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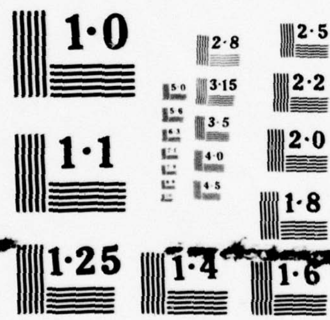

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A THEORY FOR LONGITUDINAL WAVE PROPAGATION IN A COMPOSITE ROD: A MODEL FOR LINEAR ELASTODYNAMIC BEHAVIOR OF COMPOSITE PENETRATORS

Akhilesh Maewal

Quarterly Progress Report
For the Period July 1, 1977 - September 30, 1977

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INTRODUCTION

In what follows we present a mathematical model for prediction of linear elastodynamic response of metal matrix composite penetrators. Although it is recognized that for the applications envisaged, the anelastic effects are the dominant ones, a study of linear elastic behavior is of importance inasmuch as it yields information about the composite properties required for simulation, using, for example, the HELP code[1] where material strength is not neglected.

If a rod of a fiber-reinforced composite is subjected to impact at one of its ends, longitudinal waves propagate through the rod in such a manner that on the macroscopic scale transverse normal stresses and axial shear stress are zero at least to the first degree of approximation. Thus the problem is essentially one of <u>uniaxial stress</u> rather than <u>uniaxial strain</u>. Since most of the approximate models[2,3] for longitudinal wave propagation in the direction of the fiber axis in a unidirectional fiber reinforced composite treat the case of uniaxial strain only, it is worthwhile to develop an analogous model for the case in which the primary mode of propagation induces a state of uniaxial stress.

The geometry of the system we have analyzed is shown in Figure 1. It consists of a single circular cylindrical fiber embedded in a concentric circular cylinder of the matrix whose outer surface is stress free. Although it would be more realistic to consider a rod reinforced by multiple fibers, our model leads to simpler analysis and is expected to provide adequate estimates for the gross mechanical response of the composite rod penetrator.

We first derive a binary mixture theory for the rod, using the asymptotic technique developed by Hegemier.[4] As a result of this analysis, we are also able to obtain (1) the Young's modulus for the composite in the direction of the fiber axis and (2) the Poisson's ratio ν_{xr}. These results, together with the results of the corresponding uniaxial strain analysis,[2,5] yield an estimate of one more composite property as will be shown in one of the following sections. Thus, three of the five moduli required for transversely isotropic composites can be determined using the expressions derived in this report. Although these properties have previously been obtained elsewhere,[6] our approach is somewhat different, as will be obvious from the exposition.

FORMULATION

With the geometry and coordinate system shown in Figure 1, axisymmetric motion of the rod is described by the following equations:

A. Conservation of Momentum

$$\partial_{\bar{x}}\bar{\sigma}_{xx}^{(\alpha)} + \frac{1}{\bar{r}}\partial_{\bar{r}}\left(\bar{r}\bar{\sigma}_{xr}^{(\alpha)}\right) = \bar{\rho}^{(\alpha)}\partial_{\bar{t}}^{2}u_{x}^{(\alpha)} \tag{1}$$

$$\partial_{\bar{x}}\bar{\sigma}_{xr}^{(\alpha)} + \frac{1}{\bar{r}}\partial_{\bar{r}}\left(\bar{r}\bar{\sigma}_{xr}^{(\alpha)}\right) - \frac{1}{\bar{r}}\bar{\sigma}_{\theta\theta}^{(\alpha)} = \bar{\rho}^{(\alpha)}\partial_{\bar{t}}^{2}\bar{u}_{r}^{(\alpha)} \tag{2}$$

B. Constitutive Equations

$$\begin{pmatrix} \bar{\sigma}_{xx} \\ \bar{\sigma}_{rr} \\ \bar{\sigma}_{\theta\theta} \end{pmatrix}^{(\alpha)} = \begin{pmatrix} \bar{\lambda}+2\bar{\mu} & \bar{\lambda} & \bar{\lambda} \\ \bar{\lambda} & \bar{\lambda}+2\bar{\mu} & \bar{\lambda} \\ \bar{\lambda} & \bar{\lambda} & \bar{\lambda}+2\bar{\mu} \end{pmatrix}^{(\alpha)} \begin{pmatrix} \partial_{\bar{x}}\bar{u}_{x} \\ \partial_{\bar{r}}\bar{u}_{r} \\ \bar{u}_{\bar{r}}/\bar{r} \end{pmatrix}^{(\alpha)} \tag{3}$$

$$\bar{\sigma}_{xr}^{(\alpha)} = \bar{\mu}^{(\alpha)}\left(\partial_{\bar{r}}\bar{u}_{x}^{(\alpha)} + \partial_{\bar{x}}\bar{u}_{r}^{(\alpha)}\right) \tag{4}$$

C. Interface Conditions

$$\left.\begin{array}{l} \bar{u}_x^{(1)} = \bar{u}_x^{(2)}, \ \bar{u}_r^{(1)} = \bar{u}_r^{(2)} \\ \bar{\sigma}_{xr}^{(1)} = \bar{\sigma}_{xr}^{(2)}, \ \bar{\sigma}_{rr}^{(1)} = \bar{\sigma}_{rr}^{(2)} \end{array}\right\} \text{ at } \bar{r} = \bar{r}_1 \qquad (5)$$

D. Free Surface Conditions

$$\bar{\sigma}_{xr}^{(2)} = 0, \ \ \bar{\sigma}_{rr}^{(2)} = 0 \quad \text{at } \bar{r} = \bar{r}_2 \qquad (6)$$

E. Appropriate Boundary and Initial Data

In the above set of equations, which defines a well posed boundary value problem in $(\bar{x},\bar{r},\bar{t})$ plane, we have used the superscript α to identify the fiber (α=1) and the matrix (α=2). The object of the subsequent analysis is to eliminate the $\bar{r}$ coordinate under the assumption that composite microdimension $\bar{r}_2$ is much smaller than signal wave length $\bar{\Lambda}$ for problems of interest. For this purpose we introduce the definitions:

$\bar{\Lambda}$ = Typical signal wavelength

$\bar{E}_{(m)}$ = Composite Young's modulus in the axial direction

$\bar{\rho}_{(m)}$ = Composite density

$$\bar{c}^2_{(m)} = \bar{E}_{(m)}/\bar{\rho}_{(m)}, \ \bar{t}_{(m)} = \bar{\Lambda}/\bar{c}_{(m)} \qquad (7)$$

$$\varepsilon = \bar{r}_2/\bar{\Lambda}$$

$$(x,r,t) = (\bar{x}/\bar{\Lambda}, \ \bar{r}/\bar{r}_2, \ \bar{t}/\bar{t}_{(m)})$$

$$(\lambda,\mu)^{(\alpha)} = (\bar{\lambda},\bar{\mu})^{(\alpha)}/\bar{E}_{(m)}$$

$$\rho^{(\alpha)} = \bar{\rho}^{(\alpha)}/\bar{\rho}_{(m)}$$

$$(\sigma_{xx},\varepsilon\sigma_{xr},\sigma_{\theta\theta},\sigma_{rr})^{(\alpha)} = (\bar{\sigma}_{xx},\bar{\sigma}_{xr},\bar{\sigma}_{\theta\theta},\bar{\sigma}_{rr})^{(\alpha)}/\bar{E}_{(m)}$$

$$(u_x, \epsilon u_r)^{(\alpha)} = (\bar{u}_x, \bar{u}_r)^{(\alpha)}/\bar{\Lambda}$$

It is noted here that at this stage of analysis, the appropriate mixture modulus $\bar{E}_{(m)}$ and density $\bar{\rho}_{(m)}$ are not known; however, they will be determined in a later section.

In terms of the variables defined in Eq. (7), the initial boundary value problem (1-6) reduces to the following equations.

A. Conservation of Momentum

$$\partial_x \sigma_{xx}^{(\alpha)} + \frac{1}{r}\partial_r\left(r\sigma_{xr}^{(\alpha)}\right) = \rho^{(\alpha)}\partial_t^2 u_x^{(\alpha)} \tag{8}$$

$$\epsilon^2\partial_x\sigma_{xr}^{(\alpha)} + \frac{1}{r}\partial_r\left(r\sigma_{rr}^{(\alpha)}\right) - \frac{\sigma_{\theta\theta}^{(\alpha)}}{r} = \epsilon^2\rho^{(\alpha)}\partial_t^2 u_r^{(\alpha)} \tag{9}$$

B. Constitutive Equations

$$\begin{pmatrix}\sigma_{xx}\\ \sigma_{rr}\\ \sigma_{\theta\theta}\end{pmatrix}^{(\alpha)} = \begin{pmatrix}\lambda+2\mu & \lambda & \lambda\\ \lambda & \lambda+2\mu & \lambda\\ \lambda & \lambda & \lambda+2\mu\end{pmatrix}^{(\alpha)}\begin{pmatrix}\partial_x u_x\\ \partial_r u_r\\ u_r/r\end{pmatrix}^{(\alpha)} \tag{10}$$

$$\epsilon^2\sigma_{xr}^{(\alpha)} = \mu^{(\alpha)}\left(\partial_r u_x + \epsilon^2\partial_x u_r\right)^{(\alpha)} \tag{11}$$

C. Interface Conditions

$$\left.\begin{array}{l} u_r^{(1)} = u_r^{(2)},\ u_x^{(1)} = u_x^{(2)} \\ \sigma_{rr}^{(1)} = \sigma_{rr}^{(2)},\ \sigma_{xr}^{(1)} = \sigma_{xr}^{(2)} \end{array}\right\} \text{ at } r = r_1 \tag{12}$$

D. Free Surface Conditions

$$\sigma_{rr}^{(2)} = 0, \ \sigma_{xr}^{(2)} = 0 \text{ at } r = 1 \tag{13}$$

MIXTURE THEORY

To derive an approximate theory that includes the effect of material inhomogeneity, we first define averaged and partial quantities by

$$f^{(\alpha a)}(x,t) = \frac{1}{A^{(\alpha)}} \int_{\Omega^{(\alpha)}} f^{(\alpha)}(x,r,t)\, dA \tag{14}$$

$$f^{(\alpha p)}(x,t) = n^{(\alpha)} f^{(\alpha a)}(x,t), \tag{15}$$

for a dependent variable f, where

$$\Omega^{(1)} \equiv [0,r_1], \ \Omega^{(2)} \equiv [r_1,1] \tag{16}$$

$$A^{(1)} = \pi r_1^2, \quad A^{(2)} = \pi - A^{(1)} \tag{17}$$

$$n^{(\alpha)} = A^{(\alpha)}/\pi \ . \tag{18}$$

Obviously, the quantity $n^{(\alpha)}$ denotes the volume fraction of the α constituent. If the axial momentum equation (8) is now averaged according to (14) and the relevant interface and free surface conditions are utilized, we obtain the following equation for conservation of average axial momentum:

$$\partial_x \sigma_{xx}^{(1p)} - \rho^{(1p)} \partial_t^2 u_x^{(1a)} = -P \tag{19}$$

$$\partial_x \sigma_{xx}^{(2p)} - \rho^{(2p)} \partial_t^2 u_x^{(2a)} = P \tag{20}$$

where P is an interaction term reflecting the axial momentum transfer from the matrix to the fiber, and is given by

$$P \equiv 2r_1 \, \sigma_{xr}^{(\alpha)}(x,r,t) \tag{21}$$

Equations (19) and (20), which are basic to the approximate analysis of wave propagation in a composite rod, are to be complemented by appropriate constitutive relations for partial stresses $\sigma_{xx}^{(\alpha p)}$ and the interaction term P. Such relations are most easily obtained through an asymptotic scheme which shall now be pursued.

Asymptotic Expansions

The system (8-13) contains the parameter ε^2, which is square of the ratio of rod radius to the signal wavelength. For problems of interest here, this quantity is much smaller than unity. Hence we expand all the dependent variables in a power series in this parameter; thus, we write

$$h(x,r,t,\varepsilon) = \sum_{n=0}^{\infty} \varepsilon^{2n} \, h_{(2n)}(x,r,t) \tag{22}$$

for any dependent variable h. If the expansion (22) is substituted into (8-13), a hiearchy of models is obtained by retaining the terms corresponding to different values of n. The simplest theory can be derived from the zeroth order system, the first equation of which is obtained from (11). Thus, we have

$$\partial_r u_{x(0)}^{(\alpha)} = 0 \tag{23}$$

so that

$$u_{x(0)}^{(\alpha)}(x,r,t) = u_{x(0)}^{(\alpha)}(x,t) \tag{24}$$

From the zeroth order expansion of the radial momentum equation (9) and the necessary constitutive equations, interface and free surface conditions we obtain the following:

In-Plane Stress Problem

$$\partial_r \sigma^{(\alpha)}_{rr(0)} + \frac{1}{r}\left(\sigma^{(\alpha)}_{rr(0)} - \sigma^{(\alpha)}_{\theta\theta(0)}\right) = 0 \qquad (25)$$

$$\begin{pmatrix} \sigma^{(\alpha)}_{rr(0)} \\ \\ \sigma^{(\alpha)}_{\theta\theta(0)} \end{pmatrix} = \lambda^{(\alpha)} \partial_x u^{(\alpha)}_{x(0)} \begin{pmatrix} 1 \\ \\ 1 \end{pmatrix} + \begin{pmatrix} \lambda+2\mu & \lambda \\ \\ \lambda & \lambda+2\mu \end{pmatrix}^{(\alpha)} \begin{pmatrix} \partial_r u_r \\ \\ u_r/r \end{pmatrix}^{(\alpha)} \qquad (26)$$

$$\sigma^{(2)}_{rr(0)} = 0 \quad \text{at} \quad r = 1 \qquad (27)$$

$$\sigma^{(1)}_{rr(0)} = \sigma^{(2)}_{rr(0)}; \; u^{(1)}_{r(0)} = u^{(2)}_{r(0)} \quad \text{at} \quad r = r_1 \qquad (28)$$

If the boundary value problem (25-28) is solved subject to the condition that $u^{(1)}_{r(0)}$ be bounded at $r = 0$, we obtain

$$u^{(1)}_{r(0)} = B^{(1)} r \qquad (29)$$

$$u^{(2)}_{r(0)} = \frac{A^{(2)}}{r} + B^{(2)} r \qquad (30)$$

where

$$A^{(2)} = n^{(1)} \left[B^{(1)} - B^{(2)} \right] \qquad (31)$$

and

$$B^{(\alpha)} = \sum_{\beta=1}^{2} b_{\alpha\beta} \partial_x u^{(\beta)}_{x(0)} \qquad (32)$$

The elements of the matrix $b_{\alpha\beta}$ in Eq. (32) can be expressed in terms of the material properties by the following equations:

$$b_{11} = \lambda^{(1)} \left[\lambda^{(2)} + \mu^{(2)} \left(1 + n^{(1)}\right)\right]/2D$$

$$b_{12} = n^{(2)}\lambda^{(2)}\mu^{(2)}/2D, \quad b_{21} = n^{(1)}\lambda^{(1)}\mu^{(2)}/2D \qquad (33)$$

$$b_{22} = \lambda^{(2)} \left(\mu^{(1)} + \mu^{(2)}n^{(2)} + \lambda^{(1)}\right)/2D$$

where

$$D = -\left[\left(\lambda^{(1)} + \mu^{(1)}\right) \left\{\lambda^{(2)} + \mu^{(2)} \left(1 + n^{(1)}\right)\right\} + n^{(2)}\mu^{(2)} \left(\lambda^{(2)} + \mu^{(2)}\right)\right] \qquad (34)$$

Using (29-31) we can obtain constitutive relations for the partial stresses in the following manner. First, from (10) we have:

$$\sigma^{(\alpha)}_{xx(0)} = \left(\lambda+2\mu\right)^{(\alpha)} \partial_x u^{(\alpha)}_{x(0)} + \lambda^{(\alpha)} \frac{1}{r} \partial_r \left(r u^{(\alpha)}_{r(0)}\right) \qquad (35)$$

so that on using (14,15) we obtain

$$\sigma^{(2p)}_{xx(0)} = \left(\lambda+2\mu\right)^{(2p)} \partial_x u^{(2a)}_{x(0)} + 2\lambda^{(2)} \left[u^{(2)}_{r(0)}(x,1,t) - r_1 u^{(2)}_{r(0)}(x,r_1,t)\right] \qquad (36a)$$

$$\sigma^{(1p)}_{xx(0)} = \left(\lambda+2\mu\right)^{(1p)} \partial_x u^{(1a)}_{x(0)} + 2\lambda^{(1)} \left[r_1 u^{(1)}_{r(0)}(x,r_1,t)\right] \qquad (36b)$$

If (29-32) are now substituted into (36), we finally obtain

$$\sigma_{xx(0)}^{(2p)} = \sum_{\beta=1}^{2} c_{\alpha\beta}\, \partial_x u_{x(0)}^{(\beta a)} \tag{37}$$

where

$$c_{\alpha\beta} = \delta_{\alpha\beta}\, (\lambda+2\mu)^{(\alpha p)} + 2\lambda^{(\alpha)} n^{(\alpha)} b_{\alpha\beta}\ , \quad \text{no sum on } \alpha, \tag{38}$$

with $\delta_{\alpha\beta}$ denoting the Kronecker delta. It can easily be shown that the matrix $c_{\alpha\beta}$ is symmetric, i.e.,

$$c_{12} = c_{21} \tag{39}$$

AXIAL SHEAR PROBLEM AND THE INTERACTION TERM

To obtain the constitutive relation for the interaction term P, we use the zeroth order expansion for the axial momentum equation, which is

$$\partial_x \sigma_{xx(0)}^{(\alpha)} + \frac{1}{r}\, \partial_r \left(r \sigma_{xr(0)}^{(\alpha)} \right) = \rho^{(\alpha)} \partial_t^2 u_{x(0)}^{(\alpha)}, \tag{40}$$

To (40) we append the necessary constitutive equations, interface and free surface conditions:

$$\sigma_{xr(0)}^{(\alpha)} = \mu^{(\alpha)} \left[\partial_r u_{x(2)}^{(\alpha)} + \partial_x u_{r(0)}^{(\alpha)} \right] \tag{41}$$

$$\sigma_{xr(0)}^{(1)} = \sigma_{xr(0)}^{(2)} = \frac{P}{2r_1} \quad \text{at} \quad r = r_1 \tag{42}$$

$$\sigma_{xr(0)}^{(2)} = 0 \quad \text{at} \quad r = 1 \tag{43}$$

It is noted here that in (42) we have used the definition (21) with $\sigma_{rx}^{(\alpha)}$ replaced by $\sigma_{rx(0)}^{(\alpha)}$.

We now use (35,41) in (40) to obtain differential equations for $u_{x(2)}^{(\alpha)}$, which turn out to be

$$\frac{\mu^{(\alpha)}}{r} \partial_r \left[r \, \partial_r u^{(2)}_{x(2)} \right] = Q^{(\alpha)}(x,t) \tag{44}$$

where $Q^{(\alpha)}$ represents terms in (40) which are independent of the radial coordinate. We can, however, relate $Q^{(\alpha)}$ to the interaction term by integrating (44) over $\Omega^{(\alpha)}$ and using (42), (43) and (29) through (31). The final result of this procedure is

$$Q^{(\alpha)} = -(-1)^{\alpha} \frac{P}{n^{(\alpha)}} - 2\mu^{(\alpha)} \partial_x B^{(\alpha)} \tag{45}$$

Equation (44) can now be solved in conjunction with (41-43) and (45) to yield $u^{(\alpha)}_{x(2)}$ and, consequently, the displacement fields $u^{(\alpha)}_x$ correct to $O(\varepsilon^2)$. Thus, we obtain

$$u^{(1)}_x = u^{(1)}_{x(0)} + \varepsilon^2 H_1(x,t) + \frac{\varepsilon^2}{4} \left(\frac{P}{\mu^{(1)} n^{(1)}} - 2 \, \partial_x B^{(1)} \right) r^2 \tag{46a}$$

$$u^{(2)}_x = u^{(1)}_{x(0)} + \varepsilon^2 H_1(x,t) + \varepsilon^2 \left[\frac{P}{4} \left\{ \frac{1}{\mu^{(1)}} - \frac{1}{\mu^{(2)} n^{(2)}} \left(\ln n^{(1)} - n^{(1)} - \ln r^2 + r^2 \right) \right\} - \frac{1}{2} \partial_x B^{(2)} \left(n^{(1)} \ln n^{(1)} - n^{(1)} - n^{(1)} \ln r^2 + r^2 \right) + \frac{1}{2} \partial_x B^{(1)} \left(n^{(1)} \ln n^{(1)} - n^{(1)} - n^{(1)} \ln r^2 \right) \right] \tag{46b}$$

where $H_1(x,t)$ is an undetermined function. To proceed further, (46) is averaged according to (14) to yield

$$u_x^{(1a)} = u_{x(0)}^{(1)} + \varepsilon^2 H_1 + \varepsilon^2 \frac{n^{(1)}}{8} \left[\frac{P}{\mu^{(1)} n^{(1)}} - 2\,\partial_x B^{(1)}\right] \tag{47a}$$

$$u_x^{(2a)} = u_{x(0)}^{(1)} + \varepsilon^2 H_1 + \varepsilon^2 \left[\frac{P}{4}\left\{\frac{1}{\mu^{(1)}} - \frac{1}{\mu^{(2)} n^{(2)}}\left(\frac{\ell n\, n^{(1)}}{n^{(2)}} + \frac{3-n^{(1)}}{2}\right)\right\} - \frac{1}{2}\,\partial_x B^{(2)}\left(\frac{n^{(1)}}{n^{(2)}}\,\ell n\, n^{(1)} + \frac{1+n^{(1)}}{2}\right) + \frac{1}{2}\,\partial_x B^{(1)}\left(\frac{n^{(1)}}{n^{(2)}}\,\ell n\, n^{(1)}\right)\right] \tag{47b}$$

From (47) we obtain the desired expression for P in terms of average displacements, i.e.,

$$P = \frac{u_x^{(2a)} - u_x^{(1a)}}{\varepsilon^2 \gamma} + \sum_{\alpha=1}^{2} \partial_x B^{(\alpha)} \zeta^{(\alpha)} \tag{48}$$

where

$$\gamma = \frac{1}{8}\left[\frac{1}{\mu^{(1)}} - \frac{1}{\mu^{(2)} n^{(2)}}\left(3-n^{(1)} + \frac{2\,\ell n\, n^{(1)}}{n^{(2)}}\right)\right] \tag{49a}$$

$$\zeta^{(1)} = -\frac{1}{4\gamma}\left[n^{(1)}\left(1 + \frac{2\,\ell n\, n^{(1)}}{n^{(2)}}\right)\right] \tag{49b}$$

$$\zeta^{(2)} = +\frac{1}{4\gamma}\left(1 + n^{(2)} + \frac{2n^{(1)}\,\ell n\, n^{(1)}}{n^{(2)}}\right) \tag{49c}$$

In order to complete the formulation we replace $u_{x(0)}^{(\alpha)}$ in (32) by the corresponding averages, i.e., by $u_x^{(\alpha a)}$; this procedure furnishes

$$P = \frac{u_x^{(2a)} - u_x^{(1a)}}{\varepsilon^2 \gamma} + \sum_{\alpha=1}^{2} \xi^{(\alpha)} \frac{\partial^2 u_x^{(\alpha a)}}{\partial x^2} \tag{50}$$

where

$$\xi^{(\alpha)} = \sum_{\beta=1}^{2} \zeta^{(\beta)} b_{\beta\alpha} \tag{51}$$

In a similar fashion, we drop the subscript zero from the variables in (37), thus, obtaining,

$$\sigma_{xx}^{(\alpha p)} = \sum_{\beta=1}^{2} c_{\alpha\beta} \partial_x u_x^{(\beta a)} \tag{52}$$

COMPOSITE PROPERTIES

The foregoing analysis completes the construction of an approximate theory for longitudinal wave propagation in a composite rod, which is given by the equations (19)-(20), (33)-(34) and (49) through (52). To obtain the mixture Young's modulus from these equations we eliminate from them all of the variables except $u_x^{(1a)}$, to obtain

$$\left[\left(c_{11} + 2c_{12} + c_{22}\right) \partial_x^2 - \left(\rho^{(1p)} + \rho^{(2p)}\right) \partial_t^2 + 0\,(\varepsilon^2)\right] u_x^{(1a)} = 0 \tag{53}$$

Equation (53) suggests the definition of mixture density according to

$$\bar{\rho}_{(m)} = \bar{\rho}^{(1p)} + \bar{\rho}^{(2p)} \tag{54}$$

We choose the mixture modulus such that the coefficient of ∂_x^2 in (53) is unity. This choice, together with (7, 33, 34 and 38), furnishes

$$\bar{E}_{(m)} = \left(\bar{\lambda}^{(1)} + 2\bar{\mu}^{(1)}\right) n^{(1)} + \left(\bar{\lambda}^{(2)} + 2\bar{\mu}^{(2)}\right) n^{(2)} - \bar{E}^*/\bar{D} \tag{54}$$

where

$$\begin{aligned}\bar{E}^* = {} & n^{(1)}\bar{\lambda}^{(1)} \left[\bar{\lambda}^{(1)} \left\{\bar{\lambda}^{(2)} + \bar{\mu}^{(2)} \left(1 + n^{(1)}\right)\right\} \right. \\ & \left. + \bar{\lambda}^{(2)}\mu^{(2)}n^{(2)}\right] + n^{(2)}\bar{\lambda}^{(2)} \left[\bar{\lambda}^{(1)}\bar{\mu}^{(2)}n^{(1)} \right. \\ & \left. + \bar{\lambda}^{(2)} \left(\bar{\lambda}^{(1)} + \bar{\mu}^{(1)} + \bar{\mu}^{(2)}n^{(2)}\right)\right] \\ \bar{D} = {} & \left(\bar{\lambda}^{(1)} + \bar{\mu}^{(1)}\right) \left[\bar{\lambda}^{(2)} + \bar{\mu}^{(2)} \left(1 + n^{(1)}\right)\right] \\ & + \bar{\mu}^{(2)}n^{(2)} \left(\bar{\lambda}^{(2)} + \bar{\mu}^{(2)}\right)\end{aligned} \tag{55}$$

To obtain one of the mixture Poisson's ratios, we first note that the stress-strain relations for a transversely isotropic material are given, in part, by

$$\begin{pmatrix} e_{xx} \\ e_{\theta\theta} \\ e_{rr} \end{pmatrix} = \frac{1}{E_x} \begin{pmatrix} 1 & -\nu_{xr} & -\nu_{xr} \\ -\nu_{xr} & E_x/E_r & -\nu_r E_x/E_r \\ -\nu_{xr} & -\nu_r E_x/E_r & E_x/E_r \end{pmatrix} \begin{pmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{rr} \end{pmatrix} \tag{56}$$

Thus, if an infinitely long circular cylinder of unit radius is subjected to uniaxial stress,

$$u_r(x,1) = - \nu_{xr}\partial_x u_x \tag{57}$$

On the other hand, for the composite cylinder, we have, from (39)-(31), that

$$u_{r(0)}^{(2)}(x,1,t) = \sum_{\alpha=1}^{2} \sum_{\beta=1}^{2} n^{(\alpha)} b_{\alpha\beta} \partial_x u_{x(0)}^{(\beta)} \tag{58}$$

If we now set

$$\partial_x u_{x(0)}^{(1)} = \partial_x u_{x(0)}^{(2)} = \partial_x u_x \tag{59}$$

we obtain

$$u_{r(0)}^{(2)}(x,1,t) = \left[\sum_{\alpha=1}^{2} \sum_{\beta=1}^{2} n^{(\alpha)} b_{\alpha\beta} \right] \frac{\partial u_x}{\partial x} \tag{60}$$

A comparison of (57) and (60) suggests the definition of equivalent Poisson's ratio ν_{xr} for the composite by

$$\nu_{xr} = - \sum_{\alpha=1}^{2} \sum_{\beta=1}^{2} n^{(\alpha)} b_{\alpha\beta} \tag{61}$$

Finally, on using (33-34) and reverting to dimensional material properties, we obtain

$$\nu_{xr} = \left[\bar{\lambda}^{(1)} \left(\bar{\lambda}^{(2)} + 2\bar{\mu}^{(2)} \right) n^{(1)} + \bar{\lambda}^{(2)} \left(\bar{\lambda}^{(1)} + \bar{\mu}^{(1)} + \bar{\mu}^{(2)} \right) n^{(2)} \right] / 2\bar{D} \tag{62}$$

where $\bar{D}$ is given by (55).

As was mentioned in the introduction, we can obtain one more combination of material properties by using the results of the corresponding uniaxial strain analysis.[2,5] To do so, we have, from (56) that, in case of uniaxial strain,

$$e_{xx} = \frac{1}{E_x^{(e)}} \sigma_{xx} \tag{63}$$

where

$$E_x^{(e)} = 1/\left[\frac{1}{E_x}\left\{1 - \frac{2\nu_{xr}^2}{1-\nu_r}\frac{E_r}{E_x}\right\}\right] \tag{64}$$

For the composite, the axial Young's modulus E_x is given by $\overline{E}_{(m)}$ in (54) and the modulus under uniaxial strain is obtained from[2,5]

$$E_x^{(e)} = \overline{E}_{(m)}^{(e)} = \left(\overline{\lambda}^{(1)} + 2\overline{\mu}^{(1)}\right) n^{(1)} + \left(\overline{\lambda}^{(2)} + 2\overline{\mu}^{(2)}\right) n^{(2)} - \left(\overline{\lambda}^{(1)} - \overline{\lambda}^{(2)}\right)^2 / E \tag{65}$$

where

$$E = \frac{1}{n^{(1)}n^{(2)}} \left[\left(\overline{\lambda}^{(2)} + \overline{\mu}^{(2)}\right) n^{(1)} + \left(\overline{\lambda}^{(1)} + \overline{\mu}^{(1)}\right) n^{(2)} + \overline{\mu}^{(2)}\right] \tag{66}$$

Thus, using (64,65) we obtain

$$(1-\nu_r)^{-1} E_r = \overline{E}_{(m)} \left(1 - \frac{\overline{E}_{(m)}}{\overline{E}_{(m)}^{(e)}}\right) / 2\nu_{rx}^2 \tag{67}$$

Thus, from the results given here, we can determine the combination $(1-\nu_r)^{-1}E_r$ for the composite. In closing this section, we note that if, following [7], we set

$$\nu_r = n^{(1)}\nu^{(1)} + n^{(2)}\nu^{(2)} \tag{68}$$

where $\nu^{(\alpha)}$ denotes the Poisson's ratio for the α constituent, we can use (67) to obtain the transverse Young's modulus, E_r, of the composite. Thus the only composite property that remains to be determined is the axial shear modulus to be used in

$$\sigma_{rx} = 2\mu_x e_{rx} \tag{69}$$

We can calculate μ_x from the expression given in [6], i.e., from

$$\mu_x = \bar{\mu}^{(2)} \frac{\bar{\mu}^{(1)}(1+n^{(1)}) + \bar{\mu}^{(2)} n^{(2)}}{\bar{\mu}^{(1)} n^{(2)} + \bar{\mu}^{(2)}(1+n^{(1)})} \tag{70}$$

Thus, all the elastic moduli of the composite can be calculated from the expressions (54, 62, 67, 68 and 70).

NUMERICAL RESULTS

Using the expressions given here for the composite properties, we have calculated the elastic moduli and Poisson's ratios for five sets of material combinations as a function of the fiber volume fraction. The constituent properties used for these calculations are shown in Table 1. The calculated composite properties are given in Tables 2 through 6. From these tables it can easily be concluded that calculation of the Poisson's ratio ν_{xr} by using the rule of mixtures, which was used for computing ν_r, does not entail any significant loss of accuracy.

From the results shown in these tables, various elastic wave speeds can very easily be calculated since the density of the composite is obtained simply by the rule of mixtures. Although the dispersion curve for time harmonic longitudinal waves in the composite rod can also be computed using the theory derived in this report, we have chosen not to do so since dispersive phenomena are not expected to be important factors in materials selection for metal matrix composite penetrators.

CONCLUDING REMARKS

A binary mixture theory has been derived for propagation of longitudinal waves in a composite rod, which is proposed a simple model for metal matrix composite penetrators. Theory includes the effect of material inhomogeneity and exhibits dispersive character typical of composite materials.

Expressions for computing the composite material properties have been presented, and have been used to calculate the composite moduli for five sets of material combinations as a function of fiber volume fraction.

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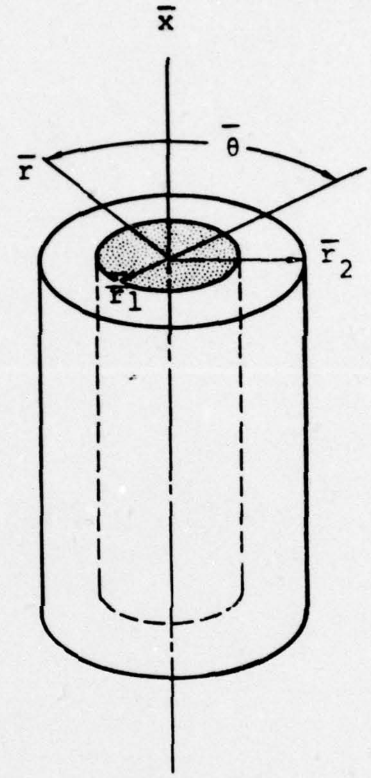

Figure 1. Geometry and coordinate system.

TABLE 1

CONSTITUENT ELASTIC PROPERTIES

	Young's Modulus (GPa)	Poisson's Ratio
Aluminum 6061-T6	69	0.33
Steel 1010	200	0.29
Lead	14	0.43
SiC on W Substrate	427	0.19
Aluminum FP	345	0.27
Thornel 50 Gr	393	0.20
Tungsten Wire	407	0.28

TABLE 2

COMPOSITE ELASTIC PROPERTIES
ALUMINA FP FIBERS IN ALUMINUM MATRIX

Fiber Volume Fraction	Young's Moduli E_x (GPa)	E_r	Poisson's Ratio ν_r	ν_{xr}	Axial Shear Modulus μ_x (GPa)
0.2	124	97	0.318	0.316	34
0.4	180	129	0.306	0.303	45
0.6	235	171	0.294	0.291	62
0.8	290	235	0.282	0.280	88

TABLE 3

COMPOSITE ELASTIC PROPERTIES

THORNEL 50 Gr FIBERS IN ALUMINUM MATRIX

Fiber Volume Fraction	Young's Moduli E_x (GPa)	E_r	Poisson's Ratio ν_r	ν_{xr}	Axial Shear Modulus μ_x (GPa)
0.2	134	103	0.304	0.300	35
0.4	199	140	0.278	0.272	47
0.6	264	189	0.252	0.246	66
0.8	328	264	0.226	0.222	98

TABLE 4

COMPOSITE ELASTIC PROPERTIES

SiC (ON W SUBSTRATE) FIBERS IN STEEL MATRIX

Fiber Volume Fraction	Young's Moduli (GPa) E_x	E_r	Poisson's Ratio ν_r	ν_{xr}	Axial Shear Modulus μ_x (GPa)
0.2	246	237	0.270	0.268	91
0.4	291	277	0.250	0.247	107
0.6	336	321	0.230	0.227	126
0.8	382	370	0.210	0.208	150

TABLE 5
COMPOSITE ELASTIC PROPERTIES
TUNGSTEN WIRE FIBERS IN STEEL MATRIX

Fiber Volume Fraction	Young's Moduli (GPa) E_x	E_r	Poisson's Ratio ν_r	ν_{xr}	Axial Shear Modulus μ_x (GPa)
0.2	241	230	0.288	0.288	89
0.4	283	264	0.286	0.286	102
0.6	324	303	0.284	0.284	118
0.8	366	350	0.282	0.282	136

TABLE 6
COMPOSITE ELASTIC PROPERTIES
ALUMINA FP FIBERS IN LEAD MATRIX

Fiber Volume Fraction	Young's Moduli (GPa) E_x	E_r	Poisson's Ratio ν_r	ν_{xr}	Axial Shear Modulus μ_x (GPa)
0.2	80	39	0.398	0.395	7
0.4	146	59	0.366	0.362	11
0.6	213	90	0.334	0.330	17
0.8	278	150	0.302	0.299	33