

MANAGEMENT STRATEGIES IN FIXED-STRUCTURE MODELS  
OF COMPLEX ORGANIZATIONS II

BY

ALAN E. GELFAND and CRAYTON C. WALKER

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MANAGEMENT STRATEGIES IN FIXED-STRUCTURE MODELS  
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Alan E. Gelfand and Crayton C. Walker

1. Introduction

In this paper we examine consensus as a management control strategy in complex organizations. We consider how different degrees of consensus may interact with two other management strategies, namely, management by exception and management by priority, to influence the tractability of organizations' behavior.

The background for this paper is found in a previous publication [9]. In the earlier work we introduced the interpretation of an organizational control system as a network of binary switching elements. Each element has a fixed number of inputs and computes a binary output according to a function that represents its response to input information. We interpret management by exception as the extent to which the function has identical output values, and management by priority as the extent to which individual inputs are effective in producing identical output. Our findings are that these two strategies can

control overall system behavior. We also focused on the problem of providing modeled organizational behavior that is on the average within useful, empirically reasonable ranges. Additionally we sought to provide results useful to the organizational designer or intervener who must work in the face of uncertainty as to both the detailed structure of organizations and the functioning of its individual parts.

In the earlier paper we suggest that management by exception and management by priority can be modeled by the rigorously defined concepts internal homogeneity and forcibility respectively. We also mention that the idea of control by consensus or aggregation of information can be conveniently interpreted using the notion of a threshold. We now develop this idea. "Consensus", when there can be more or less of it, implies the existence of a metric on which the amount of evidence or disposition "for" or "against" something can be scaled. Neither internal homogeneity nor forcibility provide such a measure. Internal homogeneity simply indicates the behavior of a net element in an aggregate sense, no account being taken of the specific conditions giving rise to the behavior. Forcibility reflects the efficacy of elements' inputs individually, no account being taken of possible joint effects among inputs.

Control by consensus, in addition, invokes the idea of a level, at or above which some course of

action is taken or some specific condition maintained. That is, control by consensus can be interpreted as a sufficiency condition.

In noting the importance of a metric and the sufficiency condition in the concept "consensus", we are naturally led to model it with threshold functions. Such functions show both the aggregative scaling of input values and the sufficiency condition for constant output. Conventional threshold functions [2], however, often used to model nerve cells, typically interpret "0" as "off" and "1" as "on", and provide both necessary and sufficient conditions for setting output values. In addition, conventional threshold functions typically set the "on" condition for above threshold input configurations. These restrictions seem excessive for our purposes. In a managerial environment it might be useful to take no action given sufficient evidence, or to take an action on finding a substantial lack of something (say, productivity), in the input. It might also be useful to be able to take the above threshold action, in certain circumstances, at below threshold input values. The notion of a threshold which we formalize in the next section, allows these possibilities. Hence our definition extends the more familiar concept of a threshold [2] and may appropriately be called an extended threshold. With this understood we will shorten the terminology to just threshold.

In passing we note that where all inputs "on" achieve the same action as all inputs "off" we have a map which arguably is not a threshold function. That is, where the action to be taken or the condition to be set is the same for both extremes of amount of information, it can be objected that we are dealing with a different concept. However, separating out such maps does not change the conclusions we will reach; nor do the separated maps themselves behave differently in the large with respect to internal homogeneity and forcibility. For these reasons, and to maintain greater generality, we will not complicate the definition of extended threshold to effect their removal.

In the next section we characterize extended threshold rigorously, and develop some counting relationships for it. Following that section we interpret extended threshold together with internal homogeneity and forcibility in managerial terms. We conclude with more general comments on the interrelationship between systems theory and organizational theory.

## 2. Definitions and Notation and some Preliminary Results

We wish formally to define the notion of a mapping on  $k$  inputs which has (extended) threshold  $\ell$  ( $1 \leq \ell \leq k$ ). Speaking casually, we may say that a mapping on  $k$  inputs has threshold  $\ell$  if whenever  $\ell$  or more inputs take on a specified value the mapping takes on a specified value.

We call the resultant mapping or output value the threshold state associated with that input value. The specified input value may be "0" or "1" and may be coupled with a threshold state of "0" or "1". In this definition  $\ell$  is the minimum number of inputs for which the statement is true since if the statement holds at  $\ell$  it would obviously hold at  $\ell+1, \ell+2, \dots, k$ .

The difficulty associated with such an informal definition may be revealed by attempting to answer the following illustrative question. Can the system ever be "on" if fewer than  $\ell$  inputs are "on"? If the answer is no we shall refer to  $\ell$  as an absolute threshold although it is not clear whether  $\ell$  or  $k-\ell$  should be called the threshold since  $k-\ell$  inputs "off" imply the system is "off". If the answer is yes we shall refer to  $\ell$  only as a threshold (for the number of "on" inputs). Every mapping must have a threshold (at the largest it would be  $k$ ) but only a subset of mappings have an absolute threshold. Because our extended conception of a threshold allows either "off" or "on" inputs to turn an element again either "off" or "on" how do we assign a threshold value,  $\ell$  to a mapping? We need to consider for a mapping  $m$ , a threshold for the number of "off" or "0" inputs which we denote by  $\ell_0(m)$  and similarly a threshold for the number of on or "1" inputs which we denote by  $\ell_1(m)$ . We then define  $\ell(m) = \min(\ell_0(m), \ell_1(m))$ .



In light of our extended definition the minimum of these two numbers is clearly the more significant value. In considering absolute thresholds we quickly discover that if  $\ell_0$  is an absolute threshold so is  $\ell_1$  and that  $\ell_0 + \ell_1 = k+1$ . We will prove shortly that for any mapping  $m$ ,  $\ell_0(m) + \ell_1(m) \geq k+1$ . Table 1 attempts to clarify the definitions and notations. For mapping  $m_1$  we have  $\ell_0(m_1) = 4$ ,  $\ell_1(m_1) = 3$  and  $\ell(m_1) = 3$ . For mapping  $m_2$  we have  $\ell_0(m_2) = 2$ ,  $\ell_1(m_2) = 4$  and  $\ell(m_2) = 2$ . For mapping  $m_3$  we have  $\ell_0(m_3) = 2$ ,  $\ell_1(m_3) = 3$  with both  $\ell_0$  and  $\ell_1$  absolute thresholds and  $\ell(m_3) = 2$ . Lastly for mapping  $m_4$  we have a situation where the notion of a threshold has little significance (as noted in the introduction) i.e.,  $\ell_0(m_4) = \ell_1(m_4) = \ell(m_4) = 4$ . The notion of a threshold for the two trivial or constant mappings is not meaningful. For convenience we define  $\ell = 0$  for both and do not consider them further in this discussion.

A brief examination of Table 1 reveals that the lexicographic ordering for a mapping is not at all convenient for establishing thresholds. A better arrangement would be to order the input rows monotonically by the number of "0"'s (hence by the number of "1"'s.) To obtain  $\ell_0$  and  $\ell_1$  from this "monotonic" ordering is quite simple. Suppose for example the input rows are arranged by increasing number of "0"'s.

If we scan up the mapping value column in this table for the first change of value ("0" to "1" or "1" to "0") and it occurs for a row having  $j$  input "0"'s then  $\ell_0 = j+1$ . If we scan down the mapping value column for the changes and it occurs for a row having  $j'$  input "0"'s (hence  $k-j'$  input "1"'s) then  $\ell_1 = k-j'+1$ .

We may immediately notice that since we are considering only nontrivial maps we must have  $j' \leq j+1$ , i.e.,  $k-\ell_1+1 \leq \ell_0$  or  $\ell_0+\ell_1 \geq k+1$ . As Lemma 1 we state and formally prove a slightly broader result.

Lemma 1: For any nontrivial mapping  $m$ ,  $\ell_0(m)+\ell_1(m) \geq k+1$  with equality i.f.f.  $\ell_0$  (and hence  $\ell_1$ ) is an absolute threshold.

Pf.: If for  $m$ ,  $\ell_0(m) = j$  (then all the input rows having  $j$  or more "0"'s will be forced to a common mapping value. The remaining rows have at most  $j-1$  "0"'s hence at least  $k-j+1$  "1"'s. Among rows having  $j-1$  "0"'s there must be at least one row with mapping value different from the common mapping value for all input rows with  $j$  or more "0"'s. Hence  $\ell_1(m) \geq k-j+1$  and thus  $\ell_0(m) + \ell_1(m) \geq k+1$ . Finally,  $\ell_1(m) = k-j+1$  (i.e.,  $\ell_0(m) + \ell_1(m) = k+1$ ) i.f.f. all rows with  $j-1$  or fewer "0"'s have a common mapping value. This mapping clearly has  $\ell_0(m) = j$  and  $\ell_1(m) = k-j+1$  as absolute thresholds.  $\square$

We note that if  $\ell(m) = 1$  then  $\ell$  is immediately an absolute threshold. Mappings having threshold  $\ell_0=1$  and

| $x_4$ | $x_3$ | $x_2$ | $x_1$ | $m_1$ | $m_2$ | $m_3$ | $m_4$ |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 1     | 1     | 1     | 1     | 1     | 1     | 0     |
| 0     | 1     | 1     | 1     | 1     | 0     | 1     | 1     |
| 1     | 0     | 1     | 1     | 1     | 1     | 1     | 0     |
| 0     | 0     | 1     | 1     | 0     | 0     | 0     | 1     |
| 1     | 1     | 0     | 1     | 1     | 1     | 1     | 1     |
| 0     | 1     | 0     | 1     | 0     | 0     | 0     | 0     |
| 1     | 0     | 0     | 1     | 0     | 0     | 0     | 1     |
| 0     | 0     | 0     | 1     | 1     | 0     | 0     | 0     |
| 1     | 1     | 1     | 0     | 1     | 1     | 1     | 1     |
| 0     | 1     | 1     | 0     | 0     | 0     | 0     | 0     |
| 1     | 0     | 1     | 0     | 0     | 0     | 0     | 1     |
| 0     | 0     | 1     | 0     | 1     | 0     | 0     | 0     |
| 1     | 1     | 0     | 0     | 1     | 0     | 0     | 0     |
| 0     | 1     | 0     | 0     | 0     | 0     | 0     | 0     |
| 1     | 0     | 0     | 0     | 0     | 0     | 0     | 1     |
| 0     | 0     | 0     | 0     | 1     | 0     | 0     | 0     |

Table 1: Illustrative Mappings on Four Inputs

corresponding threshold state 1 have been defined as noncontractible by Rosen [6]. We thus will speak more generally of mappings with  $\ell=1$  as being extended noncontractible.

We now attempt to enumerate the number of mappings having a particular threshold  $\ell$ . If  $\ell$  is an absolute

threshold the counting is simple. Given  $\ell_0$  and hence  $\ell_1 = k+1-\ell_0$  there will be two mappings having this particular  $\ell_0$  and  $\ell_1$ --the mapping having threshold state "0" on the "0" inputs, threshold state "1" on the "1" inputs and vice versa. Since  $\ell_0$  runs from 1 to k the total number of mappings having an absolute threshold is  $2k$ . The number having absolute threshold  $\ell$  is 4 if  $\ell_0 \neq \ell_1$  and 2 if  $\ell_0 = \ell_1$ . Note that  $\ell_0 = \ell_1$  implies k is odd and  $\ell = \frac{k+1}{2}$ . Only for this  $\ell$  may the number of maps having absolute threshold be 2.

More generally let  $\beta(k, \ell)$  be the number of mappings on k with threshold  $\ell$  (including those with absolute threshold  $\ell$ ). In computing  $\beta(k, \ell)$  it will be convenient if we first calculate  $\alpha(k, j, j')$  which is the number of mappings on k inputs with  $\ell_0 = j$  and  $\ell_1 = j'$ . Note that by Lemma 1,  $j+j'$  must be at least  $k+1$ . We have the following theorem.

Theorem 1:

$$(i) \alpha(k, j, j') = 2 \quad \text{if } j+j' = k+1$$

$$(ii) \alpha(k, j, j') = 2 \binom{k}{j-1} + 2 - 6 \quad \text{if } j+j' = k+2$$

$$(iii) \alpha(k, j, j') = 4 \binom{k}{j'-1} \binom{k}{j-1} \sum_{i=k-j'+2}^{j-2} \binom{k}{i} \quad \text{if } j+j' > k+2.$$

Pf.: Given  $j$  and  $j'$  fixes a common mapping value, say  $a$ , for all rows with  $j$  or more "0"'s and a common

mapping value, say  $b$ , for all rows with  $k-j'$  or fewer "0"'s. We must examine the possibilities for the remaining rows involving more than  $k-j'$  "0"'s but fewer than  $j$  "0"'s. (i) Suppose  $j+j'=k+1$ . Then  $k-j'=j-1$  and there are no rows unaccounted for. If  $a=0$  then  $b=1$  and vice versa, hence  $\alpha(k,j,j') = 2$ .

(ii) Suppose  $j+j'=k+2$ . Then  $k-j'=j-2$  and thus only rows with  $j-1$  "0"'s must be considered. There are  $\binom{k}{j-1}$  such rows. If  $a=b=0$  at least one of these rows must have mapping value 1. This may be done in  $2\binom{k}{j-1}-1$  ways. If  $a=b=1$  at least one of these rows must have mapping value 0 which again may be done in

$2\binom{k}{j-1}-1$  ways. If  $a=1$  and  $b=0$  then at least one row must have mapping value "0" and at least one row must have mapping value "1". This may be accomplished in

$2\binom{k}{j-1}-2$  ways. Similarly this is so for  $a=0$  and  $b=1$ .

Combining these possibilities we have

$\alpha(k,j,j') = 2(2\binom{k}{j-1}-1)+2(2\binom{k}{j-1}-2)$  which after simplification yields (ii).

(iii) If  $j+j'>k+2$  then  $k-j'<j-2$ . At least one row having  $j-1$  "0"'s must have mapping value  $1-a$  and at least one row having  $k-j'+1$  "0"'s ( $k-j'+1<j-1$ ) must have mapping value  $1-b$ . Rows involving more than  $k-j'+1$  but fewer than  $j-1$  "0"'s may be selected arbitrarily. Since  $a$  and  $b$  may each be "0" or "1" this allows 4 initial

choices. Combining these possibilities yields (iii).  $\square$

For any mapping  $m$  on  $k$  inputs with  $\ell_0(m) \leq \ell_1(m)$  there is a symmetrically equivalent mapping (in terms of thresholds)  $m'$  given by

$$m'(x_1, x_2, \dots, x_k) = m(1-x_1, 1-x_2, \dots, 1-x_k).$$

It is apparent that  $m'$  arises in the monotonic ordering of  $m$  by simply inverting the mapping value column and hence clearly  $\ell_1(m') = \ell_0(m) \leq \ell_1(m) = \ell_0(m')$ . This symmetry implies  $\alpha(k, j, j') = \alpha(k, j', j)$  and thus finally enables us to calculate  $\beta(k, \ell)$ .

Theorem 2:  $\beta(k, \ell) = 2 \sum_{\ell'=\ell+1}^k \alpha(k, \ell, \ell') + \alpha(k, \ell, \ell)$  where

$\alpha(k, \ell, \ell') = 0$  if  $\ell + \ell' < k + 1$ .

Pf.: The proof is contained in the above discussion.

### 3. Interrelationships Between Thresholds, Forcibility and Internal Homogeneity

The notions of internal homogeneity and forcibility have been discussed in previous articles. Walker and Ashby [8] examine internal homogeneity and its effect on system behavior while Kauffman in a series of articles, most notably [4], [5] has developed the concept of forcibility. Forcibility appears to be a very strong conceptualization with respect to determining system behavior.

To recall our earlier definitions (Part I), internal homogeneity, denoted henceforth by  $I$  is the larger of the number of "0" entries and the number of "1" entries in the table of values of a mapping, i.e.,  $I = \max(\#0\text{'s}, \#1\text{'s})$ . Hence  $2^{k-1} \leq I \leq 2^k$ .

A mapping is forcible on a given input when a given state of the input "forces" the output of the mapping to a single value regardless of the values of the other inputs. This given state is called the forcing state. If an input is forcing on both states then the mapping is either constant (trivial) or has half "1"'s and half "0"'s. In the former case all inputs are forcing on both states while in the latter case the mapping must be forcing only on the one input. Since forcibility with only one input is trivial we restrict attention to the case where the number of inputs  $k \geq 2$ . The forced value of an element is that value to which it is forcible.

If an element is forcible on more than one input line, its forced value is identical for all the inputs on which it is forcible. We denote the number of forcing inputs by  $F$ .

Enumeration of Boolean transformations by internal homogeneity and forcibility is discussed in Part I. We now turn to the extension of these enumerations to include thresholds.

Given any mapping  $m(x)$  on  $k$  inputs consider the mappings  $m' = m(1-x)$  (introduced in the previous section),

$\bar{m} = 1 - m(x)$  and  $\bar{m}' = 1 - m(1-x)$ . It is easy to verify that four mappings  $m$ ,  $m'$ ,  $\bar{m}$ , and  $\bar{m}'$  are equivalent with regard to internal homogeneity forcibility and thresholds. It is possible that this equivalence class may consist of just two elements, i.e.,  $m=m'$  or  $m = \bar{m}'$  (obviously  $m$  can't equal  $\bar{m}$ ). It is apparent that if  $I$  is odd four distinct mappings must arise and that if  $m = \bar{m}'$  then  $I = 2^{k-1}$ . It is a complicated enumeration to establish the exact number of classes,  $C(k)$ , generated by such equivalence classes. However simple upper and lower bounds are readily available from the following theorem.

$$\text{Theorem 2: } \frac{1}{4} \sum_{i=2^{k-1}}^{2^k} N_k(i) \leq C(k) \leq \frac{1}{2} \sum_{m=2^{k-2}}^{2^{k-1}} N_k(2m) + \frac{1}{4} \sum_{m=2^{k-2}}^{2^{k-1}-1} N_k(2m+1)$$

where  $N_k(i)$  is the number of mappings on  $k$  inputs with  $I=i$ .

Pf.: It is easy to see that  $N_k(i) = 2 \binom{2^k}{i}$  for

$2^{k-1} < i \leq 2^k$  and that  $N_k(2^{k-1}) = \binom{2^k}{2^{k-1}}$ . If  $I$  is odd,

say  $2m+1$ , the number of equivalence classes generated is thus  $\frac{1}{4} N_k(2m+1)$ . If  $I$  is even, say  $2m$ , the number of



classes generated is at least  $\frac{1}{4}N_k(2m)$  and at most  $\frac{1}{2}N_k(2m)$ . Combining these results we have the theorem.  $\square$

Similar equivalence classes developed for the special case of Boolean transformations with feedback have been considered in Walker and Aadryan [7] and Gelfand and Walker [3]. They show that if  $k=3$ , 88 equivalence classes arise. Theorem 2 provides bounds of  $6^4$  and 96 for this case.

When  $k$  is large surely more than one equivalence class may have the same values for  $I$ ,  $F$  and  $\ell$ . Even when  $k=2$  these three properties do not uniquely determine equivalence classes, as the example in Table 2 indicates.

| $x_2$ | $x_1$ | $m_1$ | $m_2$ |
|-------|-------|-------|-------|
| 1     | 1     | 1     | 1     |
| 0     | 1     | 1     | 0     |
| 1     | 0     | 0     | 1     |
| 0     | 0     | 0     | 0     |

Table 2: Two mappings in different equivalence classes with  $I=2$ ,  $F=1$  and  $\ell=2$ .

This example and discussion indicates that the three properties are clearly interrelated. Part I expanded upon the strong relationship between internal homogeneity and forcibility; essentially, larger  $F$  must

be accompanied by larger I. We shall eventually see that  $\ell$  is weakly related to I and inversely related to F.

Let us first examine internal homogeneity and threshold. If for a mapping  $m$  on  $k$  inputs,  $\ell(m)=1$  then clearly  $I=2^{k-1}$  (and in fact  $F=k$ ). Conversely if  $I=2^{k-1}$  it is obvious that  $\ell_0 + \ell_1 \leq k+2$  and thus  $1 \leq \ell \leq \frac{k+2}{2} = k/2+1$ . More precisely of the  $2^{k+1}$  mappings with  $I=2^{k-1}$  how many have threshold  $\ell$ ? The exact answer depends on whether  $k$  is odd or even. If  $k$  is odd,  $1 \leq \ell \leq \frac{k-1}{2} + 1$  and the number is  $4 \binom{k}{\ell-1}$ . If  $k$  is even,  $1 \leq \ell \leq \frac{k}{2}+1$  and number is  $4 \binom{k}{\ell-1}$  if  $1 \leq \ell \leq \frac{k}{2}$  and  $2 \binom{k}{k/2}$  at  $\ell = \frac{k}{2} + 1$ .

Let us now generally enumerate  $\tau(k,i,\ell)$ , the number of mappings on  $k$  inputs having internal homogeneity  $i$  and threshold  $\ell$ . It will be convenient as in Theorem 1 to calculate  $\tau(k,i,\ell_0,\ell_1)$  first. Also for convenience let

$$c = \max\left(\sum_{j=\ell_0}^k \binom{k}{j}, \sum_{j=\ell_1}^k \binom{k}{j}\right), \quad d = \min\left(\sum_{j=\ell_0}^k \binom{k}{j}, \sum_{j=\ell_1}^k \binom{k}{j}\right).$$

Recall also  $a$  and  $b$  as defined in Theorem 1.

We first note the following.

Lemma 2: (i) If  $a=b$  then  $\tau(k,i,\ell_0,\ell_1)=0$  if  $i < c+d$ .

(ii) If  $a \neq b$  then  $\tau(k,i,\ell_0,\ell_1)=0$  if  $i < c$  or

$$2^{k-1} < d.$$

Pf.: Obvious from definitions.

We are now ready to calculate  $\tau(k, i, \ell_0, \ell_1)$ . Let  $T(k, i, j, j')$  be the number of mappings on  $k$  inputs with  $I=i$ ,  $\ell_0 \leq j$  and  $\ell_1 \leq j'$ . If  $T$  is obtained  $\tau$  may be computed from  $T$  via a second order difference, i.e.,

$$\tau = \Delta_{\ell_0, \ell_1}^2(T).$$

More specifically this notation means

$$\begin{aligned} \tau(k, i, \ell_0, \ell_1) &= T(k, i, \ell_0, \ell_1) - T(k, i, \ell_0 - 1, \ell_1) \\ &\quad - T(k, i, \ell_0, \ell_1 - 1) + T(k, i, \ell_0 - 1, \ell_1 - 1). \end{aligned}$$

Theorem 3 enables us to compute  $T(k, i, \ell_0, \ell_1)$ .

$$\text{Theorem 3: Let } e_1 = \begin{pmatrix} 2^{k-(c+d)} \\ 2^{k-1} \end{pmatrix}, e_2 = \begin{pmatrix} 2^{k-(c+d)} \\ 1 \end{pmatrix},$$

$$e_3 = \begin{pmatrix} 2^{k-(c+d)} \\ i-c \end{pmatrix}, \text{ and } e_4 = \begin{pmatrix} 2^{k-(c+d)} \\ i-d \end{pmatrix} \text{ and define}$$

$$\binom{b}{a} \equiv 0 \text{ if } a > b. \text{ Then } T(k, i, \ell_0, \ell_1) = 2(e_1 + e_2 + e_3 + e_4)$$

$$\text{if } 2^{k-1} < i \leq 2^k = 2(e_1 + e_3) \text{ if } i = 2^{k-1}.$$

Pf.: Without loss of generality suppose  $\ell_0 > \ell_1$ . Then in the monotonic ordering (by number of "0"'s) at least

the last  $c$  input rows have map value fixed at  $a$  and at least the first  $c$  rows have map value fixed at  $b$  in order to insure at most  $\ell_0$  and  $\ell_1$  respectively.

Suppose that  $i$  is actually number of "1"'s in the table of mapping values. For the remaining  $2^k - (c+d)$  rows,  $e_1$  counts the number of ways we may insure at most  $\ell_0$  and  $\ell_1$  with  $a=b=1$ ,  $e_2$  indicates how many ways we may do this with  $a=b=0$ ,  $e_3$  indicates how many ways with  $a=1$ ,  $b=0$  and  $e_4$  with  $a=0$  and  $b=1$ .

Since  $i$  may be the number of "0"'s in the table of values we must multiply each  $e_i$  by 2. In the case that  $i=2^{k-1}$  we have  $e_1=e_2$ ,  $e_3=e_4$  and hence the expression for  $T$  is halved to avoid double counting.  $\square$

Lastly we may obtain  $\tau(k,i,\ell)$ .

$$\text{Theorem 4: } \tau(k,i,\ell) = 2 \sum_{\ell'=\ell+1}^k \tau(k,i,\ell,\ell') + \tau(k,i,\ell,\ell').$$

Pf.: It is obvious by symmetry considerations that

$$\tau(k,i,\ell,\ell') = \tau(k,i,\ell',\ell).$$

Our next objective is to examine forcibility and threshold. We have already observed that if for a mapping  $m$ ,  $\ell(m) = 1$  then  $m$  is forcing on all inputs. If  $\ell$ ,  $1 < \ell \leq \lfloor \frac{k+1}{2} \rfloor$  ( $\lfloor x \rfloor$  denotes the greatest integer in  $x$ ) is an absolute threshold consider any input, say the  $i$ th. For any row with  $x_i=1$  and fewer than  $\ell$  inputs equal to "1" the mapping value will differ from that of a row with at least  $\ell$  inputs equal to "1". A similar

argument holds when  $x_i=0$  so no forcing inputs are possible.

The gist of the preceding paragraph may be stated as a lemma.

Lemma 3: The intersection of the set of forcible mappings and the set of mappings with an absolute threshold is the set of extended noncontractible mappings.

Pf.: The proof is contained in the above discussion.

We now examine the more general situation. We show first that for any mapping having forcing inputs both input values must have the same threshold state or  $\max(\ell_0, \ell_1)=k$ .

Theorem 5: Consider any mapping  $m$  on  $k$  inputs with differing threshold states and with  $\ell_0(m) = j < k$  and  $\ell_1(m) = j' < k$ . Then  $m$  has no forcing inputs.

Pf.: We show that the first input can not be forcing. The proof reveals that the choice of input is arbitrary and thus that the theorem follows. Consider the row with  $x_1=1$  and  $x_i=0$ ,  $i=2, \dots, k$  and the row with  $x_1=1$ ,  $i=1, \dots, k-1$  and  $x_k=0$ . Since both  $j$  and  $j'$  are less than  $k$  we must have  $m(1,0,0, \dots, 0) \neq m(1,1,1, \dots, 1,0)$ . Similarly  $m(0,1,1, \dots, 1) \neq m(0,0,0, \dots, 0,1)$  and thus the first input can not be forcing.  $\square$

A partial converse to this result is available if we consider mappings having exactly  $f$  forcing inputs such that each forcing input has the same forcing state. For such a mapping the threshold can not exceed  $k+1-f$ .

Theorem 6: Suppose  $m$  is a mapping on  $k$  inputs which is forcing on exactly  $f$  of them and each forcing input has the same forcing state. Then  $\ell(m) \leq k+1-f$ .

Pf.: Without loss of generality we may assume that the first  $f$  inputs are forcing, that the forcing state is "1" and that the forced value is "1". If we examine any row having  $k+1-f$  or more "1"'s at least one of them must be associated with one of the first  $f$  inputs and thus  $m=1$ . Hence  $\ell_1(m) \leq k+1-f$  and therefore  $\ell(m) \leq k+1-f$ .  $\square$

We note two obvious corollaries.

Corollary 1: If  $f'$  of the  $f$  forcing inputs have forcing state "1" and  $f-f'$  having forcing state "0" then

$$\ell(m) \leq k+1-\max(f', f-f').$$

Corollary 2: If  $m$  is forcing on exactly  $k-1$  inputs and each forcing input has the same forcing state, then  $\ell(m) = 2$ .

Table 3 illustrates the need for all forcing inputs to have the same forcing state. Both mappings  $m_1$  and  $m_2$  have inputs 1 and 2 as forcing. However  $m_1$  satisfies the conditions of Theorem 6 (and Corollary 2) and has  $\ell(m_1)=2$ . Mapping  $m_2$  does not and has  $\ell(m_2)=3$  (in agreement with Corollary 1).

| $x_3$ | $x_2$ | $x_1$ | $m_1$ | $m_2$ |
|-------|-------|-------|-------|-------|
| 1     | 1     | 1     | 1     | 1     |
| 0     | 1     | 1     | 1     | 1     |
| 1     | 0     | 1     | 1     | 1     |
| 0     | 0     | 1     | 1     | 1     |
| 1     | 1     | 0     | 1     | 0     |
| 0     | 1     | 0     | 1     | 0     |
| 1     | 0     | 0     | 0     | 1     |
| 0     | 0     | 0     | 0     | 1     |

Table 3: An Illustration of Theorem 6

We next turn to the calculation of  $\sigma(k, f, \ell)$ , the number of mappings on  $k$  inputs with exactly  $f$  forcing inputs and threshold  $\ell$ . The expression we develop is extremely awkward to calculate and will be presented via a lemma and two theorems. The reasons for the complexity were touched upon in the discussion surrounding the previous theorem, i.e., forcing inputs need not have the same forcing state. As a result we must calculate  $\rho(k, \ell_0, \ell_1, r, s)$ , the number of maps on  $k$  inputs with thresholds  $\ell_0, \ell_1$  respectively and  $r$  inputs having forcing state "1",  $s$  inputs having forcing state "0".

We first note the following lemma.

Lemma 4:  $\rho(k, \ell_0, \ell_1, r, s) = 0$  unless both  $r \leq \ell_0$  and  $s \leq \ell_1$ .

Pf.: Using the same argument as in Theorem 6 and Corollary 1 we see that  $\ell_0 \leq k+1-s$ ,  $\ell_1 \leq k+1-r$ . Hence  $s \leq k+1-\ell_0$ ,  $r \leq k+1-\ell_1$ . If  $\ell_1 < s$  then  $\ell_1 < k+1-\ell_0$ , i.e.,  $\ell_1 + \ell_0 < k+1$  which is impossible. Similarly for  $\ell_0 < r$ .  $\square$

Lemma 3 resolves the calculation of  $\rho$  when  $\ell_0 + \ell_1 = k+1$ , i.e., only  $\ell = 1$  is possible and all inputs force. Thus  $\rho(k, 1, k, k, 0) = \rho(k, k, 1, 0, k) = 2$ . Furthermore since

$$\alpha(k, \ell_0, \ell_1) = \sum_{\substack{s \leq k+1-\ell_0 \\ r \leq k+1-\ell_1}} \rho(k, \ell_0, \ell_1, r, s)$$

we may calculate  $\rho(k, \ell_0, \ell_1, 0, 0)$  by subtraction if we have obtained  $\rho(k, \ell_0, \ell_1, r, s)$  for any  $r, s$  with  $\max(r, s) \geq 1$ .

As we shall see, it is most convenient to first calculate  $R'(k, \ell_0, \ell_1, r, s)$  which is the number of mappings on  $k$  inputs having thresholds  $\ell_0$  and  $\ell_1$  with at least the first  $r$  inputs having forcing state "1" and at least the next  $s$  inputs having forcing state "0". Lemma 5 calculates  $R'$ , Theorem 7 shows how  $R'$  may be adjusted to yield  $\rho$  and finally Theorem 8 obtains  $\sigma$  from  $\rho$ .

Lemma 5: Case (i)  $r \geq 1$ ,  $s \geq 1$ .

$$\binom{k-(r+s)}{\ell_0 - r - 1}$$

If  $\ell_0 + \ell_1 = k+2$ ,  $R'(k, \ell_0, \ell_1, r, s) = 2 \binom{k-(r+s)}{\ell_0 - r - 1} - 1$ .



If  $l_0 + l_1 > k+2$ ,  $R'(k, l_0, l_1, r, s) =$

$$2 \binom{k-(r+s)}{l_0-r-1} (2^{-1}) \binom{k-(r+s)}{l_1-s-1} (2^{-1}) \sum_{i=k-r-l_1+2}^{l_0-r-2} \binom{k}{i}.$$

Case (ii)  $r \geq 1$ ,  $s=0$ .

If  $l_0 = k$ ,  $l_0 + l_1 = k+2$ ,  $R'(k, k, 2, r, 0) = 2^{k-r+2} - 6$ .

If  $l_0 = k$ ,  $l_0 + l_1 > k+2$ ,  $R'(k, k, l_1, r, 0) =$

$$4 \binom{k-r}{l_1-1} (2^{-1}) \sum_{i=k-r-l_1+2}^{k-r-2} \binom{k}{i}.$$

If  $l_0 < k$ , same expressions as in case (i).

Case (iii)  $s \geq 1$ ,  $r=0$ .

If  $l_1 = k$ ,  $l_0 + l_1 = k+2$ ,  $R'(k, 2, k, 0, s) = 2^{k-s+2} - 6$ . If  $l_1 = k$ ,

$l_0 + l_1 > k+2$ ,  $R'(k, l_0, k, 0, s) =$

$$4 \binom{k-s}{l_0-1} (2^{-1}) \sum_{i=k-l_1+2}^{l_0-2} \binom{k}{i}.$$

If  $l_0 < k$ , same expressions as in case (i).

Pf.: The proof of this lemma is quite similar to that of Theorem 1. We only prove cases (i) and (ii) since case (iii) is symmetrically equivalent to case (ii).

(1)  $r \geq 1, s \geq 1$  implies in the lexicographic order, the first  $2^k - 2^{k-r}$  rows (i.e., those which have a "1" for any of the first  $r$  inputs) are determined at, say  $a$ . Also the last  $2^{k-r} - 2^{k-r-s}$  rows must as well be determined at this  $a$  (i.e., the forced value is unique and these are the remaining rows having a "0" in at least one of the next  $s$  inputs). Note that since the first and last rows have map value  $a$ , both input values must have the same threshold state,  $a$ . Consider the undetermined rows. All have  $x_1 = x_2 = \dots = x_r = 0, x_{r+1} = x_{r+2} = \dots = x_{r+s} = 1$ . If  $\ell_0 = r$  or  $\ell_1 = s$  all of these rows must also be determined to have map value  $a$  and thus the trivial map results. Hence we take  $r < \ell_0, s < \ell_1$ . To have threshold  $\ell_0$  and  $\ell_1$ , for the remaining  $k-(r+s)$  inputs whenever  $\ell_0 - r$  or more are "0" or whenever  $\ell_1 - s$  or more are "1" the map value must again be  $a$ . To insure exactly  $\ell_0, \ell_1$  we must look among these  $k-(r+s)$  inputs at rows with exactly  $\ell_0 - r - 1$  of them at "0" and at rows with exactly  $\ell_1 - s - 1$  of them at "1" (i.e.,  $k-(r+s) - (\ell_1 - s - 1) = k - r - \ell_1 + 1$  of them at "0").

If  $\ell_0 + \ell_1 = k + 2$  then  $k - r - \ell_1 + 1 = \ell_0 - r - 1$  and for at least one of these  $\binom{k-(r+s)}{\ell_0 - r - 1}$  rows the map value must be

1- $a$ . Since  $a$  may be chosen in two ways the expression for  $\ell_0 + \ell_1 = k + 2$  follows.

If  $\ell_0 + \ell_1 > k + 2$  then  $k - r - \ell_1 + 1 < \ell_0 - r - 1$  and the second part of (i) follows as in (iii) of Theorem 1 except that again  $a$  can be chosen in two ways and  $b$  must be  $1 - a$ .

(ii)  $r \geq 1, s = 0$  implies again that the first  $2^k - 2^{k-r}$  rows are determined at say  $a$ . However with  $s = 0$  the map value for the last row is not fixed. From Theorem 5 with  $1 \leq r \leq \ell_0$  (Lemma 4), if the threshold states differ  $\ell_0$  must equal  $k$  and the last row will have map value  $1 - a$ . Hence again as in Theorem 1, if  $\ell_0 = k$  we may have  $a \neq b$  but if  $\ell_0 < k$  we must have  $a = b$ . For the former the expressions mimic cases (ii) and (iii) of Theorem 1 while for the latter the same argument as in case (i) of this theorem is appropriate with  $s = 0$ . Hence the expressions in (ii) follow and we are done.  $\square$

Note that the completion of the rows in cases (i) and (ii) in order to fix  $\ell_0$  and  $\ell_1$  may result in more than just the first  $r$  inputs being forcing with forcing state "1" and more than just the next  $s$  inputs being forcing with forcing state "0".

Theorem 7:

$$\rho(k, \ell_0, \ell_1, r, s) = \frac{k!}{r!s!(k-(r+s))!} \sum_{\substack{0 \leq j, j' \\ j+j' \leq k-(r+s)}} (-1)^{j+j'}$$

$$\frac{(k-(r+s))!}{j!j'!(k-(r+s)-j-j')!} R'(k, \ell_0, \ell_1, r+j, s+j').$$

Pf.: The summation adjusts R' to the number of ways in which exactly the first r inputs have forcing state "1" and exactly the next s inputs have forcing state "0". The form may be established from a straightforward counting argument using symmetry in the selection of the additional j+j' forcing inputs. The details are omitted. The factorial coefficient allows the adjustment from the first r and next s inputs to an arbitrary choice of r and s from the total of k inputs.  $\square$

We finally have

Theorem 8:

$$\sigma(k, f, \ell) = 2 \sum_{\ell'=\ell+1}^k \sum_{r+s=f} \rho(k, \ell, \ell', r, s) + \sum_{r+s=f} \rho(k, \ell, \ell', r, s).$$

Pf.: In order to have exactly f forcing inputs, r+s must equal f. Since it is apparent that  $\sum_{r+s=f} \rho(k, \ell_0, \ell_1, r, s)$

$$= \sum_{r+s=f} \rho(k, \ell_1, \ell_0, r, s) \text{ the conclusion follows. } \square$$

Considering the difficulties involved in achieving Theorems 4 and 8 a theoretical enumeration of maps by  $\ell$  and I and F appears overwhelming. Instead we offer Table 4 which presents such an explicit enumeration for k=2, 3 and 4. The theoretical results for  $\ell$  and I (Theorem 4) may be verified by summing the tables over F. Similarly for  $\ell$  and F (Theorem 8) by summing over I. The tables particularly at k=3 and 4 reveal the

weak relation between  $\ell$  and I and the rather strong inverse relation between  $\ell$  and F.

In fact for general  $k$  the relationship between  $\ell$  and I must continue to be weak. We recall, using the monotonic ordering for mappings, that only an upper and lower set of input rows are determined. The remaining middle rows are all free to assume either map value. Hence if  $k$  is large and  $\ell$  is not too small I is only weakly controlled by the specification of  $\ell$ . In the situation where  $\ell$  is very small or is an absolute threshold this will not be the case but such mappings are sparse in the overall collection of possible mappings. For general  $k$  the strong inverse relationship between  $\ell$  and F also persists by reference to Theorem 6 and Corollary 1. More precisely, decreasing threshold has the effect of increasing the proportion of maps in the given threshold class that have one or more forcible inputs. For example, with  $k=4$  maps, that proportion is 0.03 at  $\ell=4$ , 0.20 at  $\ell=3$ , 0.93 at  $\ell=2$ , and 1.0 at  $\ell=1$  and  $\ell=0$ . From Table 4 it can also be seen that within threshold classes the positive relationship between internal homogeneity and forcibility noted in part I still obtains. This is a nontrivial finding in that contrary outcomes are conceivable, particularly for  $\ell=2$ . That is, extended threshold could vary the way in which internal homogeneity and forcibility are related. We find that it does not. This suggests the existence

of some mechanism common to the three measures. We return to this point in Section 5.

k=2

$\ell=2$

|   |   |   |   |   |   |    |
|---|---|---|---|---|---|----|
|   | f | 1 | 2 | 3 | 4 |    |
| 0 | 2 | - | - | - | 2 |    |
| 1 | 4 | - | - | - | 4 |    |
| 2 | - | 4 | - | - | 4 |    |
|   |   |   |   |   |   | 10 |

$\ell=1$

|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
|   | f | 1 | 2 | 3 | 4 |   |
| 0 | - | - | - | - | - |   |
| 1 | - | - | - | - | - |   |
| 2 | - | 4 | - | - | 4 |   |
|   |   |   |   |   |   | 4 |

$\ell=0$

|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
|   | f | 1 | 2 | 3 | 4 |   |
| 0 | - | - | - | - | - |   |
| 1 | - | - | - | - | - |   |
| 2 | - | - | - | 2 | 2 |   |
|   |   |   |   |   |   | 2 |

k=3

$\ell=3$

|   |    |    |    |   |   |   |   |     |
|---|----|----|----|---|---|---|---|-----|
|   | f  | 1  | 4  | 5 | 6 | 7 | 8 |     |
| 0 | 62 | 60 | 8  | - | - | - | - | 130 |
| 1 | 6  | 36 | 12 | - | - | - | - | 54  |
| 2 | -  | -  | 12 | - | - | - | - | 12  |
| 3 | -  | -  | -  | - | - | - | - | -   |
|   |    |    |    |   |   |   |   | 196 |

$\ell=2$

|       |   |    |    |    |   |    |
|-------|---|----|----|----|---|----|
| f \ 1 | 4 | 5  | 6  | 7  | 8 |    |
| 0     | 2 | 4  | -  | -  | - | 6  |
| 1     | - | 12 | 12 | -  | - | 24 |
| 2     | - | -  | 12 | -  | - | 12 |
| 3     | - | -  | -  | 12 | - | 12 |
|       | 2 | 16 | 24 | 12 | - | 54 |

$\ell=1$

|       |   |   |   |   |   |   |
|-------|---|---|---|---|---|---|
| f \ 1 | 4 | 5 | 6 | 7 | 8 |   |
| 0     | - | - | - | - | - | - |
| 1     | - | - | - | - | - | - |
| 2     | - | - | - | - | - | - |
| 3     | - | - | - | 4 | - | 4 |
|       | - | - | - | 4 | - | 4 |

$\ell=0$

|       |   |   |   |   |   |   |
|-------|---|---|---|---|---|---|
| f \ 1 | 4 | 5 | 6 | 7 | 8 |   |
| 0     | - | - | - | - | - | - |
| 1     | - | - | - | - | - | - |
| 2     | - | - | - | - | - | - |
| 3     | - | - | - | - | 2 | 2 |
|       | - | - | - | - | 2 | 2 |

$k=4$

$\ell=4$

|       |       |       |       |      |      |     |    |    |    |       |
|-------|-------|-------|-------|------|------|-----|----|----|----|-------|
| f \ 1 | 8     | 9     | 10    | 11   | 12   | 13  | 14 | 15 | 16 |       |
| 0     | 12242 | 21288 | 13814 | 6340 | 1830 | 260 | 10 | -  | -  | 55784 |
| 1     | 8     | 112   | 336   | 560  | 496  | 192 | 16 | -  | -  | 1720  |
| 2     | -     | -     | -     | -    | 24   | 48  | 24 | -  | -  | 96    |
| 3     | -     | -     | -     | -    | -    | -   | -  | -  | -  | -     |
| 4     | -     | -     | -     | -    | -    | -   | -  | -  | -  | -     |
|       | 12250 | 21400 | 14150 | 6900 | 2350 | 500 | 50 | -  | -  | 57600 |

| $\ell=3$ | $f \backslash i$ | 8   | 9    | 10   | 11   | 12   | 13  | 14  | 15 | 16 |      |
|----------|------------------|-----|------|------|------|------|-----|-----|----|----|------|
|          | 0                | 620 | 1464 | 1754 | 1496 | 734  | 156 | 6   | -  | -  | 6230 |
|          | 1                | -   | 16   | 112  | 336  | 512  | 304 | 48  | -  | -  | 1328 |
|          | 2                | -   | -    | -    | -    | 24   | 120 | 48  | -  | -  | 192  |
|          | 3                | -   | -    | -    | -    | -    | -   | 48  | -  | -  | 48   |
|          | 4                | -   | -    | -    | -    | -    | -   | -   | 12 | -  | 12   |
|          |                  | 620 | 1480 | 1866 | 1832 | 1270 | 580 | 150 | 12 | 0  | 7810 |

| $\ell=2$ | $f \backslash i$ | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |     |
|----------|------------------|---|---|----|----|----|----|----|----|----|-----|
|          | 0                | - | - | -  | 4  | 4  | -  | -  | -  | -  | 8   |
|          | 1                | - | - | -  | -  | 16 | 16 | -  | -  | -  | 32  |
|          | 2                | - | - | -  | -  | -  | 24 | 24 | -  | -  | 48  |
|          | 3                | - | - | -  | -  | -  | -  | 16 | -  | -  | 16  |
|          | 4                | - | - | -  | -  | -  | -  | -  | 16 | -  | 16  |
|          |                  | - | - | -  | 4  | 20 | 40 | 40 | 16 | -  | 120 |

| $\ell=1$ | $f \backslash i$ | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |   |
|----------|------------------|---|---|----|----|----|----|----|----|----|---|
|          | 0                | - | - | -  | -  | -  | -  | -  | -  | -  | - |
|          | 1                | - | - | -  | -  | -  | -  | -  | -  | -  | - |
|          | 2                | - | - | -  | -  | -  | -  | -  | -  | -  | - |
|          | 3                | - | - | -  | -  | -  | -  | -  | -  | -  | - |
|          | 4                | - | - | -  | -  | -  | -  | -  | 4  | -  | 4 |
|          |                  | - | - | -  | -  | -  | -  | -  | 4  | -  | 4 |

| $\ell=0$ | $f \backslash i$ | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |   |
|----------|------------------|---|---|----|----|----|----|----|----|----|---|
|          | 0                | - | - | -  | -  | -  | -  | -  | -  | -  | - |
|          | 1                | - | - | -  | -  | -  | -  | -  | -  | -  | - |
|          | 2                | - | - | -  | -  | -  | -  | -  | -  | -  | - |
|          | 3                | - | - | -  | -  | -  | -  | -  | -  | -  | - |
|          | 4                | - | - | -  | -  | -  | -  | -  | -  | 2  | 2 |
|          |                  | - | - | -  | -  | -  | -  | -  | -  | 2  | 2 |

Table 4. An enumeration of mappings by number of forcing inputs ( $f$ ), internal homogeneity ( $i$ ), and threshold ( $\ell$ ), for number of inputs  $k=1,2,3,4$ .



#### 4. Organizational Implications and Interpretations

We now examine how the relationships developed in the preceding section might be used in practice to defend the complex organization against behavioral abnormality. The manager or organizational designer intending to use these results, as we have said, would already have determined intervention in the form of detailed structural and functional specification to be inappropriate. We further assume that the he or she has decided not to manage by manipulating input span ( $k$ ). Rather, we emphasize exception, priority, and consensus over input span management because we assume that in practice many control nets will have to retain high  $k$  values.

The general way in which the inverse relationship between extended threshold and forcibility can be applied is now clear. Interpreting extended threshold in the manner suggested in our introduction as the level of information or consensus at which action (or lack thereof) is to be taken, and noting once again that increased densities of forcible mappings work generally to regularize system behavior, the manager has the option of seeking to decrease the consensus level in the appropriate organizational control net. To do so the manager might address those in charge of net elements: "Given your sources of control information as they are, try to arrange your procedures so that appropriate

action will be taken at the lowest possible amount of relevant information," or more briefly, "Be decisive, but not impetuous." Note that the latter formulation has a clear attitudinal, or dispositional component. We return to this point below.

That reducing consensus levels in a complex organization should serve to keep its behavior below certain pathological extremes may be mildly paradoxical. Some resolution of the paradox is provided by recalling that high levels of (extended) threshold do not imply consistency of response, nor, in particular, do they imply complete lack of response, at sub-threshold input. Furthermore, the manager will want it understood that consensus levels should bear a truly appropriate relation to the task at hand. They should not be set whimsically.

What might the joint relationship between the three strategies say about organizational theory and management? First, it is worth reflecting on the fact that any given mapping necessarily has definite levels of internal homogeneity, forcibility, and extended threshold. If our interpretations and point of view are at all generally valid, the organizational theory implication can be drawn that any organizational unit with a fixed functional regime operates at definite levels of management by exception, by priority, and by consensus. Since it is reasonable to think that these managerial styles may

have psychological impact on the unit's work force, the further implication is that organizational micro-climate and unit sociology may be influenced by the unit's functional regime.

If mappings (functional regimes) do affect worker psychology, this fact becomes a design consideration. It then becomes important to ask how free the organization designer is to mutually vary the three strategies. It can be seen from Table 4 that the three strategies are correlated, but the correlation is much less than perfect. Therefore it is possible to manipulate the density of forcible mappings, either directly by acting on values of management by priority, or indirectly by changing the intensity of either exception or consensus management, while at the same time allowing for some fine-tuning of, say, organizational climate, by modifying the intensity of the remaining strategy or strategies.

An important datum for intervention design in our scheme is given by Kauffman [5]. He states that if 60% of the maps in a large, complex, high  $k$  net are forcible on one or more inputs, then the net behaves essentially as does the  $k=2$  net. To illustrate the combined use of strategies, let us assume that  $k=4$ , and an intervention to decrease consensus has achieved  $\ell=3$  in the net. If no information is available as to what the existing intensities of net I and F are, a reasonable estimate

of the density of forcible maps is that provided by the marginal distribution of  $F$  given  $\ell=3$ , in Table 4. That is, the predicted density of forcible maps is 20%: lower than the Kauffman criterion. If a second intervention can raise the intensity of exception management to  $I \geq 13$ , while not raising the consensus level, the manager is assured of at least 73% forcible maps in the control net.\* That is, for  $\ell=2$ ,  $I=13$  there are 73% of the maps forcible on at least one input, and for  $\ell=2$ ,  $I \geq 13$  the densities increase. The organization is now controlled to the extent that useful behaviors are a practical possibility. The manager is still free to act directly to increase management by priority, or perhaps cautiously to decrease its mean value, so as to modify the organizational climate or to accomplish other aims.

The illustration above makes use of the fact that the three strategies are imperfectly correlated. The joint correlation, however, is fairly high, and this has its own interesting implication for organizational psychology: Whatever the distinct psychological impact

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\*More correctly, he is assured of 73% forcible maps in the population from which the net maps are assumed to be a simple random sample. For large nets the population figure is a reasonable prediction as to what will prevail in a given net.

of each strategy may be, the behaviorally reasonable complex organization with arbitrary structure would appear to provide its control system workers only a limited subset of all possible psychological environments.

What might be the psychological and sociological character of each strategy be? We speculate that management by exception might impact largely on an obsession--indifference dimension, management by priority might affect status structure and hence status-related behaviors, and management by consensus might be associated with behavior on a boldness-timidity or risk-taking continuum.

Under these interpretations, the tractable high input span complex organization would appear in two basic forms. 1) Its control nets could be specifically structured to provide tractable behavior. Such organizations' control units would be able to show a wide variety of psychological environments. In this case we would expect net structure to be tightly controlled, that is, such organizations would show the presumably few easily maintained organizational forms that promote forcibility, or they would invest relatively heavily in the maintenance of a priori organizational forms. Or, 2) its control nets would not be specifically formed. Here, we would expect to see both more variety in form and less investment in structural form maintenance and

at the same time less variety in control units' psychological environments. The environments expected in these circumstances would be such as to suit control unit personnel who are largely intermediate on the obsession-indifference scale in work habits, comfortable with a modest amount of status structure in the work environment, and who can be moderate risk takers.

The decisively pathological complex organization in our scheme has high input span control nets, little capacity for organizational form maintenance, and shows control net units in which no work related status structures are found, or control unit personnel who are typically work-obsessive or low risk takers on the job.

##### 5. System Theory and Ensemble Methods in Organizational Theory

We now ask if the similarity in effect of our three strategies may be explained by some mechanism common to all three. Such an explanation is easily found. Referring to their definitions, it can be seen that each of the three strategies is scaled by reference to the extent of sameness it provides in functional output. Useful systemic effects then, are achieved by appropriately blocking information flow. Ashby has discussed the importance of such control in general, calling functions that achieve it "part-functions" [1, p. 66]. It is worth emphasizing that providing tractable, stable behavior by information blocking does not necessarily result in a moribund organization. For the complex

organization controlled by part-functions, the picture which emerges, to paraphrase Kauffman [4], is that of a system rich in part-functions, with extensive blocked paths weaving through it, leaving pockets of informationally active elements functionally isolated from one another. The part-functions and blocked paths would provide the basic tractability of the organization. Different organizational products or outcomes would correspond to different patterns of activity, either steady state or cyclic, of the isolated, active subsystems.

One of our aims in this series of papers is to illustrate the purchase that can be obtained using system theoretic ensemble methods to address the management of complexity. The simple model proposed here for complex organization control systems is almost surely inadequate in many respects. However, a significant question in any discipline is what are the properties necessary to explain what is observed. Simple models provide an especially attractive testing ground for examining this question. Our examination of these simple models illustrates the fact that structure and function do interact. That is, the effective control structure in an organization is a dynamic entity which varies over time, and which can be modified by functional changes in control elements. We have argued

that such functional change may be achieved in recognizably different ways, different, that is, both as to how the manager might achieve the changes, and as to what psychological and sociological effects might be expected from their use. We have argued that, in suitable circumstances, details of organizational control structure might not be a crucial consideration for the organizational intervener or designer. Finally, we have tried to show that simple models can provide rich real-world interpretive and explanatory possibilities.

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Part I of this effort suggests that switching net models provide useful insight into the behavior of complex organizational control systems. Equivalence relations on the responses of system elements were defined, namely internal homogeneity and forcibility. These in turn were interpreted as managerial strategies: management by exception and management by priority, respectively.

The present article introduces a further equivalence relation--extended threshold, which is interpreted as management by consensus. The notion of extended threshold and its interaction with internal homogeneity and forcibility are examined in detail. We then observe that organization-wide control may be exercised by varying strategy levels. Finally we speculate as to the psychological impact of the strategies on the work force and explore implications these interpretations have for organizational climate in complex enterprises.