



A NATURAL APPROACH FOR DETECTING CAUSAL RELATIONSHIPS IN TIME SERIES

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I. Introduction

A post hoc data analysis in hopes of identifying causal relationships between variables is one that should be regarded with a great deal of suspicion. One need merely consider the etymology of the word "malaria" to realize that unless all factors, or variables, are taken into consideration or experimentally controlled erroneous conclusions concerning cause and effect may be reached.† It is for this reason that statisticians prefer to speak in terms of "predictability" or "correlation" rather than "causation".

In economics, however, a great deal of research has recently been published concerning causal realtionships between economic variables. The basis for this research stems from a definition of causality given by C. W. J. Granger (1964). Basically this definition states that if one can predict the future values of one variable X, using all past values of all other variables

in this system better than when predicting future X_t using all past values of all other variables

in this system except those of Y_t , then Y_t causes

 X_t . It is important to note that this definition

contains a universe of variables related in some system. In reality, of course, it is impossible to measure all the factors in the system. Hence it may appear that Y_t is causing X_t when indeed

a third variable is causing both X_t and Y_t .

Granger is careful to point this out in his discussion of the definition.

The inability to measure all variables in the universe certainly exists in economics. However, when an economic theory indicates that a change in one variable should cause a change in another, it may be necessary to identify a statistical relationship between these variables, a relationship which could quite possibly be causal but could not be proven. Thus, though some critics may argue the use of the word "causality", we shall use this term with the understanding of what it connotes in this situation. Wholesale use of techniques for identifying casuality withcut sound economic reasons for justifying such techniques should be avoided.

Granger's definition of causality is important in that not only does it appeal to our intuition as a reasonable definition, it also lends itself to statistical analyses for causality detection. The appropriate analysis to be used, though, is not apparent. In the next section we propose some analyses that seem natural in this situation but which should not be used because of certain deficiencies. The elimination of these deficiencies leads to what we propose for causality which is given in the fourth section. This discussion will consist of a universe of only two variables, however, the extension to more than two variables follows readily.

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II. The Process of Elimination

In Granger's definition one is concerned with being able to predict one variable from past values of itself and another variable. As a criterion of goodness of the prediction Granger employs mean squared error. Thus one's initial attempt at determining if Y_t causes X_t would be to regress present X_t on past X_t and Y_t

according to

$$x_{t} = a_{1}x_{t-1} + \dots + a_{n}x_{t-n} + b_{1}Y_{t-1}$$

+...+ $b_{m}Y_{t-m} + \epsilon_{t}$, (2.1)

where ε_t -NID(0, σ^2). A significant F-ratio for testing H_i: b_i = 0, i = 1, ..., m, would imply that Y_t causes X_t. This result, though, would surely be suspect because in most economic time series a definite trend exists with respect to time. This trend in time present in both series could manifest itself in the significance of the coefficients in the Y_t series when indeed Y_t

does not cause X.

The next consideration then would be the removal of or an adjustment for the time variable. In the terminology of time series analysis our aim becomes the transformation of the original series to stationary time series. We use the term "stationary" to describe a time series with constant mean having autocovariance a function only of the lag between observations. This condition on the autocovariance function implies also constant variance. Departure from constant variance may be corrected via some transformation as the logarithm. As indicated by Box and Jenkins (1976) stationarity with respect to the mean can be achieved by suitably differencing the individual series involved. For example, if a linear trend is present in the data then one would consider the series $U_t = X_t - X_{t-1}$. Had the trend had a single periodic component then a second difference would be required, namely,

 $t = U_t - U_{t-1}$

We return then to the regression model (2.1) where we realize now that the X_{t} and Y_{t} are the

results of the differencing necessary to produce stationarity. New problems arise, however, that cause the results of a classical regression analysis based on the F-test to be suspect. The standard assumptions for the F-test are that the values of the dependent variable are normally and independently distributed and that the values of the independent variables are fixed. In our situation the values of the "dependent" variable are not necessarily independent (and, in fact, are most often correlated). Furthermore

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the lock of the pendence is not due to correlated errors (which could be adjusted for) but due to the fact that the "independent" variables are not fixed but random. In fact the "dependent" variable at time t is one of the "independent" variables at some future time t + k. It is this structure that arouses questions concerning the distribution of the regression statistic and thus the validity of the test.

It should also be noted that this regression technique at this point fails for another important reason. It has been shown [Jenkins-Watts, p. 338, (1968)] that even if X_{t} and Y_{t} are inde-

pendent processes having autoregressive structure that spurious cross-correlations are possible. These spurious cross-correlations could manifest themselves in significant values for some of the Y_{t} coefficients in (2.1) implying causality when in fact none exists.

III. A More General Model

Since the regression model (2.1) has been shown to have serious deficiences in the ensuing analyses, we next consider two different models and the relationship between them. The models to be considered are the univariate autoregressivemoving average representation for each of two series, X_t and Y_t , and the bivariate autoregressive representation of X_t and Y_t jointly. The

series X_t and Y_t are assumed to be appropriately transformed or differenced so that each series is covariance stationary. Furthermore, we shall assume that the true causal relationship has remained in the residuals X_t and Y_t , although this

assumption may be sensitive to the nature of the prefiltering (especially when nonlinear transformations to achieve stationarity are used). The univariate models are:

$$\phi_{1}(B) X_{t} = \theta_{1}(B) u_{t}$$

 $\phi_{2}(B) Y_{t} = \theta_{2}(B) v_{t}$ (3.1)

where ϕ_1 , ϕ_2 , θ_1 , and θ_2 are finite degree polynomials in B (where $B^{k}X_{t} = X_{t-k}$) having constant term I and u_{t} and v_{t} are white noise processes. The processes are strictly autoregressive if $\theta_{i}(B) = 1$, i = 1, 2, and strictly moving averages if $\phi_{i}(B) = 1$, i = 1, 2. The assumption that X_{t} and Y_{t} are stationary implies that the roots of $\phi_{i}(Z) = 0$ lie outside the unit circle. We can subsequently rewrite (3.1) as

$$x_{t} = \phi_{1}^{-1} (B) \theta_{1} (B) u_{t} = \theta_{1} (B) u_{t}$$

$$y_{t} = \phi_{2}^{-1} (B) \theta_{2} (B) v_{t} = \theta_{2} (B) v_{t},$$
(3.2)

where ϕ_1^{-1} (B) and ϕ_2^{-1} (B) are infinite series in B provided they are not identically 1.

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The bivariate autoregressive representation is given by

$$\begin{bmatrix} P(B) & Q(B) \\ R(B) & S(B) \end{bmatrix} \begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} a_t \\ b_t \end{bmatrix}$$
(3.3)

where P, Q, R, and S are polynomials of the form

$$A(B) = \sum_{k=0}^{\infty} a_k^k B^k$$

with the constant term in P and S strictly 1 and $\mathbf{a}_{\rm t}$ and $\mathbf{b}_{\rm t}$ are white noise series with

$$\mathbf{E}\left(\begin{bmatrix}\mathbf{a}_{t}\\\mathbf{b}_{t}\end{bmatrix}\begin{bmatrix}\mathbf{a}_{t+k}&\mathbf{b}_{t+k}\end{bmatrix}\right) = \left(\begin{smallmatrix} \mathbf{j} & (\text{diagonal}), k=0\\ \phi & , k\neq 0 \end{smallmatrix}\right)$$

The series are stationary if the roots of

$$\begin{array}{c|c} P(Z) & Q(Z) \\ R(Z) & S(Z) \end{array} = 0$$

lie outside the unit circle.

The relationship between these two representations can be given in terms of their white noise innovations. From (3.2) we can rewrite (3.3) as

$$\begin{bmatrix} P(B) & Q(B) \\ R(B) & S(B) \end{bmatrix} \begin{bmatrix} \Theta_1(B) & 0 \\ 0 & \Theta_2(B) \end{bmatrix} \begin{bmatrix} u_t \\ v_t \end{bmatrix} = \begin{bmatrix} a_t \\ b_t \end{bmatrix}$$

or

$$\begin{bmatrix} P(B) & \Theta_1(B) & Q(B)\Theta_2(B) \\ R(B) & \Theta_1(B) & S(B)\Theta_2(B) \end{bmatrix} \begin{bmatrix} u_t \\ v_t \end{bmatrix} = \begin{bmatrix} a_t \\ b_t \end{bmatrix}$$

which with substitutions of the form $P^*(B) = P(B) \Theta_1(B)$ yields



which in single equation becomes

$$u_t = F(B) S^*(B) a_t - F(B) Q^*(B) b_t$$

 $v_t = -F(B) R^*(B) a_t + F(B) P^*(B) b_t$, (3.4)

where $F(B) = 1/[P^*(B) S^*(B) - Q^*(B) R^*(B)]$. Thus u_t and v_t are related to one another via a_t and b_t according to (3.4). It is this pair of equations that can be used to determine the causal relationship between u_t and v_t , that is, between X_t

and Yt.

If no causal relationship exists between X_t and Y_t , then Q(B) = R(B) = 0 and (3.4) reduces to

$$u_t = a_t$$
$$v_t = b_t$$

that is, the white noise innovations u and v t

have cross correlation zero for all lags. Under a null hypothesis of no causality and assuming that a_t and b_t are Gaussian, a regression of u_t on past, present, and future v_t based on the standard test statistic which does indeed follow an F distribution should find no significant coefficients. In the case of unidirectional causality,

say Y_t causing X_t , R(B) = 0. Substituting these into (3.4) yields

$$u_{t} = \frac{1}{P^{*}(B)} a_{t} - \frac{Q^{*}(B)}{P^{*}(B) S^{*}(B)} b_{t} = A_{t} + B_{t}$$

$$v_{t} = b_{t} . \qquad (3.5)$$

Assuming for now no instantaneous causality, the operator $\frac{Q^*(B)}{P^*(B)}$ is a polynomial in B having constant term zero. Thus u_t is a function of past v_t . Rewriting the first equation in (3.5) to express v_t in terms of u_t produces

$$v_{t} = -\left[\frac{Q^{*}(B)}{P^{*}(B) S^{*}(B)}\right]^{-1} u_{t} + \frac{S^{*}(B)}{Q^{*}(B)} a_{t}.$$
 (3.6)

The operator on u_t will contain terms of the form B^{-m} , m > 0 (as well as B^{m} terms perhaps), implying that v_t will be a function of future u_t (as well as possibly present and past v_t). A forward stepwise regression finding u_t a function of only past v_t and v_t a function of future and possibly present and past u_t would thus indicate that Y_t causes X_t .

A closer look at (3.5) might arouse some concern about an ordinary least square regression of u_t on v_t . In general, the first term A_t is a moving average in a_t and the second term B_t is a moving average in b_t . Thus each term represents an autocorrelated series. The nature of this autocorrelation is such that when A_t is correlated ρ_k at lag k the second term B_t must have autocorrelation $-\rho_k$ at lag k in order that u_t be white noise. In a regression u_t on v_t it

is clear that A represents the error in the re-

gression. It is well known that such autocorrelation among the error terms can result in biased estimates of the variances of the least squares estimator of the regression coefficients (see Appendix A). This causes concern in that one might tend to reject true null hypotheses more often than the significance level of the test would indicate or one might tend to not reject a false null hypothesis when indeed he should. As indicated in Appendix A the nature of the autocorrelation of the "independent" variable determines which of the above is the situation. In the case of unidirectional causality when using a forward stepwise regression, it is easy to see that at each step the next possible "independent" regression variable to enter the model is indeed uncorrelated. In this case the estimator of the error variance is asymptotically unbiased and for small samples the bias should be negligible (our simulations using around 80 observations have indicated no consistent bias in estimates of the error variance). Hence the use of ordinary least squares in a forward stepwise regression is valid.

In the case where Y_t causes X_t (or X_t

causes Y_{t}) and instantaneous causality is

present, the polynomials $Q^*(B)$, $P^*(B)$, and $S^*(B)$ each have constant term 1. Thus u_t will be a function of present and past v_t and similarly in

(3.6) v_t a function of present and past u_t .

Hence the criterion for instantaneous causality. (It should be noted that this criterion only identifies the existence of instantaneous causality. Whether X_t causes Y_t , Y_t causes X_t , or

instantaneous feedback cannot be determined.)
 When feedback is present in the system,
both Q(B) and R(B) are nonzero polynomials in B
with constant term zero (assuming no instantaneous causality). Equation (3.4) may now be ex-

$$u_{t} = A_{1}(B) a_{t} + B_{1}(B) b_{t}$$

 $v_{t} = A_{2}(B) a_{t} + B_{2}(B) b_{t}$

where A_1 and B_2 have constant term 1 and A_2 and B_1 have constant term 0. Considering u_t as a linear combination of the v_t implies

$$= A_{1}^{(B)}A_{2}^{-1}^{(B)}v_{t} + \\ [B_{1}^{(B)} - A_{1}^{(B)}A_{2}^{-1}^{(B)}B_{2}^{(B)}] b_{t}$$

Now $A_1(B)A_2^{-1}$ contains negative powers of B as well as positive powers. Thus u_t is a function of past, future, and possible present values of v_t . A symmetric argument holds for v_t as a function of u_t . Hence the criterion for determining feedback in the system. As in the unidirectional



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case the absence of autocorrelation in the "independent" regressor variables substantiates the validity of ordinary least squares for the regressions.

IV. The Resulting Procedure

In summary, the procedure that has arisen naturally from the earlier discussion is outlined as follows:

- Individually transform (e.g., taking the natural logarithm of the observations) or difference (e.g., taking the first difference to remove a linear trend) each time series to yield covariance stationary residual series denoted by X_t and Y_t.
- Individually model each series, X_t and Y_t, as an autoregressive-moving average process according to the techniques developed by Box
 and Jenkins (1976).
- Filter X_t and Y_t according to their estimated models to yield the white noise innovations u_t and v_t respectively.
- 4. Use a forward stepwise regression of u_t on future, present, and past values of v_t and a similar regression of v_t on u_t .
- 5. Determine the causal relationship as follows: a. No causality, -- u_t is not a function of
 - any of the future, present, or past values of v_t nor is v_t a function of any such u_t .
 - b. Y_t causes X_t (and similarly for X_t causing Y_t) but not instantaneously -- u_t is a function only of past v_t and v_t is a function of future and possibly present and past u_t .
 - c. Instantaneous causality -- u_t is a function of only present and past v_t and v_t is a function of only present and past u_t .
 - d. Bi-directional causality -- ut is a function of future, present, and past vt and similarly vt a function of future, present, and past ut.

We advocate this procedure because (1) it generalizes to the multivariate case and therefore serves as a basis for the detection of causality in systems of two or more variables and (2) the use of a stepwise regression serves not only to determine the direction of causality but also to determine the causality lag. The latter is an important step in system identification. For example, consider that unidirectional causality has been detected from Y_t to X_t . From

our regression analysis we have estimated $-\frac{Q^{*}(B)}{P^{*}(B) S^{*}(B)}$ in (3.5) and from our original modelling we have estimated $\theta_{1}(B)$ and $\theta_{2}(B)$. A Box-Jenkins analysis of the residual series A_{t}

will yield an estimate of $P^*(B)$ and hence P(B). Since $S^*(B) = 1$ an estimate for $Q^*(B)$ can be obtained and hence Q(B). From this analysis, then, we can estimate the original model

P (B)	Q(B)	Xt	at	
0	S (B)	Yt	bt	

(knowing that R(B) = 0 in this case).

V. Applications of the Procedure

In order to demonstrate the application of this procedure in bivariate situations the results of a simulation and an analysis of two economic times series are given. The results concerning the economic time series are quite interesting in that they differ from any other results yet published.

For the simulation 88 observations were generated for each of two series X_t and Y_t according to

Due to the choice of coefficients the processes are stationary. A Box-Jenkins modelling of the series individually yielded

$$x_{t} = .975x_{t-1} - .297x_{t-2} + u_{t} - .257_{t-8}$$
$$y_{t} = .850y_{t-1} - .465y_{t-2} + v_{t}.$$

Upon filtering each series to yield u_t and v_t and performing forward stepwise regressions of u_t on v_t and v_t on u_t , the models obtained were

$$u_{t} = .85v_{t-2} + .28v_{t-3} + .27v_{t-4}$$

- .35v_{t-5} + error
(F_{4,64} = 11.25 R² = .41 D.W. = 2.61)
v_{t} = .36u_{t+2} + .16u_{t+3} + error

$$(F_{2,66} = 21.03 R^2 = .39 D.W. = 2.47)$$

These results indicate causality from Y, to X,.

It is also interesting to note the magnitude of the Durbin-Watson statistics indicating the anticipated serial correlation for the error terms.

Applying our technique to quarterly observations of the money supply and GNP for the period 1953 through 1974, we arrived at the following model for GNP:

$$G_t = (1-B) \ln (GNP)$$

 $(G_t - .007) = .449(G_{t-1} - .007) + E_t$
 $g_t = E_t = (G_t - .007) - .449(G_{t-1} - .007)$

That is the first differences of the natural log of GNP, which is approximately equivalent to the rate of growth from quarter to quarter (a transformation necessary to achieve stationarity) can be modeled as a simple first order autoregressive (AR) process.

For money (M1), the model is

$$M_{t} = (1 - B) \ln (M1)_{t}$$

$$(M_{t} - .004) = .63(M_{t-1} - .004) + Z_{t} - .259Z_{t-10}$$

$$m_{t} = Z_{t} = (M_{t} - .004) - .63(M_{t-1} - .004) + .259Z_{t-10}$$

That is, the first differences of the natural logs of money is described by an autoregressive - moving average (ARMA) model. The residuals of these correctly filtered series, g_t and

m_t, can now be used in a regression to test for causality.

Using a forward selection regression procedure combined with the properly prefiltered values of money and GNP, we arrived at the following models:

$$g_{t} = .49_{mt-2} + .68m_{t-3} + .42m_{t+1} - .54m_{t+3}$$
(.1850) (.1874) (.1831) (.1880)
$$R^{2} = .2775 \qquad R(4.65) = 6.2408 \qquad DW = 2.41$$

and

 $m_{t} = -.1359g_{t-3} + .1901g_{t+3}$ (.0649) (.0647)

 $R^2 = .1573$ F(2,67) = 6.2520 D.W. = 2.1097,

where the values in parentheses below the coefficients are the standard errors of the regression coefficients. In both regressions, the R^2 's are small but still significant at the .05 level. Of course, when evaluating this R^2 , it must be remembered that the values used in the regression were the residuals of previous models. Hence, the explanatory power of these models is actually quite good. Since coefficients of future values appear in both regressions significantly different from zero, we have identified a bidirectional or feedback relationship between money and income.

APPENDIX A

Consider the model

$y_t = \beta x_t + z_t$

where x and z are zero mean processes with z

having variance σ_z^2 and autocorrelation ρ_k . The variance of the ordinary least squares estimator b of β given by

$$p = (\overline{\mathbf{x}}, \overline{\mathbf{x}})_{-1} \overline{\mathbf{x}}, \overline{\mathbf{x}}$$

Var (b) =
$$(x'x)^{-1} x' Vx (x'x)^{-1}$$

where

is

$$\mathbf{v} = \sigma_{\mathbf{z}}^{2} \begin{bmatrix} 1 & \rho_{1} & \rho_{\mathbf{z}} & \cdots & \rho_{n-1} \\ \rho_{1} & 1 & \rho_{1} & \cdots & \rho_{n-2} \\ \vdots & \vdots & \vdots \\ \rho_{n-1} & \rho_{n-2} & \rho_{n-3} & \cdots & 1 \end{bmatrix}$$

It is easy to show that



+
$$2p_2 \frac{\sum_{i=1}^{n} \mathbf{x}_i \mathbf{x}_{i+2}}{\sum_{i=1}^{n} \mathbf{x}_i^2}$$



If σ_z^2 were known, then the usual least squares estimator of the variance $\frac{\sigma_z^2}{\sum\limits_{i=1}^n x_i^2}$. Hence if the

 $=\frac{\sigma_2^2}{\sum_{i=1}^{n} x_i^2}$

 $\mathbf{x}_{\mathbf{t}}$ are autocorrelated with the same sign as the

 ρ_k then the ordinary least squares estimator will underestimate the true variance of b; if auto-

correlated with opposite sign, the estimator will overestimate the variance of b.

Similarly it can be shown that the residual sum of square for least squares estimator has

expectation (n-K) σ_{z}^{2} .

In estimating σ_z^2 the same relationships hold as

above accentuating the bias in estimating the variance of b using ordinary least squares.

It should be noted, however, that if the x_t are uncorrelated, the affect of the auto-

correlated errors will likely be neglible. In fact in this case the estimators are asymptotically unbiased.

[†]The word malaria means "bad air". This term came into usage because it was thought that people contracted the disease from the air present in swampy, lowlying areas when in fact the only known carrier is the anopheles mosquito.

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