AD-A048 624 AEROSPACE CORP EL SEGUNDO CALIF ENGINEERING SCIENCE --ETC F/G 22/2
CLOSED FORM MAGNETIC QUARTER ORBIT SWITCH POINT SOLUTION.(U)
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Closed Form Magnetic Quarter Orbit Switch Point Solution

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The Aerospace Corporation
El Segundo, Calif. 90245

15 July 1977

Prepared for
SPACE AND MISSILE SYSTEMS ORGANIZATION
AIR FORCE SYSTEMS COMMAND
Los Angeles Air Force Station
P.O. Box 92960, Worldway Postal Center
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This final report was submitted by The Aerospace Corporation, El Segundo, CA 90245, under Contract F04701-76-C-0077 with the Space and Missile Systems Organization (AFSC), Los Angeles Air Force Station, P.O. Box 92960, Worldway Postal Center, Los Angeles, CA 90009. It was reviewed and approved for The Aerospace Corporation by D. J. Griep, Engineering Science Operations and J. R. Stevens, Advanced Programs. Major W. J. Walker, YATA, was the Deputy for Advanced Space Programs project manager.

This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusion. It is published only for the exchange and stimulation of ideas.

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UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

19 REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 18 SAMSO-TR-77-214	2. GOVT ACCESSION NO. 9	3. RECIPIENT'S CATALOG NUMBER Technical rept.
4. TITLE (and Subtitle) 6 CLOSED FORM MAGNETIC QUARTER Orbit SWITCH POINT SOLUTION		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER 14 TR-0077(2506-16)-1
7. AUTHOR(s) 10 L. K. Herman		8. CONTRACT OR GRANT NUMBER(s) 15 F04701-76-C-0077
9. PERFORMING ORGANIZATION NAME AND ADDRESS The Aerospace Corporation El Segundo, CA 90009		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Space and Missile Systems Organization/YAPT P.O. Box 92960, Worldway Postal Center Los Angeles, CA 90009		12. REPORT DATE 11 15 July 1977
		13. NUMBER OF PAGES 13
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 12 12p.		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Magnetic Control
Spinning Satellite
Quarter Orbit

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A closed form solution is derived for determining the location of a set of magnetic switch points, spaced 90 deg apart in the orbit, which will allow the spin axis of a satellite to be moved in any, achievable, desired direction from any initial orientation. Additional closed form equations are derived for determining the magnitude of the motion that will result for a given set of switch points. The simplicity of the calculations make the technique extremely useful for applications where savings in computer time or storage are desired and for design studies where quick solutions to many cases are required.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

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CLOSED FORM MAGNETIC QUARTER ORBIT SWITCH POINT SOLUTION

L. K. Herman
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Abstract

A closed form solution is derived for determining the location of a set of magnetic switch points, spaced 90 deg apart in the orbit, which will allow the spin axis of a satellite to be moved in any, achievable, desired direction from any initial orientation. Additional closed form equations are derived for determining the magnitude of the motion that will result for a given set of switch points. The simplicity of the calculations make the technique extremely useful for applications where savings in computer time or storage are desired and for design studies where quick solutions to many cases are required.

I. Introduction

On a spinning satellite, precession control torques can be generated by the interaction of a magnetic coil, which has its dipole aligned parallel to the spin axis, with the earth's magnetic field. The spin axis can be precessed in any direction if the sign of the magnetic moment generated by the coil is switched at appropriate points, spaced 90 deg apart in the orbit. The torque resulting from the interaction of the satellite magnetic moment, $\overline{M}_s$, and the earth's magnetic field, $\overline{B}$, is

$$\overline{T} = \overline{M}_s \times \overline{B} \qquad (1)$$

and the change in angular momentum resulting from this torque is

$$\Delta\overline{H} = \int \overline{M}_s \times \overline{B}\, dt \qquad (2)$$

This expression will be expanded to yield a closed form relationship between the location of the magnetic switch points in the orbit, the initial satellite attitude, and the desired change in attitude.

II. Magnetic Field Model

The earth's magnetic field will be represented as originating from a simple dipole rotated 11.4 deg from the geographic pole in the plane of the 110 and 290 deg east meridians (see Fig. 1). In the dipole coordinate system, which has the z-axis along the magnetic dipole axis and the y-axis in the plane of the 110 deg meridian,

$$\overline{M}^d = |\overline{M}| \begin{vmatrix} 0 \\ 0 \\ -1 \end{vmatrix} \qquad (3)$$

where $|\overline{M}| = 7.95 \times 10^{25}$ pole cm

Transforming to the nodal coordinate system (Fig. 2) which has the y-axis normal to the orbit plane and the x-axis along the orbital line of nodes yields,

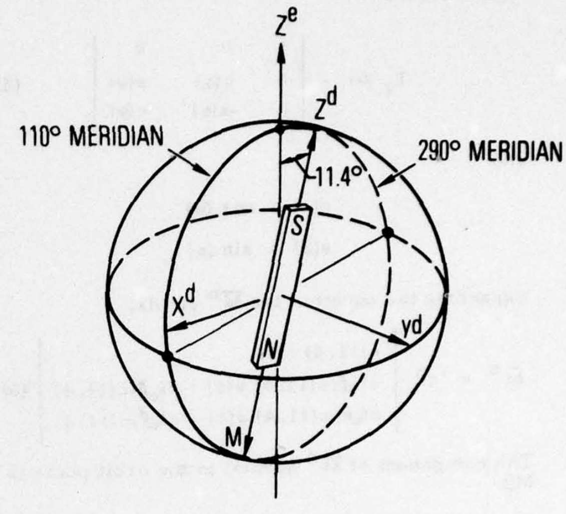

Figure 1. Dipole coordinate system

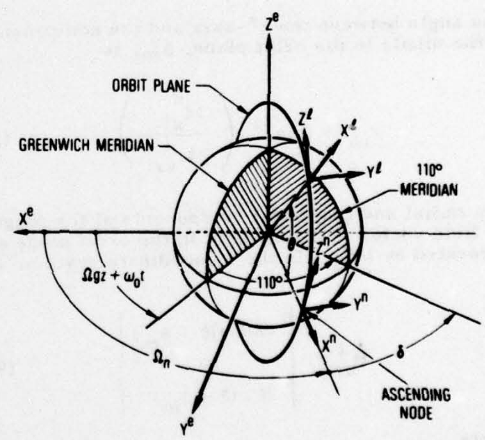

Figure 2. Coordinate frames

$$\overline{M}^n = T_x(i - 90^\circ)\, T_z(-\delta)\, T_y(11.4^\circ)\, \overline{M}^d \qquad (4)$$

where

$\delta = \Omega_{gz} + \omega_e t_g + 110^\circ - \Omega_n$ (see Fig. 2)

Ω_{gz} = Greenwich hour angle of zero

ω_e = earth rotation rate

t_g = Greenwich mean time

Ω_n = right ascension of the orbital ascending node

and $T_x(\alpha)$ represents the transformation about the x-axis through the rotation angle α, similarly for T_y and T_z, i.e.,

$$T_x(\alpha) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & c(\alpha) & s(\alpha) \\ 0 & -s(\alpha) & c(\alpha) \end{vmatrix} \quad (5)$$

and

$$c(\alpha) = \cos(\alpha)$$

$$s(\alpha) = \sin(\alpha)$$

Expanding the equation for $\overline{M}^n$ yields,

$$\overline{M}^n = |\overline{M}| \begin{vmatrix} s(11.4)\, c(\delta) \\ s(\mathcal{l})s(11.4)\, s(\delta) + c(\mathcal{l})c(11.4) \\ c(\mathcal{l})\, s(11.4)\, s(\delta) - s(\mathcal{l})c(11.4) \end{vmatrix} \quad (6)$$

The component of $\overline{M}^n$ normal to the orbit plane is M^n_y.

The magnitude of the component in the orbit plane is

$$|\overline{M}^n_{xz}| = \left[(M^n_x)^2 + (M^n_z)^2 \right]^{1/2} \quad (7)$$

The angle between the z^n-axis and the component of the dipole in the orbit plane, θ_m, is

$$\theta_m = \sin^{-1}\left(\frac{M^n_x}{|M^n_{xz}|} \right) \quad (8)$$

The radial and tangential components of the magnetic field vector $\overline{B}$ at any point in the orbit plane are generated by M^n_{xz}. In the ℓ coordinate system, $\overline{B}$, is

$$\overline{B}^{\ell}_{xz} = \begin{vmatrix} -2K\, s(\theta - \theta_m) \\ 0 \\ K\, c(\theta - \theta_m) \end{vmatrix} \quad (9)$$

where

$$K = \frac{u|M^n_{xz}|}{4\pi R^3} \quad (10)$$

R = the radial distance from the center of the earth to the point where the field is computed.

θ = the angle between the x^n-axis and the x^{ℓ}-axis, the argument of latitude.

μ = magnetic permeability

The normal component of the magnetic field at any point in the orbit plane is generated by M^n_y and is oriented along the -y-axis. The magnitude of the component is

$$K_n = \frac{u\, M^n_y}{4\pi R^3} \quad (11)$$

Therefore, the total magnetic field at any point in the orbit plane, expressed in the ℓ coordinate system is

$$\overline{B}^{\ell} = K \begin{vmatrix} -2\, s(\theta - \theta_m) \\ -K_n/K \\ c(\theta - \theta_m) \end{vmatrix} \quad (12)$$

In the nodal coordinate system, n,

$$\overline{B}^n = T_y(\theta)\, \overline{B}^{\ell} \quad (13)$$

Expansion yields,

$$\overline{B}^n = K \begin{vmatrix} -\frac{3}{2}\, s(2\theta - \theta_m) + \frac{1}{2}\, s(\theta_m) \\ -K_n/K \\ \frac{3}{2} c(2\theta - \theta_m) - \frac{1}{2}\, c(\theta_m) \end{vmatrix} \quad (14)$$

The orientation of the spin axis relative to the nodal coordinate system can be defined by the two Euler angles ϕ and ψ (see Fig. 3). The magnetic field in this coordinate system is

$$\overline{B}^a = T_x(-\psi)\, T_z(\phi)\, \overline{B}^n \quad (15)$$

Expansion yields,

$$\overline{B}^a = \begin{vmatrix} B^n_x\, c(\phi) + B^n_y\, s(\phi) \\ -B^n_x\, s(\phi)c(\psi) + B^n_y\, c(\phi)c(\psi) - B^n_z\, s(\psi) \\ -B^n_x\, s(\phi)\, s(\psi) + B^n_y\, c(\phi)\, s(\psi) + B^n_z\, c(\psi) \end{vmatrix} \quad (16)$$

III. Magnetic Control Torques

In the "a" coordinate frame, the magnetic control is aligned with the y-axis, therefore,

$$\overline{M}^a = \begin{vmatrix} 0 \\ M_s \\ 0 \end{vmatrix} \quad (17)$$

or

$$I_x = \frac{K}{2\dot{\theta}} M_s \int_{\theta_1}^{\theta_1+2\pi} \left[s(\theta_m) - 3s(2\theta - \theta_m) \right] d\theta \qquad (24)$$

$$I_x = \frac{K}{2\dot{\theta}} M_s \left[\theta s(\theta_M) + \frac{3}{2} c(2\theta - \theta_m) \right] \Bigg|_{\theta_1}^{\theta_1+2\pi} \qquad (25)$$

Substituting the limits of integration and switching the sign of M_s in each quarter orbit yields

$$\begin{aligned} I_x = \frac{K\, s(\theta_m)}{2\dot{\theta}} \Big\{ & M_s \left[(\theta_s + \pi/2) - \theta_s \right] \\ & - M_s \left[(\theta_s + \pi) - (\theta_s + \pi/2) \right] \\ & + M_s \left[(\theta_s + \tfrac{3}{2}\pi) - (\theta_s + \pi) \right] \\ & - M_s \left[(\theta_s + 2\pi) - (\theta_s + \tfrac{3}{2}\pi) \right] \Big\} \\ + \frac{3K}{4\dot{\theta}} \Big\{ & M_s \left[c(2\theta_s + \pi - \theta_m) - c(2\theta_s - \theta_m) \right] \\ & - M_s \left[c(2\theta_s + 2\pi - \theta_m) - c(2\theta_s + \pi - \theta_m) \right] \\ & + M_s \left[c(2\theta_s + 3\pi - \theta_m) - c(2\theta_s + 2\pi - \theta_m) \right] \\ & - M_s \left[c(2\theta_s + 4\pi - \theta_m) - c(2\theta_s + 3\pi - \theta_m) \right] \Big\} \end{aligned} \qquad (26)$$

Finally,

$$I_x = \frac{-6K}{\dot{\theta}} M_s\, c(2\theta_s - \theta_m) \qquad (27)$$

Let

$$I_y = \int_{t_1}^{t_2} M_s B_y^n \, dt \qquad (28)$$

or

$$I_y = -K_n M_s \int_{\theta_1}^{\theta_1+2\pi} \frac{d\theta}{\dot{\theta}} \qquad (29)$$

Evaluating I_y using the same technique used for I_x yields

$$I_y = 0 \qquad (30)$$

Let

$$I_z = \int_{t_1}^{t_2} M_s B_z^n \, dt \qquad (31)$$

or

$$I_z = \frac{K}{2\dot{\theta}} M_s \int_{\theta_1}^{\theta_1+2\pi} \left[3c(2\theta - \theta_m) - c(\theta_m) \right] d\theta \qquad (32)$$

Again, evaluating the integral using the technique used for I_x yields

$$I_z = \frac{-6K}{\dot{\theta}} M_s\, s(2\theta_s - \theta_m) \qquad (33)$$

Substitution of the integrals, evaluated with M_s switched at quarter orbit intervals, in the equation for $\overline{\Delta H}^a$ yields,

$$\overline{\Delta H}^a_{(1/4\,\text{orbit})} = \frac{6KM_s}{\dot{\theta}} \begin{vmatrix} s(\phi)\, s(\psi)\, c(2\theta_s - \theta_m) \\ -c(\psi)\, s(2\theta_s - \theta_m) \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ c(\phi)\, c(2\theta_s - \theta_m) \end{vmatrix} \qquad (34)$$

If the sign of the satellite dipole is constant over the complete orbit, the limits of integration can, without loss of generality, be replaced with 0 and 2π, and the integrals become

$$I_x = \frac{KM_s}{2\dot{\theta}} \left\{ \left[2\pi\, s(\theta_m) - 0 \right] + \frac{3}{2} \left[c(4\pi - \theta_m) - c(0 - \theta_m) \right] \right\} \qquad (35)$$

$$I_x = \frac{\pi K M_s}{\dot{\theta}} s(\theta_m) \qquad (36)$$

and

$$I_y = \frac{-K_n M_s}{\dot{\theta}} \left[2\pi - 0 \right] \qquad (37)$$

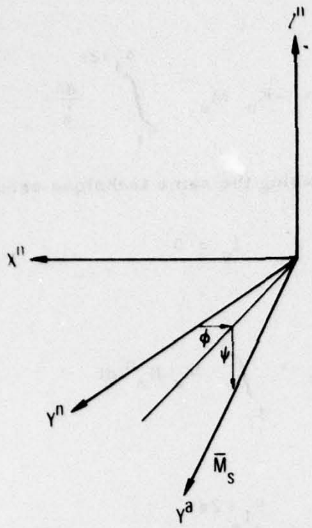

Figure 3. Euler angle definition

The torque resulting from the satellite control coil interacting with the earth's magnetic field is,

$$\overline{T}^a = \overline{M}^a_s \times \overline{B}^a \tag{18}$$

$$\overline{T}^a = \begin{vmatrix} M_s B^a_z \\ 0 \\ -M_s B^a_x \end{vmatrix} \tag{19}$$

IV. Angular Momentum

The change in angular momentum resulting from the magnetic torque is

$$\Delta\overline{H}^a = \int \overline{T}^a \, dt \tag{20}$$

Expansion yields,

$$\Delta\overline{H}^a = M_s \begin{vmatrix} -s(\phi)\, s(\theta) \int B^n_x \, dt + c(\phi)\, s(\psi) \int B^n_y \, dt \\ +c(\psi) \int B^n_z \, dt \\ 0 \\ -c(\phi) \int B^n_x \, dt - s(\phi) \int B^n_y \, dt \end{vmatrix} \tag{21}$$

The following assumptions will be utilized in evaluating the integrals.

1. The orbital radius and the orbit rate are constant. Therefore,

$$R = \text{Constant}$$

$$\theta = \int \dot{\theta} \, dt = \dot{\theta} \, \Delta t$$

2. The satellite attitude does not change significantly in one orbit. Therefore,

$$\phi \text{ and } \psi \text{ are constant}$$

3. The earth does not rotate significantly in one orbit. Therefore,

$$\theta_m \text{ is constant}$$

4. The spin axis is constantly aligned with the momentum vector due to damping.

Each of the integrals will be evaluated around an entire orbit, starting at the first quarter orbit switch point, θ_s, with M_s positive between θ_s and $\theta_s + \pi/2$ and switching signs in each subsequent quarter orbit (see Fig. 4). The first term is evaluated as follows:

$$I_x = M_s \int_{t_1}^{t_2} B^n_x \, dt \tag{22}$$

Let

$$\theta = \dot{\theta} t$$

Therefore,

$$d\theta = \dot{\theta} \, dt$$

$$dt = \frac{1}{\dot{\theta}} d\theta$$

Substitution of variables yields

$$I_x = M_s \int_{\theta_1}^{\theta_1 + 2\pi} B^n_x \frac{d\theta}{\dot{\theta}} \tag{23}$$

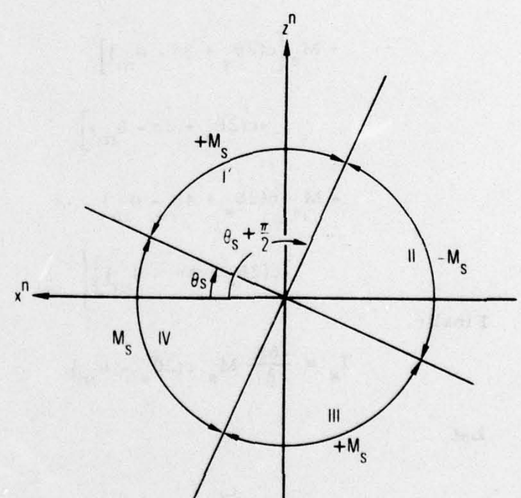

Figure 4. Quarter orbit switch points

Also,

$$I_z = \frac{KM_s}{2\dot{\theta}} \left\{ \frac{3}{2} \left[s(4\pi - \theta_m) - s(\theta_m) \right] - \left[2\pi\, c(\theta_m) - 0 \right] \right\} \tag{38}$$

And,

$$I_z = \frac{\pi\, KM_s}{\dot{\theta}}\, c(\theta_m) \tag{39}$$

Substitution of the integrals, evaluated with M_s constant over the entire orbit, in the equation for $\overline{\Delta H}^a$ yields,

$$\overline{\Delta H}^a_{(const)} = \frac{\pi KM_s}{\dot{\theta}} \left| \begin{array}{c} -s(\phi)\, s(\Psi)\, s(\theta_m) \\ -\, c(\phi)\, s(\Psi) \frac{2K_n}{K} \\ -\, c(\Psi)\, c(\theta_m) \\ \hline 0 \\ \hline -c(\phi)\, s(\theta_m) + s(\phi) \frac{2K_n}{K} \end{array} \right| \tag{40}$$

V. Spin Axis Motion

The spin axis will be colinear with the momentum vector if sufficient damping is present. Therefore, the change in the orientation of the spin axis can be determined if the change in angular momentum is known. From Fig. 3, it can be seen that for small angles,

$$\Delta\Psi = \frac{-\Delta H^a_z}{H_o} \tag{41}$$

$$\Delta\phi = \frac{-\Delta H^a_x}{H_o\, c(\Psi)} \tag{42}$$

where H_o is the nominal satellite angular momentum.

VI. Quarter Orbit Switch Points

The quarter orbit switch points, θ_s, $\theta_s + \pi/2$, and so forth, can be determined from the expressions for $\Delta\phi$ and $\Delta\Psi$ as follows:

$$\frac{\Delta\phi}{\Delta\Psi} = \frac{s(\phi)\, s(\Psi)}{c(\phi)\, c(\Psi)} - \frac{1}{c(\phi)} \tan(2\theta_s - \theta_m) \tag{43}$$

Therefore,

$$\tan(2\theta_s - \theta_m) = -c\phi \left(\frac{\Delta\phi}{\Delta\Psi} - \frac{s(\phi)\, s(\Psi)}{c(\phi)\, c(\Psi)} \right) \tag{44}$$

and

$$\theta_s = \frac{1}{2} \left[\theta_m + \tan^{-1}\left(\tan(\Psi) \sin(\phi) - \frac{\Delta\phi}{\Delta\Psi} \cos(\phi) \right) \right] \tag{45}$$

This equation defines the first quarter orbit switch point in terms of the Euler angles ϕ and Ψ, the angle θ_m, which can be determined from Eqs. (6), (7), and (8), and the ratio of the desired change in attitude $\Delta\phi/\Delta\Psi$.

When the switch points have been determined, the magnitude of the change in attitude can be determined from Eqs. (41) and (42). The required input data is ΔH^a, Eq. (34) or (40), which requires the magnitude of the satellite dipole and the orbit rate.

The units of the terms in the coefficients in the $\overline{\Delta H}^a$ equations are discussed in Appendix A. A listing of a computer program for determining switch points and resulting attitude motion is contained in Appendix B.

VII. Performance Evaluation

The performance of the closed form equations was evaluated by generating switch points and attitude change magnitudes for a number of cases, using the computer program in Appendix B. These results were compared with the outputs of a sophisticated digital simulation using a ninth order magnetic field model while switching the sign of the satellite dipole at the switch points determined by the closed form equations and integrating around an entire orbit. A summary of the results is shown in Table 1. The angular differences between orientation derived from the closed form solution and the results of the more sophisticated ninth order model are small compared to the level of performance generally achieved with magnetic control of spin stabilized satellites.

VIII. Summary

The simplicity of the calculations and the accuracy of the results make this technique extremely useful for applications where savings in computer time or storage are desired and for design studies where quick solutions to many cases may be required.

Appendix A
Computation of Coefficient

$$\frac{6KM_s}{\dot{\theta}} = \frac{6\left[\dfrac{\mu |M^n_{xz}|}{4\pi\, R^3}\right] M_s}{\dot{\theta}}$$

or

$$\frac{6KM_s}{\dot{\theta}} = 6.972852 \times 10^{-23} \frac{|M^n_{xz}|\, M_s}{\dot{\theta}\, R^3}$$

Table 1 Performance Evaluation

All angles in degrees and time in hours.

Input Conditions						Closed Form			Ninth Order		Angular Error *	
ϕ_{Avg}	Ψ_{Avg}	$\frac{\Delta\phi}{\Delta\Psi}$	SGN $\Delta\phi$	SGN $\Delta\Psi$	Time	θ_s	$\Delta\phi_c$	$\Delta\Psi_c$	$\Delta\phi_n$	$\Delta\Psi_n$	$\Delta\phi_e \mathrm{Cos}\,\Psi$	$\Delta\Psi_e$
0.0	0.0	1.0	+	+	12.0	70.45	1.01	1.01	0.96	1.00	0.05	0.01
0.0	0.0	-0.5	+	-	18.0	8.41	0.64	-1.28	0.65	-1.23	-0.01	-0.05
0.0	0.0	2.0	-	-	18.0	143.41	-1.28	-0.64	-1.23	-0.62	-0.05	-0.02
0.0	0.0	0.0	+	+	6.5	95.31	0.00	1.40	0.03	1.40	-0.03	0.00
0.0	0.0	1.E10	+	+	1.0	43.28	1.36	0.00	1.29	-0.05	-0.07	0.05
30.0	45.0	-0.33	-	+	18.0	104.21	-0.32	0.97	-0.35	0.96	0.02	0.01
30.0	45.0	3.5	-	-	18.0	140.91	-1.59	-0.45	-1.48	-0.47	-0.07	0.02
0.0	60.0	-0.30	+	-	1.0	6.63	0.39	-1.30	0.41	-1.29	-0.01	-0.01
85.0	0.0	1.E10	+	-	6.5	50.31	1.40	0.00	1.40	-0.08	0.00	0.08
0.0	85.0	-0.3	+	-	1.0	6.63	0.39	-1.30	-0.05	-1.29	0.04	-0.01
85.0	85.0	2.0	-	-	12.0	45.40	-0.02	-0.01	0.46	0.06	0.04	-0.07
85.0	85.0	1.E10	-	-	12.0	137.95	-1.43	0.00	-1.70	0.08	0.02	-0.08

*Note: Due to the Eulerian definition of ϕ and Ψ, the actual angular difference, in the ϕ direction, between the closed form and ninth order solutions is equal to ($\Delta\phi$ closed form - $\Delta\phi$ ninth order) cos (ΨAvg). This effect can be seen in the tabular results when the magnitude of the Ψ Avg increases.

Orbital Parameters

$R = 3761.0$ nmi

$\mathcal{i} = 97.71$ deg

$\Omega = 0.03$ deg

where

$\frac{6KM_s}{\dot{\theta}}$ is in ft lb sec

$|M^n_{xz}|$ is in pole cm

M_s is in pole cm (positive along the plus y^a-axis)

$\dot{\theta}$ is in $\frac{rad}{sec}$ (always positive)

R is in nautical miles (always positive)

$$\frac{\pi KM_s}{\dot{\theta}} = \frac{\pi}{6}\,\frac{6KM_s}{\dot{\theta}}$$

or

$$\frac{\pi KM_s}{\dot{\theta}} = 3.650976 \times 10^{-23}\,\frac{|M^n_{xz}|M_s}{\dot{\theta}\,R^3}$$

finally,

$$\frac{2K_n}{K} = \frac{2|M^n_y|}{|M^n_{xz}|}$$

Satellite Parameters

$M_s = 10,000.$ pole cm

$I_{Spin} = 183.$ Slug ft^2

$\omega = 2$ Rev/min

where

$|M^n_y|$ is in pole cm

References

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3. Marc Renard, "Command Laws for Magnetic Attitude Control of Spin Stabilized Earth Satellites," Journal of Spacecraft and Rockets, February 1967.

4. Masamichi Shigehara, "Geomagnetic Attitude Control of an Axisymmetric Spinning Satellite," Journal of Spacecraft and Rockets, June 1972.

Appendix B

```
      PROGRAM THETA(INPUT,OUTPUT)
      REAL MSAT, IOTA, MAGME
C
      PI = 2.*ACOS(0.)
      CDR = PI/180.
      CRD = 1./CDR
      OMEGAE = 2.*PI/86164.109
      MAGME = 7.95E25
C
C    INPUT DATA
C
      PHIMAVG = 0.*CDR
      PSIMAVG =0.*CDR
      DPHDPS = 1.0
      SGNDPHI = 1.
      SGNDPSI = 1.
      OMEGHAZ = 128.8755*CDR
      IOTA = 97.71*CDR
      R = 3761.0
      OMEGAN = .33*CDR
      PERIOD = 5769.49
      HOUR = 12.
      RMIN = 0.
      SEC = 0.
      HNOM = 183.*2.*6.*CDR
      MSAT = 1.E4
C
C
      T = HOUR*3600. + RMIN*60. + SEC
      DELTA = OMEGHAZ + OMEGAE*T + 109.33*CDR - OMEGAN
      RMKX = MAGME*(SIN(11.387*CDR)*COS(DELTA))
      RMKY = MAGME*(SIN(IOTA)*SIN(11.387*CDR)*SIN(DELTA) + COS(IOTA)*
     * COS(11.387*CDR))
      RMKZ = MAGME*(COS(IOTA)*SIN(11.387*CDR)*SIN(DELTA) - SIN(IOTA)*
     * COS(11.387*CDR))
      RMAGKXZ = SQRT(RMKX**2 + RMKZ**2)
      THETAM = ASIN(RMKX/RMAGKXZ)
      IF(RMKZ .GT. 0.) THETAM = PI - THETAM
      THETAS = .5*(THETAM + ATAN(TAN(PSIMAVG)*SIN(PHIMAVG) -
     * (DPHDPS)*COS(PHIMAVG)))
 5    CONTINUE
      IF(THETAS .LT. PI/2.) GO TO 10
      THETAS = THEATAS - PI/2.
      GO TO 5
 10   CONTINUE
      IF(THETAS .GT. 0.) GO TO 15
      THETAS = THETAS + PI/2.
      GO TO 5
 15   CONTINUE
      COEFF = 6.972853E-23*RMAGKXZ*MSAT/((2.*PI/PERIOD)*R**3)
      HAX = SIN(PHIMAVG)*SIN(PSIMAVG)*COS(2.*THETAS-THETAM) -
     * COS(PSIMAVG)*SIN(2.*THETAS-THETAM)
      HAZ = COS(PHIMAVG)*COS(2.*THETAS-THETAM)
      HAX = HAX*COEFF
      HAZ = HAZ*COEFF
      DELPSIM = -HAZ/HNOM
      DELPHIM = -HAX/(HNOM*COS(PSIMAVG))
      IF(DPHDPS .LT. 1.) GO TO 17
      IF(SGNDPHI*DELPHIM .GT. 0.) GO TO 25
      GO TO 18
 17   IF(SGNDPSI*DELPSIM .GT. 0.) GO TO 25
 18   THETAS = THETAS + PI/2.
      DELPHIM = -DELPHIM
      DELPSIM = -DELPSIM
 25   CONTINUE
C
C  OUTPUT DATA
      PHIMAVG = PHIMAVG*CRD
      PSIMAVG = PSIMAVG*CRD
      THETAMD = THETAM*CRD
      DELTAD = DELTA*CRD
      THETASD = THETAS*CRD
      DELPSIM = DELPSIM*CRD
      DELPHIM = DELPHIM*CRD
      PRINT 300
  300 FORMAT(/,21X,*PHIIM  *,5X,* PSIM *)
      PRINT 330,PHIMAVG,PSIMAVG
  330 FORMAT(5X,*    AVERAGE     *,F7.2,5X,F7.2)
      PRINT 331,DELPHIM,DELPSIM
  331 FORMAT(5X,*    DELTA       *,F7.2,5X,F7.2)
      PRINT 332 ,HOUR,RMIN,SEC
  332 FORMAT(/,5X,*HOUR = *,F7.2,7X,*MIN = *,F7.2,9X,*SEC = *,F7.2)
      PRINT 340,THETAMD,DELTAD,DPHDPS
  340 FORMAT(5X,*THETAM = *,F7.2,5X,*DELTA = *,F7.2,7X,*DPHIM/DPSIM = *
     *      ,F7.2)
      PRINT 350,THETASD,COEFF
  350 FORMAT(5X,*THETAS = *,F7.2,5X,*COEFF = *,E20.10,/)
      STOP
      END
```