Project Report

ETS-21

J. M. Sorvari

Real-Time, Intermediate-Field Astrometry

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FOR THE COMMANDER

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REAL-TIME, INTERMEDIATE-FIELD ASTROMETRY

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Group 94

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ABSTRACT

An experiment to test the accuracy of a very simple calibration scheme is described. It is concluded that a mean-offset, three-star calibration working from a catalog of 1600 reference stars can provide positional accuracy of ten seconds of arc with the ETS equipment.

CONTENTS

I. INTRODUCTION

Once a celestial object has been detected by an optical system, there remain just two quantities to be measured: brightness (as a function of wavelength and time) and position (as a function of time). Each measurement requires the establishment of a reproducible measuring system and a set of reference standards. This report deals with some aspects of the position measurement -- astrometry.

For astrometry, the measuring system can be the usual astronomical coordinate system measured with encoders on the telescope mount. Mechanical flexure and polar misalignment can be modeled and removed *via* observations of the reference standards. The irreproducible mechanical errors (hysteresis) must be made negligible by proper design of the telescope and mount. The set of reference standards is provided by a fundamental catalog of star positions such as the FK4 (ref.1). Although not a fundamental catalog, the SAO catalog (ref.2) is probably satisfactory for use as an astrometric reference in the GEODSS program (see ref.3).

There are essentially two approaches to calibration. One is to observe reference standards over the entire sky and construct a "model" to produce true positions as a function of observed positions. This global calibration procedure is unlikely to be capable of the degree of accuracy desired, primarily due to its great sensitivity to hysteresis effects. The second approach is to arrange for each field of interest to contain several reference standards and to make essentially differential corrections to measured positions. This small field calibration has been exhaustively studied (see, for example, ref.4) and is capable of a high degree of accuracy.

The preceding considerations would seem to indicate that small field astrometry was the desired approach. There are, however, some possible disadvantages to small field astrometry in the context of the ETS mission. The requirement for "several" stars in each small field of interest implies a need for a quite large catalog of standards. For each unknown object, many standard stars must be measured and extensive calculations performed on the data. In a real-time system, this can mean an undesireably large proportion of system overhead and operating time devoted to calibration.

The accuracy achieved by the measurement and calibration system is determined primarily by two sources of error: the accuracy of the astrometric calculations (the modeling error) and the accuracy with which data can be taken (the instrumental error). For the current ETS configuration, typical small field astrometry can make the modeling error much smaller than the instrumental error. In light of the problems with small field astrometry, it makes sense to consider less sophisticated calibration schemes. This Report describes the results of an experiment which addresses the question: How simple a calibration scheme may be used and still produce positional accuracy sufficient for the GEODSS program?

II. INTERMEDIATE FIELD CALIBRATION

Falling short of failing to calibrate at all, the simplest scheme is to assume

$$\alpha_0 = \alpha' + \Delta\alpha$$

$$\delta_0 = \delta' + \Delta\delta$$

where the naught subscripts refer to the true positions, and the primes refer to the observed positions. Values for $\Delta\alpha$ and $\Delta\delta$ are obtained from observations of one or more standard stars. It should be noted that even a simple displacement of the coordinate pole requires a more complicated transformation than this. The model is thus extremely crude, and would be expected to fail badly for distances in excess of 0.1 radian ($\sim$350') or near the pole.

The positions produced by this calibration technique would be expected to show effects of both the modeling error and the instrumental error. The instrumental error should be the same for all objects while the modeling error to first approximation should increase linearly with the separation between unknown object and calibration star. A plot of the error in position of the unknown (d) *vs.* separation (r) should show d increasing linearly with r for large r and "bottoming off" to some minimum value for small r. The minimum error, E, should be given by

$$E = e\sqrt{1 + 1/n}$$

where e is the instrumental error and n is the number of reference stars used.

In order to test these expectations and to evaluate the possibility of using this simple calibration scheme for GEODSS operation, nine test stars were observed. Two different reductions were used on the data: the simple averaging described above and the same averaging preceded by a correction for atmospheric refraction. There are several telescope independent corrections to a star's position which may be made entirely independently of the telescope model. Most of these, however, are either very small or have very small first derivatives and thus are just as well lumped into the model correction. Atmospheric refraction, however, has a significant first derivative and may be easily corrected, as it is a function only of the telescope elevation. The data are presented in Section III.

It will be useful to have a relationship between the average distance, r, from test star to reference star and the star density, σ. Consider stars distributed at random on the plane with density σ. A circle with radius, ρ, given by

$$\rho = 1/\sqrt{\pi\sigma}$$

will contain, on the average, one star. The mean distance from the center of a circle to all points in the circle is 2/3 of the radius, so the relation between mean distance between test object and nearest one star, r_1, is given by

$$r_1, = 2/3\sqrt{\pi\sigma} = .376/\sqrt{\sigma} .$$

If we want three reference stars, we must obviously consider a circle with three times the area. Thus, the mean distance between test object and nearest three stars, r_3, is given by

$$r_3 = \sqrt{3} r_1 = .651/\sqrt{\sigma}$$

The distribution of stars on the sky is not completely random, especially for the brighter stars, and the compilation of a catalog of reference stars will include some sort of attempt to make the distribution more uniform. It might, therefore, be argued that the assumption of randomness above is not well satisfied and that the relationships shown are inaccurate. To test the effect of order in the star distribution on the calculations, the nearest neighbor calculation has been done for the case of background stars arranged at the vertices of a square grid. The result is

$$r_1 = \{\sqrt{2} + \ln(1 + \sqrt{2})\}/6\sqrt{\sigma} = .383/\sqrt{\sigma}$$

It can be seen that even this extreme case of non-random distribution affects the numbers only minimally.

III. THE OBSERVATIONS

The nine stars used as test objects for this experiment are listed in Table 1. The positional data were obtained from reference (5) updated to 1978.0. The set of reference stars was the S-20 photometric catalog compiled at ETS (ref. 6). The positional data in this catalog are also updated data from reference (5), so both test and reference stars are in the same coordinate system.

Observations were made as follows: The telescope was driven to a test star, centered, and the time and position recorded. The automatic extinction package (ref. 7) was then used to drive the telescope to the three nearest stars read from the photometric catalog. These stars were centered and time and position recorded. Mean values for $\Delta\alpha$ and $\Delta\delta$ were calculated from the reference stars and applied to the observation of the test star. The discrepancy, d, between the corrected position of the test star and its catalog value was then calculated. All nine stars were observed on each of two nights. On one night this was done normally and on the other the mean separation was increased by using only odd numbered stars from the photometric catalog. A few stars were also observed on each of several additional nights. There is no indication that any particular night was significantly better or worse than the others. The calculation was carried out using both the mean of the three nearest stars and also using only the single nearest star.

TABLE I

TEST STARS FOR INTERMEDIATE FIELD CALIBRATION

	RA	DEC	GC
1	$15^h37^m02\overset{s}{.}0$	$40^\circ25'27''$	21032
2	17 37 04.6	60 46 04	23944
3	17 42 06.7	-21 40 29	24030
4	18 08 47.8	03 06 57	24764
5	19 06 35.6	32 27 58	26340
6	19 48 15.5	70 12 44	27471
7	21 04 21.2	05 52 12	29451
8	22 11 01.0	56 43 45	31070
9	23 42 52.8	29 14 21	32954

As mentioned in Section II, it makes sense to correct the raw data for atmospheric refraction before proceeding with the data reduction. This was done for all nine stars on the night when all catalog stars were used and for Star 7 on each of three different nights. The correction was made in both the three star and one star versions of the reduction. The discrepancy, d, in the test object position was then compared to the corresponding value calculated without first making the refraction correction. The mean value of the ratio of d (with refraction correction) to d (without refraction correction) was 1.24. This is a rather surprising result, but one which has been found by other workers as well (L. G. Taff, private communication). Only data reduced without the refraction correction is referred to in the remainder of this Report.

Table II lists the data for the observations. The primed star numbers refer to the observations made on the "odd only" night. One observation of Star 4 could not be reduced in the one star version because the raw data was inadvertently destroyed after the three star reduction had been carried out. Also listed are the azimuth and elevation at which each test star was observed. Figure 1 shows a plot of d for the three star reduction as a function of azimuth and elevation. The only corelation apparent is a mild tendency toward larger d in the

TABLE II

POSITION DISCREPANCIES AND MEAN DISTANCES

TO REFERENCE STARS FOR TEST STARS

			3 Star		1 Star	
	a	e	d	r	d	r
1	299^{o}	35^{o}	47"	448'	13"	209'
1	294	63	24	448	5	209
2	339	48	24	386	24	323
2	340	49	14	386	9	323
3	222	22	13	495	13	310
4	178	59	9	183	-	-
4	236	44	6	183	8	76
5	273	70	20	414	6	133
5	274	67	21	414	22	133
6	352	53	58	410	19	89
6	352	53	52	410	5	89
7	179	62	10	192	17	138
7	174	62	10	192	11	138
7	108	34	8	192	18	138
8	023	64	16	483	19	328
8	041	39	13	483	19	328
9	086	55	13	471	15	276
1'	296	43	21	843	34	506
2'	343	51	43	801	106	495

TABLE II (Continued)

	a	e	3 Star d	3 Star r	1 Star d	1 Star r
3'	212°	28°	23"	535'	14"	310'
4'	222	52	3	185	14	76
5'	265	81	29	817	21	615
6'	359	54	29	591	43	511
7'	147	58	35	465	11	138
8'	034	58	13	629	16	328
9'	080	44	43	924	69	815

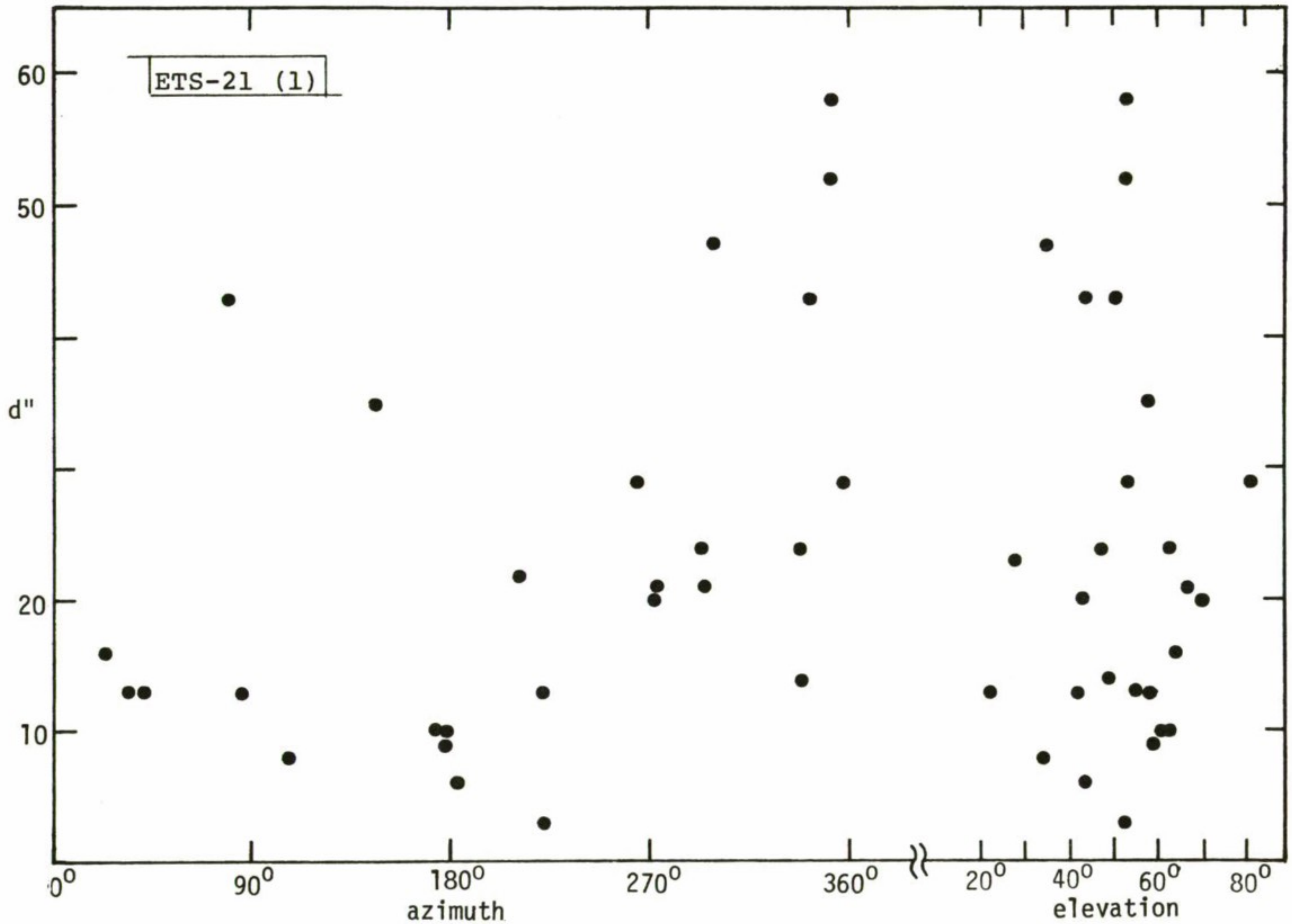

Fig. 1. The position discrepancy (d, in seconds of arc) of test stars as a function of azimuth and elevation.

north. Figure 2 shows a plot of d as a function of r. this graph is the basis for the comments in Section IV. Data for the 3 star reduction and for the 1 star reduction are plotted together, but with different symbols.

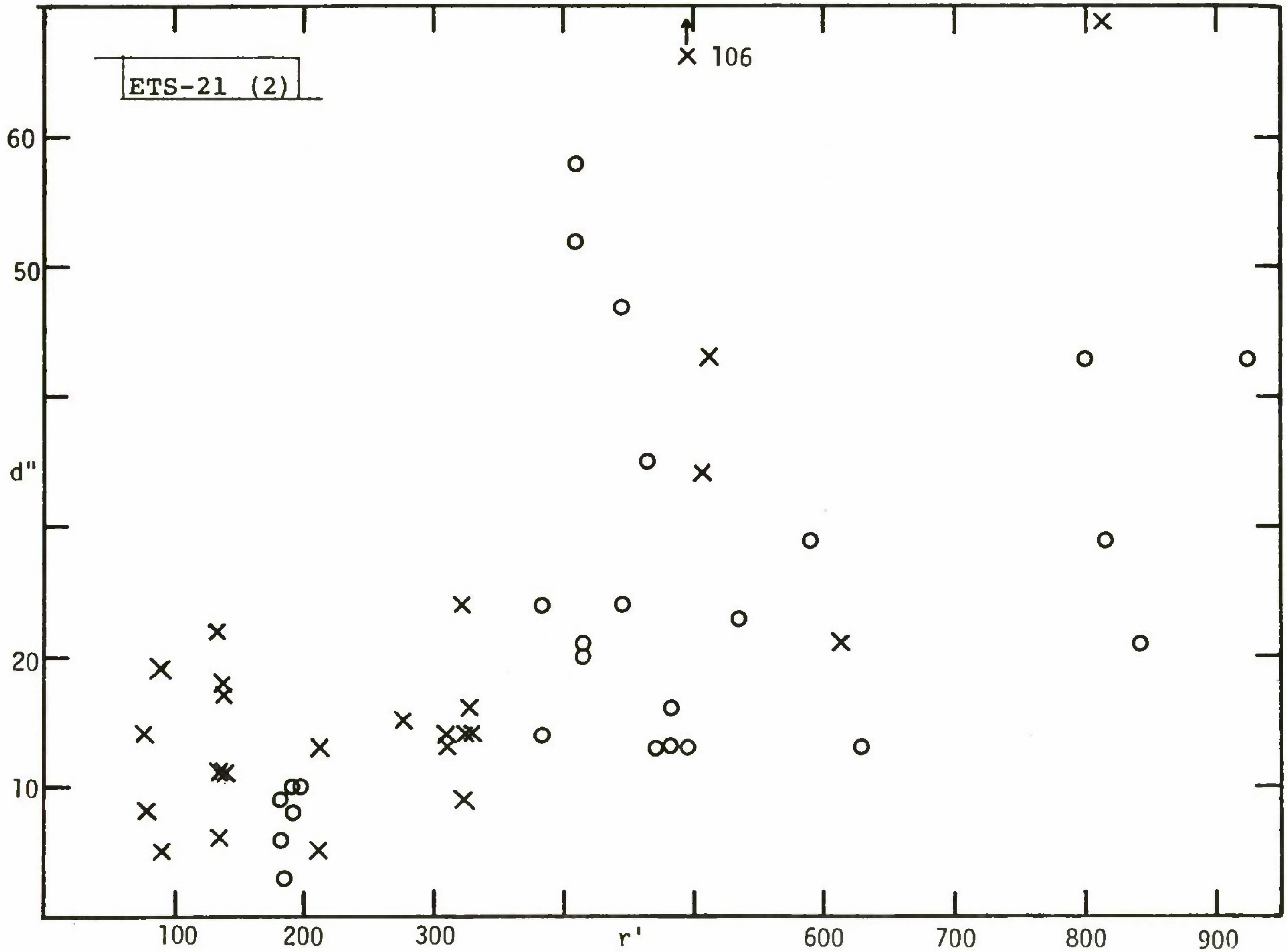

Fig. 2. The position discrepancy (d, in seconds of arc) of test stars as a function of mean separation between test and reference stars (r, in minutes of arc). Crosses refer to one star reductions and circles to three star reductions.

IV. CONCLUSIONS

The data obtained here are insufficient for a thorough analysis of the errors involved in the mean offset calibration. Nonetheless, several conclusions may be reached upon examining Figure 2. In this analysis, the "target" accuracy is taken to be ten seconds of arc.

Our expectations regarding the discrepancy as a function of mean separation seem to be borne out. The value of d appears to increase linearly with r for large r, and to reach a minimum for small r. The data indicate an instrumental error of approximately eight seconds of arc, consistent with the known properties of the equipment and with other studies (L. G. Taff, private communication).

The calibration using a single star does not appear capable of reaching the target accuracy. Use of three stars does appear capable of reaching target accuracy if the mean separation is less than about 200'. This translates into a required star density of 0.038 star per square degree, or about 1600 stars distributed over the entire sky.

One way of evaluating the proposed calibration scheme is to ask if it can provide sufficient accuracy. For the current parameters the answer is clearly yes. If the target accuracy is made smaller without an improvement in instrumental accuracy, the answer is clearly no. It is not presently possible to decide what the result would be if both target accuracy and instrumental accuracy were improved.

Another way of evaluating the calibration scheme is to ask if it can provide the best possible accuracy. Because the instrumental accuracy and target accuracy are approximately the same, the answer is still yes. Again, it is unclear what the answer would be if instrumental accuracy were improved.

The ultimate test of these conclusions is obviously the installation in RTS of a mean-offset, three-star calibration working from a reference catalog of 1600 stars. Observations with this system would also provide data which could resolve the uncertainties in the scaling of results with improvement in the instrumental accuracy.

REFERENCES

(1) W. Fricke and A. Kopff, Vierter Fundamental Katalog (Publ. Astron. Rechen-Institut,[Heidelberg, W. Germany, Number 10, 1963]).

(2) Smithsonian Astrophysical Observatory, Star Catalog (Publ. Smithsonian Inst., Nr 4562, Washington, D.C., 1966).

(3) L. G. Taff, "The Handling and Uses of the SAO Catalog," Project Report ETS-6, Lincoln Laboratory, M.I.T. (5 November 1976), DDC AD-A035871/3.

(4) L. G. Taff, "Astrometry in Small Fields," Technical Note 1977-2, Lincoln Laboratory, M.I.T. (14 June 1977), DDC AD-A043568.

(5) A. Becvar, Atlas of the Heavens-II Catalog 1950.0, (Sky Publishing Corp., Cambridge, Mass., 1964).

(6) J. M. Sorvari, "Magnitudes of Stars on the S-20 System," Project Report ETS-19, Lincoln Laboratory, M.I.T. (14 September 1977).

(7) J. M. Sorvari and C. E. Beane, "Automatic Real-Time Extinction Measurement," Project Report ETS-17, Lincoln Laboratory, M.I.T. (12 September 1977).

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