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THE FORMATION AND STABILITY OF LONGITUDINAL ROLL VORTICES IN SHEAR FLOWS

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13. ABSTRACT

Longitudinal roll vortices occur frequently in shear flows. Although they are most commonly driven by body forces (either centrifugal or buoyant), they are also observed during transition in homogeneous boundary layers. Under the above grant, various investigations have been made of their formation and, for the case of thermally driven vortices, their own instability. This report summarizes the results obtained. More detailed results can be found in the journal publications referred to in the report.

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2. Personnel Associated with the Grant

Robert E. Kelly, Principal Investigator

Prakash Iyer, Ph.D. awarded March, 1973
(thesis title: "Instabilities in Buoyance-Driven Boundary Layer Flows over Inclined Surfaces")

P. Amar, graduate student

Richard M. Clever, postdoctoral scholar

Ferdinand Hendriks, postdoctoral scholar

3. Publications Reporting Research Carried out under the Grant

1. P. A. Iyer and R. E. Kelly, "The Stability of the Laminar Free Convection Flow Induced by a Heated, Inclined Plate," Int. J. Heat and Mass Transfer, v. 17 (1974), 517-525.

2. F. Hendriks, Appendix to "Nonlinear Wave Interactions in Shear Flows, Part 2" (by J. R. Usher and A. D. D. Craik), J. Fluid Mech. v. 70 (1975), 458-461.

3. R. M. Clever and F. H. Busse, "Instabilities of Longitudinal Convection Rolls in an Inclined Layer," to be published in J. Fluid Mech.

4. R. M. Clever, F. H. Busse, and R. E. Kelly, "Instabilities of Longitudinal Convection Rolls in Couette Flow," to be published in Zeit. angew. Math. Physik.

5. R. E. Kelly and R. M. Clever, "Primary and Secondary Instabilities in Unstably Stratified Shear Flows," to be published in Proc. RAND 2nd Boundary — Layer Transition Workshop.

6. R. E. Kelly, "The Onset and Development in Shear Flows: A Review," to be published in *Proc. Int. Conf. on Physical Chemistry and Hydrodynamics* (Oxford, 1977; to be published by Hemisphere Publ. Corp.)

(Copies of all articles have been submitted to ARO.)

4. The Formation and Stability of Longitudinal Roll Vortices in Shear Flows

Longitudinal roll-like vortices are fairly common in shear flows. Such vortices have their axes in the direction of the mean flow and are spatially periodic in the spanwise direction. They are most commonly associated with instabilities due to body forces, e.g., centrifugal forces in the case of Görtler vortices in boundary layer flows over concave surfaces, or gravity in the case of flows over heated surfaces. An illustration of the vortices for the latter case is given in Figure 1. They are frequently observed in the unstable planetary boundary layer by the formation of cloud rows which can form at the tops of the rather straight updraft regions where moist air is advected upwards and condenses. A schematic illustration is given in Figure 2, which is Figure 5 of [1]. Cloud rows are especially common in the tropics, where thermal convection is prevalent. They have been observed to extend in length for up to 500 km, and their ratio of height to wavelength is close to what one would predict on the basis of linear Rayleigh-Bénard convection theory. This is rather remarkable from a basic fluid mechanical point of view because the Rayleigh and Reynolds numbers for the planetary boundary layer, based on molecular diffusivities, are very large, and one would therefore expect fully turbulent flow. On a small scale, of course, turbulence does occur, as can be seen from the features of individual clouds. On a large scale, however, extraordinary regularity occurs. Indeed, no better example of "large scale organization" in turbulent flows can be given. Also, this seems to be one situation where the use of an eddy viscosity is truly meaningful when examining the large scale dynamics.

Longitudinal vortices can also be observed in homogeneous boundary layers on straight surfaces during the process of transition [3]. This phenomenon has been explained by Benney [4] as arising from the interaction of two and three-dimensional unstable Tollmien-Schlichting waves. Benney assumed that the waves have the same downstream wavenumber and frequency, which cannot be strictly true. One of the original goals of the present research program was to do a more accurate calculation in which this assumption would be relaxed and in which true boundary layer velocity profiles would be used. However, such a calculation has now been done by Antar and Collins [5] (see also [6], [7]), who have shown that the resulting vortex system acquires a temporal periodicity once the difference in frequencies between the two disturbances is taken into account. We tried to address the more basic question concerning what determines the observed wavelength of the longitudinal vortices, i.e., are there preferred wave interactions which give rise to the vortices? (The Lin-Benney theory concerned any two disturbances and was aimed at explaining how a vortex structure can arise rather than predicting the wavelength of the structure.) A promising theory was advanced in 1971 by Craik [8], in which resonant interaction between some three-dimensional waves and unstable two-dimensional waves was suggested as leading to rapid development of three-dimensionality in the boundary layer. Because Craik did the numerical analysis for a "straight line" profile (constant shear in the boundary layer), F. Hendriks [9] did a more refined numerical analysis under the grant, using a Blasius velocity profile for the mean flow. He concluded that strong resonant interaction could indeed occur. At a fixed value of Reynolds number, he found that the amplitude coefficients in the interaction equations were such that the strongest interaction occurs when the wavenumber of the two-dimensional wave is close to (but <u>not</u> exactly equal to) the most unstable two-dimensional wave. This suggests that an estimate of the observed three-dimensionality can indeed be made by determining the wavenumber of the three-dimensional wave which can interact resonantly with the most unstable two-dimensional wave. Further work on this topic will require consideration of the dependence of the Reynolds number on downstream distance.

The other work done under the grant has concerned the formation and instability of longitudinal roll vortices which occur due to buoyancy (i.e., unstable stratification). Some work on this topic was being done by P. Iyer at the initiation of the grant and was continued under it. This research concerned the instability of the free convection boundary layer on the upper surface of an inclined, heated plate. For the vertical case, only propagating, two-dimensional ("transverse") waves are unstable. As the plate is inclined away from the vertical, however, the longitudinal vortex mode can occur due to the component of gravity normal to the unstably stratified boundary layer. Sparrow and his co-workers [10,11] had observed in experiments that two-dimensional waves were predominant for angles of tilt less than 14°, that both waves and longitudinal vortices were observed to be equally strong for angles between 14° and 17°, and that the vortices predominate for still higher angles. We decided to see whether these results could be correlated with theoretical rpedictions based on linear stability theory. We first determined the angle of tilt at which both modes of instability start at the same station along the plate. This angle turned out to be only 4°, however, and so this criterion is unreliable, based on the observations. We next determined the amplitude ratio of each instability for various angles of tilt and distances downstream from the edge of the plate. By "amplitude ratio", I mean the ratio of the amplitude of an unstable disturbance at a downstream station to some arbitrary value taken at the initial point of instability. In particular, we examined the amplitude ratios for those downstream stations at which the instabilities were first observed in the experiments. For an angle of 16°, the ratio for each mode was found to be nearly equal, and so the theoretical prediction is in accord with the experimental observation. Even though the two-dimensional wave becomes unstable only after the longitudinal vortex, the growth rate of the wave soon becomes larger and so it "catches up" in terms of amplitude. For an angle of 9°, the amplitude ratio of the two-dimensional wave was about nine times greater than that for the longitudinal vortex mode, whereas for 35°, the amplitude ratio of the longitudinal vortex was six times greater than that of the two-dimensional wave. These results are shown in Figures 3, 4 and 5 (taken from [12]) and indicate further that linear stability theory is applicable to the initial stages of growth, if not to the neutral point. In order to say that it predicts the neutral point, more careful experiments using controlled disturbances should be done.

After I returned from sabbatical leave during 1973-74 at Imperial College (during which time only some funds for computing were used), I decided to concentrate on the study of longitudinal vortices in unstably stratified shear flows. There were several reasons for this decision. First of all, various investigators (e.g. [13]) demonstrated that nonparallel flow effects were important for the Blasius boundary layer linear stability problem, and so the already difficult nonlinear problem appeared on its way to becoming still more difficult. Secondly, Dr. Richard Clever expressed an interest in working with me. Dr. Clever did his thesis in the area of thermal convection, and so an extension of his work to the shear flow problem seems likely to produce good results. Lastly, interest had been shown by the Department of Defense in the possibility of stabilizing boundary layers in water against shear instabilities by heating. This idea rests on the fact that the viscosity of water decreases with temperature, and so a fuller and more stable boundary layer profile results. However, it ignores the fact that thermal convection might also occur (at least, on an upwards facing surface), and none of the analyses (c.f. [14]) had taken the possibility of thermal convection into account. Hence, it seems to be a good time to investigate thermal convection in shear flows more thoroughly.

While on this grant, Dr. Clever has completed work which has led to two papers [15, 16]. This first concerns free convection flow in an inclined channel whose walls have different temperatures, while the second concerns thermal convection in Couette flow. (The first problem had been initiated while Dr. Clever was working with Professor Busse at UCLA and was finished under the ARO grant.) The principal aim in both papers was to examine the stability of the longitudinal convection rolls as the Rayleigh and Reynolds numbers increase. For zero Reynolds number, it is known that the rolls can develop three-dimensionality as the Rayleigh number increases. For convection in a shear flow, however, very little is known, although Avsec [17] in 1937 published a note showing the development of waviness on the rolls in a laboratory experiment when a channel flow is superimposed on the convection.

Two modes of instability of the longitudinal vortices are possible for the case of Couette flow. The first is called the "wavy" instability and consists of a sinuous undulation of the rolls which propagates at the mean flow speed (i.e., it is a stationary wave for the antisymmetric Couette flow investigated). The second is termed the "oscillatory" instability and consists of waves which propagate along the rolls with a wavespeed different from the mean flow speed. In order to calculate the stability boundaries for each mode, the rolls themselves and the distortion of the mean flow due to the rolls must be calculated for arbitrary Rayleigh number. This was done by a Galerkin technique. This nonlinear solution was then perturbed in a linear manner, and the stability characteristics of the rolls were determined. Extensive use of the UCLA computing facilities was made.

Only some results for the case of Couette flow will be given. The mean distortion of the shear flow by the rolls is shown in Figure 6. It is clear that the folls cause a pronounced boundary layer structure (with a consequent rise in wall shear stress) to occur as the Rayleigh number (Ra) increases. The neutral stability curves for the development of three-dimensional rolls are shown in Figure 7. It is clear that the boundaries are strongly dependent on the Reynolds number (Re). In fact, the actual mode of instability is determined by how the Rayleigh and Reynolds numbers are changed relative to each other. For Re > 200, however, the wavy mode is predominant and sets in slightly above the critical Rayleigh number for the initial formation of the rolls (1707.8). Thus, strictly two-dimensional rolls in Couette flow are rather special. Three-dimensional rolls are more common at higher Reynolds numbers. It remains to be seen if this is true of other flows.

Once the rolls become three-dimensional, they are able to extract energy from the mean shear flow, rather than being driven only by buoyancy. Calculations of the energy budget for the disturbed rolls have shown that such energy exchange occurs. In this sense, the two-dimensional rolls act in a way as a "roughness" agent.

As a result of my study of thermal convection in shear flows, I have also written a review paper on the subject which contains 67 references [18]. The paper surveys results which have been obtained in a number of distinct fields (engineering, meteorology, geophysics, etc.). One conclusion, however, is that no report exists of the instability of a Blasius boundary layer on a heated plate. It would be very desirable to have a thorough experimental and theoretical investigation made of this fundamental problem.

5. References

1. R. G. Fleagle, "Bomex: An Appraisal of Results," Science v. 176 (1972), 1079.

2. J. P. Kuettner, "Cloud Bands in the Earth's Atmosphere," Tellus v. 23 (1971), 404.

3. P. S. Klebanoff, K. D. Tidstrom, and L. M. Sargent, "The Three-Dimensional Nature of Boundary-Layer Instability," J. Fluid Mech. v. 12 (1962), 1.

4. D. J. Benney, "A Nonlinear Theory for Oscillations in a Parallel Flow," J. Fluid Mech. v. 10 (1961), 209.

5. B. N. Antar and F. G. Collins, "Numerical Calculation of Finite Amplitude Effects in Unstable Laminar Boundary Layers," Phys. Fluids v. 18 (1975), 289.

6. B. N. Antar, "Second Order Effects in Unstable Boundary Layers on a Flat Plate," Ph.D. thesis in Aerospace Studies at the University of Texas at Austin, 1974 (University Microfilms No. 74-14, 668).

7. G. Nelson and A. D. D. Craik, "Growth of Streamwise Vorticity in Unstable Boundary Layers," Phys. Fluids v. 20 (1977), 698.

8. A. D. D. Craik, "Non-linear Resonant Instability in Boundary Layers," J. Fluid Mech. v. 50 (1971), 393.

9. F. Hendriks, Appendix to "Nonlinear Wave Interactions in Shear Flows, Part 2," (by J. R. Usher and A. D. D. Craik), J. Fluid Mech., v. 70 (1975), 437.

10. E. M. Sparrow and R. B. Husar, "Longitudinal Vortices in Natural Convection Flow on Inclined Plates," J. Fluid Mech. v. 37 (1969), 251.

11. J. R. Lloyd and E. M. Sparrow, "On the Instability of Natural Convection Flow on Inclined Plates," J. Fluid Mech. v. 42 (1970), 465.

12. P. A. Iyer and R. E. Kelly, "The Stability of the Laminar Free Convection Flow Induced by a Heated Inclined Plate," Int. J. Heat Mass Trans. v. 17 (1974), 517.

13. W. S. Saric and A. H. Nayfeh, "Nonparallel Stability of Boundary-Layer Flows," Phys. Fluids v. 18 (1975), 945.

14. E. Reshotko, "Boundary-Layer Stability and Transition," Ann. Rev. Fluid Mech. v. 8 (1976), 311.

15. R. M. Clever and F. H. Busse, "Instabilities of Longitudinal Convection Rolls in an Inclined Layer," to be published in J. Fluid Mech.

16. R. M. Clever, F. H. Busse, and R. E. Kelly, "Instabilities of Longitudinal Convection Rolls in Couette Flow," to be published in Zeit. angew. Math. Physik.

17. D. Avsec, "Sur les Formes Ondulees des Tourbillons en Bandes Longitudinals," Comptes Rendus Acad. Sci. v. 204 (1937), 167.

18. R. E. Kelly, "The Onset and Development of Rayleigh-Benard Convection in Shear Flows: A Review," to be published in Proc. Int. Conf. Phys. Chem. and Hydrodynamics (Oxford, 1977; to be published by Hemisphere Publ. Corp.).

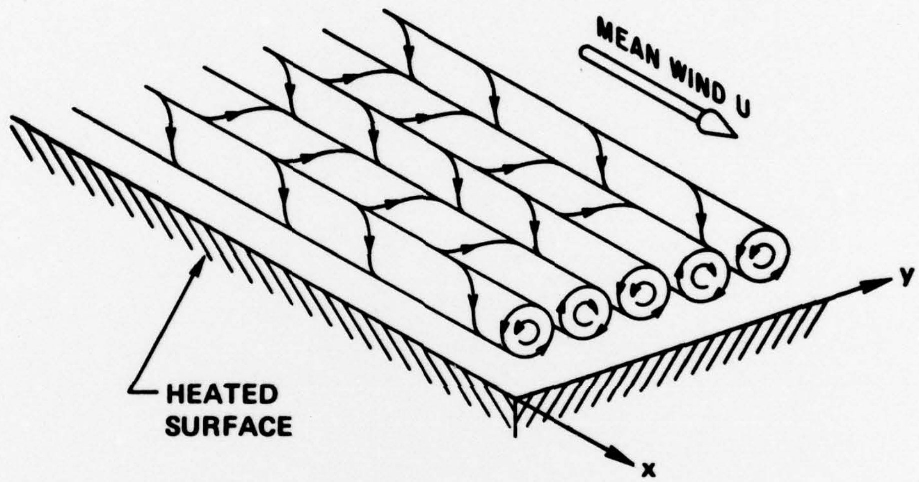

Figure 1. Schematic of Longitudinal Roll Vortices in a Flow Over a Heated Horizontal Surface.

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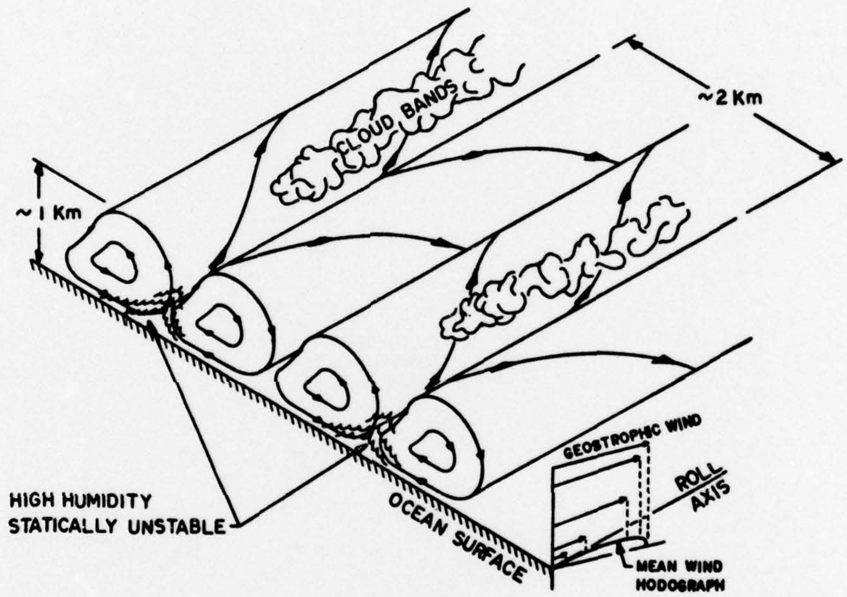

Figure 2. Schematic of Longitudinal Roll Vortices in the Oceanic Planetary Boundary Layer (Fig. 5 of [1]; copyright 1972 by the American Association for the Advancement of Science).

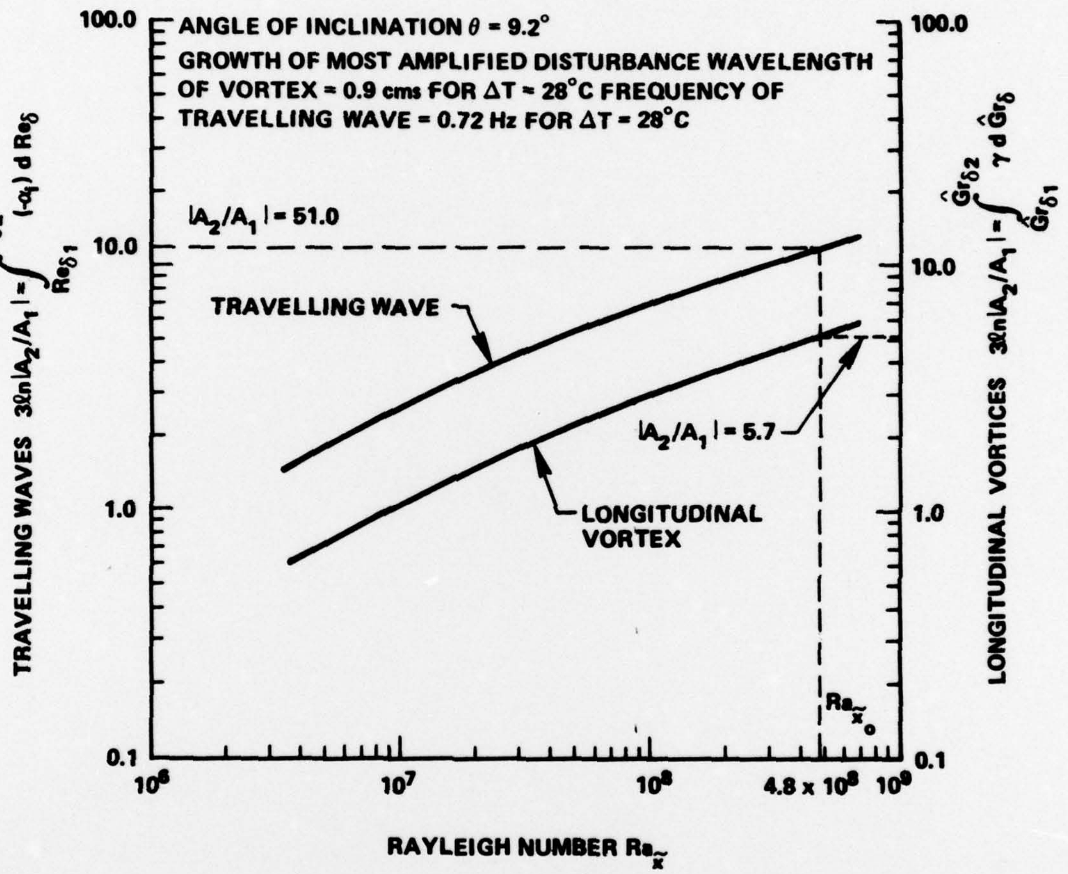

Figure 3. Amplitude Ratios for Waves and Longitudinal Vortices in Unstable Free Convection Boundary Layer Flow on a Tilted Surface; Angle of Tilt = 9.5°.

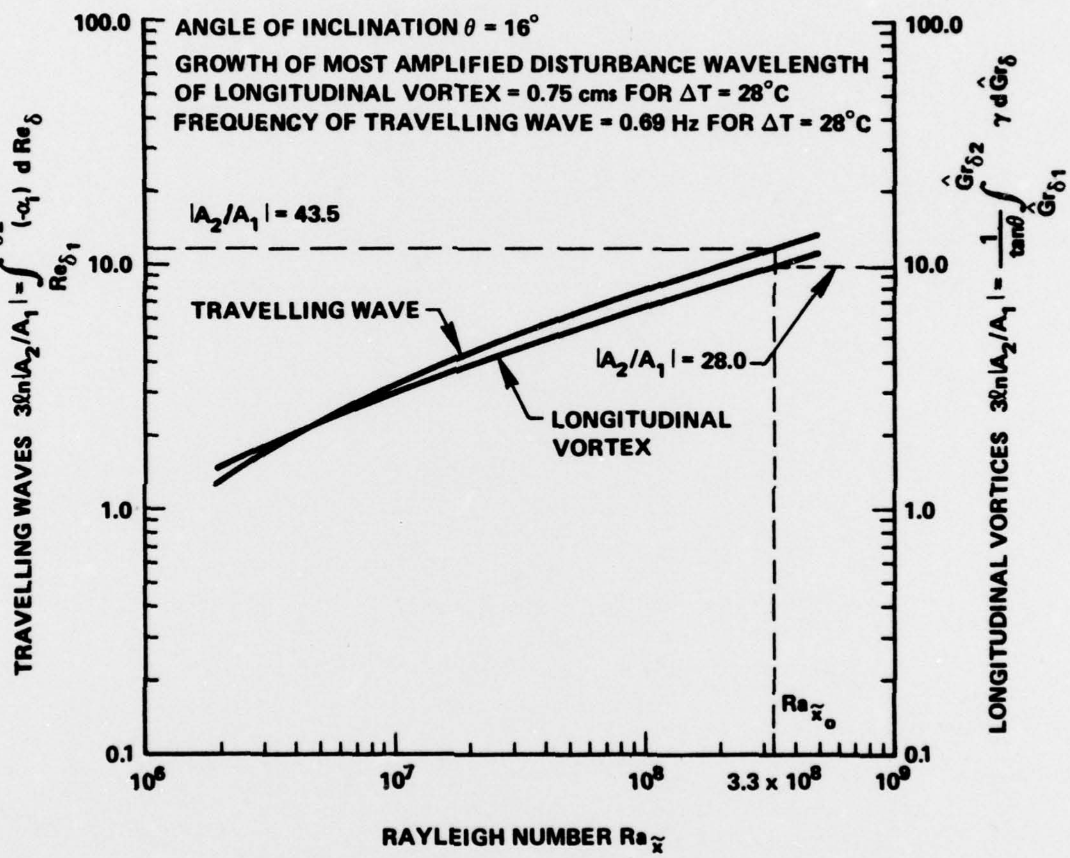

Figure 4. Amplitude Ratios for Waves and Longitudinal Vortices in Unstable Free Convection Boundary Layer Flow on a Tilted Surface; Angle of Tilt = 16°.

ANGLE OF INCLINATION θ = 35°

GROWTH OF MOST AMPLIFIED DISTRUBANCES (AMPLITUDE RATIOS FOR LONGITUDINAL VORTICES WITH WAVELENGTHS BETWEEN 0.48 cms AND 0.68 cms FOR ΔT = 28°C) FREQUENCY OF TRAVELLING WAVE = 0.87 Hz FOR ΔT = 28°C

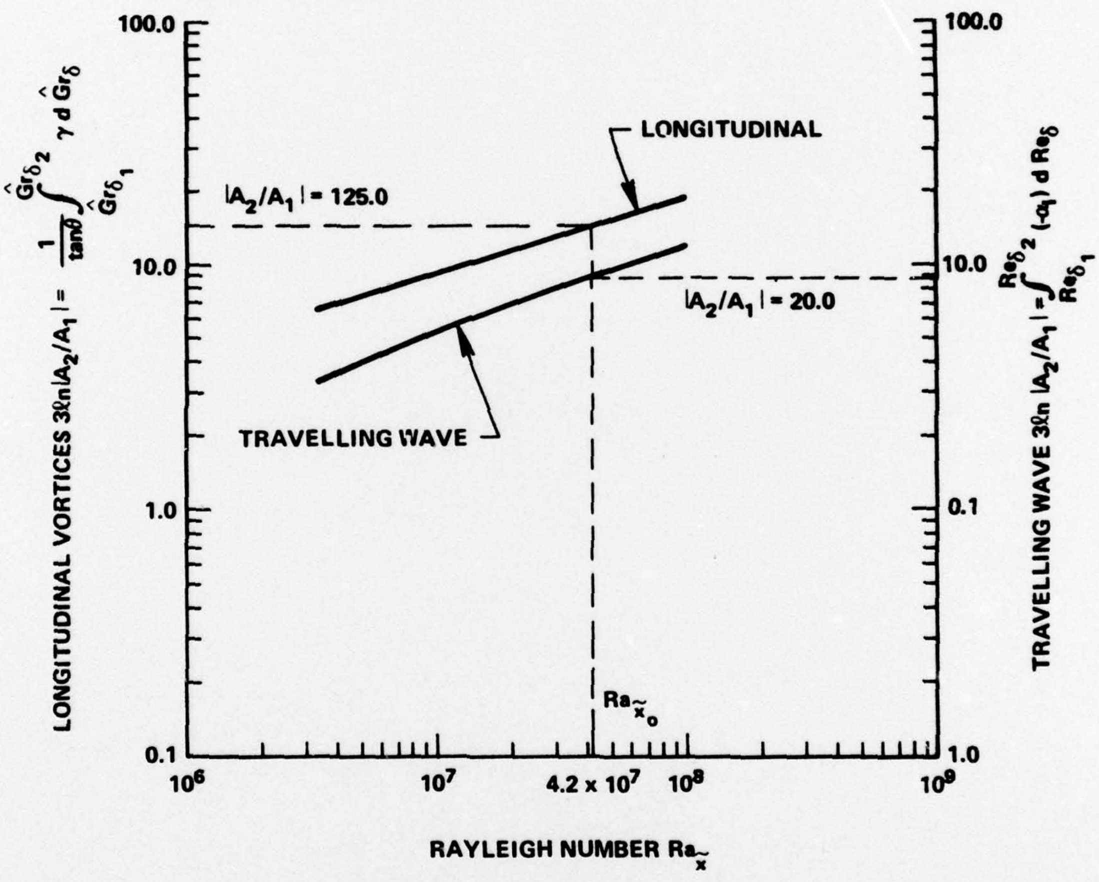

Figure 5. Amplitude Ratios for Waves and Longitudinal Vortices in Unstable Free Convection Boundary Layer Flow on a Tilted Surface; Angle of Tilt = 35°.

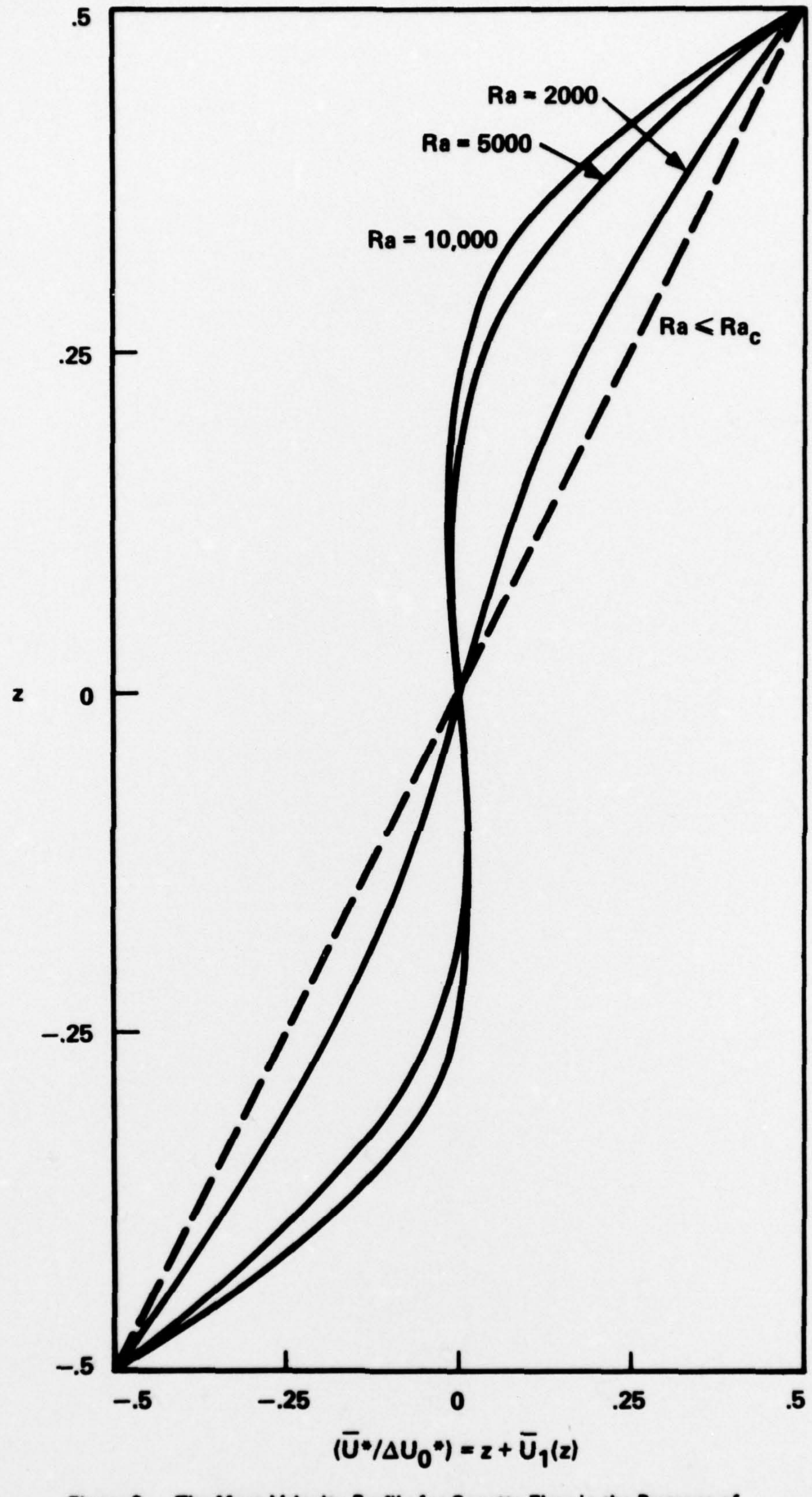

Figure 6. The Mean Velocity Profile for Couette Flow in the Presence of Longitudinal Convection Rolls as a Function of Rayleigh Number (Prandtl Number = 0.7).

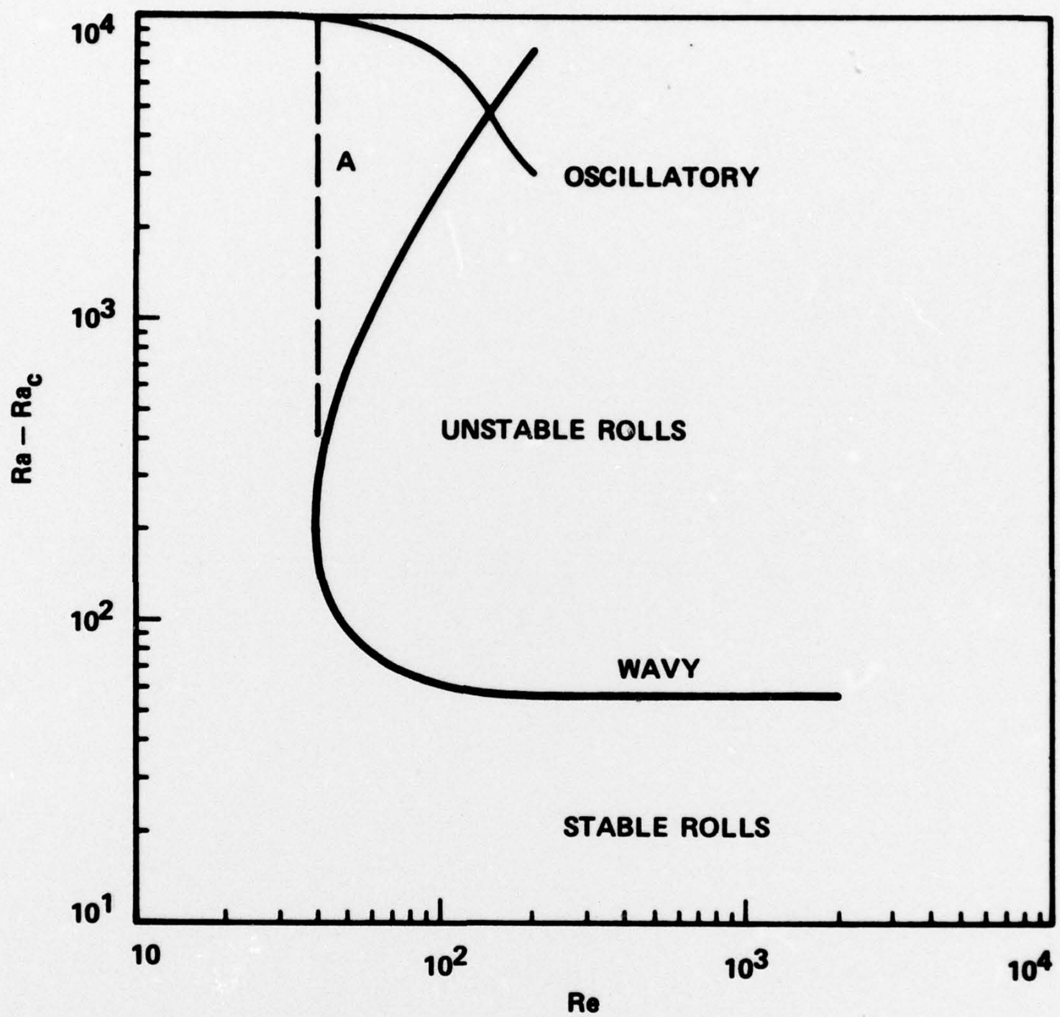

Figure 7. The Neutral Curves for Instability of Longitudinal Convection Rolls in Couette Flow as a Function of Rayleigh (Ra) and Reynolds (Re) Numbers (Prandtl Number = 0.7).