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RESEARCH REPORT

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DEVELOPING THE TECHNOLOGY OF PROBABILISTIC INFERENCE: Aggregating by Averaging Reduces Conservatism

> LEE C. EILS, III DAVID A. SEAVER WARD EDWARDS

Sponsored by: Advanced Research Projects Agency Department of Defense

MONITORED BY: ENGINEERING PSYCHOLOGY PROGRAMS OFFICE OF NAVAL RESEARCH CONTRACT NO. N00014-76-C-0074, ARPA

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AUGUST 1977

SSRI RESEARCH REPORT 77-3

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6 DEVELOPING THE TECHNOLOGY OF PROBABILISTIC INFERENCE: AGGREGATING BY AVERAGING REDUCES CONSERVATISM 9) Technical right. Oct 76- Sep 77, Research Report 77-3 August 1977 31p. JAN 20 1810 10 Lee C./Eils, III, David A./Seaver Ward/Edwards Social Science Research Institute

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15) NODD14-76-C- PP74

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Summary

A relatively large body of research indicates that people are conservative processors of probabilistic information. Recent attention has focussed on two possible explanations of this phenomenon. The misaggregation hypothesis depicts conservatism as an inability to properly combine the information in sequences of data. The other explanation suggests conservatism is the result of a response bias: the avoidance of extreme odds or probability judgments.

Two experiments explored the use of a specific response, average certainty, that was devised to thwart conservatism caused by either a response bias or a certain form of misaggregation. Use of appropriate instructions and response scales made the average certainty judgments good subjective assessments of the arithmetic mean likelihood ratio which could then be used in the appropriate form of Bayes' Theorem to calculate posterior odds. These judgments seemed unlikely to be affected by a response bias since extreme responses were not needed. In addition, research has suggested that people are more likely to aggregate information by averaging than by adding or multiplying, so misaggregation may be exhibited only in specific forms of aggregation and may not be present in averaging.

The results of Experiment I indicated that average certainty judgments were both more orderly and more veridical than cumulative certainty judgments of the type usually obtained in probabilistic inference tasks. The cumulative judgments were very conservative while the average certainty judgments were only slightly radical. Experiment II indicated that average certainty judgments and individual likelihood ratio judgments were

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both more orderly and veridical than cumulative certainty judgments but that they did not differ significantly from each other in either orderliness or veridicality. A second factor, the diagnosticity level of the data was also found to influence the veridicality of obtained judgments. Regardless of the method of aggregation employed, estimates became more veridical as the data became more diagnostic. Since these studies were undertaken only to see if average certainty judgments are an effective way to reduce conservatism, they do not directly test what causes conservatism. However, some implications concerning the nature of conservatism are discussed, as are the implications for the technology of probabilistic inference.

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Acknowledgment

This research was supported by the Advanced Research Projects Agency of the Department of Defense and was monitored by the Office of Naval Research under Contract N00014-76-C-0074 and subcontract from Decisions and Designs, Inc.

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I. Introduction

A relatively large body of literature (see reviews by DuCharme, 1969; Slovic and Lichtenstein, 1971) supports the assertion that people are conservative processors of probabilistic information. When presented with data, individuals typically revise their opinion to an extent less than prescribed by Bayes' Theorem, the formally appropriate model. In its simplest form with two hypotheses and conditionally independent data, Bayes' Theorem takes the form:

$$\Omega_{n} = \Omega_{0} \prod_{i=1}^{n} L_{i}$$
(1)

where Ω_0 is the prior odds, L_i is the likelihood ratio for the ith datum, and Ω_n is the revised (posterior) odds.

Three hypotheses have been advanced to explain conservative human inference (Edwards, 1968). The misaggregation hypothesis depicts conservatism as an inability to properly combine the information present in sequences of data, although each single datum is judged accurately. A second hypothesis--the misperception hypothesis--argues that people aggregate information properly (that is, according to Bayes' Theorem), but incorrectly diagnose the information in a single datum. The final explanation considers conservatism the result of a response bias: the avoidance of extreme odds or probability judgments.

Recent attention has focussed primarily on the misaggregation and response bias hypotheses, since Wheeler and Edwards (1975) demonstrated

rather convincingly that misperception played little, if any, role in producing conservatism. DuCharme (1970) provides the main support for the response bias explanation. He found aggregated posterior odds judgments to be near veridical for non-extreme odds, while unaggregated likelihood ratio judgments were conservative when the true likelihood ratio was relatively extreme. Wheeler and Edwards refute some of DuCharme's findings, providing persuasive support for the misaggregation hypothesis. Still they conclude that

> "in all likelihood misperception, misaggregation, and response biases all contribute to conservatism. The real questions of importance then becomes finding the manner in which each phenomenon contributes to conservatism and the best way of avoiding or compensating for this nonoptimal behavior." (p. 10)

One strategy that could lead to reduction or elimination of the bias against extreme responses is to remove the need for extreme responses: that is, to find some type of judgment that conveys the necessary information but for which the numerical response is not extreme. Consideration of these requirements for posterior odds judgments suggests some sort of average judgment. The responses required for average judgments have the advantage that the aggregated judgment falls within the range of the inputs to the judgment, as opposed to the usual posterior odds judgments in which the veridical response will involve multiplication of likelihood ratios, typically making the response much larger than the inputs.

Reviewing alternative forms of Bayes' Theorem (equation 1) suggests two possible types of judgments that convey all the information necessary and that are also averages of some sort. The geometric mean of the likelihood ratios is one possibility. However, geometric means seem to be

a difficult judgment for people to make. In addition, if subjects substituted arithmetic means for the geometric means--a seemingly likely occurrence--the results would be biased away from conservatism since the arithmetic mean is always larger than the geometric mean.

Fortunately, a second possibility exists. Taking base ten logarithms in equation (1) yields

$$\log \alpha_n = \log \alpha_0 + \sum_{i=1}^n \log L_i$$
 (2)

The judgment that would play a role in equation (2) equivalent to that played by the geometric mean likelihood ratio is the arithmetic mean log likelihood ratio. If subjects assess arithmetic mean log likelihood ratios, AMLL's, the posterior odds will be

$$\Omega_{n} = \Omega_{0} 10^{n (AMLL)}$$
(3)

A review of descriptive studies exploring how people process information lends additional support to the use of the arithmetic mean log likelihood ratio as a normative procedure for processing probabilistic information. People have been shown to use averaging rather than adding or multiplying as a method for combining information in a wide variety of contexts, for example in determining the overall value of products (Troutman and Shanteau, 1976), in deciding how well they would like a person described by personality trait adjectives (Andersen, 1965), and in predicting a criterion number on the basis of two cue numbers (Lichtenstein, Earle,

and Slovic, 1975).

Probably the most relevant descriptive studies are those of Shanteau (1975) and Troutman and Shanteau (1977) in which they used a task very similar to that employed in conservatism experiments. In both studies, an averaging model provided a much better fit to the data than did a multiplying model, which is the appropriate model if subjects actually process information in a manner consistent with Bayes' Theorem.

Certainly these descriptive results should be considered in designing normative procedures. If indeed, as seems likely, people do tend to average information, then a normative procedure taking advantage of this tendency may produce better results than one which requires an alternative form of processing. Perhaps conservatism results from the specific manner in which subjects are required to aggregate information. That is, people may not be accustomed to using addition or multiplication as aggregation procedures for processing information.

The present experiments explore the possibility of using subjective judgments of the arithmetic mean log likelihood ratio to reduce conservatism in aggregated judgments of certainty. These aggregated judgments are also compared against unaggregated likelihood ratio judgments.

II. Experiment I

II.1. Method

II.1.1. Apparatus. The stimuli for Experiment I were 6.5 inch (16.51 cm) pick-up sticks painted yellow on one end and blue on the other. The

length of yellow (or, because of symmetry, the length of blue) was the random variable. The sticks shown subjects were hypothetically drawn from two normally distributed populations differing only in mean length of yellow. One population (the predominantly yellow) had a mean of 4.05 inches (10.287 cm) of yellow and the other (the predominantly blue) a mean of 2.45 inches (6.223 cm). The populations shared a common standard deviation of 1.0 inches (2.54 cm).

Each stick was displayed on a white rectangular card. Eight sequences of four sticks were randomly selected for use as stimuli. Half the sequences were drawn from the predominantly blue population while the other half were drawn from the predominantly yellow population, so prior odds were always 1:1.

The population characteristics were displayed to the subjects by means of two random histograms, each representing a sample of 99 sticks from one of the two populations. The lengths displayed had been carefully chosen to accurately represent the populations. The displays were actual size and colors, and, on each, the population mean was displayed by a heavy black horizontal line at the appropriate position. Both displays were present throughout the experiment. All responses were made on sheets of paper which contained space at the top for the subject to check the more likely population along with four logarithmically spaced scales ranging from 1:1 odds (designated the "uncertainty" point) to odds of 10,000:1 and 1:10,000.

<u>II.1.2. Subjects.</u> Twenty naive subjects, run in groups of three or four, were drawn from an introductory undergraduate psychology course taught at the University of Southern California.

II.1.3. Procedure. Subjects were randomly assigned to one of two groups: one group made average certainty judgments, while the other judged cumulative certainty. For the group judging average certainty, the subjects were presented with the first stick and told to indicate, at the top of the scale, their choice of the more likely population, after which they were to designate, on the first scale column, the odds corresponding to their subjective certainty. As the second stick was placed alongside the first, subjects were told that their responses should represent a judgment as to the "average of, rather than the total of, their certainty". Each subject was told to use, as a reference in responding, the position of the point on the first scale column. Detailed examples emphasizing the process that should take place in arriving at an average certainty judgment were provided. Subjects were told that this procedure would continue with the addition of the third and fourth sticks, after which a new response sheet would be provided for a new sequence of four sticks. Each subject was also told that the same process was employed, but that direction changed toward the bottom half of the scale in the event that the subject changed his or her belief about which population was more likely to have produced the sticks in view.

The instructions given to each subject in the group judging cumulative certainty differed from those described above in one important way. As the second stick was placed alongside the first, the subject was asked to represent his or her cumulative certainty on the second scale column. This cumulative certainty was to be judged relative to the certainty after seeing the previous stick and was to be represented by moving distance (away from the "uncertainty" line) on the second scale column. Examples

of this procedure were provided in the instructions.

Having read through one set of instructions with the subjects, the experimenter provided four sequences of three sticks each as examples. Subjects responded to the example stimuli and general feedback was provided. Then eight sequences of four sticks were presented and 32 point placements were made on eight sets of four scale columns.

II.2. Results and Discussion

The data were first subjected to a logarithmic transformation and the average certainty judgment responses in logarithmic form were each multiplied by n, the number of sticks on which the judgment was based. All analyses were performed on the log transformed responses and the dependent variable was the log posterior odds that were inferred from the subjects' responses to the sequences of sticks.

For each subject, a regression analysis of inferred log posterior odds on veridical log posterior odds was performed. The correlation coefficients and slopes from these analyses are presented in Table 1. Judgments in the average certainty condition were significantly more orderly (t(18) = 3.975p < .001 on Fisher-z transformed correlation coefficients) as reflected in the mean correlation coefficients of .896 and .689 for the average and cumulative certainty response groups respectively. Posterior odds inferred from the averaging condition tended to be slightly radical (mean slope = 1.264), while the posterior odds obtained from the cumulative condition were extremely conservative (mean slope = 0.283). This difference was significant, t (18) = 6.115, p < .001, in the predicted direction. Although average certainty judgments were slightly radical, they were much closer to veridical than the cumulative certainty judgments.

	Ave	rage Certainty	Cumulative	Certainty
	r	b	r	b
	. 882	1.068	. 598	0.164
	.934	1.543	. 809	0.371
	.922	0.951	. 898	0.509
-	. 808	0.900	.864	0.356
	.925	0.501	. 748	0.329
	.934	0.898	.415	0.330
	. 871	1.899	.479	0.108
	.909	1.463	.601	0.147
	. 882	2.110	.571	0.155
	. 896	1.295	.906	0.358
mean	. 896	1.264		0, 283
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TABLE 1

Correlations and Slopes for Inferred Posterior Odds

The results of this study were encouraging for the use of average certainty judgments as a means of assessing subjective certainty. Subjects apparently can make this type of judgment and, at least under the conditions of this experiment, make such judgments quite well. Since the feasibility of this type of judgment seems to have been established, further questions need to be pursued. Specifically, do these results generalize to other levels of data diagnosticity? And how does the veridicality of average certainty judgments compare with that of non-aggregated likelihood ratio judgments? These questions are addressed in Experiment II.

III. Experiment II

III.1. Method

The method used in Experiment II was the same as that used in Experiment I with two exceptions: three levels of data diagnosticity were used, and a third type of uncertainty judgment--single-datum likelihood ratios--was also examined. The two pairs of normal distributions used, in addition to the original pair had mean yellow lengths of 4.35 inches (11.149 cm) and 3.75 inches (9.525 cm) for the predominantly yellow populations, and 2.15 inches (5.461 cm) and 2.75 inches (6.985 cm) for the predominantly blue populations. Thus, the three levels of data diagnosticity, d' = $(m_1 - m_2)/\sigma$, were 2.2, 1.6, and 1.0.

A three by three factorial design was created by crossing the three types of uncertainty judgments with the three levels of data diagnosticity. Ninety subjects, ten per cell, were randomly assigned to the experimental conditions and were run in groups of four or five. A few subjects were

run individually to obtain equal cell sizes. The subjects were from an introductory psychology course at the University of Southern California in which participation in several experiments was required.

III.2. Results

A regression analysis was performed on the individual data of each subject. For subjects making average or cumulative certainty judgments, inferred log posterior odds were regressed on veridical log posterior odds, while for subjects making likelihood ratio judgments, subjective likelihood ratios were regressed on veridical likelihood ratios. The correlation coefficients and slopes from these analyses are presented in Table 2.

Results of analyses of variance performed on the slopes and Fisher-z transformed correlation coefficients are shown in Tables 3 and 4 respectively. Inspection of the cell means of the slopes, plotted in Figure 1(a), indicated little difference between the likelihood ratio judgments (mean = 1.359) and the average certainty judgments (mean = 1.366), both being slightly radical. The cumulative certainty judgments, however, were again extremely conservative (mean = .365). Slopes generally tended to decrease as d' increased.

The significant interaction was unexpected, but may be due to a problem on the first day of experimentation. Elimination of the five subjects run the first day (average certainty judgments at d' = 1.6) eliminated the interaction as illustrated in Figure 1(b). These five subjects all had higher slopes than subsequent subjects run in this condition. Perhaps these subjects did not correctly understand the nature of the judgments they were asked to make.

TABLE 2

Correlations and Slopes for Average Certainty, Cumulative Certainty, and Likelihood Ratio Judgments

d' = 1.0

	Aver	age Certainty	Cumulativ	e Certainty	Individual	Likelihood Ratio
	r	b	r	b	r	Ъ
_	.498	1.194	. 893	0.834	.741	1.878
	.351	1.449	.613	0.420	.831	1.800
	.844	2.538	.852	0.565	.778	1.985
	. 566	1.443	.640	0.282	.900	2.585
	.863	2.143	.913	0.726	.828	1.976
	.240	0.461	.870	0.385	.753	1.188
	.842	2.414	.838	0.518	.850	1.811
	.819	1.560	.752	0.407	.464	1.038
	.908	1.291	. 729	0.507	. 896	2.052
	.761	1.586	.712	0.310	.763	1.352
mean —	.669	1.608	.781	.495	.780	1.767
tandard eviation	.238	.619	.107	.176	.125	.459

TABLE 2 (continued)

Correlations and Slopes for Average Certainty,

Cumulative Certainty, and Likelihood Ratio Judgments

d' = 1.6

A	verage	Certainty	Cumulative	Certainty	Individual Like	lihood Ratio
	r	b	r	b	r	b
	.822	2.583	.931	0.582	.572	0.623
	.932	2.606	.576	0.055	.797	1.335
	.928	1.788	.716	0.250	.855	1.110
	.773	1.580	.749	0.237	.642	0.877
	.912	2.051	.739	0.239	.811	1.780
	. 864	0.965	.575	0.139	.901	2.120
	.939	0.667	.758	0.405	.904	1.703
	.914	0.877	. 393	0.099	.819	1.034
	. 860	0.948	.877	0.256	.751	-0.333
	.848	1.341	.646	0.160	.875	1.110
mean	. 879	1.541	. 696	.242	. 793	1.136
tandard	.055 n	.705	.156	.155	.110	.687

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TABLE 2

(continued)

Correlations and Slopes for Average Certainty,

Cumulative Certainty, and Likelihood Ratio Judgments

d' = 2.2

Ave	rage	Certainty	Cumulative	Certainty	Individual	Likelihood Ratio
2	r	b	r	b	r	Ь
	.776	0.785	.813	0.587	.841	1.542
	.830	0.643	.689	0.355	.901	1.156
	.813	0.915	.838	0.464	.929	1.036
	. 896	0.964	.881	0.472	.927	1.328
	. 813	1.359	.641	0.267	.959	1.374
	. 889	1.107	.750	0.412	.938	1.335
	.701	1.197	. 564	0.210	. 528	0.601
	.950	0.869	.786	0.364	. 799	1.033
	.676	0.770	.608	0.113	. 890	1.155
	. 847	0.914	.779	0.323	. 806	1.195
mean	. 819	.952	. 735	.357	.852	1.176
standard eviation	.085	.215	.105	.138	.127	.257

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	_		-

ANOVA of Slopes

Source Sums of Squares df Mean Squares P Within Cells 15.731 0.194 81 Aggregation Method 19.931 2 9.996 0.001 Diagnosticity Level 3.346 2 1.673 0.001 Aggregation x Diagnosticity 2.070 4 0.518 0.038

TABLE 4

ANOVA of Correlations

Sums of Squares	df	Mean Squares	p
8.224	81	0.102	
0.746	2	0.373	0.030
0.401	2	0.201	0.146
1.540	4	0.385	0.007
	<u>Sums of Squares</u> 8.224 0.746 0.401 1.540	Sums of Squares df 8.224 81 0.746 2 0.401 2 1.540 4	Sums of SquaresdfMean Squares8.224810.1020.74620.3730.40120.2011.54040.385

The average certainty and likelihood ratio judgments also appeared to be more orderly than the cumulative certainty judgments as measured by the correlations. Data diagnosticity did not significantly affect the correlations.

IV. Discussion

The results of these experiments suggest that aggregated judgments of

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Figure 1: Mean Slopes

certainty need not be conservative. Past results have invariably shown cumulative judgments to be conservative, but experimental procedures have generally allowed subjective judgments to be veridical only if the aggregation method was multiplication. In some cases, proper use of the response scale could make the aggregation additive. For example, if the response scale presents logarithmically spaced odds, the normative aggregation method will be additive in distances on the scale.

However, in this study use of averaging as an aggregation method was demonstrated to be a viable method that could be taught to subjects and not result in conservatism. Specifically, subjects were asked to make arithmetic mean log likelihood ratio judgments, although of course, these precise words were never used. The posterior odds resulting from the judgments were very near veridical; while posterior odds, judged in the usual way by asking for cumulative certainty, were very conservative.

Either or both of two factors may account for the lack of conservatism in the average judgments. The aggregated responses necessary for the average judgments fall within the range of the single datum likelihood ratios that are inputs to the aggregation, and therefore, are not extreme responses. Thus, the bias against extreme responses may be reduced or eliminated. The second factor is simply the aggregation process that subjects need to employ to be veridical. In many situations people have been shown to use averaging rather than some other method of aggregating multiple pieces of information. In particular, the work by Shanteau (Shanteau, 1975; Troutman and Shanteau, 1977) has shown that people average the information in multiple samples of data on a task very similar to the one used in this study.

Since averaging appears to be the natural method of aggregation found in these descriptive studies, it seems reasonable that normative information processing based on averaging would outperform other methods of aggregation, a hypothesis consistent with the results of this study.

These findings have implications for the further development of the technology of inference. Because people typically have proved to be conservative processors of information, researchers have looked for methods of obtaining the desired information from people without the elicited judgments being affected by conservatism. Probabilistic information processing (PIP) systems were developed for this specific purpose (Edwards, Phillips, Hays, and Goodman, 1968). In a PIP system, people make only likelihood ratio judgments, a task this study and others (Wheeler and Edwards, 1975) show they can perform quite well. Bayes' Theorem is then used to combine the likelihood ratio judgments in the proper manner to produce posterior odds.

Since people are able to judge likelihood ratios quite accurately, why even consider average certainty judgments as an alternative? This study certainly did not show that average certainty judgments are superior to likelihood ratio judgments. The reason for considering an alternative to likelihood ratio judgments is that a problem may arise in applying PIP systems in real world contexts. The people assessing the likelihood ratios will typically have access to feedback about the posterior odds that are calculated from their likelihood ratios. Goodman (1973), in a reanalysis of data from five studies exploring methods of eliciting judgments about uncertain events, concludes that feedback about the implications of judgments makes them less extreme and is probably the most powerful variable

controlling the extremeness of the judgments. Thus, even a PIP system may be susceptible to conservatism in real-world applications. This problem seems less likely to characterize judgments of average certainty due to the very nature of the elicited judgments. Should further research confirm feedback produced conservatism in PIP systems, average certainty judgments may prove to be a useful alternative to PIP.

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Robert G. Gough, Major, USAF Associate Professor Department of Economics, Geography and Management USAF Academy, CO 80840 Chief, Systems Effectiveness Branch Human Engineering Division Attention: Dr. Donald A. Topmiller Wright-Patterson AFB, OH 45433

Aerospace Medical Division (Code RDH) Attention: Lt. Col. John Courtright Brooks AFB, TX 78235

Other Institutions

The Johns Hopkins University Department of Psychology Attention: Dr. Alphonse Chapanis Charles and 34th Streets Baltimore, MD 21218

Institute for Defense Analyses Attention: Dr. Jesse Orlansky 400 Army Navy Drive Arlington, VA 22202

Director, Social Science Research Institute University of Southern California Attention: Dr. Ward Edwards Los Angeles, CA 90007

Perceptronics, Incorporated Attention: Dr. Amos Freedy 6271 Variel Avenue Woodland Hills, CA 91364

Director, Human Factors Wing Defense and Civil Institute of Environmental Medicine P.O. Box 2000 Downsville, Toronto Ontario, Canada

Stanford University Attention: Dr. R.A. Howard Stanford, CA 94305

Montgomery College Department of Psychology Attention: Dr. Victor Fields Rockville, MD 20850

General Research Corporation Attention: Mr. George Pugh 7655 Old Springhouse Road McLean, VA 22101

Oceanautics, Incorporated Attention: Dr. W.S. Vaughan 3308 Dodge Park Road Landover, MD 20785

Director, Applied Psychology Unit Medical Research Council Attention: Dr. A.D. Baddeley 15 Chaucer Road Cambridge, CB 2EF England

Department of Psychology Catholic University Attention: Dr. Bruce M. Ross Washington, DC 20017 Stanford Research Institute Decision Analysis Group Attention Dr. Allan C. Miller III Menlo Park, CA 94025

Human Factors Research, Incorporated Santa Barbara Research Park Attention: Dr. Robert R. Mackie 6780 Cortona Drive Goleta, CA 93017

University of Washington Department of Psychology Attention: Dr. Lee Roy Beach Seattle, WA 98195

Eclectech Associates, Incorporated Post Office Box 179 Attention: Mr. Alan J. Pesch North Stonington, CT 06359

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1. REPORT NUMBER 2. GOVT ACCESSION 001855-3-T	NO. 3. RECIFIENT'S CATALOG NUMBER
	5. TYPE OF REPORT & PERIOD COVE
Developing the Technology of Probabilistic	
Inference: Aggregating by Averaging Reduces	Technical 10/76 - 9/77
Conservatism	6. PERFORMING ORG. REPORT NUMBE
7. AUTHOR(9)	8. CONTRACT OR GRANT NUMBER(+)
Lee C. Eils, III, David A. Seaver, and	Prime Contract N00014-76-
Ward Edwards	
	Subcontract 76-0308-0715
9. PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT, PROJECT, TA
Iniversity of Southern California	
University of Southern Carifornia	
11. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE
Advanced Research Projects Agency	August, 1977
1400 Wilson Blvd.	13. NUMBER OF PAGES
Arlington, Virginia 22209	
14. MONITORING AGENCY NAME & ADDRESS(II dillorent from Controlling United	BICIACCTETED
Suite 100, 7900 Westnark Drive	UNCLASSIFIED
McLean, Virginia 22101	154. DECLASSIFICATION DOWN GRADIN
(under contract from Office of Naval Research)	SCHEDULE
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that was devised to thwart conservatism caused by either a response bias or a certain form of misaggregation. Use of appropriate instructions and response scales made the average certainty judgments good subjective assessments of the arithmetic mean likelihood ratio which could then be used in the appropriate form of Bayes' Theorem to calculate posterior odds. These judgments seemed unlikely to be affected by a response bias since extreme responses were not needed. In addition, research has suggested that people are more likely to aggregate information by averaging than by adding or multiplying, so misaggregation may be exhibited only in specific forms of aggregation and may not be present in averaging. The results of Experiment To indicated that average certainty judgments

The results of Experiment I indicated that average certainty judgments were both more orderly and more veridical than cumulative certainty judgments of the type usually obtained in probabilistic inference tasks. The cumulative judgments were very conservative while the average certainty judgments were only slightly radical. Experiment II indicated that average certainty judgments and individual likelihood ratio judgments were both more orderly and veridical than cumulative certainty judgments but that they did not differ significantly from each other in either orderliness or veridicality. A second factor, the diagnosticity level of the data was also found to influence the veridicality of obtained judgments. Regardless of the method of aggregation employed, estimates became more veridical as the data became more diagnostic. Since these studies were undertaken only to see if average certainty judgments are an effective way to reduce conservatism, they do not directly test what causes conservatism. However, some implications concerning the nature of conservatism are discussed, as are the implications for the technology of probabilistic inference.

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