A Mathematical Model of Sound Velocity for the Indian Ocean

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<th>Recipient's Catalog Number</th>
<th>Type of Report &amp; Period Covered</th>
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<th>Contract or Grant Number(s)</th>
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<th>Distribution Statement (of this Report)</th>
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<th>Supplementary Notes</th>
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<td>Oct 1975</td>
<td>14</td>
<td>Approved for public release; distribution unlimited.</td>
<td>(Continue on reverse side if necessary and identify by block number)</td>
<td>Oceanography, Acoustics, Indian Ocean, Sound Velocity</td>
<td>Describes a mathematical model of sound velocity for the Indian Ocean that can predict sound velocity at depths between 200 and 2450m within 2 m/sec RMS. It is used in computer generation of acoustic province charts.</td>
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INTRODUCTION

An objective procedure for producing a three-dimensional mathematical model of sound velocity in the oceans has been developed by the U. S. Naval Oceanographic Office. This model contains the necessary statistical characteristics to form the basis for acoustic province delineation. The mathematical procedures necessary to generate a primary three-dimensional model of sound velocity containing horizontal spatial frequency components in the vertical plane are being included in the compilation of such products as NAVOCLANO's ASW Prediction Area Charts.

The accuracy of this modeling system for predicting sound velocity at depth between 200 m and 2450 m at any latitude and longitude within the model area has been demonstrated to be well within ± 2 m/sec. RMS. The significance of this modeling concept has been demonstrated through rapid computer generation of acoustic province charts as mentioned above, as well as a sound velocity envelope for each delineated acoustic province.

The validity of the model has been proven in the Indian Ocean (Arabian Sea), as elsewhere, where good agreement was found between sound velocity profiles generated by the model and measured cruise data. The mean difference between the model and the measured data in the Arabian Sea was 0.43 m/sec.
DESCRIPTION OF THE SOUND VELOCITY MODEL

An objective procedure for producing a three-dimensional mathematical model of sound velocity and other oceanographic parameters has been developed by the U. S. Naval Oceanographic Office. This model contains the necessary statistical characteristics to form the basis for the prediction of detection ranges, acoustic survey design, and acoustic province delineation.

The model contains the mathematical procedures necessary to generate a primary three-dimensional model of sound velocity containing horizontal spatial frequency components down to 60 nautical mile wavelengths with 400-meter wavelength resolution in the vertical plane. Multiple tests have shown this model accurate within 2 meters/second RMS in predicting sound velocity at depth between 200 and 2450 meters at any latitude and longitude within the model area.

This model functions as follows:

1. The latitudes and longitudes of the corner points of a polygon inscribing the area of interest define the search area from which sound velocity profiles will be extracted from the On-Station (OSTA) data tapes. These tapes are a packed listing of all NODC station files through 1970 and contain sound velocity computed by Wilson's 1960 equation.
2. A 1° mercator grid is constructed to contain the search polygonal area and provide for a 2° overlap into adjacent areas which assures continuity of contours into adjacent areas.

3. Each OSTIA Station is tested to assure conformance to input selection criteria.

4. Accepted stations meeting minimum depth requirements but failing to reach the desired maximum sample depth are augmented by sound velocity extrapolated from surrounding data to the desired maximum depth.

5. Accepted profiles are subjected to cubic spline interpolation to yield a final profile from 200 to 2450 meters with values at every 50 meters (46 levels) at each 1° grid point.

6. A seventh degree least squares orthogonal polynomial is then fitted to each final profile and the coefficients retained.

7. The resultant sets of orthogonal polynomial coefficients are fitted into a 30 x 30 minute grid by employing a three-phase algorithm which interpolates random data. Phase one partially fills the desired grid by assigning input values to the closest grid point. If more than one point lines near a given grid point, its value will be a weighted average of these input values. Phase two uses a modified version of an algorithm developed by Shepard (1968) to expand this partial grid by completing boundaries and adding enough interior values to allow splining. Phase three than completes the grid using a bicubic-spline algorithm of Davis (1970) applied twice and averaged.
8. The gridded orthogonal polynomial coefficients through the fourth degree are prefitted for smoothing, and the magnitude of the gradients are computed and contoured to yield secondary province boundaries from selected values of the gradients of the coefficients.

9. Polynomial curves are computed from the resultant linear correlation coefficients to yield minimum, mean, and maximum sound velocity. The geographical positions of grid points and the related linear correlation coefficients and groups of the latter characterize the average and type shape of the sound velocity profiles for each secondary province.

10. The Navy Interim Surface Ship Sonar Prediction Model (NISSM) computes for each secondary sound velocity province, the 50% probability of detection ranges in kiloyards, and minimum, mean, and maximum sound velocity profiles constructed from the given linear correlation coefficients as well.

11. The distribution of detection range variability within the secondary provinces is analyzed to yield the final selection of primary sound velocity provinces.

MODEL FOR THE ARABIAN SEA

An example of the output of this model is demonstrated in Standard Navy Ocean Area and Region IN-1 (the Arabian Sea) where one can delineate areas with similar coefficients and therefore with similar sound velocity characteristics. Figure 1 indicates that this sea can be divided into six primary sound velocity provinces including the Gulf of Oman and Persian Gulf. Figures 2 through 7 depict the minimum, mean, and maximum sound velocity structure in each province. It should be noted that each province has a distinctly different structure.
A comparison of sound velocity profiles generated by the model for this area with measured cruise data indicates good agreement (Cocke, 1974). Sixteen stations occupied in the Indian Ocean during the southwest monsoon were compared to the model and the mean difference was found to be as low as 0.43 m/sec. (0.4 ft/sec.). Figure 8 (after Cocke), which illustrates comparisons of cruise data with the model, demonstrates the model’s validity.

Detailed analyses of the effects of the northeast and southwest monsoons on the sound velocity structure in the Arabian Sea (and Indian Ocean) is contained in Fenner and Bucca (1972). In addition, tabulations of sound velocity at 100-meter intervals below 1,000 meters, as well as information on critical depths for the Arabian Sea are available in Audet (1975).

SUMMARY

A mathematical model has been developed which can predict sound velocity at depth between 200 and 2450 meters at any latitude and longitude within the model area.

Application of this model to the Arabian Sea resulted in the delineation of six distinct acoustic provinces.

Comparisons of sound velocity profiles generated by the model for each province with measured cruise data indicate that the model is valid. The mean difference between the model and measured data in the Arabian Sea is only 0.43 m/sec.
FIGURE 1  ACOUSTIC PROVINCES IN INDIAN OCEAN REGION 1
FIGURE 2  ANNUAL SOUND VELOCITY PROFILES IN PROVINCE 1
FIGURE 3  ANNUAL SOUND VELOCITY PROFILES IN PROVINCE 2
FIGURE 5  ANNUAL SOUND VELOCITY PROFILES IN PROVINCE 4
FIGURE 6  ANNUAL SOUND VELOCITY
PROFILES IN PROVINCE 5
FIGURE 7  ANNUAL SOUND VELOCITY PROFILES IN PROVINCE 6
Figure 8. Average S.V. Profile Comparisons: Provinces 3 and 4
REFERENCES


DATE: October 1975  

SUBJECT: A MATHEMATICAL MODEL OF SOUND VELOCITY FOR THE INDIAN OCEAN

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