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TRANSIENT RESPONSE OF SHELLS WITH INTERNALLY ATTACHED STRUCTURE--ETC(U)
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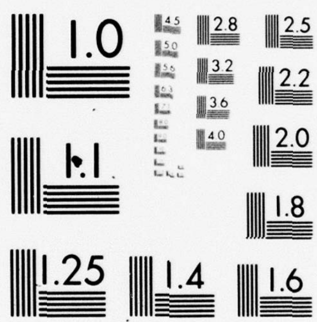

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ABSTRACT

Equations of motion are derived for the transient response, to shock wave, of a submerged shell with internal structures. A substructuring procedure, which does not require calculation of a system stiffness matrix, is employed to obtain these equations in a general manner for arbitrary internal structures approximated bv finite elements.

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I INTRODUCTION

In an earlier paper (Ref. [1]), equations of motion for the transient response, to a shock wave, of a submerged shell with internally attached structures were obtained by a substructuring procedure. Several quantities in these equations, relating to the attachments, were only broadly defined, and determined in detail only in an ad hoc manner for specific appendages. In this report, matrix expressions for these quantities are defined which permit their computation for arbitrary internal structures approximated by finite elements.

The present report addresses the same basic problem as Ref. [1], but does not consider the details of the fluid loading on the shell, which may be found in Ref. [1]. The effects of external loads applied to the internal structures, which were not considered in Ref. [1], are included here.

The substructuring procedure described herein is intended for use in transient response problems. Unlike many other substructuring schemes, which are used primarily to solve steady-state problems (see, e.g., Ref. [2]), the present method does not involve the modes and natural frequencies of the combined system (shell and appendages). In addition, a system stiffness matrix is not required, since the present technique employs the interaction forces and moments at the junctions of the shell and the attached structures.

II GENERAL KINEMATICS

Consider a shell S of arbitrary geometry (see Fig. 1). Let n, 2 and 3 denote mutually perpendicular directions, respectively normal and tangent to the middle surface of the shell. If d_n, d_2 and d_3 denote corresponding displacement components of this surface, a 3 by 1 displacement matrix $\underline{d}(s, t)$ may be defined, in which s denotes an arbitrary point on the middle surface, t is time, and underlining indicates a matrix; d_n is assumed positive inward. Corresponding to this displacement matrix, the 3 by 1 jth shell mode $\underline{\phi}_j(s)$ has components ϕ_j^n, ϕ_j^2 and ϕ_j^3, with which are associated a natural frequency ω_j and a generalized mass

$$\mu_j = \int_A m\underline{\phi}_j^T \underline{\phi}_j \, dA \tag{1}$$

in which m denotes the mass per unit of surface area A of the middle surface of the shell and the superscript T denotes a transpose. The shell modes, which satisfy the orthogonality condition

$$\int_A m \, \underline{\phi}_k^T \underline{\phi}_j \, dA = 0 \quad \text{if} \quad k \neq j \tag{2}$$

may be used as expansion functions for the shell displacements, i.e.,

$$\underline{d}(s, t) = \sum_{k=1}^{\nu_S} q_{Sk}(t) \, \underline{\phi}_k(s) \tag{3}$$

in which the $q_{Sk}(t)$ are generalized coordinates for the shell and ν_S is the number of modes of the shell considered.

Assume that an elastic structure σ, entirely within the shell S, is attached to S at points $s_I (I = 1, 2,..., N_A)$, as shown in Fig. 2. Let σ be approximated by finite elements with interface (attached) nodal points I, coinciding with s_I, and interior (unattached) nodal points $\alpha(\alpha = 1, 2,...,N_U)$, corresponding to material points of σ not in contact with S. The finite element representation of σ is assumed to include rotational degrees of freedom.

At each interface point I, certain components of displacement and/or rotation of σ must match corresponding quantities of S, depending on the type of support. Thus, at an interface point I, define the vector

$$\underline{X}^{A}_{\sigma I} = \begin{bmatrix} \bar{\underline{X}}_{\sigma I} \\ \hline \hat{\underline{X}}_{\sigma I} \end{bmatrix} \tag{4}$$

where the upper partition, an $\bar{N}_I$ by 1 matrix, contains those physical degrees of freedom of σ which must match corresponding physical degrees of freedom of S, and the lower partition, an $\hat{N}_I$ by 1 matrix, contains the remaining physical degrees of freedom of σ active at interface point I.*)

Since the interior points of σ are not connected to S, define the vector of physical degrees of freedom of σ at an interior point α as

$$\underline{X}^{U}_{\sigma\alpha} = \hat{\underline{X}}_{\sigma\alpha} \tag{5}$$

It may be observed in Eqs. (4) and (5), and in what follows, that the symbol - above a quantity indicates a quantity associated with physical degrees of freedom of σ constrained to move with S, while the symbol ^ above a quantity indicates a quantity associated with physical degrees of freedom of σ not

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*) At a simple support, e.g., the displacements of σ and S must be equal, but the rotations of σ and S are not equal. Thus, assuming two active physical degrees of freedom, the displacement of σ is located in the upper partition of Eq. (4), and the rotation of σ is located in the lower partition of Eq. (4).

constrained to move with S.

If the appropriate definitions of Eqs. (4) and (5) are applied to all nodal points of σ, the physical degrees of freedom of σ may be arranged in the form

$$\underline{X}_{\sigma} = \begin{bmatrix} \bar{\underline{X}}_{\sigma} \\ \hline \hat{\underline{X}}_{\sigma} \end{bmatrix} \tag{6}$$

in which the upper partition, an $\bar{N}$ by 1 matrix, contains the $\bar{\underline{X}}_{\sigma I}$ terms from all interface points [Eq. (4)] and the lower partition, an $\hat{N}$ by 1 matrix, contains the $\hat{\underline{X}}_{\sigma I}$ terms from all interface points [Eq. (4)] and the $\hat{\underline{X}}_{\sigma\alpha}$ terms from all interior points [Eq. (5)].

In the sequel, the elements in the upper partition of Eq. (6) will be referred to as "constrained physical degrees of freedom of σ", while the elements in the lower partition will be referred to as "unconstrained physical degrees of freedom of σ". The matrix $\underline{X}_{\sigma}$ has size N by 1, where

$$N = \bar{N} + \hat{N} \tag{7}$$

is the total number of active physical degrees of freedom of σ.

From Eq. (3), the shell displacements at an interface point I may be written as

$$\underline{d}(s_I, t) \equiv \underline{d}_I = \sum_{k=1}^{\nu_S} q_{Sk}(t)\, \underline{\phi}_k(s_I) \equiv \underline{\phi}_I \underline{q}_S \tag{8}$$

where $\underline{\phi}_I$ is a 3 by ν_S matrix with elements $\underline{\phi}_k(s_I)$ and $\underline{q}_S$ is a ν_S by 1 matrix containing the generalized coordinates of the shell. Similarly, the rotations of the shell cross section at an interface point I may be expressed as

$$\underline{\beta}_I \equiv \sum_{k=1}^{\nu_S} q_{Sk}(t)\, \underline{\phi}_k^R(s_I) \equiv \underline{\phi}_I^R\, \underline{q}_S \tag{9}$$

in which $\underline{\phi}_k^R(s_I)$ denotes the 3 by 1 matrix of rotations about the n, 2 and 3 axes in the mode $\underline{\phi}_k$ and $\underline{\phi}_I^R$ is a 3 by ν_S matrix with elements $\underline{\phi}_k^R(s_I)$.

Continuity of appropriate components of displacement and rotation of shell and substructure at an interface point I requires that

$$\bar{\underline{X}}_{\sigma I} = \underline{D}_I \begin{bmatrix} \underline{d}_I \\ \hdashline \underline{\beta}_I \end{bmatrix} \tag{10}$$

where $\underline{D}_I$, an $\bar{N}_I$ by 6 matrix, is a local rotational coordinate transformation which transforms quantities expressed in the S-coordinate system into the σ-coordinate system. Each row of $\underline{D}_I$ contains at most three non-zero terms.*) Use of Eqs. (8) and (9) permits Eq. (10) to be written in the form

$$\bar{\underline{X}}_{\sigma I} = \underline{C}_I \, \underline{q}_S \tag{11}$$

where $\underline{C}_I$ is an $\bar{N}_I$ by ν_S matrix given by

$$\underline{C}_I = \underline{D}_I \begin{bmatrix} \underline{\phi}_I \\ \hdashline \underline{\phi}_I^R \end{bmatrix} \tag{12}$$

Application of Eq. (11) to all interface points, with reference to Eq. (6) for proper ordering, permits the continuity between shell and substructure to be written as

$$\bar{\underline{X}}_{\sigma} = \underline{C} \, \underline{q}_S \tag{13}$$

in which $\underline{C}$ is an $\bar{N}$ by ν_S matrix having the form

*) The coordinate transformation array in Eq. (10) is used for convenience in presentation. Actually, only a 3 by 3 coordinate transformation array is required at each interface point, the rows of this array applying to both displacements and rotations.

$$\underline{C} = \begin{bmatrix} \underline{C}_1 \\ \vdots \\ \underline{C}_I \\ \vdots \\ \underline{C}_{N_A} \end{bmatrix} \tag{14}$$

Equation (13) expresses the constrained physical degrees of freedom of σ in terms of the free-free modes of S.*)

As suggested in Ref. [3], the response of σ may be taken as the superposition of its static response to the actual motion of its supports and the dynamic response with respect to fixed supports, i.e.**),

$$\hat{\underline{X}}_\sigma = \hat{\underline{\phi}}_\sigma \, \underline{q}_\sigma + \hat{\underline{g}} \, \bar{\underline{X}}_\sigma \tag{15}$$

In Eq. (15), $\hat{\underline{\phi}}_\sigma$ is the $\hat{N}$ by ν_σ fixed modal matrix, where ν_σ denotes the number of fixed-base modes, and $\underline{q}_\sigma$ is the ν_σ by 1 matrix of corresponding generalized coordinates. The $\hat{N}$ by $\bar{N}$ matrix $\hat{\underline{g}}$ is obtained by considering the static response of σ to an arbitrary support motion (support "settlement") when the nodal loads corresponding to the unconstrained physical degrees of freedom vanish. In such a case, Eq. (15) becomes

$$(\hat{\underline{X}}_\sigma)_{static} = \hat{\underline{g}} \, (\bar{\underline{X}}_\sigma)_{static} \tag{16}$$

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*) Equation (13) replaces Eq. (5) of Ref. [1]:

$$\underline{d}_\sigma(s_j, t) = \underline{d}(s_j, t)$$

**) Equation (15) replaces Eq. (4) of Ref. [1]:

$$\underline{d}_\sigma(s, t) = \sum_i q_{\sigma i}(t) \, \underline{\phi}_{\sigma i}(s) + \underline{g}(s, t)$$

As shown in Ref. [2], the matrix $\hat{\underline{g}}$ (sometimes termed the "constraint modes") may be obtained by considering the linear static "force-displacement" relation for σ:

$$\underline{K}(\underline{X}_\sigma)_{\text{static}} = \underline{R} \tag{17}$$

in which $\underline{K}$ is the N by N unconstrained stiffness matrix for the substructure and $\underline{R}$ is the corresponding vector of nodal forces and moments. For the determination of $\hat{\underline{g}}$, Eq. (17) may be partitioned to correspond to Eq. (6), yielding

$$\left[\begin{array}{c|c} \underline{k}_{11} & \underline{k}_{12} \\ \hline \underline{k}_{21} & \underline{k}_{22} \end{array}\right] \left[\begin{array}{c} \bar{\underline{X}}_\sigma \\ \hline \hat{\underline{X}}_\sigma \end{array}\right]_{\text{static}} = \left[\begin{array}{c} \bar{\underline{R}} \\ \hline \underline{0} \end{array}\right] \tag{18}$$

since the only nodal loads are the reactions. It then follows from the lower partition of Eq. (18) that

$$(\hat{\underline{X}}_\sigma)_{\text{static}} = -\underline{k}_{22}^{-1}\,\underline{k}_{21}\,(\bar{\underline{X}}_\sigma)_{\text{static}} \tag{19}$$

where the superscript -1 indicates an inverse. A comparison of Eqs. (19) and (16) shows that

$$\hat{\underline{g}} = -\underline{k}_{22}^{-1}\,\underline{k}_{21} \tag{20}$$

Thus, the elements of $\hat{\underline{g}}$ are the static displacements and rotations (including rigid-body motion), corresponding to the unconstrained physical degrees of freedom of σ, which result from successive unit displacements and rotations, corresponding to the constrained physical degrees of freedom of σ, at attachment points I when all other constrained physical degrees of freedom of σ are set equal to zero.

The use of Eq. (13) permits Eq. (15) to be written as

$$\hat{\underline{X}}_\sigma = \hat{\underline{\Phi}}_\sigma\,\underline{q}_\sigma + \hat{\underline{g}}\,\underline{C}\,\underline{q}_S \tag{21}$$

As suggested in Ref. [2], Eqs. (13) and (21) may be combined into the single matrix equation

$$\underline{X}_\sigma = \underline{T}^C \underline{q} \tag{22}$$

in which $\underline{X}_\sigma$ is given by Eq. (6), the transformation matrix $\underline{T}^C$ has the form

$$\underline{T}^C = \left[\begin{array}{c|c} \underline{C} & \underline{0} \\ \hline \hat{\underline{g}}\,\underline{C} & \hat{\underline{\phi}}_\sigma \end{array}\right] \tag{23}$$

and the array of generalized coordinates is given by

$$\underline{q} = \left[\begin{array}{c} \underline{q}_S \\ \hline \underline{q}_\sigma \end{array}\right] \tag{24}$$

In Eq. (22), $\underline{X}_\sigma$ is an N by 1 matrix, $\underline{q}$ is a ν by 1 matrix, with

$$\nu = \nu_S + \nu_\sigma \tag{25}$$

and $\underline{T}^C$ is an N by ν matrix. Equation (22) may be written in subscript notation as

$$X_{\sigma k} = \sum_{j=1}^{\nu} T^C_{kj}\, q_j \quad (k = 1, 2, \ldots, N) \tag{26}$$

III EQUATIONS OF MOTION OF SHELL

The kinetic and potential energies of the combination of shell S and internal structure σ may be written as

$$T = \frac{1}{2} \sum_{j=1}^{\nu_S} \mu_j \dot{q}_{Sj}^2 + \frac{1}{2} \sum_{j=1}^{N} M_j \dot{X}_{\sigma j}^2 \tag{27}$$

$$V = \frac{1}{2} \sum_{j=1}^{\nu_S} \mu_j \omega_j^2 q_{Sj}^2 + \frac{1}{2} \sum_{j=1}^{N} \sum_{k=1}^{N} K_{jk} X_{\sigma j} X_{\sigma k} \tag{28}$$

in which the $X_{\sigma j}$ are given by Eq. (26), the K_{jk} are the elements of the partitioned stiffness matrix of Eqs. (17) and (18), and the M_j are the elements of the partitioned mass matrix of σ from an assumed lumped mass formulation*). The mass matrix must be partitioned to correspond to Eq. (6), i.e.,

$$\underline{M} = \left[\begin{array}{c|c} \underline{m}_{11} & \underline{0} \\ \hline \underline{0} & \underline{m}_{22} \end{array}\right] \tag{29}$$

where the upper left partition, an $\bar{N}$ by $\bar{N}$ matrix, and the lower right partition, an $\hat{N}$ by $\hat{N}$ matrix, are diagonal.

The equations of motion of the shell may be derived from Lagrange's equations:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_{Sj}}\right) + \frac{\partial V}{\partial q_{Sj}} = Q_{Tj} + \hat{Q}_{Sj} \quad (j = 1,2,\ldots,\nu_S) \tag{30}$$

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*) If Eq. (26) is substituted into Eqs. (27) and (28), one obtains the equations which replace Eqs. (6) and (7) of Ref. [1]:

$$T(\dot{q}_{Sj},\dot{q}_{\sigma j}) = \frac{1}{2} \sum_i \mu_i \dot{q}_{Si}^2 + T_\sigma(\dot{q}_{\sigma j},\dot{q}_{Sj})$$

$$V(q_{Sj},q_{\sigma j}) = \frac{1}{2} \sum_i \mu_i \omega_i^2 q_{Si}^2 + V_\sigma(q_{\sigma j},q_{Sj})$$

in which the Q_{Tj} are the generalized forces corresponding to the total fluid pressure p_T acting on the surface of the shell (see Fig. 1) and the $\hat{Q}_{Sj}$ are the generalized forces from the action of σ on S, exclusive of the contribution derivable from the kinetic and potential energies. As shown in the Appendix, the $\hat{Q}_{Sj}$ are the elements of the matrix

$$\underline{\hat{Q}}_S = \underline{C}^T \, \underline{\hat{g}}^T \, \underline{\hat{R}} \tag{31}$$

where $\underline{\hat{R}}$, an $\hat{N}$ by 1 matrix, is the vector of known applied nodal loads acting on σ (see Fig. 2).

It follows from Eqs. (26) and (24) that

$$\frac{\partial X_{\sigma k}}{\partial q_{Sj}} = \frac{\partial \dot{X}_{\sigma k}}{\partial \dot{q}_{Sj}} = T^C_{kj} \qquad \begin{array}{l}(j = 1, 2, \ldots, \nu_S; \\ \ k = 1, 2, \ldots, N)\end{array} \tag{32}$$

which may be seen to reference terms in the upper and lower left partitions of Eq. (23). Thus, the use of Eqs. (27), (28), and (32) leads to

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_{Sj}}\right) = \mu_j \, \ddot{q}_{Sj} + \sum_{k=1}^{N} T^C_{kj} \, M_k \, \ddot{X}_{\sigma k} \tag{33}$$

$$\frac{\partial V}{\partial q_{Sj}} = \mu_j \omega_j^2 \, q_{Sj} + \sum_{k=1}^{N} T^C_{kj} \sum_{i=1}^{N} K_{ki} \, X_{\sigma i} \tag{34}$$

The substitution of Eqs. (31), (33), and (34) into Eq. (30), with reference to Eqs. (18), (23), and (29), results in

$$\mu_j \, \ddot{q}_{Sj} + \mu_j \omega_j^2 \, q_{Sj} = Q_{Tj} - P_j \qquad (j = 1, 2, \ldots, \nu_S) \tag{35}$$

where the P_j are the elements of the vector

$$\underline{P} = \underline{\bar{P}} + \underline{\hat{P}} \tag{36}$$

whose addends are defined as

$$\bar{\underline{P}} = \underline{C}^T [\underline{m}_{11} \ddot{\bar{\underline{X}}}_\sigma + \underline{k}_{11} \bar{\underline{X}}_\sigma + \underline{k}_{12} \hat{\underline{X}}_\sigma] \tag{37}$$

$$\hat{\underline{P}} = \underline{C}^T \hat{\underline{g}}^T [\underline{m}_{22} \ddot{\hat{\underline{X}}}_\sigma + \underline{k}_{21} \bar{\underline{X}}_\sigma + \underline{k}_{22} \hat{\underline{X}}_\sigma - \hat{\underline{R}}] \tag{38}$$

The bracketed term in Eq. (38) may be recognized as the physical equations of motion of σ. Hence, the contribution of Eq. (38) vanishes, and the equations of motion of S become

$$\mu_j \ddot{q}_{Sj} + \mu_j \omega_j^2 q_{Sj} = Q_{Tj} + Q_{\sigma j} \tag{39}$$

in which the generalized forces $Q_{\sigma j}$, corresponding to the forces and moments exerted by σ on S, are

$$Q_{\sigma j} = - \sum_{k=1}^{\bar{N}} C_{kj} (M_k \ddot{X}_{\sigma k} + \sum_{i=1}^{N} K_{ki} X_{\sigma i}) \tag{40}$$

which result from the dynamic reactions

$$R_{\sigma k} = -(M_k \ddot{X}_{\sigma k} + \sum_{i=1}^{N} K_{ki} X_{\sigma i}) \tag{41}$$

exerted by σ on S*).

Using Eqs. (22) to (24), the equations of motion of S may now be written in the form

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*) Equations (40) and (41) are generalizations of specific examples given by Eqs. (17), (18); (22), (23); (A1), (A2); and (A12), (A13), (A16) of Ref. [1].

$$\sum_{k=1}^{\nu_S} (\mu_k \delta_{jk} + \sum_{i=1}^{\bar{N}} C_{ij} M_i C_{ik}) \ddot{q}_{Sk} + \mu_j \omega_j^2 q_{Sj} = Q_{Tj} - \sum_{i=1}^{\bar{N}} C_{ij} F_i \quad (j = 1, 2, \ldots, \nu_S) \tag{42}$$

where δ_{jk} is the Kronecker delta and the F_i, the elastic forces and moments exerted by σ on S and vice-versa, are the elements of the $\bar{N}$ by 1 matrix

$$\underline{F} = (\underline{k}_{11} + \underline{k}_{12} \hat{\underline{g}}) \underline{C} \underline{q}_S + \underline{k}_{12} \hat{\underline{\phi}}_\sigma \underline{q}_\sigma \tag{43}$$

The effect of inertia forces appears in the double summation in Eq. (42) It may be observed that the equations of motion of the shell have stiffness coupling only between shell and substructure.

In the equations of motion of the shell, the contributions of the inertia forces and the elastic reactions of σ on S may be given a simple physical interpretation by referring to Fig. 3. Figure 3a shows an idealized finite element model of a substructure σ attached to a shell S, and Figs. 3b and 3c show the manner in which Eqs. (42) and (43) treat the connection of σ to S. The effect of inertia forces results from the rigidly attached point mass of Fig. 3b, while the effect of the elastic reactions of σ on S results from the "spring force" of Fig. 3c.

IV EQUATIONS OF MOTION OF SUBSTRUCTURE

The equations of motion of the substructure σ may be obtained from Lagrange's *equations*:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_{\sigma j}}\right) + \frac{\partial V}{\partial q_{\sigma j}} = \hat{Q}_{\sigma j} \qquad (j = 1, 2, \ldots, \nu_\sigma) \tag{44}$$

in which the $\hat{Q}_{\sigma j}$ are the generalized forces corresponding to the known applied nodal loads acting on σ. As shown in the Appendix, the $\hat{Q}_{\sigma j}$ are the elements of the matrix

$$\underline{\hat{Q}}_\sigma = \underline{\hat{\phi}}_\sigma^T \underline{\hat{R}} \tag{45}$$

It follows from Eqs. (26) and (24) that

$$\frac{\partial X_{\sigma k}}{\partial q_{\sigma j}} = \frac{\partial \dot{X}_{\sigma k}}{\partial \dot{q}_{\sigma j}} = T^C_{k,\, j+\nu_S} \qquad \begin{matrix}(j = 1, 2, \ldots, \nu_\sigma; \\ k = 1, 2, \ldots, N)\end{matrix} \tag{46}$$

which may be seen to reference terms in the upper and lower right partitions of Eq. (23). Since the upper right partition of Eq. (23) is a null matrix, the use of Eqs. (27), (28), and (46) leads to

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_{\sigma j}}\right) = \sum_{k=1}^{\hat{N}} \hat{\phi}_{\sigma kj} M_{k+\bar{N}} \ddot{X}_{\sigma k+\bar{N}} \tag{47}$$

$$\frac{\partial V}{\partial q_{\sigma j}} = \sum_{k=1}^{\hat{N}} \hat{\phi}_{\sigma kj} \sum_{i=1}^{N} K_{k+\bar{N},i} X_{\sigma i} \tag{48}$$

The substitution of Eqs. (45), (47), and (48) into Eq. (44) results in

$$\sum_{k=1}^{\hat{N}} \hat{\phi}_{\sigma kj}\left(M_{k+\bar{N}} \ddot{X}_{\sigma k+\bar{N}} + \sum_{i=1}^{N} K_{k+\bar{N},i} X_{\sigma i} - \hat{R}_k\right) = 0 \tag{49}$$

which may be written in matrix form as

$$\underline{\hat{\phi}}_\sigma^T [\underline{0} \mid \underline{m}_{22}] \begin{bmatrix} \ddot{\bar{\underline{X}}}_\sigma \\ \hline \ddot{\hat{\underline{X}}}_\sigma \end{bmatrix} + \underline{\hat{\phi}}_\sigma^T [\underline{k}_{21} \mid \underline{k}_{22}] \begin{bmatrix} \bar{\underline{X}}_\sigma \\ \hline \hat{\underline{X}}_\sigma \end{bmatrix} = \underline{\hat{\phi}}_\sigma^T \underline{\hat{R}} \tag{50}$$

If Eqs. (22) to (24) are substituted into Eq. (50), and the matrix products are carried out, there results

$$\begin{aligned} &\underline{\hat{\phi}}_\sigma^T \underline{m}_{22} \underline{\hat{\phi}}_\sigma \ddot{\underline{q}}_\sigma + \underline{\hat{\phi}}_\sigma^T \underline{k}_{22} \underline{\hat{\phi}}_\sigma \underline{q}_\sigma + \underline{\hat{\phi}}_\sigma^T (\underline{k}_{21} + \underline{k}_{22} \underline{\hat{g}}) \underline{C} \underline{q}_S \\ &+ \underline{\hat{\phi}}_\sigma^T \underline{m}_{22} \underline{\hat{g}} \underline{C} \ddot{\underline{q}}_S = \underline{\hat{\phi}}_\sigma^T \underline{\hat{R}} \end{aligned} \tag{51}$$

In view of Eq. (20), which defines the constraint modes, the third term of Eq. (51) vanishes. Moreover, as discussed in Ref. [2], the fixed-base modes of σ are the solutions of the eigenvalue problem

$$\underline{k}_{22} (\underline{\hat{\phi}}_\sigma)_j = \omega_{\sigma j}^2 \underline{m}_{22} (\underline{\hat{\phi}}_\sigma)_j \tag{52}$$

where $\omega_{\sigma j}$ is the natural frequency of the jth mode, and hence are orthogonal. Thus, Eq. (51) may be simplified to yield the equations of motion of σ :

$$\underline{\mu}_\sigma \ddot{\underline{q}}_\sigma + \underline{\kappa}_\sigma \underline{q}_\sigma = \underline{\hat{\phi}}_\sigma^T \underline{\hat{R}} - \underline{\mu}_\sigma \underline{G} \underline{C} \ddot{\underline{q}}_S \tag{53}$$

where $\underline{\mu}_\sigma$ and $\underline{\kappa}_\sigma$ are diagonal matrices whose main diagonal terms are, respectively, the generalized masses and generalized stiffnesses of σ, and

$$\underline{G} = \underline{\mu}_\sigma^{-1} \underline{\hat{\phi}}_\sigma^T \underline{m}_{22} \underline{\hat{g}} \tag{54}$$

is a ν_σ by $\bar{N}$ matrix whose columns are the expansion coefficients of a modal series representation of the constraint modes. In subscript notation, Eq. (53) becomes*)

$$\ddot{q}_{\sigma j} + \omega_{\sigma j}^2 q_{\sigma j} = \sum_{k=1}^{\bar{N}} \left[\frac{1}{\mu_{\sigma j}} \hat{\phi}_{\sigma kj} \hat{R}_k - G_{jk} \sum_{i=1}^{\nu_S} C_{ki} \ddot{q}_{Si}\right] \quad (j = 1, 2, \ldots, \nu_\sigma) \tag{55}$$

It may be observed that the equations of motion of the substructure σ have inertia coupling only between shell and substructure.

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*) Equation (55) may be recognized as a generalization of Eq. (A8) of Ref. [1].

V CONCLUDING REMARKS

The equations of motion for the transient response of a shell S with internal structure σ are given by Eqs. (42), (43), (53), and (54). It is important to note that no total system (S + σ) modal stiffness matrix is required in the present formulation, as is required when eigenvalues and eigenvectors for the entire system are sought (see, e.g., Ref. [2]). It should also be noted that, in view of the stiffness coupling between S and σ in Eqs. (42) and (43), the modal mass matrix to be inverted in the solution of a transient response problem has size ν_S by ν_S, determined solely by the number of modes of the shell employed in the solution. Formulations with inertia coupling between shell and substructure require the size of the modal mass matrix to be inverted to increase as the total number of modes ν increases.

The physical stiffness and mass matrices of σ, $\underline{K}$ and $\underline{M}$ of Eqs. (18) and (29), respectively, may be obtained by modifying a finite element computer code such as SAPIV (Ref. [4]). The constraint modes may be determined by means of Eq. (20), or by utilizing a solution option available with SAPIV. Hence, it is possible to evaluate all of the matrices required in Eqs. (42) and (43) by using only those rows of $\underline{K}$ and only those main diagonal terms of $\underline{M}$ which correspond to the constrained physical degrees of freedom of σ. Once the expansion coefficients of the constraint modes [Eq. (54)] and the modal coefficients for the interaction forces and moments [Eq. (43)] have been evaluated, the constraint modes and the matrices $\underline{K}$ and $\underline{M}$ may be eliminated from the final form of the equations of motion of S and σ. This elimination and the absence of a system modal stiffness matrix will reduce the computer core (central memory) required to solve a given problem.

The present formulation will also reduce the amount of computer time (central processor time) required to set up the matrices in the equations of motion, as compared with that required by a method employing a system modal stiffness matrix. By way of example, if the formulation of Ref. [2] is used for a transient response problem, the matrices

$$\underline{K}^C = \underline{T}^{CT} \underline{K}\, \underline{T}^C \tag{56}$$

$$\underline{M}^C = \underline{T}^{CT} \underline{M}\, \underline{T}^C \tag{57}$$

must be evaluated. The computer time required to evaluate Eq. (56) may be excessive, especially if the problem under consideration requires many modes ν and/or many physical degrees of freedom of σ, and if $\underline{K}$ is not well banded. It should be pointed out that the rearrangement of physical degrees of freedom of σ to produce the partitioning of Eq. (18) usually results in a poorly banded matrix. For a lumped mass formulation, the computer time required to evaluate Eq. (57) is not significant compared with that needed to evaluate Eq. (56). Thus, a reduction in the amount of computer time required to set up the equations of motion is obtained by the elimination of a system modal stiffness matrix. A reduction in solution time is also quite probable.

The free-free modes of the shell S may be determined by means of a computer code such as BOSOR4 (Ref. [5]), while SAPIV may, of course, be used to obtain the fixed-base modes of the substructure σ. If a consistent mass formulation is desired for σ, the equations of Sections III and IV may be modified in a straightforward manner. However, in such a case, SAPIV, which is based on a lumped mass formulation, must be replaced by a computer code employing a consistent mass formulation. Although the equations derived in Sections III and IV deal with one piece of internal structure, the present method is easily extended to include any number of internal appendages.

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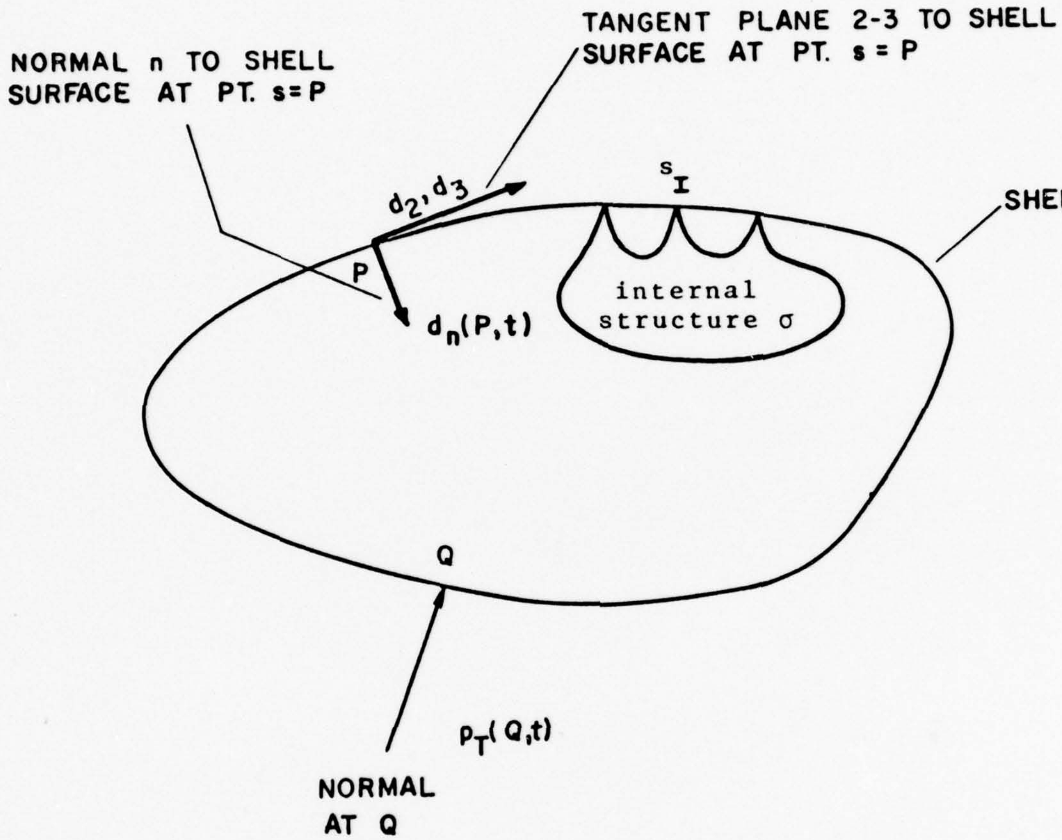


TANGENT PLANE 2-3 TO SHELL SURFACE AT PT. s = P
NORMAL n TO SHELL SURFACE AT PT. s=P
d_2, d_3
s_I
SHELL S
P
$d_n(P,t)$
internal structure σ
Q
$p_T(Q,t)$
NORMAL AT Q

FIG. I

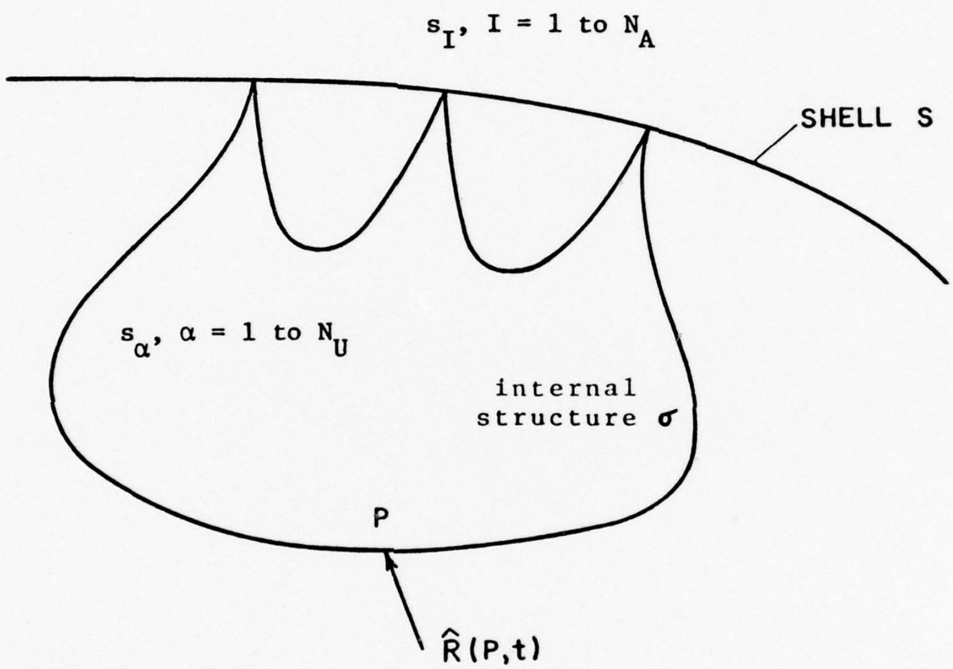
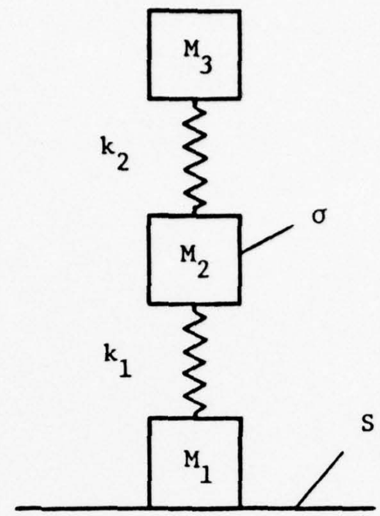
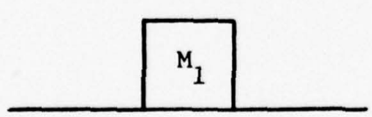

s_I, I = 1 to N_A
SHELL S
s_α, α = 1 to N_U
internal structure σ
P
$\hat{R}(P,t)$

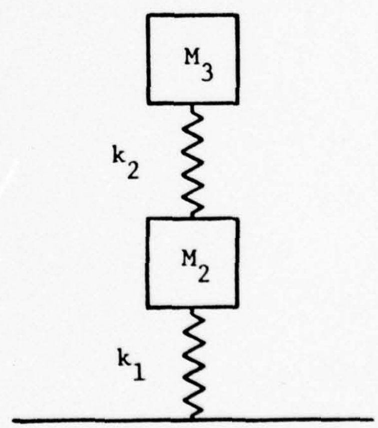
FIG. 2



(a) SUBSTRUCTURE ATTACHED TO SHELL



(b) RIGIDLY ATTACHED MASS



(c) ELASTIC REACTION

FIG. 3

APPENDIX - GENERALIZED FORCES DUE TO APPLIED LOADS ON SUBSTRUCTURE

The virtual work due to the known applied nodal loads on substructure σ may be written as

$$\delta W = (\delta \hat{\underline{X}}_\sigma)^T \hat{\underline{R}} \tag{A1}$$

in which $\delta \hat{\underline{X}}_\sigma$ is the $\hat{N}$ by 1 matrix of virtual displacements and $\hat{\underline{R}}$ is the $\hat{N}$ by 1 matrix of applied loads. In Eq. (A1), both the virtual displacements and the applied loads correspond to the unconstrained physical degrees of freedom of σ. From Eq. (21), it follows that

$$\delta \hat{\underline{X}}_\sigma = \hat{\underline{\phi}}_\sigma \, \delta \underline{q}_\sigma + \hat{\underline{g}} \, \underline{C} \delta \underline{q}_S \tag{A2}$$

so that Eq. (A1) may be expressed as

$$\delta W = \delta \underline{q}_\sigma^T (\hat{\underline{\phi}}_\sigma^T \hat{\underline{R}}) + \delta \underline{q}_S^T (\underline{C}^T \hat{\underline{g}}^T \hat{\underline{R}}) \tag{A3}$$

The virtual work of Eq. (A1) may be written in terms of generalized forces as

$$\delta W = \delta \underline{q}^T \hat{\underline{Q}} \tag{A4}$$

where $\underline{q}$ is given by Eq. (24) and

$$\hat{\underline{Q}} = \begin{bmatrix} \hat{\underline{Q}}_S \\ \hline \hat{\underline{Q}}_\sigma \end{bmatrix} \tag{A5}$$

is a ν by 1 matrix partitioned to correspond to Eq. (24). The use of Eqs. (24) and (A5) in Eq. (A4) results in

$$\delta W = \delta \underline{q}_S^T \hat{\underline{Q}}_S + \delta \underline{q}_\sigma^T \hat{\underline{Q}}_\sigma \tag{A6}$$

Since the variations of the generalized coordinates are arbitrary, a comparison of Eqs. (A3) and (A6) yields

$$\hat{\underline{Q}}_S = \underline{C}^T \, \hat{\underline{g}}^T \, \hat{\underline{R}} \tag{A7}$$

$$\hat{\underline{Q}}_\sigma = \hat{\underline{\phi}}_\sigma^T \, \hat{\underline{R}} \tag{A8}$$

The components of generalized force associated with the generalized coordinates of the shell S are given by Eq. (A7), and those associated with the generalized coordinates of the substructure σ are given by Eq. (A8).

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Equations of motion are derived for the transient response, to a shock wave, of a submerged shell with internal structures. A substructuring procedure, which does not require calculation of a system stiffness matrix, is employed to obtain these equations in a general manner for arbitrary internal structures approximated by finite elements.

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