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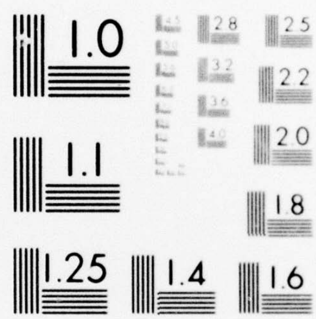

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10 Loren R. McMurray

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STABILITY DIAGRAM FOR AN ADAPTIVE RECURSIVE FILTER

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STABILITY DIAGRAM FOR AN ADAPTIVE RECURSIVE FILTER

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Abstract

Adaptive digital filters are being used and proposed for various applications. The adaptive recursive filter by Feintuch [1] is an example. A stability triangle is developed analytically for this nonlinear adaptive recursive filter using a frozen-time viewpoint. There are reasons to believe that the average stability triangle is correlated to the true stability region for all modes of operation. Computer simulation of two cases of the adaptive filter in the coefficient evaluation mode shows that the stability triangle predicts the stable region with reasonable accuracy for the cases considered.

1. Introduction

Adaptive digital filters have been investigated for a variety of applications, especially in the area of signal processing. The adaptive transversal (nonrecursive) filter is well known in this area [4, 5]. Adaptive recursive filters have also been investigated [1, 3].

The stability of adaptive filters is of interest to those who intend to use them. The stability of adaptive recursive filters are especially difficult to determine analytically since the filter is nonlinear and the stability criteria are functions of several quantities.

The stability of the adaptive recursive filter defined by Feintuch [1] has been examined analytically. Inequalities defining a triangle are derived. The derivation is simple with a minimum of assumptions, but it does use a frozen-time viewpoint. Consequently the triangular area tends to be a necessary but not a sufficient condition for stability. The stability triangle concept is checked for one mode of operation of the adaptive recursive filter by simulation of two specific examples. The triangle predicted the stable region with reasonable accuracy.

2. Stability Triangle

Feintuch [1] defines the following adaptive recursive filter:

$$y(n) = \sum_{k=0}^{NF-1} a_k(n)\, x(n-k) + \sum_{m=1}^{NB} b_m(n)\, y(n-m) \qquad (1)$$

$$\epsilon(n) = r(n) - y(n) \qquad (2)$$

$$a_k(n+1) = a_k(n) + \mu_F\, \epsilon(n)\, x(n-k) \qquad (3)$$

$$b_m(n+1) = b_m(n) + \mu_B\, \epsilon(n)\, y(n-m) \qquad (4)$$

where *n* is the iteration number, $x(n)$ is the input, $r(n)$ is a reference quantity, $\epsilon(n)$ is the error in the output, $a_k(n)$ and $b_m(n)$ are the adapting feed forward and feedback weights, and μ_F and μ_B are the feed forward and feedback gain constants.

The stability criteria will be derived by considering one instant of time rather than the normal approach of taking a limit as the number of iterations becomes very large. The advantage of freezing time is the ease in which results about a nonlinear filter can be obtained. The disadvantage is that the stability criteria will not be complete. But in dealing with nonlinear equations, a partial solution which provides a starting point for a detailed analysis is much better than no solution.

Assume that $y(n_o)$ and $\epsilon^2(n_o)$ have just been computed using Eq (1) and (2). By substituting (1) into (2), $\epsilon(n_o)$ is given by Eq (5)

$$\epsilon(n_o) = r(n_o) - \sum_{k=0}^{NF-1} a_k(n_o)\, x(n_o - k) - \sum_{m=1}^{NB} b_m(n_o)\, y(n_o - m) \qquad (5)$$

i.e., $\epsilon^2(n_o)$ is a function of present and past inputs, of past outputs, and of present weights. Figure 1 shows the error squared for the case of one feed forward and one feedback weight. The error squared is a parabolic cylinder. If $\epsilon^2(n_o)$ is non zero, it can be reduced by modifying the present weights, see Figure 1. In particular, let the weights be modified proportional to the negative of the first derivative.

$$\tilde{a}_k(n_o) = a_k(n_o) - 0.5\, \mu_F\, \partial\epsilon^2(n_o)/\partial a_k(n_o) \qquad (6)$$

$$\tilde{b}_m(n_o) = b_m(n_o) - 0.5\, \mu_B\, \partial\epsilon^2(n_o)/\partial b_m(n_o) \qquad (7)$$

Since changing $a_k(n_o)$ and $b_m(n_o)$ do not affect $r(n_o)$, $x(n_o-k)$, nor $y(n_o-m)$ of Eq (5),

$$\tilde{a}_k(n_o) = a_k(n_o) + \mu_F\, \epsilon(n_o)\, x(n_o-k) \qquad (8)$$

$$\tilde{b}_m(n_o) = b_m(n_o) + \mu_B\, \epsilon(n_o)\, y(n_o-m) \qquad (9)$$

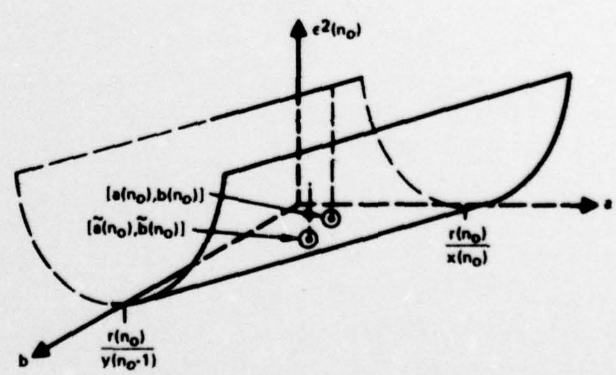

Figure 1. Error Squared for $y(n) = a(n)\, x(n) + b\, m\, y(n-1)$

Two observations are to be made. First, $\tilde{a}_k(n_o)$ and $\tilde{b}_m(n_o)$ can be used as initial estimates for $a_k(n_o+1)$ and $b_m(n_o+1)$, respectively. Repeating this process for each value of *n* results in Eq (3) and (4). This can be considered as another derivation of Eq (3) and (4) since the derivation differs from that given in [1].

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The second observation is that the values of μ_F and μ_B must be constrained to lie within certain bounds if $\epsilon^2(n_o)$ is to be reduced by using $\tilde{a}_k(n_o)$ and $\tilde{b}_m(n_o)$. Let $\tilde{\epsilon}^2(n_o)$ denote the value of the error squared when using the modified weights, and let $\epsilon^2(n_o)$ denote its value when using the unmodified initial weights. One wants

$$\tilde{\epsilon}^2(n_o) < \epsilon^2(n_o) \tag{10}$$

Rewriting Eq (5) using $\tilde{a}_k$ and $\tilde{b}_m$ and then substituting Eq (8) and (9) for their values gives

$$\tilde{\epsilon}(n_o) = \epsilon(n_o)\left\{1 - \mu_F \sum_{k=0}^{NF-1} x^2(n_o-k) - \mu_B \sum_{m=1}^{NB} y^2(n_o-m)\right\} \tag{11}$$

It follows that

$$0 < \mu_F \sum_{k=0}^{NF-1} x^2(n_o-k) + \mu_B \sum_{m=1}^{NB} y^2(n_o-m) < 2 \tag{12}$$

is equivalent to inequality (10).

This is the main stability criterion, and several observations will be made about it. To simplify the notation, define $\Gamma(n)$ as

$$\Gamma(n) = \mu_F \sum_{k=0}^{NF-1} x^2(n-k) + \mu_B \sum_{m=1}^{NB} y^2(n-m) \tag{13}$$

The derivation of inequality (12) does not specify the nature of the inputs and outputs, e.g., deterministic and/or statistical, and hence should apply to both.. Furthermore, the mode of operation is not specified, e.g., signal enhance or interference cancel, and hence should apply to all of them.

Two check points exist on the stability criterion. If NB is zero, Eq (1), (2) and (3) become the well known (non recursive) adaptive transversal filter. Inequality (12) becomes

$$0 < \mu_F \sum_{k=0}^{NF-1} x^2(n-k) < 2 \tag{14}$$

which coincides with a sufficient stability criterion of the transversal filter [4]. Also, if x(n) and r(n) are zero for $n > n_1 - NF$, then the filter is recursive only and goes into a turn off state if stable. A sufficient condition for y(n) to converge to zero for this situation is given in [2], namely if

$$\sum_{M=1}^{NB} b_m^2(n_1) \le \gamma < \frac{1}{NB} \tag{15}$$

and if

$$0 \le \mu_B \, NB \, \mathrm{Max}\{y^2(n_1 - m) \mid m = 1,2,\ldots, NB\} \le 2 \tag{16}$$

This is reasonably consistent with

$$0 < \mu_B \sum_{m=1}^{NB} y^2(n-m) < 2 \tag{17}$$

Inequality (12) requires either μ_F or μ_B be positive, but does not require both to be positive. It is common practice to set the outputs and the weights to zero as initial values for the adaptive filter. Under this initialization process, $\Gamma(n)$ is dominated for the first few iterations by $\mu_F \Sigma x^2(n-k)$ and μ_F must be positive if $\Gamma(n)$ is to be positive. If μ_B is negative, then the lower and upper bound on $\Gamma(n)$ can be exceeded, but if μ_F and μ_B are both positive, then the lower bound on $\Gamma(n)$ is inherently satisfied. For these reasons the constraints

$$\mu_F > 0 \tag{18}$$

$$\mu_B > 0 \tag{19}$$

will be added. Inequalities (12), (18) and (19) define a triangle in the (μ_F, μ_B) plane, and this triangle will be denoted the stability triangle.

The above derivation of the stability criterion (12) does not require the gains μ_F and μ_B to be constant. They need only to be independent of the weights. They may be functions of the iteration n and indices k and m. The general stability criterion would then be

$$0 < \sum_{k=0}^{NF-1} \mu_{F,k}(n)\, x^2(n-k) + \sum_{m=1}^{NB} \mu_{B,m}(n)\, y^2(n-m) < 2 \tag{20}$$

In summary, a general stability triangle has been derived. Modifying the weights using values of μ_F and μ_B from this triangle will reduce the present error. One could expect this triangular area to be correlated with the true stability region of a given application.

3. Application to the Coefficient Evaluation Mode

The stability triangle uses the philosophy that if the present error is reduced the stability of the filter is enhanced. It seems reasonable, but verification is left to a computer simulation solving Eq (1) - (4) for the transient response of a given situation.

The filter can be used in several modes of operation: signal enhancing, interference cancelling, coefficient evaluation, etc. To test the theory, two test cases were chosen from the coefficient evaluation mode, see Figure 2. The unknown fixed filter is a transversal or recursive filter with fixed weights whose values are unknown.

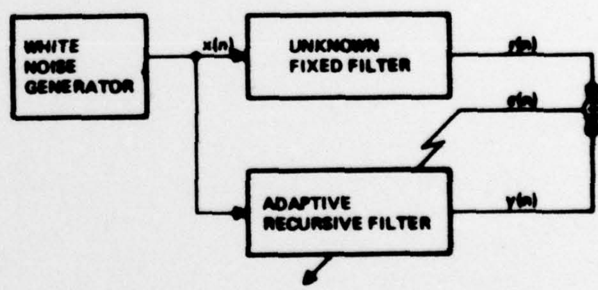

Figure 2. Coefficient Evaluation Mode

This mode of operation was chosen as it has several advantages for the test to be performed. If the adaptive filter is at least as long as the fixed filter with respect to NF and NB, the adaptive filter error squared will converge to zero if stable. This allows a simple accurate numerical test for stability. Also by keeping the filter lengths short, the amount of computation was minimized.

The white noise generator was Gaussian, mean zero and variance of one. At the start of each run the adaptive filter was reset to zero. Stability was defined by

$$\epsilon^2(n) \begin{cases} < 1.0 \times 10^{-30} & \text{stable} \\ > 1.0 \times 10^{+30} & \text{unstable} \end{cases} \tag{21}$$

iteration was continued until one of the two states was reached for some value of n. Since $\epsilon(n)$ appears in the weight update equations, it was assumed that further iterations would not reverse the result and the run was terminated. After monitoring the transient response of $\epsilon^2(n)$ on many runs, it is believed that the approach was adequate.

The two cases considered were:

$$\text{Case I:} \quad r(n) = 0.0273\, x(n) - 0.0371\, x(n-1) + 0.0252\, x(n-2) + 0.8926\, r(n-1) - 0.3984\, r(n-2) \tag{22}$$

$$\text{Case II:} \quad r(n) = x(n-2) \tag{23}$$

For both cases, NF = 5 and NB = 4 for the adaptive filter. Several thousand Monte Carlo runs were made for each case. The results are shown in Figures 3 and 4 by contours of 10 percent, 50 percent and 90 percent probability of being stable. These contours are density functions. A (μ_F, μ_B) point picked from the 10 percent contour will on the average result in the adaptive filter being stable 10 times out of every 100 Monte Carlo runs.

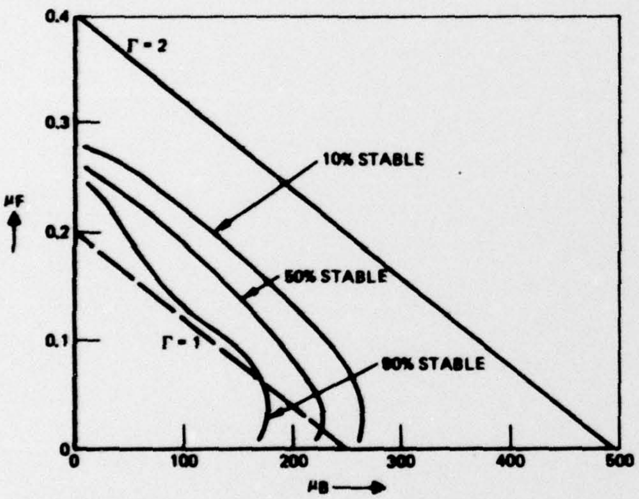

Figure 3. Stability Diagram for Case I

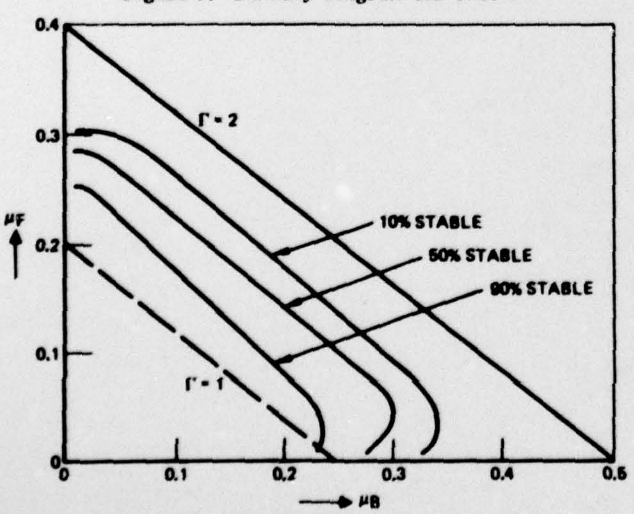

Figure 4. Stability Diagram for Case II

The average stability triangle was determined in the following manner. Γ of Eq (13) varies from iteration to iteration. In order to get some average triangle, it was assumed that

$$NF\, \sigma_x^2 \doteq \sum_{k=0}^{NF-1} x^2(n-k) \tag{24}$$

$$NB\, \sigma_y^2 \doteq \sum_{m=1}^{NB} y^2(n-m) \tag{25}$$

so that

$$\Gamma \doteq \mu_F\, NF\, \sigma_x^2 + \mu_B\, NB\, \sigma_y^2 \tag{26}$$

The variance of x(n) is known to be unity from the white noise generator. If the filter is stable,

$$\sigma_y^2 \doteq \sigma_r^2 \tag{27}$$

near convergence. Driving the fixed filter by the white noise gave a measured σ_r^2 of 0.001 (analytical results not normally available gave 0.001015) for case I, and for case II, $\sigma_r^2 = \sigma_x^2$. Thus

$$\Gamma = 5\,\mu_F + 0.004\,\mu_B \qquad \text{Case I} \tag{28}$$

$$\Gamma = 5\,\mu_F + 4\,\mu_B \qquad \text{Case II} \tag{29}$$

The lines for $\Gamma = 1$ and $\Gamma = 2$ are shown on Figures 3 and 4.

As can be seen, the contours of constant stability tend to be parallel to a line of constant Γ. With very little information, the stable region has been predicted by the average stability triangle with reasonable accuracy.

There are "edge" effects in the contour lines near the μ_F and μ_B axes. These effects are not explained by the stability triangle theory.

A variety of (μ_F, μ_B) values were tried. Whenever $\Gamma > 2$, or $\mu_F < 0$, or $\mu_B < 0$ the adaptive filter was unstable.

If NF and NB has been larger (smaller), then according to the stability criterion (12) the stable region would be smaller (larger) and the probability of a given (μ_F, μ_B) point being stable would decrease (increase). A spot check indicated this was true when NF and NB were made larger, and it was partially true when NF and NB were made smaller. For small NB, Eq (25) is not a good approximation, and this is suspected of being the cause of the limited agreement.

Figures 3 and 4 show that for these two cases the true stability region is not sharply defined. It has a transition region from high to low probability of being stable.

4. Summary

A stability triangle has been derived analytically on fundamental principles. It is expected that there would be a correlation between an average stability triangle and the true stability region. The theory should be applicable whenever an average triangle can be determined. Computer simulations of two cases in the coefficient evaluation mode showed that it did apply there.

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