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ARO 12048.8-M

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BOUNDARY VALUE PROBLEMS AND PERIODIC SOLUTIONS
OF NONLINEAR ORDINARY, FUNCTIONAL AND
PARTIAL DIFFERENTIAL EQUATIONS

FINAL REPORT

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August 10, 1977

U. S. Army Research Office

Grant Nos. DAH-C-04-74-C-0208
DAH-C-04-75-G-0175
DAAG-29-76-G-0186

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER P-12048M	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER <i>rept.</i>
4. TITLE (and Subtitle) BOUNDARY VALUE PROBLEMS AND PERIODIC SOLUTIONS OF NONLINEAR ORDINARY, FUNCTIONAL AND PARTIAL DIFFERENTIAL EQUATIONS.		5. TYPE OF REPORT & PERIOD COVERED Final June 15, 1974 - June 14, 1977
6. AUTHOR(s) Klaus Schmitt		6. PERFORMING ORG. REPORT NUMBER 15 Jun 74 - 14 Jun 77
7. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Mathematics University of Utah Salt Lake City, Utah 84112		8. CONTRACT OR GRANT NUMBER(s) DAH-C-84-74-C-0208 DAH-C-84-75-G-0175 DAAG-29-76-G-0186
9. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) <i>18 ARO</i>		12. REPORT DATE 8 Aug 77
13. DISTRIBUTION STATEMENT (of this Report) <i>19 12048.8-M</i> Approved for public release; distribution unlimited.		14. SECURITY CLASS. (of this report) Unclassified
15. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) NA		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE NA
16. SUPPLEMENTARY NOTES The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.		
17. KEY WORDS (Continue on reverse side if necessary and identify by block number) nonlinear differential equations, elliptic equations, diffusion equations, Green's function, coincidence theorems, fixed point theorems, existence theory, periodic solutions, nodal problem.		
18. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report summarizes the research findings of a three year period of G. B. Gustafson and the author in the areas of ordinary and partial differential equations (linear and nonlinear theory). In particular the results obtained are concerned with properties of Green's functions and matrices defined by linear differential operators and their implication to the study of nonlinear problems, with existence theory for periodic solutions of systems of nonlinear ordinary and elliptic partial differential equations, with nonlinear boundary value problems of Dirichlet and Neumann type for elliptic partial differential equations, with properties of nonlinear		

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Block 20. diffusion equations such as the existence of maximal and minimal solutions of initial- and initial boundary value problems and connectedness properties of solution sets, and finally with abstract coincidence and fixed point theorems which lend themselves to the study of nonlinear problems for differential equations. *↖*

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During the period covered by this report 22 research papers (the items listed in the bibliography of this report) have been written by G. B. Gustafson and this author (some of them jointly with others who were not supported by the three grants). Our research has mainly been concerned with certain qualitative aspects of the theory of nonlinear ordinary and partial differential equations (of elliptic or parabolic type) and with abstract fixed point and coincidence theorems which lend themselves to study the mentioned aspects of nonlinear differential equations. The research results obtained are discussed in the following categories:

- (a) Fixed point and coincidence theorems for nonlinear operator equations.
- (b) Ranges of nonlinear operators and applications to boundary value problems for equations with noninvertible linear part.
- (c) Boundary value problems for and periodic solutions of nonlinear elliptic partial differential equations.
- (d) Results for nonlinear parabolic partial differential equations.
- (e) The study of coupled systems of nonlinear ordinary differential equations.
- (f) Conjugacy results for linear equations with positive matrices.
- (g) The study of Green's functions for linear ordinary differential equations with applications to nonlinear problems.
- (h) The study of conjugacy functions for higher order equations.

We now proceed to describe the results obtained.

(a) Fixed point theorems have been obtained for nonzero solutions of coupled operator equations which are of the type of Krasnosel'skii's compression and expansion theorems. ([10]). These results are used together with a classical Galerkin method to study numerical computation of solutions of the nodal problem for sub and super linear equations ([4]).

In studying equations of the form $Lx = Nx$ where L is an invertible linear operator many classical fixed point theorems are available, in [15], [17] we consider the case where L is not necessarily invertible but a Fredholm map of index zero and obtain extensions of some of the classical fixed point results, such as Rothe's, Altman's and Krasnosel'skii's. These results are applied to several situations where periodic solutions of nonlinear equations are of interest.

(b) In [1] and [16] we consider nonlinear ordinary differential equations with noninvertible linear part and nonlinearities which are not linearizable (such as switches, etc.) and study bifurcation phenomena and surjectivity results for such operators.

(c) The papers [18], [20], [21] are concerned with nonlinear elliptic partial differential equations and boundary value problems for such problems. In [21] we obtain sufficient conditions in order that a nonlinear Schrödinger equation have periodic solutions. In [20] we show how to obtain existence results for boundary value problems for partial differential equations by ordinary differential equations techniques, in particular we show that the "method of lines" works in obtaining existence results. In [20] we survey the literature on Nagumo type upper and lower solution results for elliptic partial differential equations and show how these results may be used to study nonlinear eigenvalue problems and maximal and minimal solutions of such problems.

(d) The problem of finding invariant sets for nonlinear diffusion equations is an important one. In [5], [6], we give sufficient conditions in order that certain convex sets be invariant for solutions of initial boundary value problems. We also study connectedness properties of the solution set and obtain maximal and minimal solutions of such problems.

(e) In [14] we present a theory for the existence of periodic solutions of coupled systems of second order nonlinear ordinary differential equations which implies many results obtained during the past two decades. [22] presents a theory of boundary value problems for infinite systems of ordinary differential equations which are defined in some convex set of a Banach space and exhibit the delicate interplay between the geometry of the underlying Banach space and the nonlinearities involved.

(f) Coupled systems of the form $x'' + A(t)x = 0$, where $A(t)$ is a positive matrix are important in many applications of differential equations. In [20] we present a conjugacy theory for such equations which is based on the famous spectral theory for positive operators of Krein and Rutman. Applications to bifurcation problems for nonlinear problems are obtained as a byproduct.

(g) The research finding of papers [2], [3], [7], [8] and [11] are concerned with multipoint boundary value problems for linear and nonlinear higher order differential equations, particularly with questions of existence, uniqueness, integral representation of solutions, continuous dependence of integral kernels (Green's functions) and continuous dependence on given data.

(h) The papers [12] and [13] are concerned with higher order equations (linear ordinary). Explicit coefficient criteria for the comparison of solutions of linear (or nonlinear) equations are given. The results are modelled after the Sturm theory for second order equations and fill some gaps in the theory of Volterra integral equations. Also an example is given of a fourth order linear equation for which certain conjugate point functions are discontinuous. This fact is intimately connected with some lower order bifurcation phenomena associated with fourth order equations.

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Unclassified

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