDE  225    C
EXTRUDE  226    C ***********************************************************************
EXTRUDE  227    C CALCULATE INITINAL YIELD STRESS.
EXTRUDE  228    C ***********************************************************************
EXTRUDE  229    C
EXTRUDE  230          CALL HARD(0.,YIELD)
EXTRUDE  231          WRITE(6,2010) YIELD
EXTRUDE  232          WRITE(6,2011)ITER
EXTRUDE  233    C
EXTRUDE  234    C ***********************************************************************
EXTRUDE  235    C     READ AND PRINT OF NODAL POINT DATA
EXTRUDE  236    C ***********************************************************************
EXTRUDE  237    C
EXTRUDE  238          L=0
EXTRUDE  239          IF(NPRINT.EQ.0) GO TO 60
EXTRUDE  240          WRITE (6,1114)
EXTRUDE  241          WRITE (6,2004)
EXTRUDE  242       60 READ  (5,1002) N,CODE(N),R(N),Z(N)
EXTRUDE  243       90 IF(NUMNP-N) 100,110,60
EXTRUDE  244      100 WRITE (6,2009) N
EXTRUDE  245          CALL EXIT
EXTRUDE  246      110 CONTINUE
EXTRUDE  247          YSTART=(Z(1)+Z(NIPTS+2))/2.-0.001
EXTRUDE  248          YMIN=Z(NUMNP)
EXTRUDE  249          REXIT=R(NUMNP)
EXTRUDE  250          IF(NPRINT.EQ.0) GO TO 120
EXTRUDE  251          WRITE (6,2002) (K,CODE(K),R(K),Z(K),K=1,NUMNP)
```

```
EXTRUDE  252    C
EXTRUDE  253    C ***************************************************************
EXTRUDE  254    C      READ AND PRINT SPECIAL CONDITIONS ALONG BOUNDARY
EXTRUDE  255    C      NRF = TOTAL NUMBER OF NODAL POINTS AT WHICH FORCES ARE DESIRED
EXTRUDE  256    C      NF(I) = SEQUENTIAL NUMBER OF NODAL POINTS AT WHICH FORCES ARE DESIRED
EXTRUDE  257    C      RR(I)=R-COORDINATE OF FLOW LINES TO BE CONSTRUCTED AT ENTRANCE.
EXTRUDE  258    C ***************************************************************
EXTRUDE  259    C
EXTRUDE  260      120 READ(5,1003)       (NF(I),I=1, NRF)
EXTRUDE  261          IF(NUMPC .LE. 0) GO TO 440
EXTRUDE  262          WRITE(6,1007)
EXTRUDE  263    C
EXTRUDE  264    C ***************************************************************
EXTRUDE  265    C SEE EXPLANATION OF INPUT DATA CARDS PREPARATION
EXTRUDE  266    C ***************************************************************
EXTRUDE  267    C
EXTRUDE  268          DO 441 I=1, NUMPC
EXTRUDE  269          READ(5,1005) IJBC(I,1),IJBC(I,2),(PSBC(I,J),J=1,4)
EXTRUDE  270      441 WRITE(6,1008)IJBC(I,1),IJBC(I,2),(PSBC(I,J),J=1,4)
EXTRUDE  271      440 CONTINUE
EXTRUDE  272          WRITE(6,1006) (NF(I),I=1,NRF)
EXTRUDE  273          READ (5,1001) (RR(I),I=1,NTIMES)
EXTRUDE  274          WRITE(6,1013) (RR(I),I=1,NTIMES)
EXTRUDE  275    C
EXTRUDE  276    C ***************************************************************
EXTRUDE  277    C       READ AND PRINT OF ELEMENT PROPERTIES
EXTRUDE  278    C ***************************************************************
EXTRUDE  279    C
EXTRUDE  280          N=0
EXTRUDE  281      130 READ (5,1003) M,(IEL(M,I),I=1,4)
EXTRUDE  282      140 N=N+1
EXTRUDE  283          IF (M-N) 170,170,150
EXTRUDE  284      150 DO 160 J=1,4
EXTRUDE  285      160 IEL(N,J)=IEL(N-1,J)+1
EXTRUDE  286      170 IF (M-N) 180,180,140
EXTRUDE  287      180 IF (NUMEL-N) 190,190,130
EXTRUDE  288      190 CONTINUE
EXTRUDE  289    C
EXTRUDE  290          IF(NPRINT.EQ.0) GO TO 210
EXTRUDE  291          WRITE (6,2001)
EXTRUDE  292          DO 205 N=1,NUMEL
EXTRUDE  293      205 WRITE (6,2003) N,(IEL(N,I),I=1,4)
EXTRUDE  294    C
EXTRUDE  295    C ***************************************************************
EXTRUDE  296    C      DETERMINE BAND WIDTH
EXTRUDE  297    C ***************************************************************
EXTRUDE  298    C
EXTRUDE  299      210 J=0
EXTRUDE  300          DO 240 N=1,NUMEL
EXTRUDE  301          DO 240 I=1,4
EXTRUDE  302          DO 230 L=1,4
EXTRUDE  303          KK=IABS(IEL(N,I)-IEL(N,L))
EXTRUDE  304          IF(KK-J) 230,230,220
EXTRUDE  305      220 J=KK
EXTRUDE  306      230 CONTINUE
EXTRUDE  307      240 CONTINUE
EXTRUDE  308          MBAND=3*J+3
EXTRUDE  309          MDIAG=1
EXTRUDE  310      250 WRITE (6,1122) NEQ,MBAND,MDIAG
EXTRUDE  311    C
EXTRUDE  312    C ***************************************************************
EXTRUDE  313    C FOR EACH ELEMENT, ASSIGN THE MEAN PRESSURE VALUE TO THE NODAL POINT
EXTRUDE  314    C OF THE HIGHEST NUMBER AMONG THE FOUR CORNER NODAL POINTS.
EXTRUDE  315    C ***************************************************************
EXTRUDE  316    C
EXTRUDE  317          NSTOP=0
EXTRUDE  318          DO 370 J=1,NUMEL
EXTRUDE  319          MID=MAX0(IEL(J,1),IEL(J,2),IEL(J,3),IEL(J,4))
EXTRUDE  320          CODE(MID)=CODE(MID)+10.
EXTRUDE  321          IF(CODE(MID).LT.20.) GO TO 370
EXTRUDE  322          WRITE(6,1012) MID
EXTRUDE  323          NSTOP=1
EXTRUDE  324      370 CONTINUE
EXTRUDE  325          IF(NSTOP.EQ.1) STOP
EXTRUDE  326    C
EXTRUDE  327     1000 FORMAT(1H1)
EXTRUDE  328     1001 FORMAT(8F10.0)
EXTRUDE  329     1002 FORMAT (I5,F5.0,5F10.0)
EXTRUDE  330     1003 FORMAT(16I5)
EXTRUDE  331     1005 FORMAT(2I5,4F10.0)
EXTRUDE  332     1006 FORMAT(// * THE NODAL POINTS AT WHICH FORCE CALCULATIONS ARE DESIR
EXTRUDE  333         1ED* // 20I5)
EXTRUDE  334     1007 FORMAT(1H1,15X, 39H LINEARLY DISTRIBUTED BOUNDARY STRESSES/
EXTRUDE  335         1 /    * NODE-I...NODE-J...PRESSURE-I...PRESSURE-J*
EXTRUDE  336         2  *          SHEAR I          SHEAR J*)
EXTRUDE  337     1008 FORMAT(1H ,2I5,4E15.5)
```

```
EXTRUDE  338     1012 FORMAT(/* NODAL POINT ERROR FOR POINT NO. *,I5,*.......IT CONTAINS
EXTRUDE  339         1 MORE THAN ONE ELEMENT INFORMATION*/)
EXTRUDE  340     1013 FORMAT(/* THE STARTING R-COORDINATES FOR STRAIN CALCULATIONS,*//
EXTRUDE  341         1  11F9.4,/)
EXTRUDE  342     1114 FORMAT(1H1, *   NODAL POINT INFORMATION*,//)
EXTRUDE  343     1122 FORMAT(/// * NNUMBER OF EQUATIONS  =*, I4/
EXTRUDE  344         1           * BANDWIDTH             =*, I4/
EXTRUDE  345         2           * DIAGONAL ELEMENTS     =*, I4 )
EXTRUDE  346     2000 FORMAT(1H  12A6/
EXTRUDE  347         1 30H0 NUMBER OF NODAL POINTS------ I3 /
EXTRUDE  348         2 30H0 NUMBER OF ELEMENTS---------- I3 /)
EXTRUDE  349     2001 FORMAT (1H1,40H ELEMENT NO.      I     J     K     L     )
EXTRUDE  350     2002 FORMAT (I12,F12.2,2F12.3)
EXTRUDE  351     2003 FORMAT (1I13,8I6,1I12)
EXTRUDE  352     2004 FORMAT (/ ,* NODAL POINT         TYPE  R-ORDINATE  Z-ORDINATE*)
EXTRUDE  353     2009 FORMAT (26H0NODAL POINT CARD ERROR N= I5)
EXTRUDE  354     2010 FORMAT(// * INITIAL YIELD STRESS = *, F15.2//)
EXTRUDE  355     2011 FORMAT(///*  MAXIMUM NUMBER OF ITERATIONS ALLOWED=*,I3)
EXTRUDE  356          RETURN
EXTRUDE  357          END
```

```
EXTRUDE  359          SUBROUTINE PLAST(R,Z,UR,UZ,CODE,IEL,YY,STS,TEPS,EPS,NF,FPUR,
EXTRUDE  360         1IJBC,PSBC,FR,FZ,SDIE,RR,B,A,FST,ALANDA,GVECTR,NQ,NEL,NPC,
EXTRUDE  361         1NB2,FLIMIT,ITCONT,NTIMES,NIPTS,NJPTS,NMAX)
EXTRUDE  362     C
EXTRUDE  363     C ***************************************************************************
EXTRUDE  364     C  PLAST IS THE CONTROLLING SUBROUTINE OF THE MATRIX METHOD
EXTRUDE  365     C ***************************************************************************
EXTRUDE  366     C
EXTRUDE  367          COMMON/GENCON/NUMNP,NUMEL,HED(12),VOL,NEQ,NS,ITERNO,ISTOP,
EXTRUDE  368         1YIELD,MBAND,TEST,MDIAG,NBF,NUMPC,ITER,NBF2,NSCALE,NPUNCH,NPRINT,
EXTRUDE  369         2  NCHECK,ACFINI
EXTRUDE  370          COMMON /WALL/ THETA,FT,TANTH
EXTRUDE  371          COMMON /DIMEN/ M1,M2,M3,M4,M5,M6,M7,M8,M9,M10,M11,M12,M13,M14
EXTRUDE  372          DIMENSION R(1),Z(1),UR(1),UZ(1),CODE(1),IEL( NEL ,1),YY(1)
EXTRUDE  373         1,STS(6,1),TEPS(5,1),EPS(5,1),NF(1),FPUR(1),IJBC(NPC,1),RR(1),
EXTRUDE  374         2PSBC( NPC ,1),FR(1),FZ(1),SDIE(1),B(1),A( NQ,1),FST(NB2 ,1)
EXTRUDE  375          DIMENSION ALANDA(1),GVECTR(1)
EXTRUDE  376     C
EXTRUDE  377     C***************************************************************************
EXTRUDE  378     C INITIALIZED ALL VARIABLES
EXTRUDE  379     C GVECTR(N) IS USED FOR STORING THE AMOUNT OF VELOCITY AND MEAN STRESS
EXTRUDE  380     C             TO BE MODIFIED FROM THE RESULT OF ITERATION,THESE VALUES
EXTRUDE  381     C             ARE REDUCED BY DECCELARATION COEFFICIENT IN ORDER TO ENSURE
EXTRUDE  382     C             THE CONVERGENCE OF THE SOLUTION.
EXTRUDE  383     C TEPS(5,N)=TOTAL EFFECTIVE STRAIN
EXTRUDE  384     C A, B, STAND FOR MATRICES IN EQUATION AX=B, AFTER SOLVINF THIS EQUATION
EXTRUDE  385     C         THE X VECTOR IS THEN STORED IN B
EXTRUDE  386     C YY(N)= EFFECTIVE STRESS
EXTRUDE  387     C TEPS(I,N)= TO STORE THE PREVIOUS VALUE OF YY(N) FOR CHECKING
EXTRUDE  388     C             CONVERGENCE FOR WORK-HARDENING MATERIAL
EXTRUDE  389     C TEPS(2,N)=DIFFERENCE OF EFFCTIVE STRESS BETWEEN THE PREVIOUS AND NEW
EXTRUDE  390     C             SOLUTIONS.(FOR WORK-HARDENING MATERIALS)
EXTRUDE  391     C ITST= INDEX FOR CONVERGENCE OF SOLUTIONS(ITST=2 INDICATES THE ITRATION
EXTRUDE  392     C          DOES NOT CONVERGENT)
EXTRUDE  393     C FSQRI,FF= TOTAL SQUARE NORM OF EQUATIONS IN MINIMIZATION
EXTRUDE  394     C FSQRS,FFFF= SQUARE NORM OF EQUATIONS CORRESSPONDING TO
EXTRUDE  395     C                INCOMPRESSIBILITY CONDITIONS(VOLUMN CONSISTANCE)
EXTRUDE  396     C DIFF=FF-FSQRI, DIFFERENCE OF TOTAL SQUARE NORMS OF PREVIOUS AND
EXTRUDE  397     C           PRESENT ITERATIONS
EXTRUDE  398     C***************************************************************************
EXTRUDE  399     C
EXTRUDE  400          DO 998 I=1,M14
EXTRUDE  401      998 A(I)=0.0
EXTRUDE  402          DO 999 N=1,NEQ
EXTRUDE  403          ALANDA(N)=0.
EXTRUDE  404      999 GVECTR(N)=0.
EXTRUDE  405          DO 442 N=1, NUMEL
EXTRUDE  406          YY(N)=1.0
EXTRUDE  407          DO 442 I=1, 5
EXTRUDE  408      442 TEPS(I,N)=0.
EXTRUDE  409          FSQR1=0.
EXTRUDE  410          FSQRS=0.
EXTRUDE  411          FFFF=0.
EXTRUDE  412          ITST=1
EXTRUDE  413          FF=0.
EXTRUDE  414          DIFF=0.
EXTRUDE  415     C
EXTRUDE  416     C***************************************************************************
EXTRUDE  417     C  READ THE INPUT VELOCITY FIELD
EXTRUDE  418     C***************************************************************************
EXTRUDE  419     C
EXTRUDE  420          READ(5,1017) (UR(I), UZ(I), I=1, NUMNP)
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EXTRUDE   421    C
EXTRUDE   422    C*************************************************************************
EXTRUDE   423    C DOUBLE CHECK THE INPUT DATA
EXTRUDE   424    C*************************************************************************
EXTRUDE   425    C
EXTRUDE   426          DO 9889 I=1,NUMNP
EXTRUDE   427          IF(CODE(I).EQ.1..OR.CODE(I).EQ.11..OR.CODE(I).EQ.3..OR.CODE(I)
EXTRUDE   428         1    .EQ.13.) UR(I)=0.0
EXTRUDE   429          IF(CODE(I).EQ.5..OR.CODE(I).EQ.15.) UR(I)=UZ(I)*TANTH
EXTRUDE   430     9889 CONTINUE
EXTRUDE   431          WRITE(6,1020)
EXTRUDE   432          WRITE(6,1017) (UR(I), UZ(I), I=1, NUMNP)
EXTRUDE   433    C
EXTRUDE   434    C*************************************************************************
EXTRUDE   435    C ITER=TOTAL NUMBER OF ITERATIONS ASSIGNED.
EXTRUDE   436    C ITERNO =  AN INDEX FOR QUAC1(SEE SUBROUTINE STIFF AND COMMENT BELOW)
EXTRUDE   437    C*************************************************************************
EXTRUDE   438    C
EXTRUDE   439          ITERNO=1
EXTRUDE   440          DO 2000 KIT=1,ITER
EXTRUDE   441          INOCON=KIT
EXTRUDE   442    C
EXTRUDE   443    C*************************************************************************
EXTRUDE   444    C CHECK IF IT IS FIRST ITERATION
EXTRUDE   445    C IF FIRST ITERATION, ITERNO=1,PERFORM QUAD1.   OTHERWISE JUST READ
EXTRUDE   446    C RESULTS OF QUAD1 FROM TAPE 1.
EXTRUDE   447    C*************************************************************************
EXTRUDE   448    C
EXTRUDE   449      579 IF(KIT-1) 599,599,598
EXTRUDE   450    C
EXTRUDE   451    C*************************************************************************
EXTRUDE   452    C NWKHRD IS AN INDEX  TO SUBROUTINE STIFF
EXTRUDE   453    C NWKHRD=1 JUST CALCULATE STRAIN RATE IN SUBROUTINE STIFF
EXTRUDE   454    C NWKHRD=0 DOING WHOLE THING IN SUBROUTINE STIFF
EXTRUDE   455    C*************************************************************************
EXTRUDE   456    C
EXTRUDE   457      599 NWKHRD=0
EXTRUDE   458          IF(ITCONT.NE.1) NWKHRD=1
EXTRUDE   459          CALL STIFF (R,Z,UR,UZ,CODE,IEL,YY,STS,EPS,NF,FPUR,FST,
EXTRUDE   460         1IJBC,PSBC,A,B,NEL,NO,NPC,NB2,ALANDA,NWKHRD)
EXTRUDE   461          ITERNO=2
EXTRUDE   462          IF(ITCONT.EQ.1) GO TO 593
EXTRUDE   463    C
EXTRUDE   464    C*************************************************************************
EXTRUDE   465    C UNNECESSARY TO CALCULATE TOTAL EFFECTIVE STRAIN FOR NON-HARDENING
EXTRUDE   466    C MATERIALS EXCEPT AT FINAL OUTPUT
EXTRUDE   467    C*************************************************************************
EXTRUDE   468    C
EXTRUDE   469          CALL STRAINS(R,Z,UR,UZ,IEL,EPS,TEPS,A(M1),A(M2),A(M3),A(M4),A(M5),
EXTRUDE   470         1  A(M6),A(M7),A(M8),A(M9),A(M10),A(M11),A(M12),RR,A(M13),NIPTS,
EXTRUDE   471         2  NJPTS,NTIMES,NEL,NUMNP,NMAX)
EXTRUDE   472    C
EXTRUDE   473    C*************************************************************************
EXTRUDE   474    C CALCULATE NORMALIZED EFFECTIVE STRESS (FOR WORK-HARDENING)
EXTRUDE   475    C*************************************************************************
EXTRUDE   476    C
EXTRUDE   477          DO 220 N=1, NUMEL
EXTRUDE   478          CALL HARD(TEPS(5,N),YY(N))
EXTRUDE   479          YY(N)=YY(N)/YIELD
EXTRUDE   480      220 TEPS(1,N)=YY(N)
EXTRUDE   481          NWKHRD=0
EXTRUDE   482          CALL STIFF (R,Z,UR,UZ,CODE,IEL,YY,STS,EPS,NF,FPUR,FST,
EXTRUDE   483         1IJBC,PSBC,A,B,NEL,NO,NPC,NB2,ALANDA,NWKHRD)
EXTRUDE   484      593 CALL MODIFY (CODE,A,B,NUMNP,NEQ,MBAND)
EXTRUDE   485      598 CONTINUE
EXTRUDE   486    C
EXTRUDE   487    C*************************************************************************
EXTRUDE   488    C SOLUTION FOR BANDED SYMMETRIC MATRIX
EXTRUDE   489    C*************************************************************************
EXTRUDE   490    C
EXTRUDE   491          CALL TRIA(NEQ,MBAND,A)
EXTRUDE   492          CALL BACKS(NEQ,MBAND,A,B)
EXTRUDE   493    C
EXTRUDE   494    C*************************************************************************
EXTRUDE   495    C SET CORRESSPONDING VALUE FOR INCLINED SPECIAL BOUNDARY CONDITIONS
EXTRUDE   496    C*************************************************************************
EXTRUDE   497    C
EXTRUDE   498          DO 769 N=1, NUMNP
EXTRUDE   499          IZ=3*N-1
EXTRUDE   500          IR=IZ-1
EXTRUDE   501      769 IF(CODE(N).EQ.5..OR.CODE(N).EQ.15.) B(IR)=B(IZ)*TAN(THETA)
EXTRUDE   502    C
EXTRUDE   503    C*************************************************************************
EXTRUDE   504    C IF FIRST ITERATION, SET INITIAL STEP LENGTH ACCORDING TO
EXTRUDE   505    C DECCELARATION COEFF. ASSIGNED. OTHERWISE FIND THE BEST DECCELARATION
EXTRUDE   506    C COEFF. FROM INFORMATION OF PREVIOUS ITERATION.
EXTRUDE   507    C*************************************************************************
EXTRUDE   508    C
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EXTRUDE  509      507 C4=0.0
EXTRUDE  510          DO 509 I=1,NUMNP
EXTRUDE  511          IZ=3*I-1
EXTRUDE  512          IR=IZ-1
EXTRUDE  513      509 C4=C4+B(IZ)*B(IZ)+B(IR)*B(IR)
EXTRUDE  514          CORNOM=SQRT(C4)
EXTRUDE  515          IF(KIT.EQ.1) GO TO 551
EXTRUDE  516       67 ACOEF=1.0
EXTRUDE  517      510 IF((ACOEF*CORNOM).LT.STEPLH) GO TO 550
EXTRUDE  518      522 ACOEF=0.9*ACOEF
EXTRUDE  519          GO TO 510
EXTRUDE  520      551 ACOEF=ACFINI
EXTRUDE  521          STEPLH=ACOEF*CORNOM
EXTRUDE  522          STEPOR=STEPLH
EXTRUDE  523      550 FF=FSQR1
EXTRUDE  524          FFFF=FSQR5
EXTRUDE  525      C
EXTRUDE  526      C*************************************************************************
EXTRUDE  527      C  CALCULATE FORCES AT NODAL POINTS
EXTRUDE  528      C*************************************************************************
EXTRUDE  529      C
EXTRUDE  530           CALL CFORCE(NP,FR,FZ,FST,FPUR,B,MBAND,NBF,NBF2)
EXTRUDE  531           WRITE(6,1007)
EXTRUDE  532           WRITE(6,1016) KIT
EXTRUDE  533      C
EXTRUDE  534      C*************************************************************************
EXTRUDE  535      C STS(6,N)=MEAN STRESS AT PRESENT SOLUTION.
EXTRUDE  536      C ALANDA(IR)=MEAN STRESS FROM PREVICUS SOLUTION. IF FIRST ITERATION,
EXTRUDE  537      C            THESE VALUES ARE THE SAME AS STS(6,N)
EXTRUDE  538      C*************************************************************************
EXTRUDE  539      C
EXTRUDE  540           DO 133 N=1, NUMEL
EXTRUDE  541           I=MAX0(IEL(N,1),IEL(N,2),IEL(N,3),IEL(N,4))
EXTRUDE  542           IR=3*I
EXTRUDE  543      112  STS(6,N)=B(IR)
EXTRUDE  544           IF(KIT.EQ.1) ALANDA(IR)=B(IR)
EXTRUDE  545           GVECTR(IR)=B(IR)-ALANDA(IR)
EXTRUDE  546      133  B(IR)=0.
EXTRUDE  547      C
EXTRUDE  548      C*************************************************************************
EXTRUDE  549      C  CALCULATION OF SOLUTION NORM FOR VELOCITIES
EXTRUDE  550      C SNORM= NORM OF SOLUTION VECTOR OF VELOCITIES
EXTRUDE  551      C ENORM= NORM OF ERROR VECTOR OF VELOCITIES
EXTRUDE  552      C*************************************************************************
EXTRUDE  553      C
EXTRUDE  554           SNORM = 0.
EXTRUDE  555           DO 134 I=1, NUMNP
EXTRUDE  556           IZ=3*I-1
EXTRUDE  557           IR=IZ-1
EXTRUDE  558           IN=IZ+1
EXTRUDE  559           GVECTR(IR)=B(IR)
EXTRUDE  560           GVECTR(IZ)=B(IZ)
EXTRUDE  561           SNORM = SNORM + UR(I)*UR(I) + UZ(I)*UZ(I)
EXTRUDE  562      C
EXTRUDE  563      C*************************************************************************
EXTRUDE  564      C MODIFIY THE VELOCITY FIELDS AND MEAN STRESSES
EXTRUDE  565      C*************************************************************************
EXTRUDE  566      C
EXTRUDE  567           ALANDA(IN)=ALANDA(IN)+GVECTR(IN)*ACOEF
EXTRUDE  568           UR(I)=UR(I)+B(IR)*ACOEF
EXTRUDE  569           UZ(I)=UZ(I)+B(IZ)*ACOEF
EXTRUDE  570      134  CONTINUE
EXTRUDE  571           ENORM=CORNOM
EXTRUDE  572           SNORM = SQRT(SNORM)
EXTRUDE  573           ESNORM=ENORM/SNORM
EXTRUDE  574      C
EXTRUDE  575      C *************************************************************************
EXTRUDE  576      C PRINT SOLUTION NORM OF VELOCITIES, VELOCITY DISTRIBUTIONS AND NODAL-
EXTRUDE  577      C POINT FORCES
EXTRUDE  578      C *************************************************************************
EXTRUDE  579      C
EXTRUDE  580           IF(NBF .LE. 0) GO TO 125
EXTRUDE  581           DO 123 I=1, NBF
EXTRUDE  582           FR(I)=FR(I)*YIELD
EXTRUDE  583           FZ(I)=FZ(I)*YIELD
EXTRUDE  584      123  CONTINUE
EXTRUDE  585      125  CONTINUE
EXTRUDE  586           WRITE(6,1015) SNORM,ENORM,ESNORM
EXTRUDE  587           WRITE(6,1006) KIT,ACOEF
EXTRUDE  588           DO 439 I=1, NUMNP
EXTRUDE  589           IZ=3*I-1
EXTRUDE  590           IR=IZ-1
EXTRUDE  591           WRITE(6,1002) I,B(IR),B(IZ),UR(I),UZ(I),R(I),Z(I)
EXTRUDE  592      439  CONTINUE
EXTRUDE  593           WRITE(6,1010)
EXTRUDE  594           DO 140 I=1, NBF
EXTRUDE  595           SHEAR=FZ(I)*COS(THETA) + FR(I)*SIN(THETA)
EXTRUDE  596      140  WRITE(6,1012) NF(I),FR(I),FZ(I) ,SHEAR
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EXTRUDE  597    C
EXTRUDE  598    C ***********************************************************************
EXTRUDE  599    C CONSTRUCT THE FLOW LINES AND CARRY OUT THE INTEGRATION OF THE STRAIN
EXTRUDE  600    C RATES TO DETERMINE THE TOTAL STRAIN DISTRIBUTION FOR WORK-HARDENING
EXTRUDE  601    C MATERIAL. THIS STEP IS PERFORMED ONLY AT FINAL ITERATION FOR NON-
EXTRUDE  602    C WORKHARDENING MATERIAL
EXTRUDE  603    C ***********************************************************************
EXTRUDE  604    C
EXTRUDE  605      563 IF(ESNORM.LE.FLIMIT.OR.KIT.EQ.ITER) GO TO 56
EXTRUDE  606          IF(ITCONT.EQ.1) GO TO 55
EXTRUDE  607      56  IF(ITCONT.EQ.1) GO TO 58
EXTRUDE  608          NWKHRD=1
EXTRUDE  609          CALL STIFF (R,Z,UR,UZ,CODE,IEL,YY,STS,EPS,NF,FPUR,FST,
EXTRUDE  610         1IJBC,PSBC,A,B,NEL,NO,NPC,NB2,ALANDA,NWKHRD)
EXTRUDE  611      58  CALL STRAINS(R,Z,UR,UZ,IEL,EPS,TEPS,A(M1),A(M2),A(M3),A(M4),A(M5),
EXTRUDE  612         1  A(M6),A(M7),A(M8),A(M9),A(M10),A(M11),A(M12),RR,A(M13),NIPTS,
EXTRUDE  613         2  NJPTS,NTIMES,NEL,NUMNP,NMAX)
EXTRUDE  614          IF(ITCONT.EQ.1) GO TO 55
EXTRUDE  615          DO 230 N=1,NUMEL
EXTRUDE  616          CALL HARD(TEPS(5,N),YY(N))
EXTRUDE  617          YY(N)=YY(N)/YIELD
EXTRUDE  618    C
EXTRUDE  619    C ***********************************************************************
EXTRUDE  620    C TEPS(1,N)= PREVIOUS EFFECTIVE STRESS
EXTRUDE  621    C TEPS(2,N)=DIFFERENCE OF EFFCTIVE STRESS BETWEEN THE PREVIOUS AND NEW
EXTRUDE  622    C           SOLUTIONS.(FOR WORK-HARDENING MATERIALS)
EXTRUDE  623    C ***********************************************************************
EXTRUDE  624    C
EXTRUDE  625          TEPS(2,N)=YY(N)-TEPS(1,N)
EXTRUDE  626      230 TEPS(1,N)=YY(N)
EXTRUDE  627    C
EXTRUDE  628    C ***********************************************************************
EXTRUDE  629    C  PRINT THE STRESS AND STRAIN-RATE DISTRIBUTIONS AND THE EFFECTIVE
EXTRUDE  630    C  STRAIN DISTRIBUTION.
EXTRUDE  631    C ***********************************************************************
EXTRUDE  632    C
EXTRUDE  633       55 CONTINUE
EXTRUDE  634          WRITE(6,1007)
EXTRUDE  635          WRITE(6,1005) KIT
EXTRUDE  636          DO 222 N=1, NUMEL
EXTRUDE  637          CONST=2.*YY(N)/(3.*EPS(5,N))
EXTRUDE  638          STS(1,N)=EPS(1,N)*CONST
EXTRUDE  639          STS(2,N)=EPS(2,N)*CONST
EXTRUDE  640          STS(3,N)=EPS(3,N)*CONST
EXTRUDE  641          STS(4,N)=EPS(4,N)*CONST/2.
EXTRUDE  642          DO 132 J=1, 3
EXTRUDE  643      132 STS(J,N)=STS(J,N)+STS(6,N)
EXTRUDE  644          STS(5,N)=EFSTRS(STS(1,N),STS(2,N),STS(3,N),STS(4,N))
EXTRUDE  645          WRITE(6,1004) N, ( EPS(I,N),I=1,5),TEPS(5,N),(STS(I,N),I=1,6)
EXTRUDE  646      222 CONTINUE
EXTRUDE  647    C
EXTRUDE  648    C ***********************************************************************
EXTRUDE  649    C CHECK THE CONVERGENCE OF NEW SOLUTION, IF IT IS NOT, THEN REDUCE
EXTRUDE  650    C ACCEL. COEFF. AND CORRESPONDING STEP LENGTH AND CHECK IT AGAIN
EXTRUDE  651    C FOR FIRST ITERATION, THIS STEP IS OMITTED AND JUST COMPUTE THE TOTAL
EXTRUDE  652    C SQUARE NORM OF EQUATIONS AND INCOMPRESSIBILITY CONDITIONS
EXTRUDE  653    C ***********************************************************************
EXTRUDE  654    C
EXTRUDE  655      564 NWKHRD=0
EXTRUDE  656          CALL STIFF (R,Z,UR,UZ,CODE,IEL,YY,STS,EPS,NF,FPUR,FST,
EXTRUDE  657         1IJBC,PSBC,A,B,NEL,NO,NPC,NB2,ALANDA,NWKHRD)
EXTRUDE  658          CALL MODIFY (CODE,A,B,NUMNP,NEQ,MBAND)
EXTRUDE  659    C
EXTRUDE  660    C ***********************************************************************
EXTRUDE  661    C COMPUTE TOTAL SQUARE NORM OF EQUATIONS(FSQR1) AND SQUARE NORM OF
EXTRUDE  662    C INCOMPRESSIBILITY CONDITIONS(FSQR5)
EXTRUDE  663    C ***********************************************************************
EXTRUDE  664    C
EXTRUDE  665          FSQR5=0.0
EXTRUDE  666          FSQR1=0.0
EXTRUDE  667          DO 501 N=1,NUMNP
EXTRUDE  668          II=3*N
EXTRUDE  669          FSQR1=FSQR1+B(II)*B(II)
EXTRUDE  670          FSQR5=FSQR5+B(II)*R(II)
EXTRUDE  671          DO 891 JJ=2,3
EXTRUDE  672          II=II-1
EXTRUDE  673          BLANII=B(II)
EXTRUDE  674          JA=JJ
EXTRUDE  675          JK=II+JA-1
EXTRUDE  676      888 IF(JA.GT.MBAND) GO TO 889
EXTRUDE  677          BLANII=BLANII-A(II,JA)*ALANDA(JK)
EXTRUDE  678          JA=JA+3
EXTRUDE  679          JK=JK+3
EXTRUDE  680          GO TO 888
EXTRUDE  681      889 IF(N.GT.1) GO TO 792
EXTRUDE  682          GO TO 891
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*XTRUDE   683      792 DO 893 J=3,II,3
EXTRUDE   684          KK=II+1-J
EXTRUDE   685          IF(KK.GT.MBAND) GO TO 891
EXTRUDE   686      893 BLANII=BLANII-A(J,KK)*ALANDA(J)
EXTRUDE   687      891 FSQR1=FSQR1+BLANII*BLANII
EXTRUDE   688      501 CONTINUE
EXTRUDE   689          IF(KIT.EQ.1) GO TO 788
EXTRUDE   690    C
EXTRUDE   691    C ***********************************************************************
EXTRUDE   692    C COMPARE SQUARE NORM OF FUNCTIONS WITH PREVIOUS ONE.
EXTRUDE   693    C ***********************************************************************
EXTRUDE   694    C
EXTRUDE   695          DIFF=FF-FSQR1
EXTRUDE   696          IF(FSQR1-FF) 571,571,572
EXTRUDE   697    C
EXTRUDE   698    C ***********************************************************************
EXTRUDE   699    C CASE FOR SQUARE NORM OF NEW SOLUTION LARGER THAN PREVIOUS ONE, REDUCE
EXTRUDE   700    C ACCEL. COEFF. BY HALF AND COMPARE AGAIN.
EXTRUDE   701    C ***********************************************************************
EXTRUDE   702    C
EXTRUDE   703      572 IF((ABS(DIFF)/FF).LE.1.E-3) GO TO 444
EXTRUDE   704          STEPLH=0.5*STEPLH
EXTRUDE   705          ACOEF=ACOEF*0.5
EXTRUDE   706          INOCON=INOCON+1
EXTRUDE   707          IF(INOCON-(KIT+5))581,581,582
EXTRUDE   708      581 DO 595 I=1,NUMNP
EXTRUDE   709          IZ=3*I-1
EXTRUDE   710          IR=IZ-1
EXTRUDE   711          IN=IZ+1
EXTRUDE   712          ALANDA(IN)=ALANDA(IN)-GVECTR(IN)*ACOEF
EXTRUDE   713          UR(I)=UR(I)-GVECTR(IR)*ACOEF
EXTRUDE   714      595 UZ(I)=UZ(I)-GVECTR(IZ)*ACOEF
EXTRUDE   715          WRITE(6,1155) STEPLH,FSQR1,FF,DIFF,FSQR5,FFFF
EXTRUDE   716          WRITE(6,1030) ACOEF
EXTRUDE   717          DO 772 I=1,NUMNP
EXTRUDE   718          IZ=3*I-1
EXTRUDE   719          IR=IZ-1
EXTRUDE   720          WRITE(6,1002)I,B(IP),B(IZ),UR(I),UZ(I)
EXTRUDE   721      772 CONTINUE
EXTRUDE   722          IF(ITCONT.EQ.1) GO TO 564
EXTRUDE   723          DO 561 N=1,NUMEL
EXTRUDE   724      561 YY(N)=TEPS(1,N)-TEPS(2,N)*0.5
EXTRUDE   725          GO TO 564
EXTRUDE   726    C
EXTRUDE   727    C ***********************************************************************
EXTRUDE   728    C IF CONTIMUOUS REDUCING DECCEL. COEFF. OF FIVE TIMES, THE SOLUTION
EXTRUDE   729    C STILL NOT CONVERGENT, THEN PUNCH AND PRINT THE RESULT AND STOP THE
EXTRUDE   730    C PROGRAM.
EXTRUDE   731    C ***********************************************************************
EXTRUDE   732    C
EXTRUDE   733      582 WRITE(6,1135) STEPLH,FSQR1,FF,DIFF,FSQR5,FFFF
EXTRUDE   734          WRITE(6,1056)
EXTRUDE   735          ITST=2
EXTRUDE   736          GO TO 600
EXTRUDE   737    C
EXTRUDE   738    C ***********************************************************************
EXTRUDE   739    C CASE FOR SQUARE NORM OF NEW SOLUTION LESS THAN PREVIOUS ONE, IF
EXTRUDE   740    C CONVERFENCE CHARACTER IS IN GOOD SITUATION, THEN INCREASE STEP
EXTRUDE   741    C LENGTH FOR NEXT ITERATION.
EXTRUDE   742    C ***********************************************************************
EXTRUDE   743    C
EXTRUDE   744      571 FSQR2=(STEPOR/STEPLH)*(FSQR1-FF)+FF
EXTRUDE   745          IF(FSQR2-0.965*FF) 574,574,575
EXTRUDE   746      575 STEPLH=0.8*STEPLH
EXTRUDE   747          GO TO 444
EXTRUDE   748      574 IF(FSQR2-0.900*FF) 576,576,444
EXTRUDE   749      576 STEPLH=1.2*STEPLH
EXTRUDE   750      444 CONTINUE
EXTRUDE   751          STEPOR=STEPLH
EXTRUDE   752          WRITE(6,1155) STEPLH,FSQR1,FF,DIFF,FSQR5,FFFF
EXTRUDE   753      788 CONTINUE
EXTRUDE   754    C
EXTRUDE   755    C ***********************************************************************
EXTRUDE   756    C STOP ITERATIONS AND PUNCH RESULTS IF (ERROR NORM)/(SOLUTION NORM)
EXTRUDE   757    C REACH THE ASSIGNED ACCURACY.
EXTRUDE   758    C ***********************************************************************
EXTRUDE   759    C
EXTRUDE   760          IF(ESNORM.LE.FLIMIT) GO TO 600
EXTRUDE   761    C
EXTRUDE   762    C ***********************************************************************
EXTRUDE   763    C PUNCH SOLUTIONS AT EVERY FIFTH ITERATION BUT DOES NOT STOP THE
EXTRUDE   764    C ITERATIONS. PUNCH SOLUTIONS AT FINAL ITERATION.
EXTRUDE   765    C ***********************************************************************
EXTRUDE   766    C
EXTRUDE   767          K1=KIT/5
EXTRUDE   768          K2=K1*5
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EXTRUDE  769          IF(K2.GE.KIT) GO TO 600
EXTRUDE  770          IF(KIT.EQ.ITER) GO TO 600
EXTRUDE  771          GO TO 660
EXTRUDE  772     600  PUNCH 1017, (UR(I),UZ(I), I=1, NUMNP)
EXTRUDE  773          NN=3*NUMNP
EXTRUDE  774          IF(NPUNCH .EQ. 0) GO TO 660
EXTRUDE  775          PUNCH 1017, (TEPS(5,N),N=1,NUMEL)
EXTRUDE  776          PUNCH 1017, ( (EPS(I,N),I=1,5), N=1,NUMEL)
EXTRUDE  777          PUNCH 1017,((STS(I,N),I=1,6),N=1,NUMEL)
EXTRUDE  778          DO 100 I=1,NBF
EXTRUDE  779          K=NF(I)
EXTRUDE  780     100  PUNCH 1012,K,UR(K),UZ(K),FR(I),FZ(I)
EXTRUDE  781     660  CONTINUE
EXTRUDE  782          IF(ITST.EQ.2) GO TO 2205
EXTRUDE  783          IF(ESNORM.LE.FLIMIT) GO TO 2205
EXTRUDE  784    2000  CONTINUE
EXTRUDE  785    C
EXTRUDE  786    C *****************************************************************
EXTRUDE  787    C  PRINT THE FINAL STRAIN-RATES, THE INCOMPRESSIBILITY AND THE
EXTRUDE  788    C  INTEGRATED EFFECTIVE STRAIN DISTRIBUTION.
EXTRUDE  789    C *****************************************************************
EXTRUDE  790    C
EXTRUDE  791     2205 CONTINUE
EXTRUDE  792          WRITE(6,1019) KIT
EXTRUDE  793          DO 500 N=1,NUMEL
EXTRUDE  794          SUM=EPS(1,N)+EPS(2,N)+EPS(3,N)
EXTRUDE  795      500 WRITE(6,1003) N,(EPS(I,N),I=1,5),SUM,TEPS(5,N)
EXTRUDE  796          WRITE(6,2222) STEPLH
EXTRUDE  797    C
EXTRUDE  798     1002 FORMAT(I10,2F13.7,10X, 2F13.7, 10X, 2F13.7)
EXTRUDE  799     1003 FORMAT(I7,11F11.6)
EXTRUDE  800     1004 FORMAT(I7,12F10.6)
EXTRUDE  801     1005 FORMAT(1H1,* STRAIN RATE-STRESS SOLUTION AT ITER.  NUMBER  =*,I4//
EXTRUDE  802         1  * EL. NO..R-STRAIN..Z-STRAIN..TH-STRAIN.RZ-STRAIN.EF-STRAIN.TOT.E
EXTRUDE  803         2FF....R-STRESS..Z-STRESS.TH-STRESS.RZ-STRESS.EF-STRESS.AVG-STRESS.
EXTRUDE  804         3.*,/,11X,*RATE*,6X,*RATE*,6X,*RATE*,6X,*RATE*,6X,*RATE*,6X,*STRAIN
EXTRUDE  805         4*/)
EXTRUDE  806     1006 FORMAT(///  30X, * VELOCITY     SOLUTION AT ITERATION NUMBER =*,I4
EXTRUDE  807         1/,30X,* DECELARATION COEFFICIENT=* F13.8,
EXTRUDE  808         1/// 15X,*CORRECTED AMOUNT OF VEL.*,15X,*NEW SOLUTION*,21X,*COORDIN
EXTRUDE  809         1ATES*/,24X,*IN..*,29X,*OF*
EXTRUDE  810         2    /* NODAL NO.      R-VELOCITY     Z-VELOCITY              R-VELOCITY
EXTRUDE  811         3    Z-VELOCITY          R-COORD             Z-COORD*)
EXTRUDE  812     1007 FORMAT(1H1)
EXTRUDE  813     1010 FORMAT( //    * NODAL POINT FORCE*//
EXTRUDE  814         1*.......N.P.........R-FORCE.......Z-FORCE......SHEAR FORCE ON DIE
EXTRUDE  815         2  SURFACE...*)
EXTRUDE  816     1012 FORMAT(I9,4F17.5)
EXTRUDE  817     1015 FORMAT( //  50X, * NORM OF SOLUTION VECTOR =*, F13.8
EXTRUDE  818         1          /  50X, * NORM OF   ERROR  VECTOR =*, F13.8
EXTRUDE  819         2          /  50X, * FRACTIONAL  NORM        =*, F13.8)
EXTRUDE  820     1016 FORMAT(/// 65X, *    VELOCITY  SOLUTION AT ITERATION NUMBER =*,I4)
EXTRUDE  821     1017 FORMAT(8F10.6)
EXTRUDE  822     1019 FORMAT(1H1,*FINAL STRAIN RATE SOLUTION AT ITER.NUMBER = *,I4,//
EXTRUDE  823         1 * EL. NO...R-STRAIN...Z-STRAIN..TH-STRAIN..RZ-STRAIN..EF-STRAIN..
EXTRUDE  824         2.....SUM.......TOT-EF-STRAIN*,/,12X,*RATE*,7X,*RATE*7X,*RATE*
EXTRUDE  825         3,7X,*RATE*,7X,*RATE*/)
EXTRUDE  826     1020 FORMAT(//,* THE INITINAL GUESS( OR INPUT DATA) OF VELOCIT DISTRIBU
EXTRUDE  827         1TION*,/,*    FROM FIRST NODAL POINT(R-VEL., Z-VEL.) TO LAST NODAL
EXTRUDE  828         2 POINT(R-VEL., Z-VEL.)*)
EXTRUDE  829     1030 FORMAT(///,25X,*THE ABOVE VELOCITY DISTRIBUTION DOES NOT CONVERGE*
EXTRUDE  830         1/,25X,*REDUCE DECCELARATION COEFF. WITH THE FOLLOWING VELOCITY DIS
EXTRUDE  831         1TRIBUTIONS*/,25X,*AND CALCULATE ALL NORMS AGAIN*
EXTRUDE  832         1/,25X,*NEW DECCELARATION COEFFICIENT=*,F13.8,
EXTRUDE  833         1 // 15X,*CORRECTED AMOUNT OF VEL.*,15X,*NEW SOLUTION*/,24X,*IN..*,
EXTRUDE  834         129X,*OF*,/,
EXTRUDE  835         2    * NODAL NO.      R-VELOCITY     Z-VELOCITY              R-VELOCITY
EXTRUDE  836         3    Z-VELOCITY*)
EXTRUDE  837     1056 FORMAT(///,20X,*SOLUTION DOES NOT CONVERGENT*)
EXTRUDE  838     1155 FORMAT(////,25X,*STEP LENGTH FOR NEXT ITERATION=*F13.8,
EXTRUDE  839         1          /,25X,*TOTAL SQUARE NORM OF ALL EQUATIONS AT PRESENT SOL
EXTRUDE  840         1UTION=*E16.6,
EXTRUDE  841         1          /,25X,*TOTAL SQUARE NORM OF ALL EQUATIONS FROM PREVIOUS
EXTRUDE  842         1SOLUTION=*E16.8,
EXTRUDE  843         1      /,25X,*DIFFERENCE=*E16.8,
EXTRUDE  844         1//,25X,*SQUARE NORM OF PRESENT VALUE IN VOL.CONSISTANCE=*E16.8,
EXTRUDE  845         1 /,25X,*SQUARE NORM OF PREVIOUS VALUE IN VOL.CONSISTANCE=*E16.8)
EXTRUDE  846     2222 FORMAT(//,20X,*FINAL STEPLH=* F14.6)
EXTRUDE  847    C
EXTRUDE  848          RETURN
EXTRUDE  849          END
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EXTRUDE  851          SUBROUTINE HARD(STRAIN,YIELD)
EXTRUDE  852    C
EXTRUDE  853    C ***************************************************************
EXTRUDE  854    C SPECIFY THE MATERIAL PROPERTIES FOR WORK-HARDENING MATERIAL (STRESS-
EXTRUDE  855    C STRAIN RELATIONSHIP) FOR NON-HARDENING MATERIAL, REPLACE #YIELD# CARD
EXTRUDE  856    C AND SET YIELD=THE CONSTANT YIELD STRESS.
EXTRUDE  857    C ***************************************************************
EXTRUDE  858    C
EXTRUDE  859          B=0.262E-4
EXTRUDE  860          AN=0.06
EXTRUDE  861          YIELD=(1.+STRAIN/B)**AN
EXTRUDE  862          RETURN
EXTRUDE  863          END



EXTRUDE  865          SUBROUTINE STIFF (R,Z,UR,UZ,CODE,IEL,YY,STS,EPS,NF,FPUR,FST,
EXTRUDE  866         1IJBC,PSBC,A,B,NEL,NO,NPC,NB2,ALANDA,NWKMRD)
EXTRUDE  867    C
EXTRUDE  868    C ***************************************************************
EXTRUDE  869    C CALCULATION STIFFNESS MATRIX FOR ENTIRE SYSTEM.
EXTRUDE  870    C ***************************************************************
EXTRUDE  871    C
EXTRUDE  872          COMMON/GENCON/NUMNP,NUMEL,HED(12),VOL,NEQ,NS,ITERNO,ISTOP,
EXTRUDE  873         1YIELD,MBAND,TEST,MDIAG,NBF,NUMPC,ITER,NBF2,NSCALE,NPUNCH,APRINT,
EXTRUDE  874         2  NCHECK,ACFINI
EXTRUDE  875          COMMON /BALL/ THETA,FT,TANTH
EXTRUDE  876          COMMON /STFMAT/ P(9,9),H(9)
EXTRUDE  877          DIMENSION ALANDA(1)
EXTRUDE  878          DIMENSION R(1),Z(1),CODE(1),UR(1),UZ(1),IEL( NEL ,1),P(1),A( NO,1)
EXTRUDE  879         1,EPS(5,1),STS(6,1),NF(1),FST( NB2 ,1),FPUR(1),IJBC( NPC ,1),
EXTRUDE  880         2 PSBC(NPC,1),RR(4),ZZ(4),IXY(9),UU(8),YY(7),ID(2)
EXTRUDE  881    C
EXTRUDE  882    C ***************************************************************
EXTRUDE  883    C INITIALIZED A AND B MATRICES (FOR EQUATION Ax=B)
EXTRUDE  884    C BECAUSE BANDED SYMMETRIC PROPERTY OF THE STIFFNESS MATRIX A, THE
EXTRUDE  885    C STORAGE OF A IS IN A SQURE ARRAY.
EXTRUDE  886    C ***************************************************************
EXTRUDE  887    C
EXTRUDE  888          NSTOP=0
EXTRUDE  889          DO 50 N=1, NEQ
EXTRUDE  890          B(N)=0.
EXTRUDE  891          DO 50 M=1, MBAND
EXTRUDE  892       50 A(N,M)=0.
EXTRUDE  893    C
EXTRUDE  894    C ***************************************************************
EXTRUDE  895    C CONSTRUCT ELEMENT LEVEL MATRICES FOR STIFFNESS
EXTRUDE  896    C IF ITERNO.GT.1 MEANS SUBROUTINE QUAD1 HAS BEEN PERFOMED ONCE BEFORE
EXTRUDE  897    C AND IS STORED IN TAPE 1.
EXTRUDE  898    C ***************************************************************
EXTRUDE  899    C
EXTRUDE  900          IF(ITERNO.GT.1) GO TO 20
EXTRUDE  901          REWIND 1
EXTRUDE  902          DO 510 N=1,NUMEL
EXTRUDE  903          DO 100 I=1,4
EXTRUDE  904          II=IEL(N,I)
EXTRUDE  905          RR(I)=R(II)
EXTRUDE  906      100 ZZ(I)=Z(II)
EXTRUDE  907          CALL QUAD1(RR,ZZ,VOL)
EXTRUDE  908       42 IF(VOL.GT.0.) GO TO 510
EXTRUDE  909          WRITE(6,1005) N
EXTRUDE  910          NSTOP=1
EXTRUDE  911      510 CONTINUE
EXTRUDE  912       20 REWIND 1
EXTRUDE  913          IF(NSTOP .EQ. 1) STOP
EXTRUDE  914    C
EXTRUDE  915    C ***************************************************************
EXTRUDE  916    C CONSTRUCT P AND H MATRICES AT ELEMENT LEVEL.
EXTRUDE  917    C ***************************************************************
EXTRUDE  918    C
EXTRUDE  919          DO 1000 N=1, NUMEL
EXTRUDE  920          DO 60 I=1,4
EXTRUDE  921          II=IEL(N,I)
EXTRUDE  922          I2=2*I
EXTRUDE  923          I1=I2-1
EXTRUDE  924          IXY(I1)=3*II-2
EXTRUDE  925          IXY(I2)=3*II-1
EXTRUDE  926          UU(I2)=UZ(II)
EXTRUDE  927       60 UU(I1)=UR(II)
EXTRUDE  928          II=MAX0(IEL(N,1),IEL(N,2),IEL(N,3),IEL(N,4))
EXTRUDE  929          IXY(9)=3*II
EXTRUDE  930          CALL QUAD2(UU,IPLMAX,TEX,TEY,TEZ,TEXY)
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EXTRUDE  931    C
EXTRUDE  932    C ***********************************************************************
EXTRUDE  933    C NWKHRD IS AN INDEX TO INDICATE WHETHER ONLY STRAIN RATE CALCULATION IS
EXTRUDE  934    C REQUIRED OR THE WHOLE STIFFNESS MATRIX NEED TO BE CONSTRUCTED.
EXTRUDE  935    C ***********************************************************************
EXTRUDE  936    C
EXTRUDE  937          IF(NWKHRD.EC.1) GO TO 201
EXTRUDE  938    C
EXTRUDE  939    C ***********************************************************************
EXTRUDE  940    C PERFORM THE ASSEMBLY OPERATION. BECAUSE MATRIX A IS SYMMETRIC, ONLY
EXTRUDE  941    C UPPER HALF OF THE MATRIX IS CREATED. AND THE STORAGE FOR MATRIX A IS
EXTRUDE  942    C A SQUARE ARRAY BECAUSE OF BANDED SYMMETRIC PROPERTY.
EXTRUDE  943    C ***********************************************************************
EXTRUDE  944    C
EXTRUDE  945          DO 62 I=1,8
EXTRUDE  946          H(I)=H(I)*YY(N)
EXTRUDE  947          DO 62 J=1,8
EXTRUDE  948       62 P(I,J)=P(I,J)*YY(N)
EXTRUDE  949          DO 200 I=1, 9
EXTRUDE  950          II=IXY(I)
EXTRUDE  951          B(II)=B(II)-H(I)
EXTRUDE  952          DO 200 J=1, 9
EXTRUDE  953          JJ=IXY(J)-II+1
EXTRUDE  954          IF(JJ .LT. 1) GO TO 200
EXTRUDE  955          A(II,JJ)=A(II,JJ)+P(I,J)
EXTRUDE  956      200 CONTINUE
EXTRUDE  957      201 CONTINUE
EXTRUDE  958          EPS(1,N)=TEX
EXTRUDE  959          EPS(2,N)=TEY
EXTRUDE  960          EPS(3,N)=TEZ
EXTRUDE  961          EPS(4,N)=TEXY
EXTRUDE  962     1000 CONTINUE
EXTRUDE  963      563 DO 150 N=1,NUMEL
EXTRUDE  964      150 EPS(5,N)=RBAR(EPS(1,N),EPS(2,N),EPS(3,N),EPS(4,N))
EXTRUDE  965          IF(NWKHRD.EQ.1) RETURN
EXTRUDE  966    C
EXTRUDE  967    C ***********************************************************************
EXTRUDE  968    C     PREPARATION FOR FORCE CALCULATION
EXTRUDE  969    C ***********************************************************************
EXTRUDE  970    C
EXTRUDE  971          IF(NBF .LE. 0) GO TO 402
EXTRUDE  972          MBAND2=2*MBAND-1
EXTRUDE  973          DO 330 I=1, NBF
EXTRUDE  974          IZ=2*I
EXTRUDE  975          IR=IZ-1
EXTRUDE  976          DO 330 J=1, MBAND2
EXTRUDE  977          FST(IR,J)=0.
EXTRUDE  978      330 FST(IZ,J)=0.
EXTRUDE  979          DO 400 I=1, NBF
EXTRUDE  980          II=NF(I)
EXTRUDE  981          I7=3*II-1
EXTRUDE  982          IR=IZ-1
EXTRUDE  983          IIZ=2*I
EXTRUDE  984          IIR=II7-1
EXTRUDE  985          DO 401 J=MBAND, MBAND2
EXTRUDE  986          JJ=J-MBAND+1
EXTRUDE  987          FST(IIR,J)=A(IR,JJ)
EXTRUDE  988      401 FST(IIZ,J)=A(IZ,JJ)
EXTRUDE  989          DO 403 J=1, MBAND
EXTRUDE  990          NR=IR-J+1
EXTRUDE  991          NZ=IZ-J+1
EXTRUDE  992          JJ=MBAND-J+1
EXTRUDE  993          IF(NR .LE. 0) GO TO 404
EXTRUDE  994          FST(IIR,JJ)=A(NR,J)
EXTRUDE  995      404 IF(NZ .LE. 0) GO TO 403
EXTRUDE  996          FST(IIZ,JJ)=A(NZ,J)
EXTRUDE  997      403 CONTINUE
EXTRUDE  998          FPUR(IIR)=B(IR)
EXTRUDE  999      400 FPUR(IIZ)=B(IZ)
EXTRUDE 1000      402 CONTINUE
EXTRUDE 1001    C
EXTRUDE 1002    C ***********************************************************************
EXTRUDE 1003    C ADD PRESSURE BOUNDARY CONDITION. (FOR CONSTANT FRICTIONAL STRESS DIES,
EXTRUDE 1004    C THE SHEAR TRACTIONAL STRESS (FRICTIONAL STRESS) IS ADDING HERE).
EXTRUDE 1005    C ***********************************************************************
EXTRUDE 1006    C
EXTRUDE 1007          IF ( NUMPC .EQ. 0 ) GO TO 410
EXTRUDE 1008          ID(1)=2
EXTRUDE 1009          ID(2)=1
EXTRUDE 1010          RI=1.0
EXTRUDE 1011          RJ=1.0
EXTRUDE 1012          DO 420 L=1,NUMPC
EXTRUDE 1013          I=IJBC(L,1)
EXTRUDE 1014          J=IJBC(L,2)
EXTRUDE 1015          DR=R(I)-R(J)
EXTRUDE 1016          DZ=Z(I)-Z(J)
```

```
EXTRUDE 1017          DO 290 M=1,2
EXTRUDE 1018          N=ID(M)
EXTRUDE 1019          I=IJBC(L,M)
EXTRUDE 1020          J=IJBC(L,N)
EXTRUDE 1021          RI=R(I)
EXTRUDE 1022          RJ=R(J)
EXTRUDE 1023          I2=2*I-1
EXTRUDE 1024          I1=I2-1
EXTRUDE 1025          PI=PSBC(L,M)
EXTRUDE 1026          PJ=PSBC(L,N)
EXTRUDE 1027          SI=PSBC(L,M+2)
EXTRUDE 1028          SJ=PSBC(L,N+2)
EXTRUDE 1029          PM=(RI*(3.0*PI+PJ)+RJ*(PI+PJ))/12.0
EXTRUDE 1030          SM=(RI*(3.0*SI+SJ)+RJ*(SI+SJ))/12.0
EXTRUDE 1031          R1=DZ*PM+DR*SM
EXTRUDE 1032          R2=-DR*PM+DZ*SM
EXTRUDE 1033          B(I1)=B(I1)+R1
EXTRUDE 1034          B(I2)=B(I2)+R2
EXTRUDE 1035      290 CONTINUE
EXTRUDE 1036      420 CONTINUE
EXTRUDE 1037      410 CONTINUE
EXTRUDE 1038      C
EXTRUDE 1039     1005 FORMAT(// 29H ELEMENT WITH NEGATIVE AREA =, I5)
EXTRUDE 1040      C
EXTRUDE 1041          RETURN
EXTRUDE 1042          END
```

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EXTRUDE 1044          SUBROUTINE QUAD1(RR,ZZ,VOL)
EXTRUDE 1045      C
EXTRUDE 1046      C ***************************************************************************
EXTRUDE 1047      C CONSTRUCT B MATRIX AND K MATRIX (K=(TRANPOSE(B))(D)(B)), AND Q VECTOR
EXTRUDE 1048      C AT ELEMENT LEVEL FOR ALL ELEMENTS AND WRITE THESE MATRIX ON TAPE 1.
EXTRUDE 1049      C ***************************************************************************
EXTRUDE 1050      C
EXTRUDE 1051          COMMON /QUAD/ B(4,8),XX(8,8),Q(8)
EXTRUDE 1052          DIMENSION RR(4),ZZ(4),SS(4),TT(4)
EXTRUDE 1053          DATA SS/-1.,+1.,+1.,-1./, TT/-1.,-1.,+1.,+1./
EXTRUDE 1054          SQRT2=1.4142135623730?2
EXTRUDE 1055          DO 1 I=1,8
EXTRUDE 1056          Q(I)=0.
EXTRUDE 1057          DO 1 J=1,4
EXTRUDE 1058        1 B(J,I)=0.
EXTRUDE 1059          R1=RR(1)
EXTRUDE 1060          R2=RR(2)
EXTRUDE 1061          R3=RR(3)
EXTRUDE 1062          R4=RR(4)
EXTRUDE 1063          Z1=ZZ(1)
EXTRUDE 1064          Z2=ZZ(2)
EXTRUDE 1065          Z3=ZZ(3)
EXTRUDE 1066          Z4=ZZ(4)
EXTRUDE 1067          R12=R1-R2
EXTRUDE 1068          R13=R1-R3
EXTRUDE 1069          R14=R1-R4
EXTRUDE 1070          R23=R2-R3
EXTRUDE 1071          R24=R2-R4
EXTRUDE 1072          R34=R3-R4
EXTRUDE 1073          Z12=Z1-Z2
EXTRUDE 1074          Z13=Z1-Z3
EXTRUDE 1075          Z14=Z1-Z4
EXTRUDE 1076          Z23=Z2-Z3
EXTRUDE 1077          Z24=Z2-Z4
EXTRUDE 1078          Z34=Z3-Z4
EXTRUDE 1079          VOL=R13*Z24-R24*Z13
EXTRUDE 1080          IF(VOL.LE.0.) RETURN
EXTRUDE 1081      C
EXTRUDE 1082      C ***************************************************************************
EXTRUDE 1083      C CALCULATION OF TOTAL VOLUME PER RADIAN OF THE ELEMENT.
EXTRUDE 1084      C ***************************************************************************
EXTRUDE 1085      C
EXTRUDE 1086          VOL1=(Z1-Z4)*(R1*R1+R4*R4+R1*R4)
EXTRUDE 1087          VOL2=(Z3-Z2)*(R3*R3+R2*R2+R3*R2)
EXTRUDE 1088          VOL3=(Z2-Z1)*(R1*R1+R2*R2+R1*R2)
EXTRUDE 1089          VOL4=(Z4-Z3)*(R3*R3+R4*R4+R3*R4)
EXTRUDE 1090          VOLL=(VOL1+VOL2+VOL3+VOL4)/6.
EXTRUDE 1091      C
EXTRUDE 1092      C ***************************************************************************
EXTRUDE 1093      C THE FOLLOWING CONSTRUCT B AND K MATRIX BY TAKING FOUR INTEGRATION
EXTRUDE 1094      C POINTS OF THE ELEMENT.
EXTRUDE 1095      C ***************************************************************************
EXTRUDE 1096      C
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EXTRUDE 1097          DO 16 II=1,4
EXTRUDE 1098          S=SS(II)*0.577350269189626
EXTRUDE 1099          T=TT(II)*0.577350269189626
EXTRUDE 1100          XJ8=VOL+S*(R34*Z12-R12*Z34)+T*(R23*Z14-R14*Z23)
EXTRUDE 1101          XJ=XJ8/8.
EXTRUDE 1102      C
EXTRUDE 1103          SMM=1.-S
EXTRUDE 1104          SP=1.+S
EXTRUDE 1105          TM=1.-T
EXTRUDE 1106          TP=1.+T
EXTRUDE 1107          H1=0.25*SMM*TM
EXTRUDE 1108          H2=0.25*SP*TM
EXTRUDE 1109          H3=0.25*SP*TP
EXTRUDE 1110          H4=0.25*SMM*TP
EXTRUDE 1111          R=H1*R1+H2*R2+H3*R3+H4*R4
EXTRUDE 1112          G1=H1/R
EXTRUDE 1113          G2=H2/R
EXTRUDE 1114          G3=H3/R
EXTRUDE 1115          G4=H4/R
EXTRUDE 1116      C
EXTRUDE 1117      C ***************************************************************************
EXTRUDE 1118      C CONSTRUCT B MATRIX AT EACH POINT
EXTRUDE 1119      C ***************************************************************************
EXTRUDE 1120      C
EXTRUDE 1121          X1=(-R24+R34*S+R23*T)/XJ8
EXTRUDE 1122          X2=(R13-R34*S-R14*T)/XJ8
EXTRUDE 1123          X3=(R24-R12*S+R14*T)/XJ8
EXTRUDE 1124          X4=(-R13+R12*S-R23*T)/XJ8
EXTRUDE 1125          Y1=(Z24-Z34*S-Z23*T)/XJ8
EXTRUDE 1126          Y2=(-Z13+Z34*S+Z14*T)/XJ8
EXTRUDE 1127          Y3=(-Z24+Z12*S-Z14*T)/XJ8
EXTRUDE 1128          Y4=(Z13-Z12*S+Z23*T)/XJ8
EXTRUDE 1129          B(1, 1)=Y1
EXTRUDE 1130          B(1, 3)=Y2
EXTRUDE 1131          B(1, 5)=Y3
EXTRUDE 1132          B(1, 7)=Y4
EXTRUDE 1133          B(2, 2)=X1
EXTRUDE 1134          B(2, 4)=X2
EXTRUDE 1135          B(2, 6)=X3
EXTRUDE 1136          B(2, 8)=X4
EXTRUDE 1137          B(3, 1)=G1
EXTRUDE 1138          B(3, 3)=G2
EXTRUDE 1139          B(3, 5)=G3
EXTRUDE 1140          B(3, 7)=G4
EXTRUDE 1141          B(4, 1)=X1/SQRT2
EXTRUDE 1142          B(4, 2)=Y1/SQRT2
EXTRUDE 1143          B(4, 3)=X2/SQRT2
EXTRUDE 1144          B(4, 4)=Y2/SQRT2
EXTRUDE 1145          B(4, 5)=X3/SQRT2
EXTRUDE 1146          B(4, 6)=Y3/SQRT2
EXTRUDE 1147          B(4, 7)=X4/SQRT2
EXTRUDE 1148          B(4, 8)=Y4/SQRT2
EXTRUDE 1149      C
EXTRUDE 1150      C ***************************************************************************
EXTRUDE 1151      C CONSTRUCT K MATRIX AT EACH POINT (IE. XX(I,J))
EXTRUDE 1152      C ***************************************************************************
EXTRUDE 1153      C
EXTRUDE 1154          DO 6 I=1,8
EXTRUDE 1155          DO 6 J=I,8
EXTRUDE 1156          DUM1=0.
EXTRUDE 1157          DO 5 K=1,4
EXTRUDE 1158        5 DUM1=DUM1+B(K,J)*B(K,I)
EXTRUDE 1159          XX(I,J)=DUM1
EXTRUDE 1160          XX(J,I)=DUM1
EXTRUDE 1161        6 CONTINUE
EXTRUDE 1162      C
EXTRUDE 1163      C ***************************************************************************
EXTRUDE 1164      C  CONSTRUCT Q VECTOR AT EACH POINT, AND ADD ALL FOUR INTEGRATION POINTS
EXTRUDE 1165      C ***************************************************************************
EXTRUDE 1166      C
EXTRUDE 1167          XJR=XJ*R
EXTRUDE 1168          Q(1)=Q(1)+(Y1+G1)*XJ*R
EXTRUDE 1169          Q(2)=Q(2)+    X1  *XJ*R
EXTRUDE 1170          Q(3)=Q(3)+(Y2+G2)*XJ*R
EXTRUDE 1171          Q(4)=Q(4)+    X2  *XJ*R
EXTRUDE 1172          Q(5)=Q(5)+(Y3+G3)*XJ*R
EXTRUDE 1173          Q(6)=Q(6)+    X3  *XJ*R
EXTRUDE 1174          Q(7)=Q(7)+(Y4+G4)*XJ*R
EXTRUDE 1175          Q(8)=Q(8)+    X4  *XJ*R
EXTRUDE 1176      C
EXTRUDE 1177      C ***************************************************************************
EXTRUDE 1178      C WRITE K AND B MATRICES AT EACH INTEGRATION POINT ON TAPE 1
EXTRUDE 1179      C ***************************************************************************
EXTRUDE 1180      C
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EXTRUDE 1181          WRITE(1) XJR
EXTRUDE 1182          WRITE(1)((XX(I,J),J=1,8 ),I=1,8 )
EXTRUDE 1183          WRITE(1)((P(I,J),J=1,8 ),I=1,4)
EXTRUDE 1184       16 CONTINUE
EXTRUDE 1185    C
EXTRUDE 1186    C ***********************************************************************
EXTRUDE 1187    C WRITE Q VECTOR (AFTER INTEGRATION, I.E. ADDING FOUR POINTS) AND THE
EXTRUDE 1188    C TOTAL VOLUMN OF THE ELEMENT ON TAPE 1.
EXTRUDE 1189    C ***********************************************************************
EXTRUDE 1190    C
EXTRUDE 1191          WRITE(1)(Q(I),I=1,8)
EXTRUDE 1192          VOL=VOLL
EXTRUDE 1193          WRITE(1) VOL
EXTRUDE 1194    C
EXTRUDE 1195          RETURN
EXTRUDE 1196          END



EXTRUDE 1198          SUBROUTINE QUAD2(UU,IPLNAX,TEX,TEY,TEZ,TEXY)
EXTRUDE 1199    C
EXTRUDE 1200    C ***********************************************************************
EXTRUDE 1201    C READ INFORMATION FROM TAPE1 REGARDING B, K AND Q, AND CONSTRUCT P AND
EXTRUDE 1202    C H MATRICES AT ELEMENT LEVEL (I.E.P(I,J) AND H(I))
EXTRUDE 1203    C ***********************************************************************
EXTRUDE 1204    C
EXTRUDE 1205          COMMON /QUAD/ B(4,8),XX(8,8),BZERO(8)
EXTRUDE 1206          COMMON /STFMAT/ P(9,9),H(9)
EXTRUDE 1207          DIMENSION E(4),UU(8)
EXTRUDE 1208          DO 1 I=1,9
EXTRUDE 1209          H(I)=0.
EXTRUDE 1210          DO 1 J=I,9
EXTRUDE 1211        1 P(I,J)  =  0.
EXTRUDE 1212          SQRT2  =  1.4142135623730952
EXTRUDE 1213          SQRT1  =  1.2247448713915886
EXTRUDE 1214    C
EXTRUDE 1215    C ***********************************************************************
EXTRUDE 1216    C TEX= R-STRAIN RATE
EXTRUDE 1217    C TEY= Z-STRAIN RATE
EXTRUDE 1218    C TEZ= TH-STRAIN RATE
EXTRUDE 1219    C TEXY= RZ-STRAIN RATE
EXTRUDE 1220    C ***********************************************************************
EXTRUDE 1221    C
EXTRUDE 1222          TEX  =  0.
EXTRUDE 1223          TEY  =  0.
EXTRUDE 1224          TEZ  =  0.
EXTRUDE 1225          TEXY=0.
EXTRUDE 1226          DO 16 II=1,4
EXTRUDE 1227          READ(1)XJR
EXTRUDE 1228          READ(1)((XX(I,J),J=1,8 ),I=1,8 )
EXTRUDE 1229          READ(1) ((B(I,J),J=1,8 ),I=1,4)
EXTRUDE 1230          DO 8 I=1,8
EXTRUDE 1231          DUM1=0.
EXTRUDE 1232          DO 7 J=1,8
EXTRUDE 1233        7 DUM1=DUM1+XX(I,J)*UU(J)
EXTRUDE 1234        8 BZERO(I)=DUM1
EXTRUDE 1235          PZERO=0.
EXTRUDE 1236          DO 9 I=1,8
EXTRUDE 1237        9 PZERO=PZERO+UU(I)*BZERO(I)
EXTRUDE 1238    C
EXTRUDE 1239    C ***********************************************************************
EXTRUDE 1240    C PZERO=(3/2) * SQUARE OF (EFF. STRINT RATE).
EXTRUDE 1241    C IF PZERO IS TOO SMALL CORRESPONDING TO RIGID ELEMENT, SET PZERO AS A
EXTRUDE 1242    C LIMITING VALUE IN ORDER NOT TO BREAK THE STIFFNESS MATRIX
EXTRUDE 1243    C P1=(2/3)/(SQUARE OF (EFFECTIVE STRAIN RATE))
EXTRUDE 1244    C P2=(2/3)/(EFFECTIVE STRAIN RATE)
EXTRUDE 1245    C ***********************************************************************
EXTRUDE 1246    C
EXTRUDE 1247          IF(PZERO.LE.1.E-08) PZERO=1.E-08
EXTRUDE 1248          P1=1./PZERO
EXTRUDE 1249          PZERO=SQRT(PZERO)
EXTRUDE 1250          P2=PZERO*SQRT1
EXTRUDE 1251          DO 13 I=1,8
EXTRUDE 1252          DO 12 J=I,8
EXTRUDE 1253       12 P(I,J)=P(I,J)+(XX(I,J)-P1*BZERO(I)*BZERO(J))*XJR/P2
EXTRUDE 1254       13 H(I)=H(I)+BZERO(I)*XJR/P2
EXTRUDE 1255          DO 14 I=1,4
EXTRUDE 1256          E(I)=0.
EXTRUDE 1257          DO 14 J=1,8
EXTRUDE 1258       14 E(I)=E(I)+B(I,J)*UU(J)
EXTRUDE 1259          TEX=TEX+E(1)*XJR
EXTRUDE 1260          TEY=TEY+E(2)*XJR
EXTRUDE 1261          TEZ=TEZ+E(3)*XJR
EXTRUDE 1262          TEXY=TEXY+E(4)*SQRT2*XJR
EXTRUDE 1263       16 CONTINUE
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EXTRUDE 1264    C
EXTRUDE 1265          READ(1)(P(J,9 ),J=1,8 )
EXTRUDE 1266          DO 17 J=1,8
EXTRUDE 1267       17 H(9 )=H(9 )+P(J,9 )*UU(J)
EXTRUDE 1268          READ (1) VOL
EXTRUDE 1269          DO 18 I=1,9
EXTRUDE 1270          DO 18 J=I,9
EXTRUDE 1271       18 P(J,I)=P(I,J)
EXTRUDE 1272          TEX=TEX/VOL
EXTRUDE 1273          TEY=TEY/VOL
EXTRUDE 1274          TEZ=TEZ/VOL
EXTRUDE 1275          TEXY=TEXY/VOL
EXTRUDE 1276    C
EXTRUDE 1277          RETURN
EXTRUDE 1278          END



EXTRUDE 1280          FUNCTION RBAR (RX,RY,RZ,RXY)
EXTRUDE 1281    C
EXTRUDE 1282    C ***************************************************************************
EXTRUDE 1283    C CALCULATE THE EFFECTIVE STRAIN RATE
EXTRUDE 1284    C ***************************************************************************
EXTRUDE 1285    C
EXTRUDE 1286          RBAR=(RX-RY)**2+(RY-RZ)**2+(RZ-RX)**2
EXTRUDE 1287          RBAR=2.*SQRT(0.5*RBAR+3.*RXY**2/4.)/3.
EXTRUDE 1288    C
EXTRUDE 1289    C ***************************************************************************
EXTRUDE 1290    C IF THE EFFECTIVE STRAIN RATE IS TOO SMALL CORRESPONDING TO A RIGID
EXTRUDE 1291    C ELEMENT, SET IT TO A LIMITING VALUE.
EXTRUDE 1292    C ***************************************************************************
EXTRUDE 1293    C
EXTRUDE 1294          IF(RBAR.LT.1.E-04) RBAR=1.E-04
EXTRUDE 1295          RETURN
EXTRUDE 1296          END



EXTRUDE 1298          FUNCTION EFSTRS(SR,SZ,ST,SRZ)
EXTRUDE 1299    C
EXTRUDE 1300    C ***************************************************************************
EXTRUDE 1301    C CALCULATION OF EFFECTIVE STRESS.
EXTRUDE 1302    C ***************************************************************************
EXTRUDE 1303    C
EXTRUDE 1304          EFSTRS=(SR-SZ)**2+(SZ-ST)**2+(ST-SR)**2
EXTRUDE 1305          EFSTRS=SQRT(0.5*EFSTRS+3.*SRZ**2)
EXTRUDE 1306          RETURN
EXTRUDE 1307          END



EXTRUDE 1309          SUBROUTINE MODIFY (CODE,A,B,NUMNP,NEQ,MBAND)
EXTRUDE 1310    C
EXTRUDE 1311    C ***************************************************************************
EXTRUDE 1312    C  DETERMINE THE POINTS FOR WHICH CORRESPONDING COMPONENT OF X IS
EXTRUDE 1313    C  SPECIFIED EQUAL TO ZERO IN AX=B, AND CALL CONDEN FOR MATRIX
EXTRUDE 1314    C  CONDENSATION
EXTRUDE 1315    C ***************************************************************************
EXTRUDE 1316    C
EXTRUDE 1317          COMMON /WALL/ THETA,FT,TANTH
EXTRUDE 1318          DIMENSION CODE(1),A(NEQ,1),B(1)
EXTRUDE 1319          DO 121 I=1, NUMNP
EXTRUDE 1320          IL=3*I
EXTRUDE 1321          IZ=IL-1
EXTRUDE 1322          IP=IZ-1
EXTRUDE 1323          C=CODE(I)
EXTRUDE 1324    C
EXTRUDE 1325    C ***************************************************************************
EXTRUDE 1326    C  CHECK IF THIS POINT CONTAINS NO INFORMATION ABOUT MEAN STRESS
EXTRUDE 1327    C ***************************************************************************
EXTRUDE 1328    C
EXTRUDE 1329          IF(C.LT.10.) CALL CONDEN(A,B,NEQ,MBAND,IL,0.)
EXTRUDE 1330    C
EXTRUDE 1331    C ***************************************************************************
EXTRUDE 1332    C  CHECK IF THE R-VELOCITY IS SPECIFIED
EXTRUDE 1333    C ***************************************************************************
EXTRUDE 1334    C
EXTRUDE 1335          IF(C.EQ.1..OR.C.EQ.11..OR.C.EQ.3..OR.C.EQ.13.)
EXTRUDE 1336         1  CALL CONDEN(A,B,NEQ,MBAND,IP,0.)
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EXTRUDE 1337    C
EXTRUDE 1338    C ***************************************************************
EXTRUDE 1339    C  CHECK IF THE Z-VELOCITY IS SPECIFIED
EXTRUDE 1340    C ***************************************************************
EXTRUDE 1341    C
EXTRUDE 1342          IF (C.EQ.2..OR.C.EQ.12..OR.C.EQ.3..OR.C.EQ.13.)
EXTRUDE 1343         1  CALL CONDEN(A,B,NEQ,MBAND,IZ,0.)
EXTRUDE 1344    C
EXTRUDE 1345    C ***************************************************************
EXTRUDE 1346    C  CHECK IF THE POINT IS ALONG THE INCLINED BOUNDARY
EXTRUDE 1347    C ***************************************************************
EXTRUDE 1348    C
EXTRUDE 1349          IF(C.EQ.5..OR.C.EQ.15.) CALL BCMIX(A,B,NEQ,MBAND,I,THETA)
EXTRUDE 1350      121 CONTINUE
EXTRUDE 1351    C
EXTRUDE 1352          RETURN
EXTRUDE 1353          END



EXTRUDE 1355          SUBROUTINE CONDEN(A,B,NEQ,MBAND,N,U)
EXTRUDE 1356    C
EXTRUDE 1357    C ***************************************************************
EXTRUDE 1358    C PERFORM THE MATRIX CONDENSATION WHEN THE VALUE OF A COMPONENT X IN
EXTRUDE 1359    C AX=B IS SPECIFIED EQUAL TO ZERO.
EXTRUDE 1360    C ***************************************************************
EXTRUDE 1361    C
EXTRUDE 1362          DIMENSION B(NEQ),A(NEQ,1)
EXTRUDE 1363          DO 235 M=2,MBAND
EXTRUDE 1364          K=N-M+1
EXTRUDE 1365          IF(K) 235,235,230
EXTRUDE 1366      230 A(K,M)=0.0
EXTRUDE 1367      235 A(N,M)=0.0
EXTRUDE 1368          A(N,1)=1.0
EXTRUDE 1369          B(N)=U
EXTRUDE 1370          RETURN
EXTRUDE 1371          END



EXTRUDE 1373          SUBROUTINE BCMIX(A,B,NEQ,MBAND,N,THETA)
EXTRUDE 1374    C
EXTRUDE 1375    C ***************************************************************
EXTPUDE 1376    C  THIS SUBROUTINE IS TO ENSURE THAT THE VELOCITY ALONG THE DIE IS
EXTRUDE 1377    C  TANGENTIAL TO THE DIE FOR CONICAL DIES
EXTRUDE 1378    C ***************************************************************
EXTRUDE 1379    C
EXTRUDE 1380          DIMENSION B(NEQ),A(NEQ,1)
EXTRUDE 1381    C
EXTRUDE 1382    C ***************************************************************
EXTRUDE 1383    C  SINCE UR=UZ*TAN(THETA) ALONG THE DIE, A CORRESPONDING CHANGE IS MADE
EXTRUDE 1384    C  IN THE STIFFNESS EQUATIONS FOR ROWS AND COLUMNS CORRESPONDING TO
EXTRUDE 1385    C  THESE COMPONENTS...THEN THE EQUATIONS CONTAINING UR ARE ELIMINATED
EXTRUDE 1386    C ***************************************************************
EXTRUDE 1387    C
EXTRUDE 1388          NZ=3*N-1
EXTRUDE 1389          NR=NZ-1
EXTRUDE 1390          ALPA= TAN(THETA)
EXTRUDE 1391          DO 350 M=1,MBAND
EXTRUDE 1392      350 A(NR,M)=A(NR,M)*ALPA
EXTRUDE 1393          A(NR,1)=A(NR,1)*ALPA
EXTRUDE 1394          A(NR,2)=A(NR,2)*2.
EXTRUDE 1395          DO 340 M=2,MBAND
EXTRUDE 1396          KR=NR-M+1
EXTRUDE 1397          IF(KR.LE.0) GO TO 341
EXTRUDE 1398      340 A(KR,M)=A(KR,M)*ALPA
EXTRUDE 1399      341 CONTINUE
EXTRUDE 1400          B(NP) = B(NR)*ALPA
EXTRUDE 1401          DO 351 M=2,MBAND
EXTRUDE 1402          KZ=NZ-M+1
EXTRUDE 1403          IF(KZ.LE.0) GO TO 352
EXTRUDE 1404          A(KZ,M)=A(KZ,M)+A(KZ,M-1)
EXTRUDE 1405      352 CONTINUE
EXTRUDE 1406          IF(M.EQ.MBAND) GO TO 353
EXTRUDE 1407          KZ=NZ+M-1
EXTRUDE 1408          IF(KZ.GT.NEQ) GO TO 353
EXTRUDE 1409          A(NZ,M)=A(NZ,M)+A(NR,M+1)
EXTRUDE 1410      353 CONTINUE
EXTRUDE 1411      351 CONTINUE
EXTRUDE 1412          A(NZ,1)=A(NZ,1)+A(NR,2)
EXTRUDE 1413          A(NR,1)=1.0
```

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EXTRUDE 1414          DO 355 M=2,MBAND
EXTRUDE 1415          KR=NR-M+1
EXTRUDE 1416          IF(KR.LE.0) GO TO 360
EXTRUDE 1417          A(KR,M)=0.0
EXTRUDE 1418      360 CONTINUE
EXTRUDE 1419          A(NR,M)=0.0
EXTRUDE 1420      355 CONTINUE
EXTRUDE 1421          B(NZ)=B(NR)+B(NZ)
EXTRUDE 1422          B(NR)=0.0
EXTRUDE 1423      C
EXTRUDE 1424          RETURN
EXTRUDE 1425          END



EXTRUDE 1427          SUBROUTINE TRIA(NN,MM,A)
EXTRUDE 1428      C
EXTRUDE 1429      C ***************************************************************************
EXTRUDE 1430      C TRIANGULARIZATION OF GUASSIAN ELIMINATION FOR THE SOLUTION OF BANDED
EXTRUDE 1431      C SYMMETRIC MATRIX.
EXTRUDE 1432      C ***************************************************************************
EXTRUDE 1433      C
EXTRUDE 1434          DIMENSION A(NN,1)
EXTRUDE 1435     1000 N=0
EXTRUDE 1436      100 N=N+1
EXTRUDE 1437          IF(N.EQ.NN)RETURN
EXTRUDE 1438          IF(A(N,1).EQ.0.0)GO TO 100
EXTRUDE 1439          I=N
EXTRUDE 1440          MB=MIN0(MM,NN-N+1)
EXTRUDE 1441          DO 260 L=2,MB
EXTRUDE 1442          I=I+1
EXTRUDE 1443          C=A(N,L)/A(N,1)
EXTRUDE 1444          IF(C.EQ.0.0)GO TO 260
EXTRUDE 1445          J=0
EXTRUDE 1446          DO 250 K=L,MB
EXTRUDE 1447          J=J+1
EXTRUDE 1448      250 A(I,J)=A(I,J)-C*A(N,K)
EXTRUDE 1449          A(N,L)=C
EXTRUDE 1450      260 CONTINUE
EXTRUDE 1451          GO TO 100
EXTRUDE 1452          END



EXTRUDE 1454          SUBROUTINE BACKS(NN,MM,A,B)
EXTRUDE 1455      C
EXTRUDE 1456      C ***************************************************************************
EXTRUDE 1457      C BACK SUBSTITUTION FOR SOLUTION OF BANDED SYMMETRIC MATRIX
EXTRUDE 1458      C ***************************************************************************
EXTRUDE 1459      C
EXTRUDE 1460          DIMENSION A(1),B(1)
EXTRUDE 1461          MMM=MM-1
EXTRUDE 1462          N=0
EXTRUDE 1463      270 N=N+1
EXTRUDE 1464          C=B(N)
EXTRUDE 1465          IF(A(N).NE.0.0)B(N)=B(N)/A(N)
EXTRUDE 1466          IF(N.EQ.NN)GO TO 300
EXTRUDE 1467          IL=N+1
EXTRUDE 1468          IH=MIN0(NN,N+MMM)
EXTRUDE 1469          M=N
EXTRUDE 1470          DO 285 I=IL,IH
EXTRUDE 1471          M=M+NN
EXTRUDE 1472      285 B(I)=B(I)-A(M)*C
EXTRUDE 1473          GO TO 270
EXTRUDE 1474      300 IL=N
EXTRUDE 1475          N=N-1
EXTRUDE 1476          IF(N.EQ.0) RETURN
EXTRUDE 1477          IH=MIN0(NN,N+MMM)
EXTRUDE 1478          M=N
EXTRUDE 1479          DO 400 I=IL,IH
EXTRUDE 1480          M=M+NN
EXTRUDE 1481      400 B(N)=B(N)-A(M)*B(I)
EXTRUDE 1482          GO TO 300
EXTRUDE 1483      C
EXTRUDE 1484          END
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EXTRUDE 1486          SUBROUTINE CFORCE(NF,FR,FZ,FST,FPUR,B,MBAND,NBF,NBF2)
EXTRUDE 1487    C
EXTRUDE 1488    C ***********************************************************************
EXTRUDE 1489    C CALCULATION OF FORCES ON BOUNDARY NODAL POINTS
EXTRUDE 1490    C ***********************************************************************
EXTRUDE 1491    C
EXTRUDE 1492          DIMENSION NF(1),FST(NBF2,1),FPUR(NBF2),FR(1),FZ(1),B(1)
EXTRUDE 1493          IF(NBF .LE. 0) GO TO 124
EXTRUDE 1494          MBAND2=2*MBAND-1
EXTRUDE 1495          DO 121 I=1, NBF
EXTRUDE 1496          IZ=3*NF(I)-1
EXTRUDE 1497          IR=IZ-1
EXTRUDE 1498          IIZ=2*I
EXTRUDE 1499          IIR=IIZ-1
EXTRUDE 1500          SUMR=0.
EXTRUDE 1501          SUMZ=0.
EXTRUDE 1502          DO 122 J=1, MBAND2
EXTRUDE 1503          JR=IR+J-MBAND
EXTRUDE 1504          JZ=JR+1
EXTRUDE 1505          IF(JR .LE. 0) GO TO 126
EXTRUDE 1506          SUMR=SUMR+FST(IIR,J)*B(JR)
EXTRUDE 1507      126 IF(JZ .LE. 0) GO TO 122
EXTRUDE 1508          SUMZ=SUMZ+FST(IIZ,J)*B(JZ)
EXTRUDE 1509      122 CONTINUE
EXTRUDE 1510          FR(I)=SUMR-FPUR(IIR)
EXTRUDE 1511      121 FZ(I)=SUMZ-FPUR(IIZ)
EXTRUDE 1512      124 CONTINUE
EXTRUDE 1513          RETURN
EXTRUDE 1514          END
```

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EXTRUDE 1516          SUBROUTINE INTRPOL (X,Y,U,XX,YY,UU,IFLAG)
EXTRUDE 1517    C
EXTRUDE 1518    C ***********************************************************************
EXTRUDE 1519    C  INTERPOLATE THE VALUE OF UU AT COORDINATES (XX,YY) BY KNOWING THE
EXTRUDE 1520    C  VALUES OF U AT FOUR SURROUNDING POINTS.
EXTRUDE 1521    C ***********************************************************************
EXTRUDE 1522    C
EXTRUDE 1523          DIMENSION X(4),Y(4),U(4),A(4,4),COEF(4)
EXTRUDE 1524    C
EXTRUDE 1525    C ***********************************************************************
EXTRUDE 1526    C  DETERMINE THE MATRIX A, THE INVERSE OF THE INTERPOLATION MATRIX...
EXTRUDE 1527    C  IF IFLAG=1, THE A MATRIX IS ALREADY KNOWN
EXTRUDE 1528    C ***********************************************************************
EXTRUDE 1529    C
EXTRUDE 1530          IF(IFLAG.EQ.1) GO TO 100
EXTRUDE 1531          X1=X(1)
EXTRUDE 1532          X2=X(2)
EXTRUDE 1533          X3=X(3)
EXTRUDE 1534          X4=X(4)
EXTRUDE 1535          Y1=Y(1)
EXTRUDE 1536          Y2=Y(2)
EXTRUDE 1537          Y3=Y(3)
EXTRUDE 1538          Y4=Y(4)
EXTRUDE 1539          X12=X1-X2
EXTRUDE 1540          X13=X1-X3
EXTRUDE 1541          X14=X1-X4
EXTRUDE 1542          X23=X2-X3
EXTRUDE 1543          X24=X2-X4
EXTRUDE 1544          X34=X3-X4
EXTRUDE 1545          Y12=Y1*Y2*X12
EXTRUDE 1546          Y13=Y1*Y3*X13
EXTRUDE 1547          Y14=Y1*Y4*X14
EXTRUDE 1548          Y23=Y2*Y3*X23
EXTRUDE 1549          Y24=Y2*Y4*X24
EXTRUDE 1550          Y34=Y3*Y4*X34
EXTRUDE 1551          A(1,1)=+X2*Y34-X3*Y24+X4*Y23
EXTRUDE 1552          A(1,2)=-X1*Y34+X3*Y14-X4*Y13
EXTRUDE 1553          A(1,3)=+X1*Y24-X2*Y14+X4*Y12
EXTRUDE 1554          A(1,4)=-X1*Y23+X2*Y13-X3*Y12
EXTRUDE 1555          A(2,1)=-Y23+Y24-Y34
EXTRUDE 1556          A(2,2)=+Y13-Y14+Y34
EXTRUDE 1557          A(2,3)=-Y12+Y14-Y24
EXTRUDE 1558          A(2,4)=+Y12-Y13+Y23
EXTRUDE 1559          Z11=Y1*X23
EXTRUDE 1560          Z12=Y1*X24
EXTRUDE 1561          Z13=Y1*X34
EXTRUDE 1562          Z21=Y2*X13
EXTRUDE 1563          Z22=Y2*X14
EXTRUDE 1564          Z23=Y2*X34
EXTRUDE 1565          Z31=Y3*X12
EXTRUDE 1566          Z32=Y3*X14
EXTRUDE 1567          Z33=Y3*X24
EXTRUDE 1568          Z41=Y4*X12
EXTRUDE 1569          Z42=Y4*X13
EXTRUDE 1570          Z43=Y4*X23
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EXTRUDE 1571          A(3,1)=+X2*Z23-X3*Z33+X4*Z43
EXTRUDE 1572          A(3,2)=-X1*Z13+X3*Z32-X4*Z42
EXTRUDE 1573          A(3,3)=+X1*Z12-X2*Z22+X4*Z41
EXTRUDE 1574          A(3,4)=-X1*Z11+X2*Z21-X3*Z31
EXTRUDE 1575          A(4,1)=-Z23+Z33-Z43
EXTRUDE 1576          A(4,2)=+Z13-Z32+Z42
EXTRUDE 1577          A(4,3)=-Z12+Z22-Z41
EXTRUDE 1578          A(4,4)=+Z11-Z21+Z31
EXTRUDE 1579          DETER=A(1,1)+A(1,2)+A(1,3)+A(1,4)
EXTRUDE 1580        100 CONTINUE
EXTRUDE 1581          IFLAG=0
EXTRUDE 1582          IF(DETER.EQ.0.) GO TO 150
EXTRUDE 1583          DO 130 J=1,4
EXTRUDE 1584          COEF(J)=0.
EXTRUDE 1585          DO 120 I=1,4
EXTRUDE 1586        120 COEF(J)=COEF(J)+A(J,I)*U(I)
EXTRUDE 1587        130 COEF(J)=COEF(J)/DETER
EXTRUDE 1588          UU=COEF(1)+COEF(2)*XX+COEF(3)*YY+COEF(4)*XX*YY
EXTRUDE 1589          RETURN
EXTRUDE 1590        150 UU=(U(1)*(YY-Y(2))+U(2)*(Y(1)-YY))/(Y(1)-Y(2))
EXTRUDE 1591          RETURN
EXTRUDE 1592          END
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EXTRUDE 1594          SUBROUTINE STRAINS (R,Z,UR,UZ,IEL,EPS,TEPS,STR,X,Y,XARB,U,V,XX,YY,
EXTRUDE 1595         1  A,B,AA,BB,C,EE,NI,NJ,NTIMES,NEL,NNP,NMAX)
EXTRUDE 1596      C
EXTRUDE 1597      C ***************************************************************************
EXTRUDE 1598      C  SUBROUTINE FOR DETERMINATION OF FLOW PATTERN, NETWORK OF GRID
EXTRUDE 1599      C  DISTORTIONS, THE EFFECTIVE STRAIN DISTRIBUTION AND THE VALUES OF
EXTRUDE 1600      C  EFFECTIVE STRAIN FOR EACH ELEMENT.
EXTRUDE 1601      C ***************************************************************************
EXTRUDE 1602      C
EXTRUDE 1603          DIMENSION R(1),Z(1),UR(1),UZ(1),IEL(NEL,1),EPS(5,1),TEPS(5,1),
EXTRUDE 1604         1  STR(NI,1),X(NI,1),Y(1),XARB(1),U(NI,1),V(NI,1),XX(NTIMES,1),
EXTRUDE 1605         2  YY(NTIMES,1),A(1),B(1),AA(NI,1),BB(1),C(1),EE(NTIMES,1)
EXTRUDE 1606          DIMENSION RR(4),ZZ(4),URR(4),UZZ(4)
EXTRUDE 1607          COMMON /STRPATH/ STEP,YSTART,YDIE,YEXIT,YMIN,RENTER,REXIT,VEXIT
EXTRUDE 1608          COMMON /WALL/ THETA,FT,TANTH
EXTRUDE 1609          NIM1=NI-1
EXTRUDE 1610          NJM1=NJ-1
EXTRUDE 1611      C
EXTRUDE 1612      C ***************************************************************************
EXTRUDE 1613      C  FIRST ARRANGE THE AVAILABLE INFORMATION IN PROPER FORM
EXTRUDE 1614      C ***************************************************************************
EXTRUDE 1615      C
EXTRUDE 1616          K=1
EXTRUDE 1617          L=0
EXTRUDE 1618        100 L=L+1
EXTRUDE 1619          Y(L)=Z(K)
EXTRUDE 1620          DO 110 I=1,NIM1
EXTRUDE 1621          X(I,L)=R(K)
EXTRUDE 1622          U(I,L)=UR(K)
EXTRUDE 1623          V(I,L)=UZ(K)
EXTRUDE 1624        110 K=K+1
EXTRUDE 1625          X(NI,L)=X(NIM1,L)
EXTRUDE 1626          U(NI,L)=U(NIM1,L)
EXTRUDE 1627          V(NI,L)=V(NIM1,L)
EXTRUDE 1628          IF (K.GE.NNP) 150,100
EXTRUDE 1629      C
EXTRUDE 1630      C ***************************************************************************
EXTRUDE 1631      C  DETERMINE THE COORDINATES OF THE CENTERS OF THE ELEMENTS
EXTRUDE 1632      C ***************************************************************************
EXTRUDE 1633      C
EXTRUDE 1634        150 DO 160 N=1,NEL
EXTRUDE 1635          I1=IEL(N,1)
EXTRUDE 1636          I2=IEL(N,2)
EXTRUDE 1637          I3=IEL(N,3)
EXTRUDE 1638          I4=IEL(N,4)
EXTRUDE 1639          A(N)=(R(I1)+R(I2)+R(I3)+R(I4))/4.
EXTRUDE 1640        160 B(N)=(Z(I1)+Z(I2)+Z(I3)+Z(I4))/4.
EXTRUDE 1641      C
EXTRUDE 1642      C ***************************************************************************
EXTRUDE 1643      C  ARRANGE THE NEWLY DETERMINED VALUES IN A PROPER FORM
EXTRUDE 1644      C ***************************************************************************
EXTRUDE 1645      C
EXTRUDE 1646          K=1
EXTRUDE 1647          L=0
EXTRUDE 1648        165 L=L+1
EXTRUDE 1649          BB(L)=B(K)
EXTRUDE 1650          AA(1,L)=0.
EXTRUDE 1651          STR(1,L)=EPS(5,K)
EXTRUDE 1652          DO 170 I=2,NIM1
EXTRUDE 1653          AA(I,L)=A(K)
EXTRUDE 1654          STR(I,L)=EPS(5,K)
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EXTRUDE 1655      170 K=K+1
EXTRUDE 1656          AA(NI,L)=REXIT+BB(L)*TANTH
EXTRUDE 1657          IF(BB(L).GT.YDIE) AA(NI,L)=1.
EXTRUDE 1658          IF(BB(L).LT.YEXIT) AA(NI,L)=REXIT
EXTRUDE 1659          STR(NI,L)=EPS(5,K-1)
EXTRUDE 1660          IF(K.LT.NEL) GO TO 165
EXTRUDE 1661      C
EXTRUDE 1662      C ***************************************************************************
EXTRUDE 1663      C  NOW DO THE INTERPOLATION AND DETERMINE THE FLOW PATTERN
EXTRUDE 1664      C ***************************************************************************
EXTRUDE 1665      C
EXTRUDE 1666          DO 500 N=1,NTIMES
EXTRUDE 1667      C
EXTRUDE 1668      C ***************************************************************************
EXTRUDE 1669      C  SET THE COORDINATES OF THE STARTING POINT
EXTRUDE 1670      C ***************************************************************************
EXTRUDE 1671      C
EXTRUDE 1672          XX(N,1)=C(N)
EXTRUDE 1673          YY(N,1)=YSTART
EXTRUDE 1674      C
EXTRUDE 1675      C ***************************************************************************
EXTRUDE 1676      C  DETERMINE THE LOCATION OF THE PRESENT COORDINATES OF THE POINT IN
EXTRUDE 1677      C  TERMS OF THE FOUR SURROUNDING NODAL POINTS
EXTRUDE 1678      C ***************************************************************************
EXTRUDE 1679      C
EXTRUDE 1680          IFLAG=0
EXTRUDE 1681          NCOUNT=0
EXTRUDE 1682          MOLD=1
EXTRUDE 1683          LOLD=1
EXTRUDE 1684      210 NCOUNT=NCOUNT+1
EXTRUDE 1685          XXX=XX(N,NCOUNT)
EXTRUDE 1686          YYY=YY(N,NCOUNT)
EXTRUDE 1687          IF(N.EQ.NTIMES.AND.YYY.LE.YEXIT) GO TO 300
EXTRUDE 1688          IF(YYY.LE.YMIN) GO TO 300
EXTRUDE 1689          DO 220 I=MOLD,NJM1
EXTRUDE 1690      220 IF(YYY.LE.Y(I).AND.YYY.GE.Y(I+1)) M=I
EXTRUDE 1691          DO 230 I=1,NI
EXTRUDE 1692      230 XARB(I)=X(I,M)-(X(I,M)-X(I,M+1))*(Y(M)-YYY)/(Y(M)-Y(M+1))
EXTRUDE 1693          DO 240 I=1,NIM1
EXTRUDE 1694      240 IF(XXX.GE.XARB(I).AND.XXX.LE.XARB(I+1)) L=I
EXTRUDE 1695          IF(NCOUNT.EQ.1) GO TO 250
EXTRUDE 1696          IF(M.EQ.MOLD.AND.L.EQ.LOLD) GO TO 260
EXTRUDE 1697      250 RR(1)=X(L,M)
EXTRUDE 1698          RR(2)=X(L,M+1)
EXTRUDE 1699          RR(3)=X(L+1,M+1)
EXTRUDE 1700          RR(4)=X(L+1,M)
EXTRUDE 1701          ZZ(1)=ZZ(4)=Y(M)
EXTRUDE 1702          ZZ(2)=ZZ(3)=Y(M+1)
EXTRUDE 1703          URR(1)=U(L,M)
EXTRUDE 1704          URR(2)=U(L,M+1)
EXTRUDE 1705          URR(3)=U(L+1,M+1)
EXTRUDE 1706          URR(4)=U(L+1,M)
EXTRUDE 1707          UZZ(1)=V(L,M)
EXTRUDE 1708          UZZ(2)=V(L,M+1)
EXTRUDE 1709          UZZ(3)=V(L+1,M+1)
EXTRUDE 1710          UZZ(4)=V(L+1,M)
EXTRUDE 1711          GO TO 270
EXTRUDE 1712      C
EXTRUDE 1713      C ***************************************************************************
EXTRUDE 1714      C  THE POINT IS LOCATED IN THE SAME ELEMENT AS IT WAS FOR THE LAST STEP.
EXTRUDE 1715      C  HENCE THE INVERSE OF THE INTERPOLATION MATRIX IS ALREADY KNOWN
EXTRUDE 1716      C ***************************************************************************
EXTRUDE 1717      C
EXTRUDE 1718      260 IFLAG=1
EXTRUDE 1719      C
EXTRUDE 1720      C ***************************************************************************
EXTRUDE 1721      C  DETERMINE U AND V AT THE POINT ON FLOW LINE BY INTERPOLATION.
EXTRUDE 1722      C ***************************************************************************
EXTRUDE 1723      C
EXTRUDE 1724      270 CALL INTRPOL(RR,ZZ,URR,XXX,YYY,UU,IFLAG)
EXTRUDE 1725          IFLAG=1
EXTRUDE 1726          CALL INTRPOL(RR,ZZ,UZZ,XXX,YYY,VV,IFLAG)
EXTRUDE 1727      C
EXTRUDE 1728      C ***************************************************************************
EXTRUDE 1729      C  DETERMINE THE NEXT LOCATION OF POINT ON FLOW LINE
EXTRUDE 1730      C ***************************************************************************
EXTRUDE 1731      C
EXTRUDE 1732          XX(N,NCOUNT+1)=XXX+UU*STEP
EXTRUDE 1733          YY(N,NCOUNT+1)=YYY+VV*STEP
EXTRUDE 1734          LOLD=L
EXTRUDE 1735          MOLD=M
EXTRUDE 1736          IF(N.NE.NTIMES) GO TO 310
EXTRUDE 1737          XXX=REXIT+YY(N,NCOUNT+1)*TANTH
EXTRUDE 1738          IF(XXX.LT.XX(N,NCOUNT+1)) XX(N,NCOUNT+1)=XXX
EXTRUDE 1739          IF(XX(N,NCOUNT+1).GT.1.) XX(N,NCOUNT+1)=1.
EXTRUDE 1740          GO TO 310
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EXTRUDE 1741    C
EXTRUDE 1742    C ***************************************************************************
EXTRUDE 1743    C  THE POINT IS MOVING WITH EXIT VELOCITY
EXTRUDE 1744    C ***************************************************************************
EXTRUDE 1745    C
EXTRUDE 1746      300 XX(N,NCOUNT+1)=XXX
EXTRUDE 1747          YY(N,NCOUNT+1)=YYY+VEXIT*STEP
EXTRUDE 1748      310 IF(NCOUNT.LT.NMAX-1) GO TO 210
EXTRUDE 1749    C
EXTRUDE 1750    C ***************************************************************************
EXTRUDE 1751    C  DETERMINE THE VALUES OF EFFECTIVE STRAIN AT EACH POINT FOR ALL FLOW
EXTRUDE 1752    C  LINES
EXTRUDE 1753    C ***************************************************************************
EXTRUDE 1754    C
EXTRUDE 1755          NCOUNT=0
EXTRUDE 1756          EE(N,1)=0.
EXTRUDE 1757          MOLD=LOLD=1
EXTRUDE 1758      400 NCOUNT=NCOUNT+1
EXTRUDE 1759          YYY=YY(N,NCOUNT)
EXTRUDE 1760          XXX=XX(N,NCOUNT)
EXTRUDE 1761          IF(YYY.LE.VEXIT) GO TO 480
EXTRUDE 1762    C
EXTRUDE 1763    C ***************************************************************************
EXTRUDE 1764    C  DETERMINE THE LOCATION OF FOUR SURROUNDING ELEMENT CENTERS.
EXTRUDE 1765    C ***************************************************************************
EXTRUDE 1766    C
EXTRUDE 1767          NJM11=NJM1-1
EXTRUDE 1768          DO 410 I=MOLD,NJM11
EXTRUDE 1769      410 IF(YYY.LE.BB(I).AND.YYY.GE.BB(I+1)) M=I
EXTRUDE 1770          DO 420 I=1,NI
EXTRUDE 1771      420 XARB(I)=AA(I,M)-(AA(I,M)-AA(I,M+1))*(BB(M)-YYY)/(BB(M)-BB(M+1))
EXTRUDE 1772          DO 430 I=1,NIM1
EXTRUDE 1773      430 IF(XXX.GE.XARB(I).AND.XXX.LE.XARB(I+1)) L=I
EXTRUDE 1774          IF(NCOUNT.EQ.1) GO TO 450
EXTRUDE 1775          IF(M.EQ.MOLD.AND.L.EQ.LOLD) GO TO 460
EXTRUDE 1776    C
EXTRUDE 1777    C ***************************************************************************
EXTRUDE 1778    C  INTERPOLATE THE VALUES OF STRAIN-RATES.
EXTRUDE 1779    C ***************************************************************************
EXTRUDE 1780    C
EXTRUDE 1781      450 RR(1)=AA(L,M)
EXTRUDE 1782          RR(2)=AA(L,M+1)
EXTRUDE 1783          RR(3)=AA(L+1,M+1)
EXTRUDE 1784          RR(4)=AA(L+1,M)
EXTRUDE 1785          ZZ(1)=ZZ(4)=BB(M)
EXTRUDE 1786          ZZ(2)=ZZ(3)=BB(M+1)
EXTRUDE 1787          URR(1)=STR(L,M)
EXTRUDE 1788          URR(2)=STR(L,M+1)
EXTRUDE 1789          URR(3)=STR(L+1,M+1)
EXTRUDE 1790          URR(4)=STR(L+1,M)
EXTRUDE 1791          GO TO 470
EXTRUDE 1792      460 IFLAG=1
EXTRUDE 1793      470 CALL INTRPOL(RR,ZZ,URR,XXX,YYY,ESTEP,IFLAG)
EXTRUDE 1794    C
EXTRUDE 1795    C ***************************************************************************
EXTRUDE 1796    C  ADD THE INTERPOLATED STRAIN-RATE INCREMENTALLY TO THE VALUE OF
EXTRUDE 1797    C  EFFECTIVE STRAIN AT PREVIOUS LOCATION TO DETERMINE THE EFFECTIVE
EXTRUDE 1798    C  STRAIN AT NEW LOCATION.
EXTRUDE 1799    C ***************************************************************************
EXTRUDE 1800    C
EXTRUDE 1801          EE(N,NCOUNT+1)=EE(N,NCCUNT)+ESTEP*STEP
EXTRUDE 1802          LOLD=L
EXTRUDE 1803          MOLD=M
EXTRUDE 1804          GO TO 490
EXTRUDE 1805      480 EE(N,NCOUNT+1)=EE(N,NCOUNT)
EXTRUDE 1806      490 IF(NCOUNT.LT.NMAX-1) GO TO 400
EXTRUDE 1807      500 CONTINUE
EXTRUDE 1808    C
EXTRUDE 1809    C ***************************************************************************
EXTRUDE 1810    C  INTERPOLATE THE EFFECTIVE STRAIN DISTRIBUTION FOR ELEMENTS FROM VALUE
EXTRUDE 1811    C  OF EFFECTIVE STRAINS ALONG FLOW LINES.
EXTRUDE 1812    C ***************************************************************************
EXTRUDE 1813    C
EXTRUDE 1814          DO 700 N=1,NEL
EXTRUDE 1815          IF(B(N).GT.YY(1,1)) GO TO 550
EXTRUDE 1816          DO 550 J=2,NTIMES
EXTRUDE 1817          DO 520 I=1,NMAX
EXTRUDE 1818          IF(B(N).LT.YY(J,I)) GO TO 520
EXTRUDE 1819          L=I
EXTRUDE 1820          GO TO 530
EXTRUDE 1821      520 CONTINUE
EXTRUDE 1822      530 IF(XX(J,L).LT.A(N)) GO TO 550
EXTRUDE 1823          K=J
EXTRUDE 1824          GO TO 5:0
EXTRUDE 1825      550 CONTINUE
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EXTRUDE 1826      560 IFLAG=0
EXTRUDE 1827          RR(1)=XX(K-1,L)
EXTRUDE 1828          RR(2)=XX(K-1,L-1)
EXTRUDE 1829          RR(3)=XX(K,L-1)
EXTRUDE 1830          RR(4)=XX(K,L)
EXTRUDE 1831          ZZ(1)=YY(K-1,L)
EXTRUDE 1832          ZZ(2)=YY(K-1,L-1)
EXTRUDE 1833          ZZ(3)=YY(K,L-1)
EXTRUDE 1834          ZZ(4)=YY(K,L)
EXTRUDE 1835          URR(1)=EE(K-1,L)
EXTRUDE 1836          URR(2)=EE(K-1,L-1)
EXTRUDE 1837          URR(3)=EE(K,L-1)
EXTRUDE 1838          URR(4)=EE(K,L)
EXTRUDE 1839          CALL INTRPOL (RR,ZZ,URR,A(N),B(N),TEPS(5,N),IFLAG)
EXTRUDE 1840          GO TO 700
EXTRUDE 1841      650 TEPS(5,N)=0.
EXTRUDE 1842      700 CONTINUE
EXTRUDE 1843        C
EXTRUDE 1844          RETURN
EXTRUDE 1845          END
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