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WHY WAR: A MATHEMATICAL SYSTEMS APPROACH.(U)

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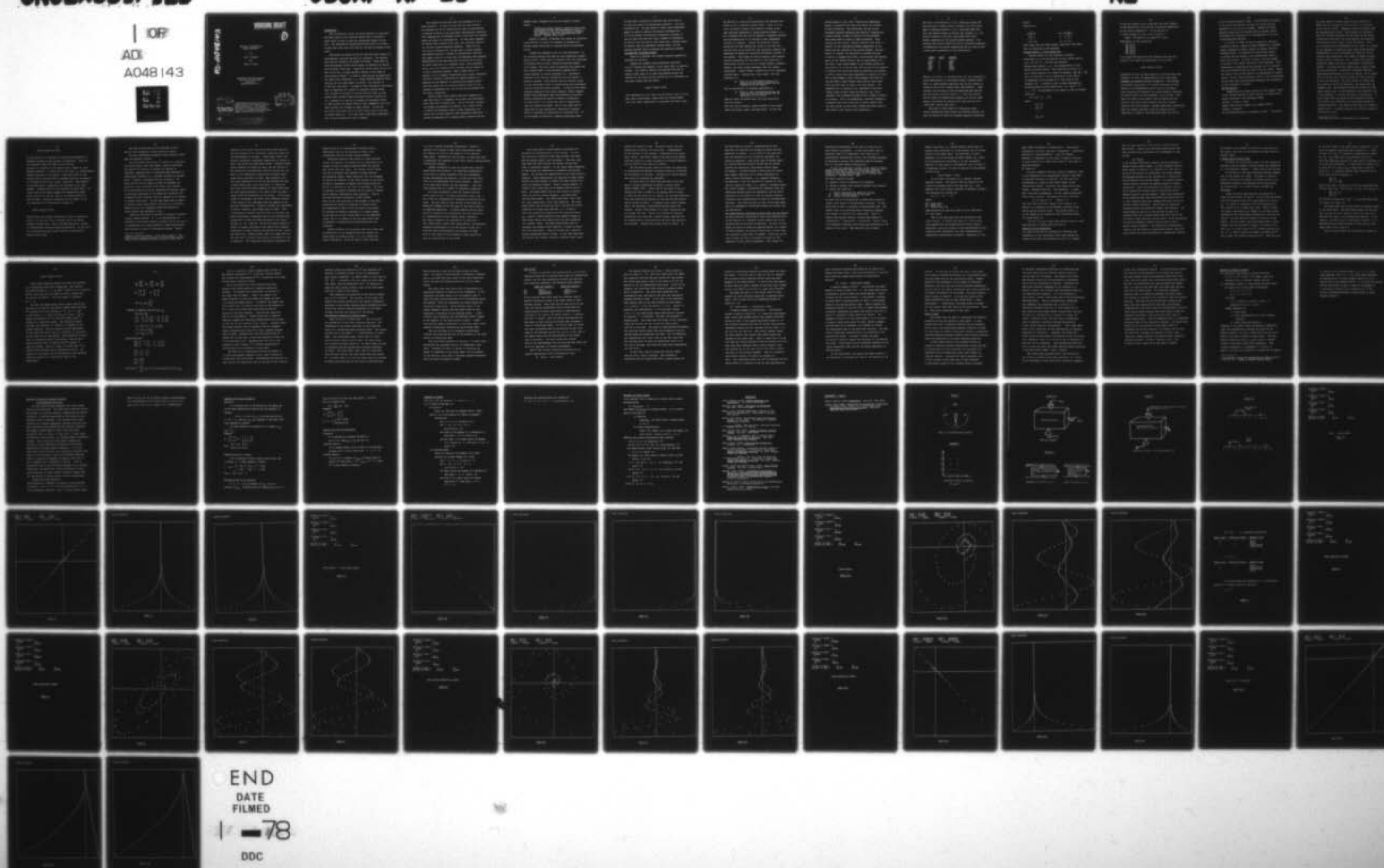

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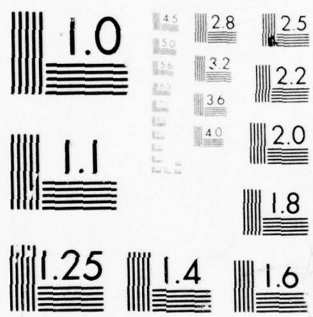

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WORKING DRAFT

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Why War: A Mathematical
Systems Approach*

by

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and

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INTRODUCTION

This introduction serves two major purposes--it describes the broad thrust of our research efforts and it provides a brief sketch of what we want our explanatory theory to look like. The introduction should provide some clues as to where we hope this first step will take us, and how we propose to get there.

The search for causes of war has been the major concern of international relations scholars for centuries. One suspects that wars are caused by all manner of things. There might be truly belligerent nations which engage in conflicts to satiate some primordal blood need. Imperialist design might account for what we can war, as might manifest destiny or the glory of some heavenly kingdom. It might be that nations systematically lie about their intentions, i.e., feign friendly behavior while awaiting for the right moment to strike. Our interest is in why wars come about. It might be that ideological differences make it impossible for nations to interpret actions which were intended as friendly in any but hostile terms, or that nations make mistakes about what other nations actually do, i.e., nations go to war due to factual or perceptual errors, or that nations are simply wrong in their judgements as to how other nations will respond to their actions. We can see that these last three sorts of causes might lead to war when no nation wishes it. It is this sort of political phenomenon that we find puzzling and wish to address.

Our concern is with wars that are accidental or in a sense unwanted. In order to deal with our three reasons for unwanted war we are faced with the task of constructing a language in which we can talk about international relations. This language and its concepts serves as our basis for talking about the causes of accidental war. In this basic language we specify what a nation looks like, what national behavior is, and how a nation generates behavior. Before we can talk about accidental war, we must have a fairly detailed and complete picture or theory of international relations. This paper serves as a first step in developing that language. Specifically in the work done here we develop the concept of state (used here in a technical sense, to be explained below, not to be confused with a nation) and our notion of a state transition function. What we have done is to develop a set of computer simulations that concern themselves with how a nation's orientation or outlook changes as a function of the behavior it receives from another nation. Thus this paper will not directly address itself to the concept of accidental war, but it will lay the ground work for such a discussion.

While we don't know exactly what this language will look like when it is complete, we do have some preferences about what form it should take. One of the major tennants of standard philosophies of science, e.g., Tarski (1941) or Hempel and Oppenheim (1948) is that conclusions of a theory are strictly deducible from antecedent conditions. A current illustration of a working social scientist who has adopted such a framework for his own efforts is Allen Newell.

> Production systems (Newell's theories) like other programming systems and mathematical theories, are complete in the sense of producing theoretical consequences that are deducable from the theory. (Newell 1973a: 517)

Similar to Newell, in meeting this aspect of scientific respectability we choose the language of mathematical systems theory with which to theorize about the phenomena of war.

A deductive language is not our only preference. In his articles dealing with what he calls production systems, Newell (1973a, 1973b) makes an argument about the advantages of building what he calls "complete processing models". In his paper, "You Can't Play 20 Questions with Nature and Win", Newell makes a very persuasive argument that a binary approach to social knowledge (i.e., hypothesis testing) is not only not necessarily the best manner in which to proceed. Given current rates of science- doing or even conceivable rates of science-doing such proceedures will not yield full blown theories. In place of the rather mindless plodding of the binary approach, Newell suggests models which completely model the phenomena of interest. Without analyzing the merits of Newell's allegations we assert that it is toward this sort of completeness that our efforts are aimed. This is the reason why we spend alot of time developing a theoretical language prior to beginning our discussion of accidental war. If we can manage to construct a complete processing model we will have a theoretical structure that will allow us to treat any aspect of international behavior -- not just accidental war. We have chosen to take a more round-about manner in which to address the notion of unwanted war. If we can succeed in building an acceptable language, we will be left with much more than a handful of hypotheses. We realize that the path we have chosen has a greater chance of failure than the hypothesis testing route, but the possible payoffs seem to outweigh the potential problems.

Introduction to Systems Theory

Allow us to motivate the systems theoretic approach to mathematical modeling.

Imagine one standard Large Midwestern University (L.M.U.) athlete who happens, in his spare time, to practice science. Our athlete, Maurice, has noticed that when he showers there seems to be some relationship between the location of the shower-control handle and the temperature of the water coming from the nozzel.

Insert Figure I Here

Most atheletes at L.M.U. don't go any further than to notice this gross relationship, fiddle with the control-handle until the shower temperature is acceptable and that's that.

But Maurice is a part-time intellectual and therefore has signed up for a political science class. Since it is an L.M.U. political science class Maurice is going to get some practical experience at doing research science, i.e., he is charged with the task of choosing a dependent variable and ascertaining how it is related to some independent variable or variables. Because the political science instructor has been through the trials of the late 60's and the riots of the early 70's the instructor realizes the importance of making his scientific discipline relevant to the student. This being the case he agrees, after several personal appearances by the athlete at the instructor's office hours (11:00 to 11:15 M-F, except Fridays, Tuesdays, Mondays, and every second Wednesday) that the "shower temperature - control handle" problem will be an acceptable research topic. Maurice does a case study. His null hypothesis is:

> Ho: There is no relationship between the location of the control handle and the temperature of the shower.

and his alternative, or research hypothesis is:

> H_1: There is some relationship between the location of the control handle and the temperature of the shower.

(Maurice knows two tailed tests are less restrictive than one tailed.)

Next day, after practice, Maurice marches to his shower armed with pencil, paper, and high hopes. He sets the control-handle to the 'cold' location and immediately begins to empathize with those who bemoan the troubles of data collection. The instructor, who has by now developed enormous enthusiasm for Maurice's original and rigorous design, assigns to Maurice his own personal work-study slave to assist in data collection. Well, leaving out much detail, Maurice discovers that there is perfect two way association between temperature of the shower and the location of the control-handle! Hot dog -- Maurice is elated, the instructor is pleased (he goes so far as to suggest that with a little polish the ensuing paper on the "Shower Project" will be publishable) etc... But alas, a more senior member of the faculty points out that with an "n" of one we can't be too sure of our results-- so Maurice, who has by now given up athletics for science, is off to survey the entire population of showers at L.M.U. NSF is good enough to spot a hot prospect when they see one and award a grant without which Maurice could never have collected data on women's and dorm showers. The candidate for a scientific law, unearthed by the pilot "Shower Study of 1972" holds and Maurice, now a Ph.D. candidate, has tremendous prospects for a career in political science. (He is in comparative, currently working on a proposal that would allow him to sample showers from the five nations of the classic Almond and Verba study to shed light on the cultural bias hypothesis.)

But alas, a new athlete at L.M U., coming up through the same political sciencey ranks as Maurice, has been encouraged to replicate Maurice's L.M.U. study and finds that only some showers behave as Maurice has claimed, i.e., one group of showers in the men's gym always yield cold showers regardless of the control-handle location. Fortunately this second student has had an extensive background in mathematics and systems engineering and can come up with a state-space explanation of the observations:

INPUTS	STATE	OUTPUTS
cold	1	cold
warm	1	warm
hot	1	hot
cold	2	cold
warm	2	cold
hot	2	cold

Maurice, of course, is insulated from this new explanation's nasty implications, by now himself being a tenured individual at a large midwest university and our new athlete is hailed as one of the 'coming breed' math modelers. (When queried, the crew down at the gym mentioned that Maurice and the upstart are both right, i.e., back when Maurice did his study all showers were hooked up to the hot water heater, but during the energy crisis some were put on a 'cold only' line to save oil.)

The point is that results of statistical input-output explorations might result in confusing results, and that the notion of state can untangle apparent unintelligibility.

Symbolically:

I	O
1	1
1	2

$\sim(I \rightarrow 0)$

I	S	O
1	1	1
1	2	2

$(I \times S \rightarrow 0)$

While input does not imply output, input plus state does. This is formalized in the appendix.

<u>Systems Theory -- A More Formal View</u>

Systems theory is a set theoretic structure. Essentially it is the delineation of sets called objects, e.g., $\{X^1, X^2, X^3 \ldots X^\alpha\}$ and the relations between those objects, i.e., the configurations of the object sets which obtain. Very generally an object is a set x^j which consists of a number of elements, $(x_1^j, x_2^j, \ldots X_n^j)$ that are referred to as the appearances of the object. An appearance can be thought of as "the value of the variable, x^j". A system is then a formal mathematical relation defined on such object sets, i.e., $S \subseteq X^1 \times X^2 \times \ldots \times X^\alpha$. An appearance of the system is then an ordered collection

$$S^1 = (x^1, x^2, \ldots x^\alpha)$$

where:

$$x_i^1 \in X^1$$
$$x_i^2 \in X^2$$
$$\vdots$$
$$x_i^\alpha \in X^\alpha$$

In the most general case S would have the cross product of the appearances of the objects as possible appearances of the system (elements $S^1 \ldots . S^n$ of the set S)

A simple example of such a cross product is:

$S \subseteq X^1 \times X^2$ where $X^1 = \{1,2\}$ and $X^2 = \{1,2,3\}$

Appearances of this system are:

$S^1 = \{1,1\}$
$S^2 = \{1,2\}$
$S^3 = \{1,3\}$
$S^4 = \{2,1\}$
$S^5 = \{2,2\}$
$S^6 = \{2,3\}$

If we graph the objects of this system we see that all points of the graph are appearances of the system:

Insert Figure II Here

Obviously if there are many objects, or an object has many elements, or both, the appearances of the system might encompass an enormous variety of behavior. (Try writing the possible appearances of a system consisting of three objects of ten appearances each if this is not clear.) Such collections of behavior can easily become as unwieldly as reality itself if some parsimony is not at hand. Hopefully the entire cross product will not obtain and the theorist will need concern himself with only a proper subset of the cross product. Systems theory gets interesting when the theorist develops rules for moving from one appearance to another. Our theorizing about war will be in this systems theoretic style. We will define the objects of the system and specify the rules for moving from one appearance of the system to another. Behavior of these posited systems will be observed and new systems, better satisfying our intuitions as to the phenomena of war will be developed and observed.

In so that we might satisfy our desire to construct a deductive theory, capable of being imbedded in a planning scheme, amenable to control theory, we develop the basic building blocks. We construct our theoretical analog to nations and the analog to the international arena in which they perform. We build 'nations' that are capable of action in the 'world' and observe their behavior. After observation we sophisticate our entities so as to preserve more of the behavior that we find in the real world. Beyond the scope of the current effort is an application of control theory. We are simply designing a model of the international system which when satisfactorily developed will be amenable to the techniques of control.

The Big Picture

Remembering the central position of the concept "state" to our systems approach we model nations as machines which behave according to the following scheme:

Outputs = f(inputs, state)

rather than the more common, e.g., Rummel (1971):

Outputs = f(inputs)

We are modeling nations as dynamical systems. Intuitively one would expect a dynamic system to have some way of addressing the question of time and here our intuitions do not lead us astray. Dynamical systems are systems that are parameterized by time. They display two functions.* The first function describes how the state of the system evolves over time while the second defines the functional relationship between (input, state) and output. Allow us to motivate what we understand as the 'state' of a nation in our approach to theorizing about inter-nation behavior. Fundamentally, we are predicating our theory on the proposition that differences with respect to degree of imperialism, wealth, militarism, etc... coupled with specific behavioral inputs can lead to differences in behavioral outputs. To illustrate this imagine that a world is made up of one dimension, a capitalist--communist dimension. Considering the behavior of only two nations, let us assume that our nations are identical except for their position on this one dimension. The first nation is located near the capitalist end of the dimension; the second near the communist end. Now suppose both nations receive the same input, namely that in a third nation at the capitalist end of the dimension, a workers rebellion is taking place. We would expect that the nation at the capitalist position will behave very differently from the nation having the communist position. Our notion of the state of a nation is exactly this -- a nation's position in a political space is the nation's state.

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* see Figure III for an illustration of a function.

Insert Figure III Here

As was noted for our approach to satisfy the properties of dynamical systems we must specify two functions. There are two functions which form what we call the big picture or a complete, in the Newell sense, theory of international behavior. A nation will at any time, t_0, simultaneously exhibit locations in two spaces. First there will be some behavior space location which corresponds directly to behavior as recorded in one of the 'events' data sets. Here behavior is recorded much as temperature was recorded in our shower example. Secondly there is some location in state space. We recall that in the shower example this amounted to either being hooked up to the hot water or not being hooked up to the hot water. These two locations serve as inputs to our two functions as is illustrated in Figure IV.

Insert Figure IV Here

Clearly these two outputs from nation 2 serve as inputs for nation 1 in the next slice of history and so the theory iterates through time. That is the big picture of a theory of international relations modeled dynamically. It can serve as a skeleton upon which future theoretic and empirical efforts can be hung.

Now that we have laid out our approach, we will discuss some assumptions we make about our nations.

Our most fundamental assumption about nations is that they are purposive systems.

At first glance the notion of a nation as a teleological system doesn't seem that unique. When we talk about the behavior of nations in ordinary (as opposed to technical) language terms we constantly make reference to teleological concepts, e.g., national interest or national goals. Consider a statement like the following: The Arabs cut-off our oil supply in order to influence our position on the resolution of the Middle East conflict. We are attributing to the oil producing Arab nations, goals (a preferred resolution of the Middle East conflict) and interpreting their behavior (the oil embargo) as an attempt to realize those goals. On the other hand, when we start to theorize about international relations in a scientific manner we do so in a language filled with notions of social forces* and correlates of war.

There are two points to be made in relation to nations as goal seeking systems: 1) It is scientifically respectable to talk about purposive systems; and 2) Not only is it respectable, it is also fruitful to think theoretically about nations in terms of teleological systems. Social

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*Rummel's (1971:48) Status - Field theory axiom 4: "Between nation attribute distances at a particular time are social forces determining dyadic behavior at that time."

scientists still carry some of the scars that were left from the slaying of the structural--functional dragon by the Ophilosophers of science. Ernest Nagel (1956) left Merton's structural--functional formulation of society in ruins, and nobody can understand Parsons. General Systems Theory embraced the notion of telos, but the version of GST practiced in international relations strips away the heart of the formulation, leaving only an empty input--output shell with which to work. Many scholars in IR talk about adaptation, but one has the feeling that most of them really aren't quite sure about it, since once they leave the broad brush approach of verbal theorizing and start getting explicit, it's back to the old input--output black box formulation of the nation. If one looks around at psychology, one finds a very different picture of the nature of the individual than was popular in the hey-days of behaviorism. Purposive systems are respectable! As Miller, Galanter, and Pribram noted in 1960: "Once a teleological mechanism could be built out of metal and glass psychologists recognized that it was scientifically respectable to admit that they had known it all along." (Miller, et. al. 1960:43) The notion of goal seeking is central to Newell and Simon's work dealing with computer simulations of human thinking and problem solving. (Newell and Simon 1972) Norbert Weiner (1961) has shown that one does not need to ascribe vital forces to an entity to call it purposive. The traditional mechanistic conceptions of behavior that we in international relations seem so comfortable with (in our theoretical work) is not incompatible with the notions of goal seeking.

Using this notion of the nation as a goal directed system, the behavior of a nation can be interpreted as an attempt to steer or control (Deutsch 1966) the environment toward some goal state. Our nations receive inputs from the environment (the behavior of other nations) and generates outputs (other behaviors) that are intended to control the behavior of the other nations in the system. We find it reasonable to suppose that foreign policy behavior is a function of two sorts of variables. The first is internal or domestic behavior and the second is foreign policy behavior exhibited by other countries. Thus we are firm believers in Rosenau's (1967) bridges. While we are firm believers in the bridges, in the work presented here domestic influences on foreign policy are summarized by what we call the state of the nation. By treating the domestic influence as a single element in our model we are not saying that it is not important, rather that it is simpler to use the state notion than to construct a complex process specification for the influence.

Another property of our nations that will strike some as unrealistic is our assumption that all nations are talked about as unitary actors, i.e., as if they were single individuals. We do not mean to imply that this is an all together acceptable formulation. Rather it absolves us of having to deal with a much messier world. The simpler our world is the easier it is to talk and think about. Hopefully we will be able, at some later date to generalize our approach to more fully reflect organizational/coordination concerns.

Another assumption is that all nations have infinite capabilities and resources. By using this assumption we do not have to concern ourselves with the relationship between development and behavior, or with wars with their roots at the competition for scarce resources. Again it is not that these factors are not important -- just that their absence buys us a degree of conceptual simplicity.

Our final assumption is that all nations strictly prefer peace to war. No nation purposively plans a war -- war is not the "Clausewitzian" extension of politics in our world. The only kinds of wars nations in our world find themselves engaged in are the types of wars where there is no purposive aggression. A nations behavior may be interpreted as aggressive and threatening, but that perception is a misperception of the sending nation's behavior. The classic example of an unwanted war is World War I. (Cf., Holsti 1965; Zinnes 1968, among others) The generally accepted interpretation of the 1914 Crisis is that of a situation where misperceptions and paranoia ran high. Later in this paper we will attempt to fully specify the role of misperceptions in our world.

Up to this point we have primarily described (or ascribed) the characteristics of our nations. We will now lay out our notions of what these nations look like. Now our nations weren't born yesterday -- they have some idea where the other nations are headed, and how they react to influence attempts. In our system these properties of the nations are summarized in conceptual forecasting models. All nations have expectations of all other nation's behavior. These expectations are expressed in these models. Nations use these models to predict how other nations will react to influence attempts. Before a nation behaves, it thinks it has a good idea of how another nation will behave. Two simple models that we will discuss in detail later can serve as useful examples: the walk-a-mile and the force models. The walk-a-mile model 'says' that if I move toward him, he will come toward me. The force model states that the only way that I can get him to move toward me is to move away from him. i.e., I have to force him toward me by showing him how strong and mean I am. Holsti's (1962) analysis of the cognitive image of John Foster Dulles very nicely fits into our conception of a nation's model. The model which Dulles used to interpret and predict Soviet behavior is called by Holsti the "bad faith model". When the Soviets were "negative" Dulles interpreted them as strong. On the other hand when the Soviets were sending "positive" behavior Dulles interpreted the Soviets as weak. The Soviet Union, from the perspective of Dulles, was always bad -- independently of the behavior it was sending. Interpreted in our framework, Dulles' "bad faith" image of the Soviet Union meshes with our notion of a model to predict or forecast behavior, and our notion of the state of a nation.

The forecasting model serves as a basis for predicting or forecasting the behavior of another nation as a function of the behavior it receives. The goals of a nation specify what sorts of responses are desirable.

At first glance it would seem that much of our work bears a strong resemblence to Rummel's status--field theory (Rummel 1971). While some of the words we use are spelled the same as Rummel's, the meaning that we attach to them and the thrust of our inquiry are very much different. The prime differences between our work and the DON project center around two point: 1) dynamic versus static systems; and 2) the functional relationship between inputs and outputs. Status--field theory is a static system and does not change with time. While it is possible to generate predictions over time with a static system, that can only be accomplished by collecting data for each time period to be predicted. Formally the system itself is static. On the other hand our system is parameterized by time. Functions are specified for moving the system from one appearance to another. It is closed in the sense that once the functions for our system are specified, the system can generate a time series with no further input. The second difference between the DON and our efforts concerns the functional relationship between inputs and outputs. The DON strategy is based upon the belief that outputs (behavior) are a linear combination of inputs (status dimensions), i.e., outputs= f(inputs). Our approach is that the behavior of a nation is a function of inputs and state, i.e., outputs = f(inputs,state). The "shower example" illustrates our arguments about the differences between these two approaches, and our above discussion of the effects of state on national behavior illustrates out interpretation of state for international relations. Thus while we both use some of the same words, there are fundamental differences between our efforts and the DON project.

<u>The Little Picture: Evolution in State Space for Two Nations</u>

We recall from our "Big Picture" above that modeling internation behavior as a dynamical system commits one to the specification of two kinds of functions. We don't do this. Rather we choose to focus our modeling efforts on a subset of these problems, the function which takes a nation from one location in state space to another. Given this as our task we might ask that a reasonable strategy for the completion of this task be presented. With respect to physiological-psychology we can look to recent work by Allen Newell for guidance. In this work Newell took as empirical datum to be explained a well known set of psychological experimental results, the Sternberg paradigm. He developed a machine that exhibited behavior matching that of the human subjects pointing out:

> We have now developed a theory of the simple Sternberg binary classification task that has modest standing. It should be possible to apply it to the experiments discussed in this symposium that make use of similar task situations. (Newell 1973a: 506)

The Newell approach becomes

1) specify a class of behavior to be examined

2) develop a machine that actually exhibits this behavior

3) employ this model to:

   i. better understand the empirical setting
   ii. assess existing theory in the area
   iii. control the environment

We would like to be in position to follow this recipe of science with respect to international relations. As luck would have it this is currently an unattainable goal. Put crudely there is no existing body of empirical findings with respect to location in a state space. While it remains that popular rhetoric includes many charges of "expansionist", "imperialist", "militarist", "facist" there is no handbook assigning scores along such dimensions to the nations of our world. This being the case we moved another step back, i.e. develop machines which begin to satisfy their intuitions as to how nations "do" or "would" evolve in this state space. One more time, to avoid ambiguity, we are not doing the whole theory, but, rather we are matching our intuitions as to what reasonable behavior in state space is with machines that we build in a simulation setting. We work on only one of the machines so here we go:

Insert Figure V here

We will model the process as a complete, general dynamical system. This entails that all objects of our system are parameterized by the same time set. (see appendix for a more formal treatment of dynamical systems.) Very generally our systems are:

$$S \subseteq X^t \times M^t \times N^t$$

where:

X^t = state space
M^t = model set
N^t = nation label set

Some discussion of each set seems to be in order here.

X^t; State Space:

This is the same space that was discussed in the above section. We define this space as a vector space. (See appendix for the formalization of vector space.) Doing this, while not crucial to the versimilitude of this primitive work, facilitates the later incorporation of mathematical optimization techniques. Dimensions of this space might reasonably be interpreted as: expansionist--isolationist; pro-east--pro-west; belligerant -- pacifistic; chauvinist-- internationalist... sorts of contina.

Assuming "n" dimensions in our space, a nation's location in space would be an n-tuple vector from X^t, the order of which has meaning.

M^t; model set

It just so happens that for a nation to behave in this or any simulation of an international system it must have an algorithm defining its behavior. We feel that this corresponds to a nation's real world "image" of the international system. Obviously such images can be very simple: Everybody hates me. Very uncertain: I really don't know what is happening. Or very sophisticated with lots of face validity. Our models will be of the form:

$$\underline{\dot{x}} \times n = f(\underline{u}, \underline{x} \times n) \qquad \text{where } n \in N$$

This says that the vector change in the location of a nation n is a function of the previous location of that nation and the control vector. The control vector, $\underline{u}$, is the change in the location of the controlling nation.

N^t; nation label set

You can't tell one nation from another without a score card and this is exactly what this set is.

<u>Capacity of the Formulation</u>

For those of you who are interested in following the mathematics of our simulations rather than hearing the things we say about the simulations allow us to suggest that you jump directly to the section "A Closer Look at What We Did". Those of you who would rather gossip than slog through matrix manipulation are recommended to read on here.

$$\underline{\dot{x}} = f(\underline{x},\underline{u})$$

Is the standard differental equation notation employed in optimum system control theory. We will interpret this as: $\dot{x}$(the change in location) is a function of x (the old location of the nation, its trajectory, its homing tendency, the changing morality of the citizens etc...) and $\underline{u}$ (the control applied by an outside power). In standard optimum systems control, controls are applied subject to certain constraints. Often the constraints are explicit such as; do not exceed this certain level of control and can be modeled directly into the problem. Other times constraints on the controls enter into the problem only implicitly via a "cost" function which says "achieve this goal, but do it without wasting resources". Both notions of constraint translate nicely into political sciency type concerns. The former might be seen as "we can ask only so much of a sacrifice from our citizenry before they will vote us out of office", while the later can be seen as "if we can achieve our goal by spending n dollars lets be sure we don't spend 2 times n dollars". The notion of the differential equation and the capacity of constrained control seem to be nicely built for doing theorizing of a complex sort wherein the values of one variable are bound up in the values of the others, only some are controllable and those only to a limited extent.

<u>A Closer Look At What We Said</u>

For those of you who would rather not wade through an exercise in matrix algebra, the main points are summarized in "A Non-Technical Summary..." which follows this section.

At the heart of the state transition function modeled here is the concept of a model. The model is used by a nation to forecast the behavior of another nation as influenced by controls (the behavior of the nation doing the influencing). As a simple example consider the following:

Assume the state space has only one dimension, S. There are two nations, N^1 and N^2. We will take the perspective of N^1 trying to move N^2 to some point on S, call it S_g. (It should be remembered that the presentation in this section is of a single time slice. When we create our system, each nation will respond to the other nation according to the illustration presented here for only one nation.) We will allow our nations to have one of two models. The first model, the conciliatory model, states roughly that the only way N^1 can get N^2 to move to N^1's goal, S_g, is to move closer to N^2's current position on S. Or, $\dot{x} = -u$, where $\dot{x}$ equals the change in position of N^2 and u equals the change in position of N^1. What this says is that the change in the other nation's behavior, $\dot{x}$, will be in the opposite direction from my behavior, u. Since N^1 wants N^2 at S_g, N^1's behavior is determined as follows: Gd = $S_g - S_n2$; where S_n2 is the position of N^2 on S. GD is the difference between N^1's goal and N^2's position, or it is the goal difference. N^1 would like N^2 to move to S_g. In order for N^2 to get to S_g it would have to move GD units on S. It therefore follows that $\dot{x}$ would be set to GD to solve for N^1's behavior, u.

$$GD = \dot{x}$$
$$GD = -1 \cdot u$$
$$u = -1 \cdot GD.$$

Thus N^1 will move -GD units on S with the expectation that N^2 will move to S_g. Thus if $S_n1 = 5$, $S_n2 = 10$, and $S_g = 8$:

$$GD = 8 - 10$$
$$GD = -2$$
$$u = 2$$

N^1 will move to position 7 ($S_n1 + u$), and then will expect N^2 to move to 8 ($S_n2 + \dot{x}$).

The other model that we will allow our nations to have is the force model. The force model states that the only way I can get the other nation to move to my goal is to show him how strong I am by moving away from him. In our notation, the force model is expressed as follows: $\dot{x} = u$. Substituting this function above, N^1 will move to 3 and expect that N^2 will move to 8.

Both of these examples are illustrated in Figure VI.

Insert Figure VI Here

These single dimension models can easily be translated to n--dimensional state space conditions. The conciliatory model becomes $\underline{\dot{x}} = -I \cdot \underline{u}$, where I equals an n x n identity matrix, and $\underline{\dot{x}}$ and $\underline{u}$ are the same as $\dot{x}$ and u, except they are vectors of length n. The force model is therefore $\underline{\dot{x}} = I \cdot \underline{u}$.

To this very simple model (which assumes that N^2 does not move on its own) we add another element which specifies N^2's behavior if N^1 did nothing: $\underline{\dot{x}} = \pm I \cdot \underline{u} + T \cdot \underline{s}_n 2$. In this model, $\underline{\dot{x}}$, I, and $\underline{u}$ are defined as above. $\underline{s}_n 2$ is simply the position of N^2 in the state space, and T is defined as a trajectory matrix. The trajectory matrix describes the behavior of a nation as a function of its current position. Thus the current location multiplied by the trajectory matrix gives an estimation or projection of the nation in one time period. This projection is based upon the assumption that N^1 did nothing. Thus if the T matrix projected that N^2 would go the N^1's goals, N^1 would do nothing. Thus in a sense, N^1 checks where N^2 is headed before deciding upon an appropriate action. The behavior of this model can be illustrated by the following numerical example of a conciliatory image of the environment:

$$S_g = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \quad S_{n^1} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad S_{n^2} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$$

$$I = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$GD = S_g - S_{n^2} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

As above, N^1 computes $\underline{u}$ such that $S_{n^2} = S_g$.

$$GD = I\underline{u} + TS_{n^2}$$

$$\begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} u^1 \\ u^2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 7 \\ 5 \end{pmatrix}$$

$$I^{-1} \cdot GD = (I^{-1} \cdot I) \cdot \underline{u} + TS_{n^2}$$

$$I^{-1} \cdot GD = \underline{u} + TS_{n^2}$$

$$\underline{u} = T \cdot S_{n^2} - I^{-1} \cdot GD$$

Substituting yields

$$\begin{pmatrix} u^1 \\ u^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 7 \\ 5 \end{pmatrix} \quad \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} u^1 \\ u^2 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} u^1 \\ u^2 \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$$

If N^1 moves to $\begin{pmatrix} 12 \\ 16 \end{pmatrix}$, $(S_{n^1} + \underline{u})$, N^1 will expect N^2 to move to S_g.

Just as a positive I matrix implies that N^1 moves in the opposite direction as N^2, a positive T matrix implies that without the intervention of N^1, N^2 would move to a state having larger coordinate values.

Up to this point the I and T matrices have been diagonal with either positive or negative unities for elements. By relaxing this restriction, we now move to our final class of models. (Since I no longer is an identity matrix, we save the symbol but change the name: I is called the impact matrix. I specifies the impact that N^1 perceives that its behavior will have on the behavior of N^2.) First let us relax the restriction calling for unities in the major diagonal -- they may now range over the set of real numbers. Without going into a detailed numerical example the following interpretation can be given to the size of the elements of the I matrix: Elements larger than unity indicate that N^1 perceives that its movement along a dimension will cause N^2 to move a greater distance than it moved. Elements less than zero specify that N^1 thinks that it must move a very great distance in order for N^2 to move much at all. Depending upon the size of the elements of the T matrix, N^2 is either accelerating or decelerating in its movements in state space.

The final bit of tinkering we do with our models is to allow off-diagonal elements of the I and T matrices to assume other than zero values. Contaminating the matrices in this way has the implication that in the case of the I matrix, movements along one dimension by N^1 will influence N^2's behavior or movement on both (in our two dimensional state space) dimensions. An impact matrix of this sort is: $\begin{bmatrix} 1 & .5 \\ .5 & 1 \end{bmatrix}$. A movement of one unit along the first dimension would, from the perspective of N^1, be expected to cause the other nation to move 1 unit on the first dimension and .5 units on the second.

This completes our exposition of the class of models that we will consider. The remainder of this paper will display the results generated by two nations interacting according to the various response models presented here and discuss the results from the perspective of the impact of model form upon the stability of the system.

Non-Technical Summary of A Closer Look...

What we did in "A Closer Look ..." was to lay out the mathematics of the state transition function. Our formulation of the state transition is built upon the notion of a forecasting model discussed above. Our nations can have one of two models -- a conciliatory or a force model. The conciliatory model "says" that if I move toward another nation in state space, the other nation will respond by moving toward me. In other words, you give up something with the expectation of getting something in return. The force model "says" that if I move away from the other nation, the other nation will move toward me. In other words, if I show the other nation my strength by moving away from him, he will follow in my direction.

Each nation has a goal for the other nation in state space. In essence a nation desires to determine a movement that it can take that will result in the other nation moving to the goal the behaving nation has for the target nation.

The models that each nation uses in determining its appropriate behavior is made up of two components: 1) how responsive the other nation is to its behavior and; 2) where the other nation would move to if the behaving nation did nothing. These two components are called impact and trajectory respectively. Large absolute values in the impact component specify that the other nation is very responsive to movements by the behaving nation. A small absolute value indicates that the behaving nation's impact on the behavior of the other nation is small. A model with positive impact coefficients is the force model, while negative coefficients indicate the conciliatory model.

The size and sign of the trajectory coefficients indicate the direction and size of the movement of the other nation if nothing were done.

Once this basic structure is laid out, " A Closer Look..." generalizes the equations into an "n" dimensional state space. Although the model is capable of employing any number of dimensions in the state space, for our purposes we use only two, since that is the most complex formulation that can easily be graphed on paper.

<u>What We Did</u>

In order to determine the characteristics of our state transition functions, we develop three types of international systems, one for each combination of force and conciliatory transition functions, and simulated their behavior:

	Model for Nation 1	Model for Nation 2
S1:	force	force
S2:	conciliatory	conciliatory
S3:	conciliatory	force

We had expected that there would be a distinct type of behavior exhibited by each of the three types of international systems. While that did not turn out to be the case, our initial presentation will be based upon the classification of model types (force or conciliatory) according to the sign of the impact matrix (- = conciliatory and + = force). Once the initial presentation has been made, we will discuss the factors that determine a force or conciliatory model. As will be seen, we do get the types of behavior that we had posited for the three types of international systems but our use of the sign of the impact matrix for the determination of the model type is incorrect. The next section will discuss some of the shortcommings with our work and some areas that we see as important for further development.

Each of the systems (classified according to our initial expectations) will be discussed in turn.

S1: Force -- Force Models

The typical behavior of a force -- force system is given in Table Ia - Id. (The first table gives the impact and trajectory matrices and goal vectors for each nation. The second table gives a plot of the behavior of the two nations in the two dimensional state space. IN all of our examples, the simulations were allowed to generate fifty pairs of behaviors. The state space dimensions are illustrative only, and hence do not have any substantive meaning attached to them. In all runs of the simulation both nations start in the same position in state space, i.e., nation 1 = (3,5) and nation 2 = (-7,2). Nation 1's movements are indicated by a '1' on the plot, nation 2 by a '2', and those points where both nation's coincide an asterisk, '*' is printed. The plots are minimum to maximum plots. This means that the values of the increments along the two axes are set so that all fifty points will fit on the plot. The orgin is respositioned accordingly. The third and fourth tables five the movements of the two nations along each dimension over time. The two dimensions are respectively the X and Y axes in the full state plot. The starting point in these one dimensional plots is at the top of the page, with each line going down representing one time unit.)

In this first type of system both nations employ what we call a 'force' strategy. This strategy is predicated upon the supposition that a target nation will respond to threatening behavior be moving toward the forceful nation. As can be seen in Table Ia this is represented in matrix form by positive entries along the major diagonal of the impact matrix. It can be seen that models of this form will cause a nation to move away from a target nation in an effort to pull him to a goal state location.

Our guess was that in a two nation world of force nations the state locations would move in opposite directions. This is born out by our simulation results in Tables Ia - Id.

S2: Conciliatory -- Conciliatory Models

A typical example of conciliatory -- conciliatory systems is given in Table IIa - d. Since the conciliatory model states that another nation will respond positively toward you only if you respond positively toward it, we had initially expected that the two nations would proceed immediately toward the goal locations and sit there. It did not turn out that way. Each nation "walked" toward the other nation, and together they moved towards the extremes of the state space. While it turns out to be the case that the initial position of the two nations vis-a-vis each other does influence their initial behavior, once both nations get on the same side of each other's goals, they move to the extremes together. Thus in a system of conciliatory nations, no nation can achieve its goal. This surprising result, as will be discussed in more detail below, is a function of how we have specified the state transition functions and should not be taken to be making assertions about a real world description of conciliatory (using the common sense notion of conciliatory) behavior.

S3: Force -- Conciliatory Models

A typical example of force -- conciliatory (or mixed models) is given in Table IIIa - d. The mixed model system illustrates a world in which one nation employs a 'force' strategy and the second employs a 'conciliatory' strategy. We would expect a forcing nation to 'pull' a conciliatory nation to the forcing nation's goal. When we examine the simulation results we see that this is roughly what happens. As nation 1 approaches the goals that nation 2 has for it, nation 2's movements become smaller and smaller. The movement of nation 2 is zero when nation 1 is at its goal. As nation 1 moves over nation 2's goal, nation 2 changes the direction of its movement in an attempt to control the concilliator back to the goal state location. The same sorts of behavioral characteristics are exhibited by the conciliatory nation. As nation A crosses the goal of nation B, nation B changes the direction of its behavior. This flip -- flop results in the sinusoidal character of the single dimensional plots and for the spiral appearance of the full plots.

As was noted above, the sign of the impact matrix is not sufficient to determine the form of the behavior of the nations. It turns out to be the case that a force model can be made to exhibit the behavior of a conciliatory model. The same holds true for the conciliatory model. Without going into the mathematics of our system of difference equations, the size and sign of the trajectory coefficients and the sign of the impact coefficients are jointly sufficient to predict the behavior of our system. The exact relationship is given in Table IV. We do get the classes of behavior exhibited by what we have called force -- force, conciliatory -- conciliatory, and mixed forms of systems -- but for reasons other than those we had anticipated. Tables Va - VIIId give illustrations of this sort.

Where To Now?

Our initial goal has been to investigate the stability properties of our two nations in state space. It soon became clear that 1) behavior stability was not definable; and 2) even if we could define behavior stability in terms of state space our nations could never (except in degenerate and uninteresting solutions) exhibit state location stability given our definition of the state transition function.

In the broader thrust of our research efforts we intend to construct peaceful international systems and then by introducing our three candidates for accidental war mentioned in the Introduction, assess their impact on national behavior. Although this paper represents just a first attempt to deal with some of these problems -- in the larger context of our research thrust we propose to represent ideological differences by specifying that the state spaces that the nations operate within are not the same for all nations. The causes of war based upon factual errors will be represented by introducing noise into the perceptions of nations. Accidental war based upon incorrect judgement of the responses to a nation's actions will be based upon incongruous models of the other nation's behavior. It is our intention to first build a perfect world having none of the perturbations mentioned above. Then by systematically introducing our candidates for accidental war, we will be in a position to determine not only if these factors do in fact cause our once stable system to break down, but also how much of a perturbation is required to disrupt the system. Since we do not link our state or orientation space to behavior, stability (defined behaviorally) in our system could not be determined -- since state space alone is not sufficient for the determination of behavior. Recalling our capitalist -- communist one dimensional world, while we would expect the behavior of the two nations to be different, there is as of yet no way to determine in what way they are different. That determination must wait until we have specified the second function machine mentioned in the Big Picture (input × state → output).

The second point mentioned above, the inability of our nations to exhibit state space stability, is a result of our development of the form of the trajectory component of the state transition function. It will be noted in Table III, the plot of the behavior of the system that took on the least extreme state space positions after fifty iterations, that the nations were oscillating around their goals, but that the oscillations were getting larger and larger. This is a result of the fact that one nation does not pose goals for the other nation. The trajectory component specifies that if a nation did nothing, the other nation will change its position by the trajectory times the current location. Thus even if both nations were sitting at the goals that each nation had for the other, both nations (assuming other than zero trajectory coefficients) will move, since neither nation realizes that both nation's goals are completely satisfied at that particular state space location configuration. Thus our immediate task is to determine alternate forms of the trajectory influence. But beyond that before we are in a position to talk about causes of accidental war we must flesh out the skeleton that we have put forth here, and make our concepts of state, state transition, output function, and behavior space explicitly operationalizeable. We have a long way to go -- but we knew it wasn't going to be easy when we started.

Appendix on Dynamic Systems *

i) A (general) time system is a system such that:

$X = A^T$, $Y = B^T \rightarrow S \subseteq A^T \times B^T$; where A and B are alphabets and T a linearly ordered time set.

ii) A dynamical system is a time system for which there are given a set Z and a pair of functions:

$$p: Z \times X \times T \rightarrow Y \times T$$

$$\phi: Z \times X \times T \times T \rightarrow Z$$

such that:

$$(\exists\ Z)\ \{p(Z,X,T) = (Y,T)\} \leftrightarrow (X,Y) \in S$$

$$p(\phi(X,Y,T,T'),X,T') = (Y,T')$$

common reference is:

Z = state space

p = state representation or system response function

ϕ = state transition function

Although to the casual reader the Mesarovic formulation might seem prohibitively rigorous the notion of the dynamic systems is in fact a common sense one and can be found in one form or another in many places. One such popular formulation that is a close cousin to dynamic systems is the Arbib (1964) finite automaton: "def; a finite automaton is a quintuple, $A = \langle I,O,S,\delta,\lambda \rangle$ where: I = finite set of inputs; O = finite set of outputs;

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* this follows closely the development by Mihalo Mesarovic in George Klir's Trends in General Systems Theory

S = finite set of internal states; $\lambda: S \times I \rightarrow S$, (next state function; and $\delta: S \times I \rightarrow O$, (next output function). An examination of the Arbib automata shows it to be a special case of the dynamic system. It is noted that present work by the authors is done in realm of development of a reasonable ϕ function and an exposition as to the utility of developing dynamic systems models of international behavior.

Appendix on Control of Dynamic Systems:
An Introduction for Poets

For many years physical phenomena have with notable success been controlled. This means that a physical process was brought to a desired condition. Remembering our systems vocabulary, a preferred appearance of the system was controlled so that it obtained. More strongly than a mere occurance of a desired appearance, physical processes have been controlled while minimizing some objective function. For example rockets might be sent to the moon while minimizing time, or energy, or total cost. A plane might be directed to land subject to a minimum number of direction changes. While success has been rather stunning for circuitry, social planning has managed to avoid direct application of optimal control techniques. It is the guess that social processes are in principle modelable that leads us to do science at all and the further suspicion that if a process is modelable we might as will cast it up in language that is amenable to control. Seen in perspective this is but the ground breaking for an enormous enterprise, the empirically useful formulation of inter-nation behavior in formal control theoretic language and the application of control theory to those formulations.

Formalizing Controllability:

Controllability is defined in reference to the objective of control. Let $S: M \times U \rightarrow Y$ be the system and $G: M \times Y \rightarrow V$ the performance function. Also, M is the control object, while U can be the set of initial states or disturbances. S is controllable in $V' \subseteq V$ over $U' \subseteq U \leftrightarrow (v)(v \in V'), (u)(u \in U'), \exists\, m \rightarrow \{v \in V' \text{ and } u \in U' \rightarrow G(m, Sm, w) = v\}$.

Appendix on Matrix Arithmetic

Addition:

i) is defined only if the matrices to be added are of the same dimension and addition of the elements is defined.

A is $m \times n$ and B is $p \times q$; we can add A+B iff $m = p$, $n = q$, and a_{ij}, b_{ij} are elements of the same field (see appendix on fields).

ii) where addition is defined $A+B = C$ implies $c_{ij} = a_{ij} + b_{ij}$.

Example:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{pmatrix}$$

$$A+B = \begin{pmatrix} 1+7 & 2+8 & 5+9 \\ 4+10 & 5+11 & 6+12 \end{pmatrix}$$

Multiplication by a Scalar

A is an element of field F where A has m rows and n columns. α is some element of field F:

$$\alpha \cdot (A) = \begin{pmatrix} \alpha \cdot a_{11} & \alpha \cdot a_{12} & \cdots & \alpha \cdot a_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha \cdot a_{m1} & \alpha \cdot a_{m2} & \cdots & \alpha \cdot a_{mn} \end{pmatrix}$$

and $\alpha \cdot (A) = (A) \cdot \alpha$.

Multiplication of Two Matrices

$C = A \cdot B$: A is an element of $F_{m,n}$; B is an element of $F_{p,q}$. Multiplication is defined only if $n = p$

and the entries are from the same field. C will be an m by q matrix with

$$c_{ij} = \sum_{k=1}^{n} a_{ik} \cdot b_{kj}$$

Example:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

$$C = A \cdot B = \begin{pmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{pmatrix}$$

Special Matrices and Operations

Transpose:

A' is defined as transpose of matrix A, let $B = A'$, then $b_{ij} = a_{ji}$ for all i,j.

Identity Matrix:

I is a square matrix (n by n) with 1's in the major diagonal and 0's every where else. $A \cdot I = I \cdot A$.

Inverse Matrix:

A is an n by n element of $F_{n,n}$; if there exists a matrix A^{-1} such that $A \cdot A^{-1} = I_{n,n} = A^{-1} \cdot A$, then A^{-1} is the inverse of matrix A.

Appendix on Fields

Let F be a set of elements: $F = \{\alpha, \beta, \gamma, \delta \ldots \}$.

F is a field if and only if:

i) addition

Given α, β, any pair of elements from F, their sum $(\alpha + \beta)$ is an element of F which is uniquely defined and:

a1) $\alpha + \beta = \beta + \alpha$, $(\alpha, \beta)(\alpha, \beta \in F)$

a2) $\alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma$, $(\alpha, \beta, \gamma)(\alpha, \beta, \gamma \in F)$

a3) there is an element in F, denoted by 0, such that $\alpha + 0 = \alpha$, $(\alpha)(\alpha \in F)$

a4) for each $\alpha \in F$, there exists an element in F, denoted by $-\alpha$, such that $\alpha + (-\alpha) = 0$ $(\alpha)(\alpha \in F)$

ii) multiplication

Given α, β (any pair of elements in F) their product is a unique element in F and:

m1) $\alpha \cdot \beta = \beta \cdot \alpha$, $(\alpha, \beta)(\alpha, \beta \in F)$

m2) $\alpha \cdot (\beta \cdot \gamma) = (\alpha \cdot \beta) \cdot \gamma$, $(\alpha, \beta, \gamma)(\alpha, \beta, \gamma \in F)$

m3) there exists an element in F denoted by 1 such that $\alpha \cdot 1 = \alpha$, $(\alpha)(\alpha \in F)$

m4) $(\alpha)(\alpha \neq 0)$, there exists an element denoted by α^{-1} such that $\alpha \cdot \alpha^{-1} = \alpha^{-1} \cdot \alpha = 1$

Addition and multiplication are related by:

$$\alpha \cdot (\beta + \gamma) = \alpha \cdot \beta + \alpha \cdot \gamma, \ (\alpha,\beta,\gamma)(\alpha,\beta,\gamma \in F)$$

Appendix on Vector Spaces

A set together with 1) addition of vectors and 2) scalar multiplication.

$$X = \{\underline{x}_1, \underline{x}_2, \underline{x}_3, \ldots\}$$

the members $(\underline{x}_1, \underline{x}_2, \underline{x}_3)$ are called vectors. X is a vector space if and only if:

a) addition

$(\underline{x},\underline{y})(\underline{x},\underline{y} \in X)$ there exists a unique vector $\underline{x} + \underline{y} \in X$

b) scalar multiplication

$(\alpha)(\alpha \in F)$, where α is a scalar and $(\underline{x})(\underline{x} \in X)$ there exists a unique vector $\alpha \cdot \underline{x} \in X$

Addition and scalar multiplication must satisfy:

i) $\underline{x} + \underline{y} = \underline{y} + \underline{x}$, $(\underline{x},\underline{y})(\underline{x},\underline{y} \in X)$

ii) $(\underline{x} + \underline{y}) + \underline{z} = \underline{x} + (\underline{y} + \underline{z})$, $(\underline{x},\underline{y},\underline{z})(\underline{x},\underline{y},\underline{z} \in X)$

iii) there exists a null vector $\underline{\theta}$ $(\underline{\theta} \in X)$ such that $\underline{x} + \underline{\theta} = \underline{x}$, $(\underline{x})(\underline{x} \in X)$

iv) $(\underline{x})(\underline{x} \in X)$, there exists a unique vector $-\underline{x}$ such that $\underline{x} + (-\underline{x}) = \underline{\theta}$

v) $\alpha \cdot (\underline{x} + \underline{y}) = \alpha \cdot \underline{x} + \alpha \cdot \underline{y}$, $(\underline{x},\underline{y})(\underline{x},\underline{y} \in X)$, and $(\alpha)(\alpha \in F)$

vi) $(\alpha + \beta) \cdot \underline{x} = \alpha \cdot \underline{x} + \beta \cdot \underline{x}$, $(\alpha,\beta)(\alpha,\beta \in F)$ and $(\underline{x})(\underline{x} \in X)$

vii) $(\alpha \cdot \beta) \cdot \underline{x} = \alpha \cdot (\beta \cdot \underline{x})$, $(\alpha,\beta)(\alpha,\beta \in F)$ and $(\underline{x})(\underline{x} \in X)$

viii) $\underline{\theta} \cdot \underline{x} = \underline{\theta}$; $1 \cdot \underline{x} = \underline{x}$

REFERENCES

Arbib, Michael (1964), Brains, Machines, and Mathematics. New York: McGraw-Hill.

Deutsch, Karl (1966), The Nerves of Government. New York: The Free Press.

Hempel, Carl, and Paul Oppenheim, "Studies in the Logic of Explanation". Philosophy of Science, 15: 135-75.

Holsti, Ole (1962), "The Belief System and National Images: A Case Study". The Journal of Conflict Resolution, 6:244-52.

__________.(1965), "The 1914 Case", American Political Science Review, 59: 365-78.

Klir, George (ed) (1972), Trends in General Systems Theory. New York: John Wiley.

Miller, G. A., E. Galanter, and K. Pribram (1960), Plans and Structure of Behavior. New York: Holt, Reinhart, and Winston.

Nagel, Ernest (1956), Logic without Metaphysics. Glencoe, Ill.: Free Press.

Newell, Allen (1973a), "Production Systems: Models of Control Structures". in William G. Chase (ed), Visual Information Processing. New York: Academic Press, pp. 436-526.

__________ (1973b), "You Can't Play 20 Questions with Nature and Win". in William G. Chase (ed), Visual Information Processing. New York: Academic Press, pp. 283-310.

Newell, Allen and Herbert Simon (1972), Human Problem Solving. New York: Prentice-Hall.

Rosenau, James (1967), Of Boundaries and Bridges: A Report on a Conference on the Interdependencies of National and International Political Systems. Princeton: Center of International Studies, Research Monograph 27.

Rummel, R. (1971), "Status Field Theory and International Relations". DON Research Report 50.

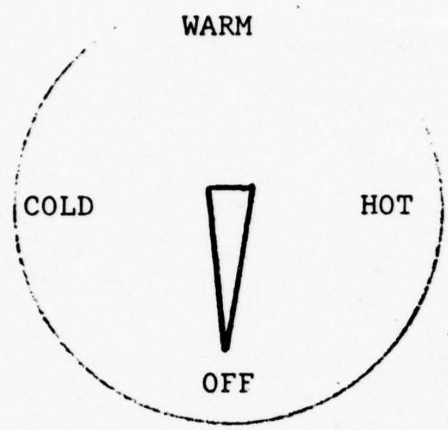
Tarski, Alfred (1941), Introduction to Logic. New York: Oxford University Press.

REFERENCES -- Page 2

Weiner, Norbert (1961) Cybernetics. New York: John Wiley.

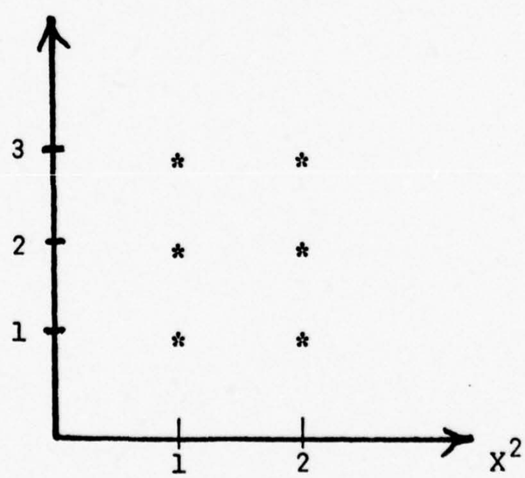
Zinnes, Dina (1968), "Expression and Perception of Hostility in Prewar Crisis: 1914", in J. David Singer (ed), Quantitative International Politics. New York: The Free Press, pp. 85-123.

FIGURE I



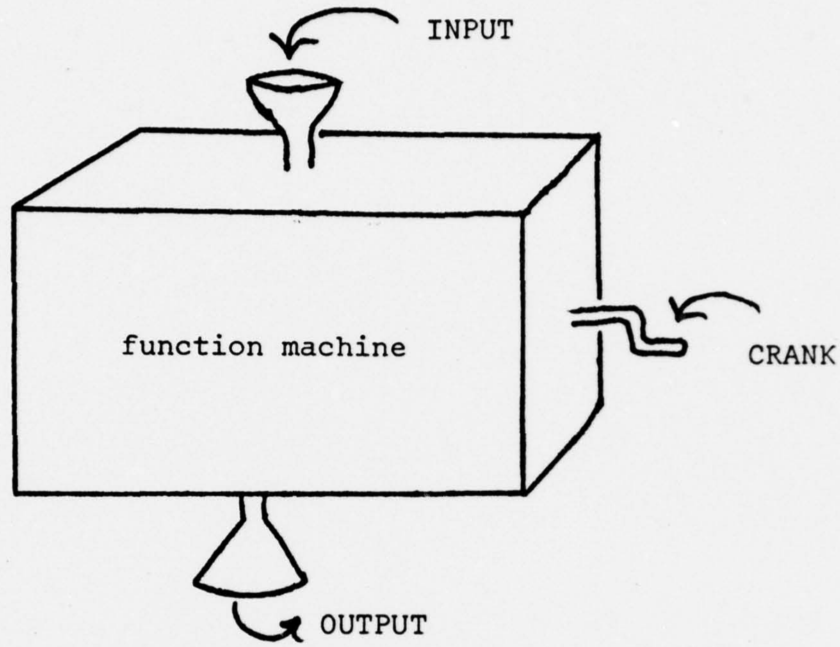
Mock up of Shower-Control Handle

FIGURE II



Cartesian Product of Objects
X^1 and X^2

FIGURE III



FUNCTION MACHINE

FIGURE IV

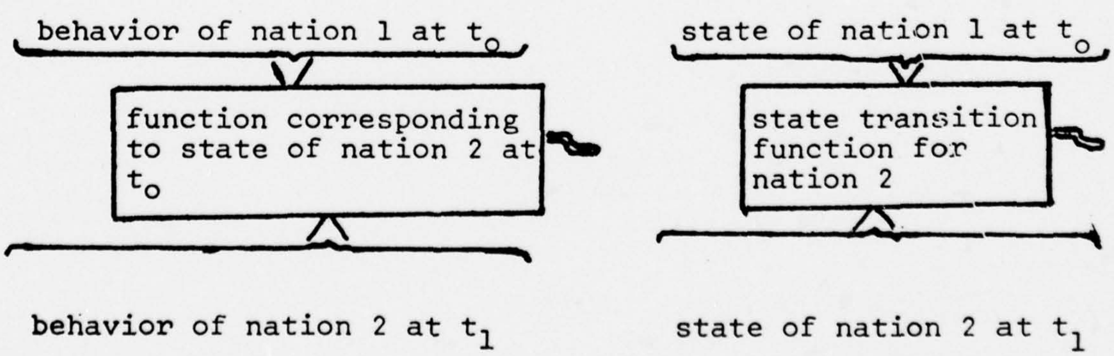


FIGURE V

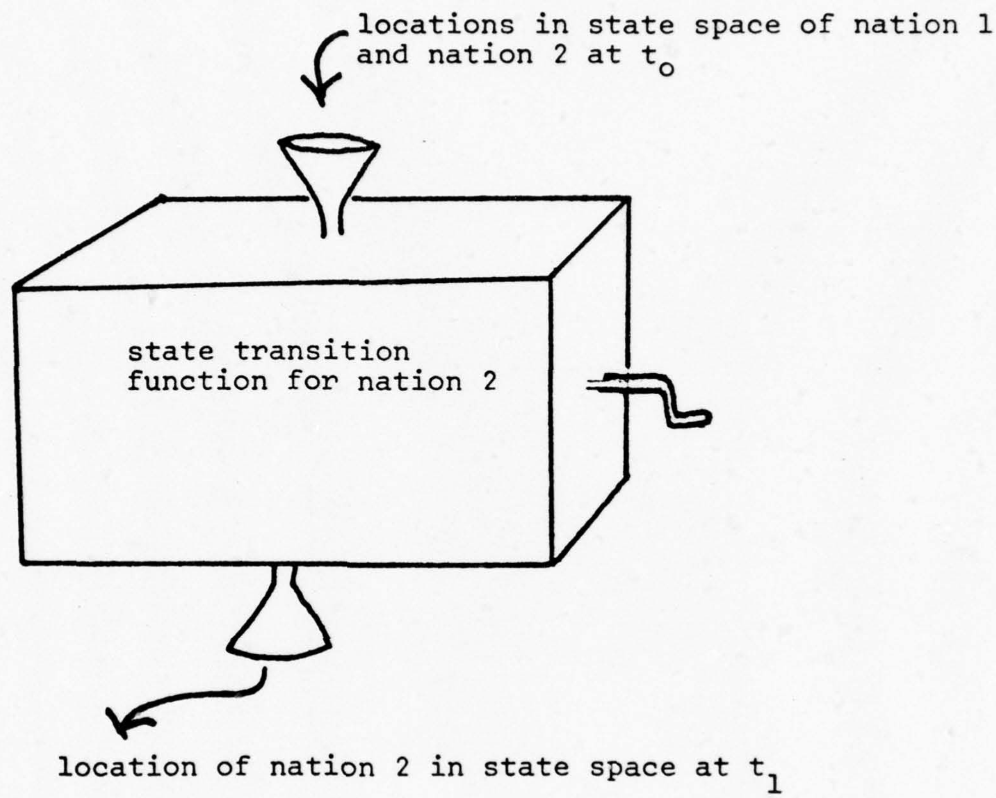
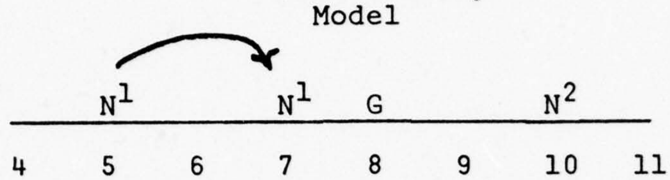
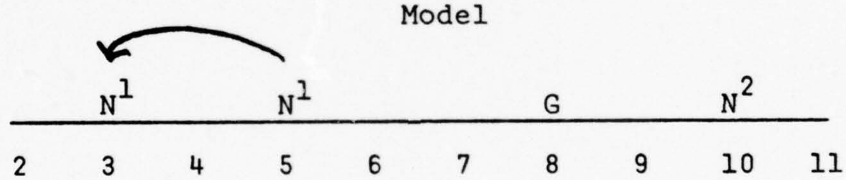

locations in state space of nation 1 and nation 2 at t_o
state transition function for nation 2
location of nation 2 in state space at t_1

FIGURE VI

Conciliatory
Model



Force
Model



```
NATION 1'S IMPACT =
  2.0000            .0
  .0                2.0000

NATION 2'S IMPACT =
  4.0000            .0
  .0                4.0000

NATION 1'S TRAJ =
 -.50000            .0
  .0               -.50000

NATION 2'S TRAJ =
 -.50000            .0
  .0               -.50000

NATION 1'S GOAL =       .0              .0
NATION 2'S GOAL =       10.000          15.000
```

FORCE -- FORCE MODELS

TABLE Ia

XMIN = -10314. XMAX = 7307.2
YMIN = -40471. YMAX = 28639.
X SCALE = 228.85 Y SCALE = 1410.4

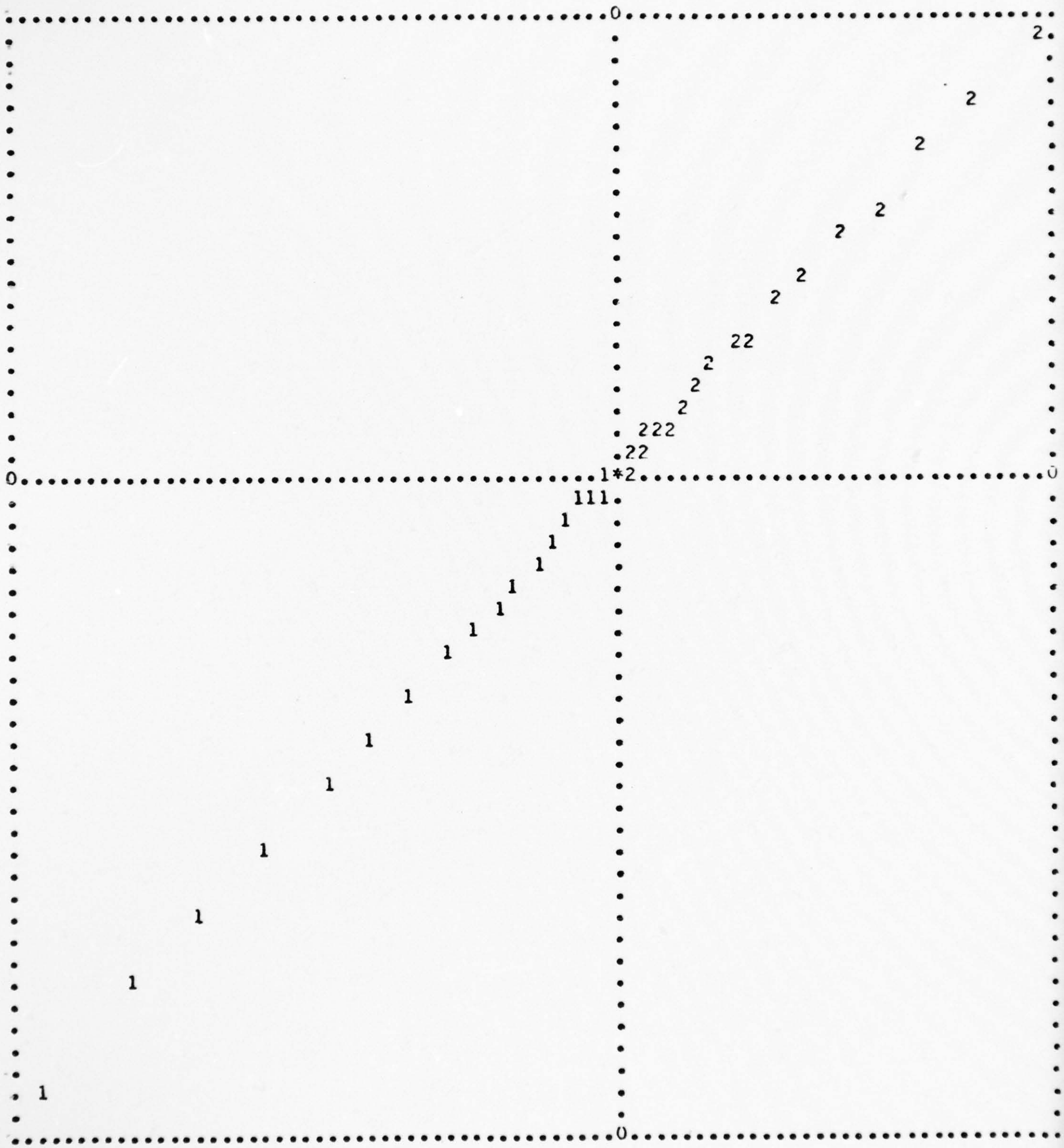


TABLE Ib

FIRST DIMENSION

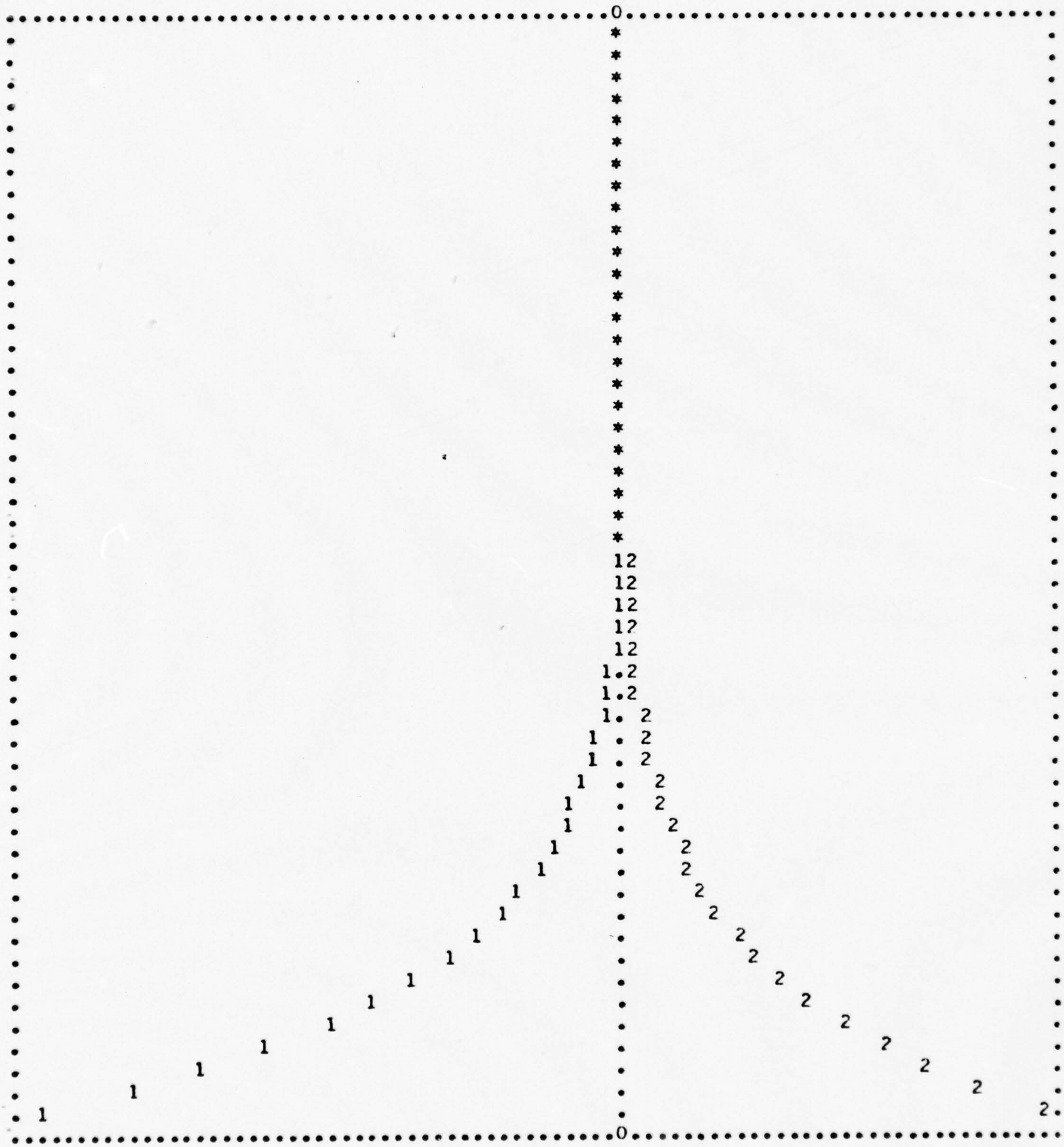


TABLE Ic

SECOND DIMENSION

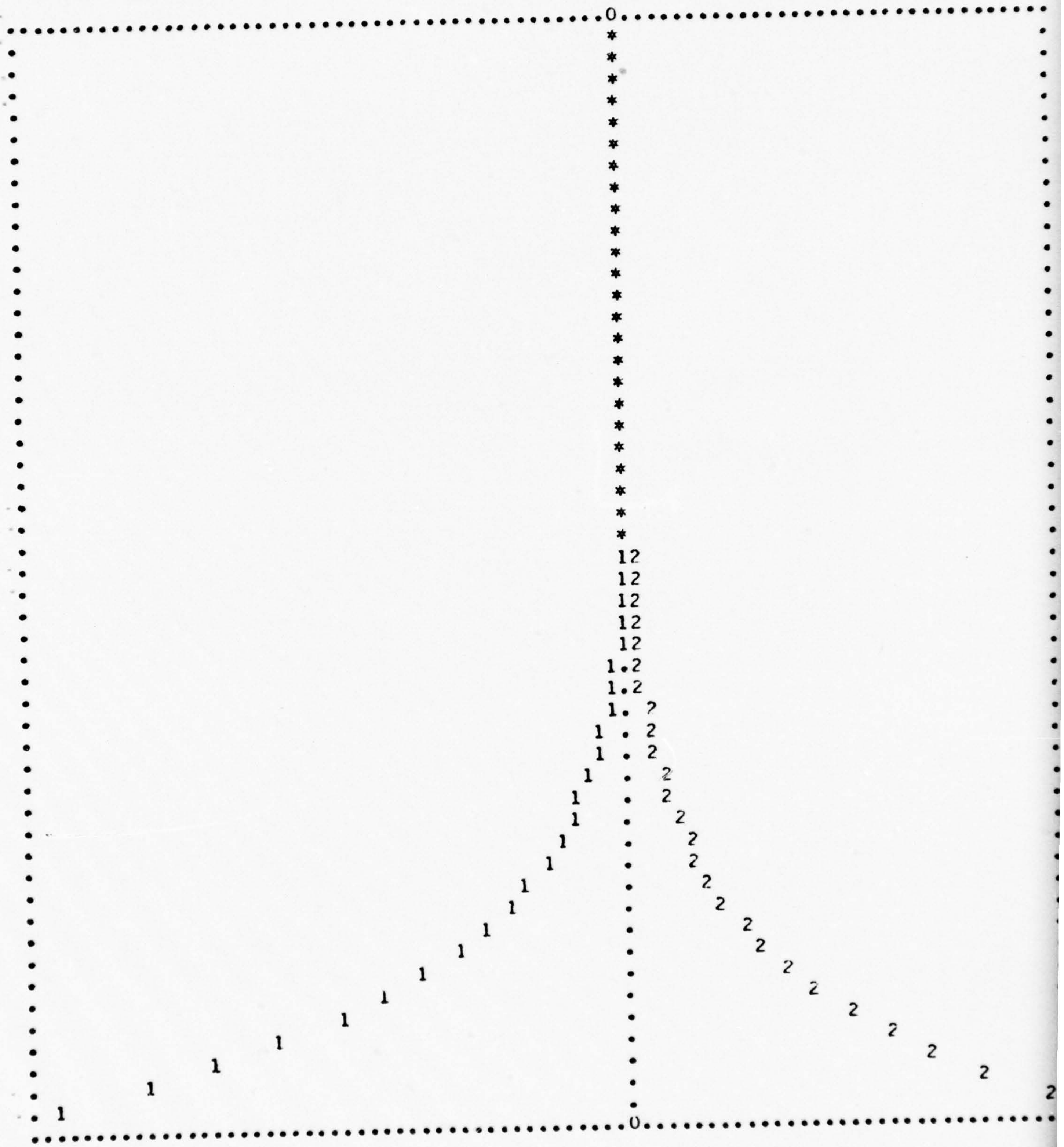


TABLE Id

```
NATION 1'S IMPACT =
 -2.0000              .0
  .0                 -2.0000

NATION 2'S IMPACT =
 -4.0000              .0
  .0                 -4.0000

NATION 1'S TRAJ =
 -.50000              .0
  .0                 -.50000

NATION 2'S TRAJ =
  4.0000              .0
  .0                 4.0000

NATION 1'S GOAL =        .0             .0
NATION 2'S GOAL =      10.000         15.000
```

CONCILIATORY -- CONCILIATORY MODLES

TABLE IIa

XMIN = -.67085E+10   XMAX = 3.0000
YMIN = 2.0000   YMAX = .91140E+10
X SCALE = .87123E+08   Y SCALE = .18600E+09

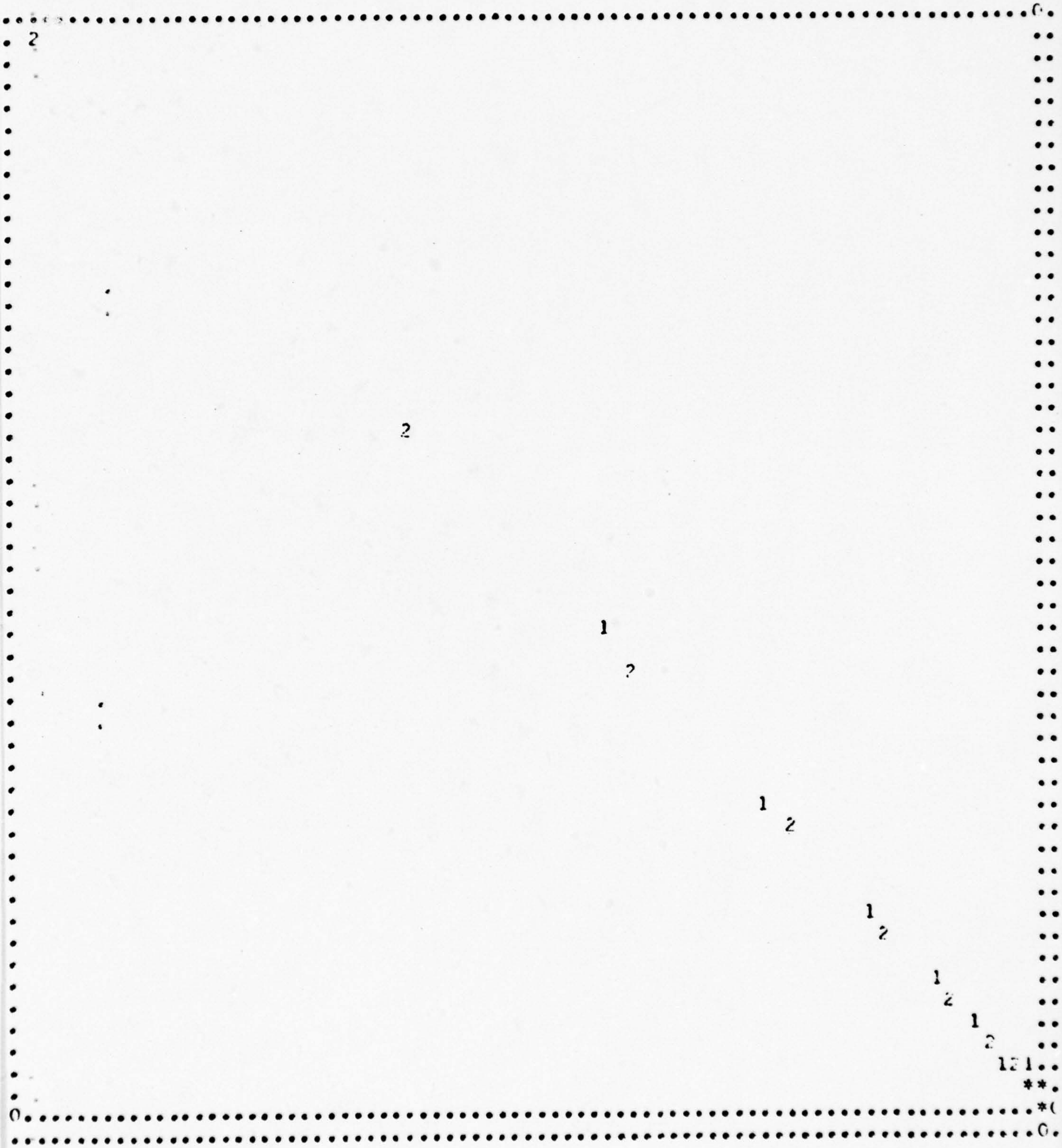


TABLE IIb

FIRST DIMENSION

TABLE IIc

FIRST DIMENSION

TABLE IIc

SECOND DIMENSION

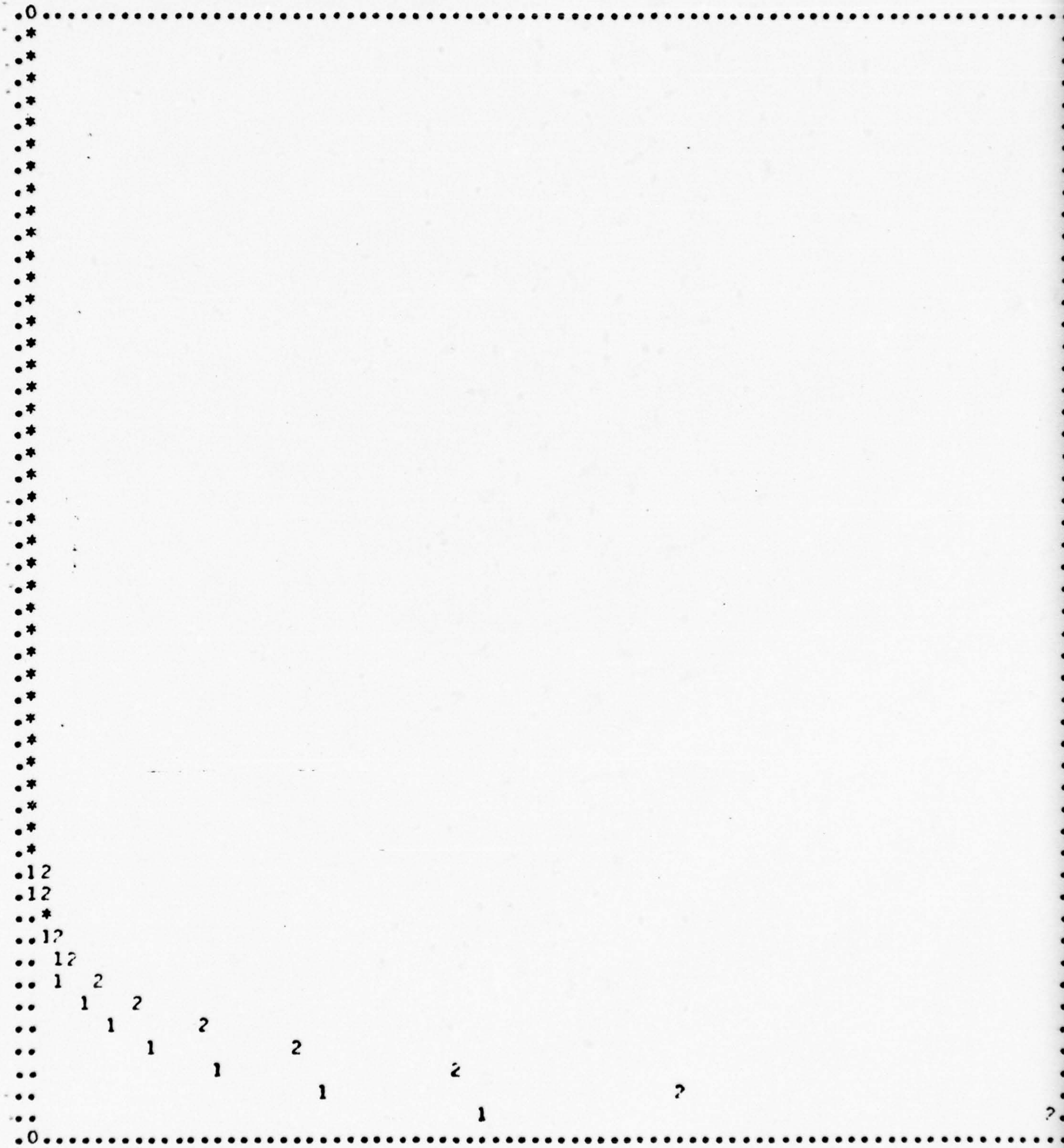


TABLE IId

"

```
NATION 1'S IMPACT =
 -4.0000              .0
   .0              -4.0000

NATION 2'S IMPACT =
  16.000              .0
   .0               16.000

NATION 1'S TRAJ =
 -4.0000              .0
   .0              -4.0000

NATION 2'S TRAJ =
 -2.0000              .0
   .0              -2.0000

NATION 1'S GOAL =        .0              .0
NATION 2'S GOAL =        10.000          15.000
```

MIXED MODELS

TABLE IIIa

XMIN = -94.390 XMAX = 51.024
YMIN = -71.505 YMAX = 25.186
X SCALE = 1.8885 Y SCALE = 1.9733

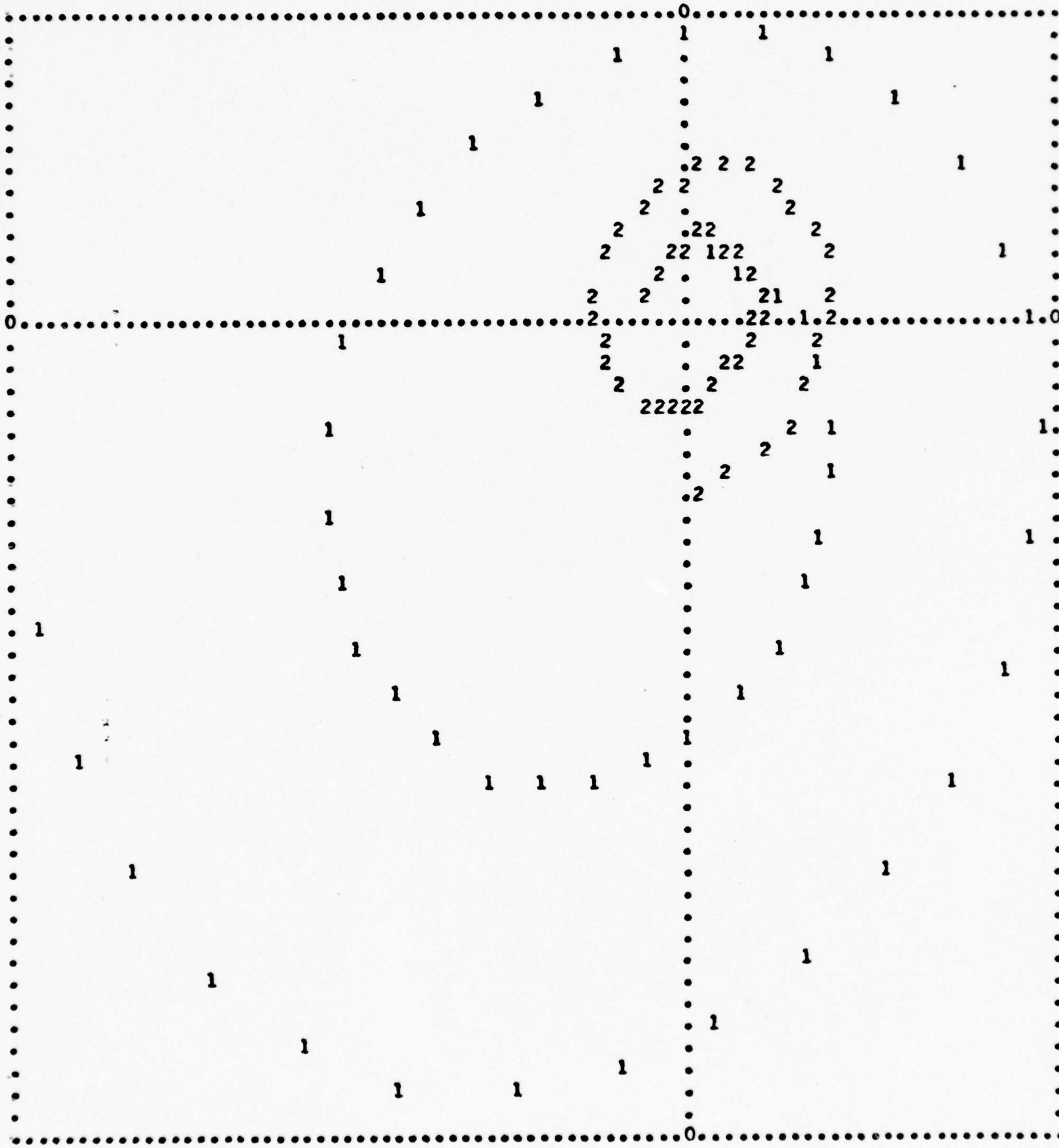


TABLE IIIb

FIRST DIMENSION

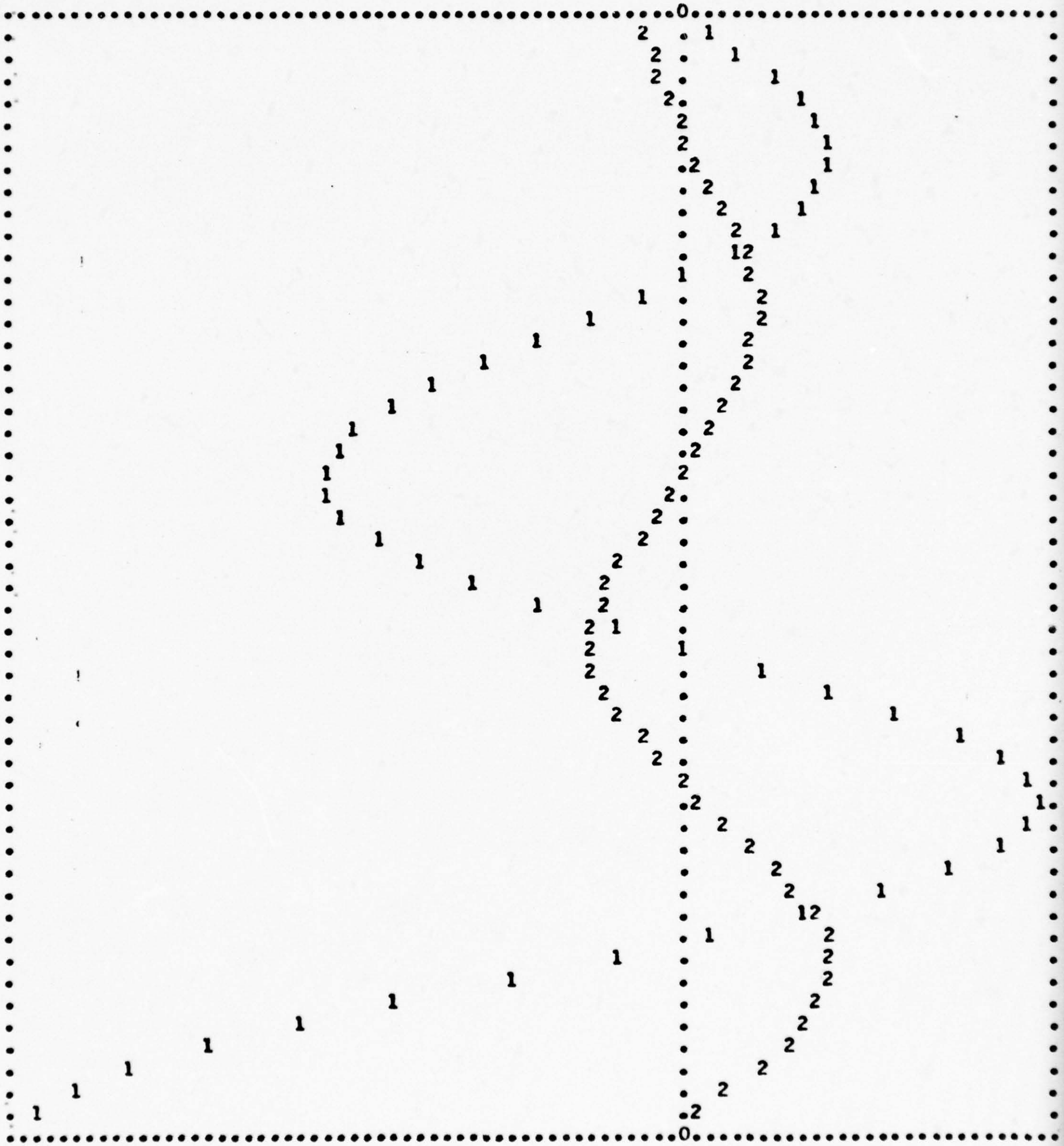


TABLE IIIc

SECOND DIMENSION

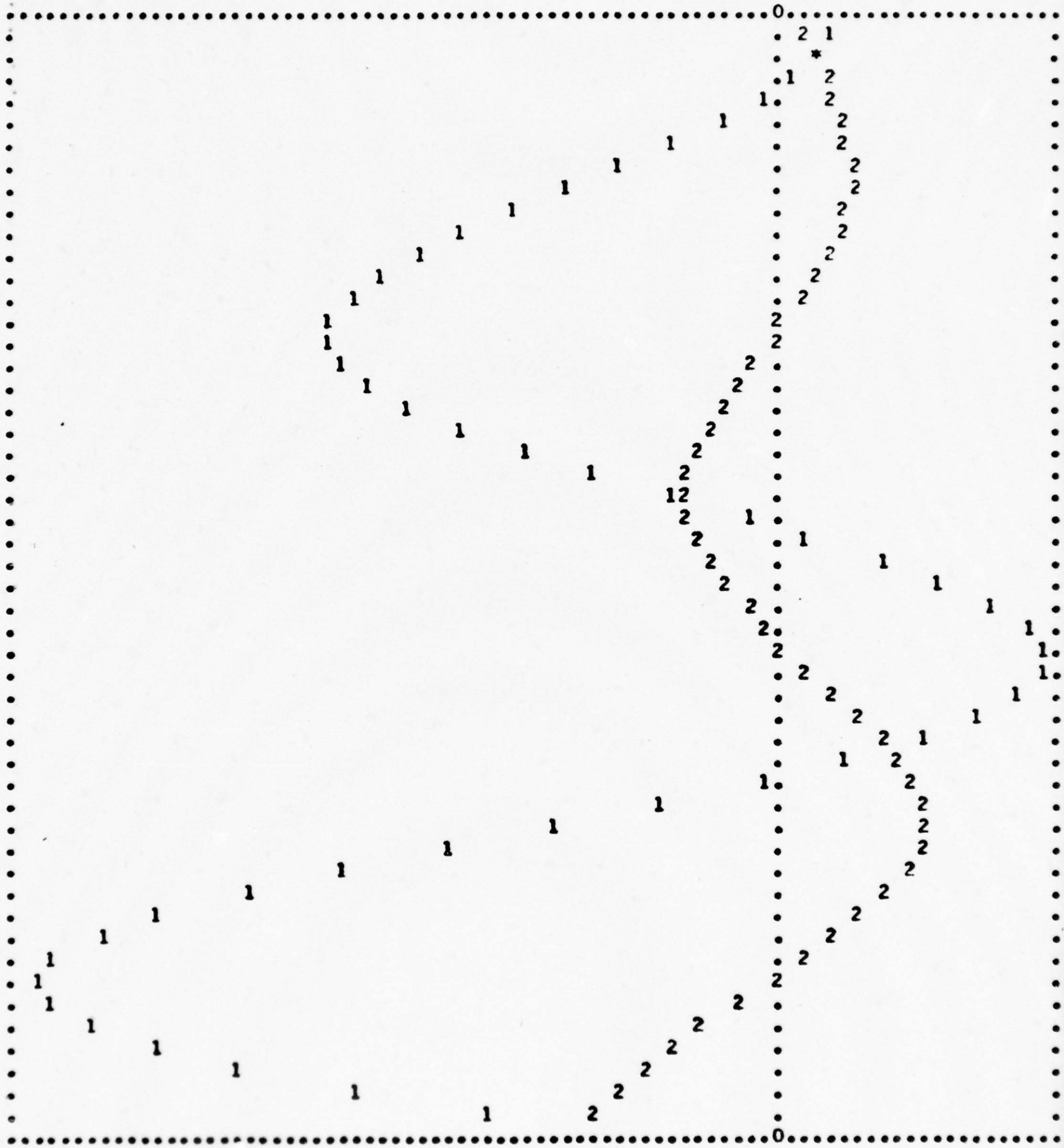


TABLE IIId

$|T| < |1|$ T = trajectory coefficient

Impact Sign	Trajectory Sign	Behavior Type
+	+	force
+	-	force
-	+	conciliatory
-	-	conciliatory

$-1 > T \geq 1$

Impact Sign	Trajectory Sign	Behavior Type
+	+	force
+	-	conciliatory
-	+	conciliatory
-	-	force

In the case where the trajectory is -1, the nation's behavior is a linear function of the goal:

$$-G \cdot I^{-1}$$

TABLE VI

```
NATION 1'S IMPACT =
  2.0000             .0
  .0                 2.0000

NATION 2'S IMPACT =
  4.0000             .0
  .0                 4.0000

NATION 1'S TRAJ =
 -.50000             .0
  .0                -.50000

NATION 2'S TRAJ =
 -2.0000             .0
  .0                -2.0000

NATION 1'S GOAL =        .0             .0
NATION 2'S GOAL =        10.000         15.000
```

FORCE BEHAVING AS MIXED

TABLE Va

```
NATION 1'S IMPACT =
  2.0000             .0
  .0                 2.0000

NATION 2'S IMPACT =
  4.0000             .0
  .0                 4.0000

NATION 1'S TRAJ =
 -.50000             .0
  .0                -.50000

NATION 2'S TRAJ =
 -2.0000             .0
  .0                -2.0000

NATION 1'S GOAL =       .0            .0
NATION 2'S GOAL =       10.000        15.000
```

FORCE BEHAVING AS MIXED

TABLE Va

XMIN = -61.351 XMAX = 41.568
YMIN = -78.842 YMAX = 64.430
X SCALE = 1.3366 Y SCALE = 2.9239

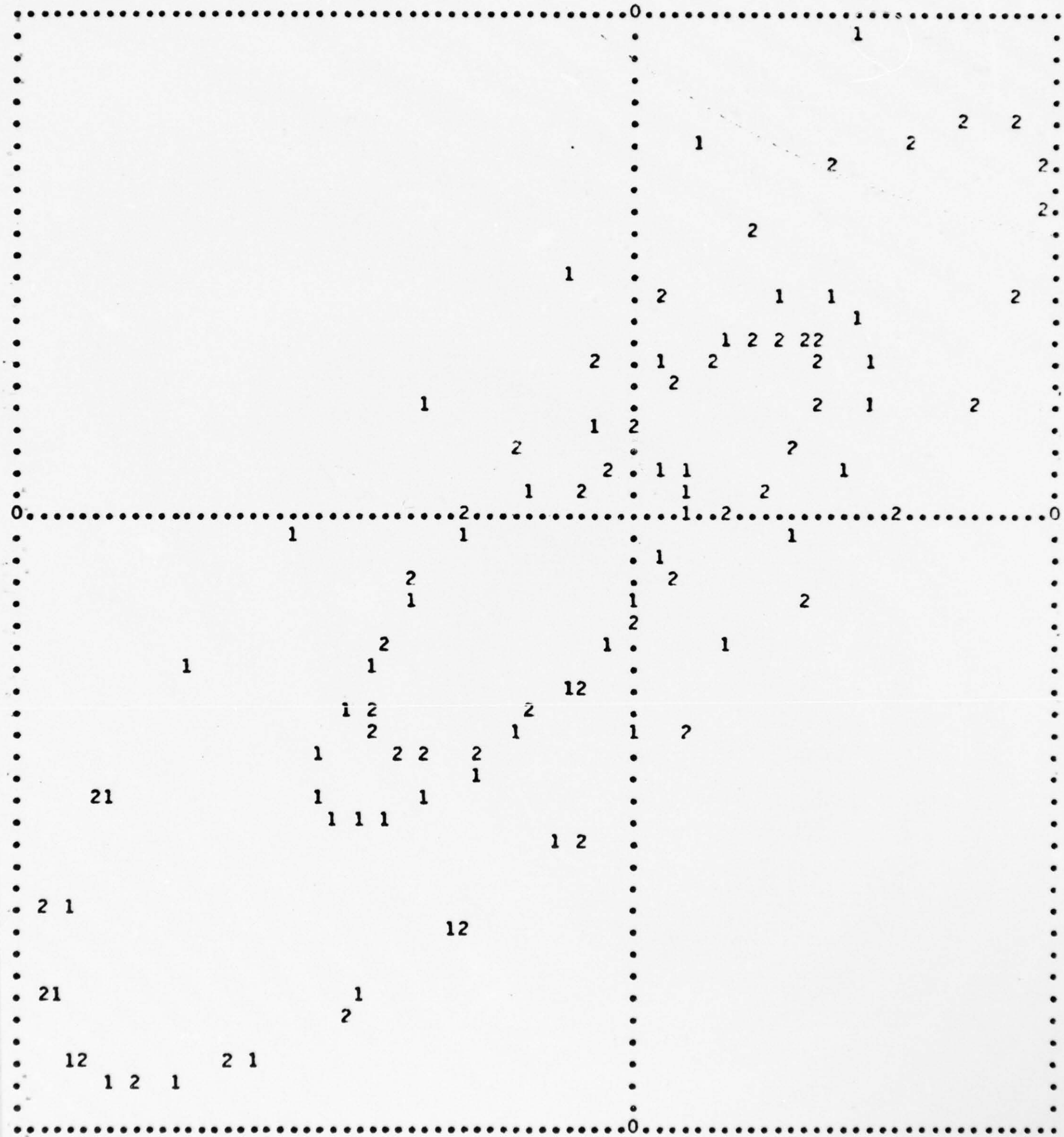


TABLE Vb

FIRST DIMENSION

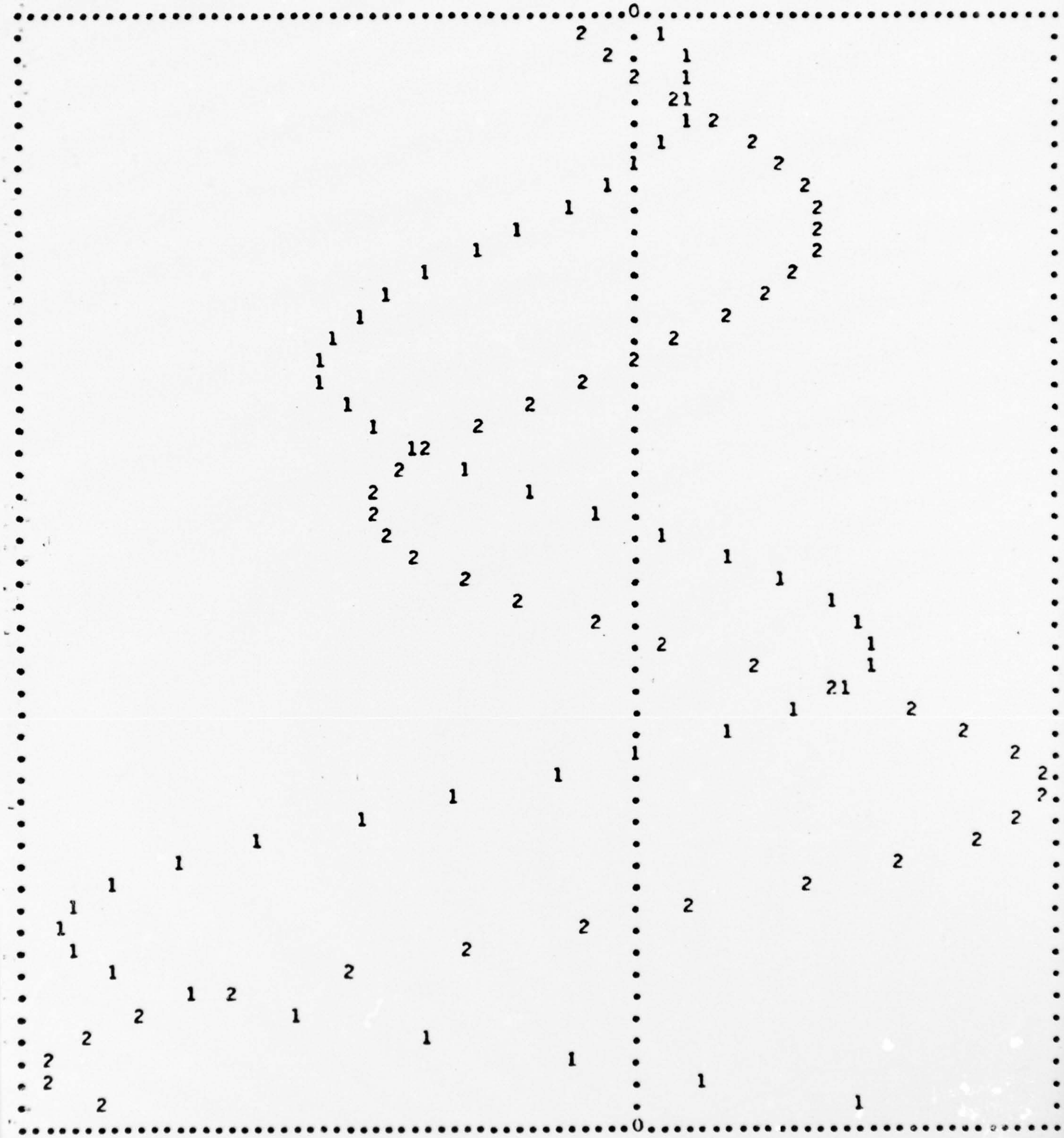


TABLE Vc

SECOND DIMENSION

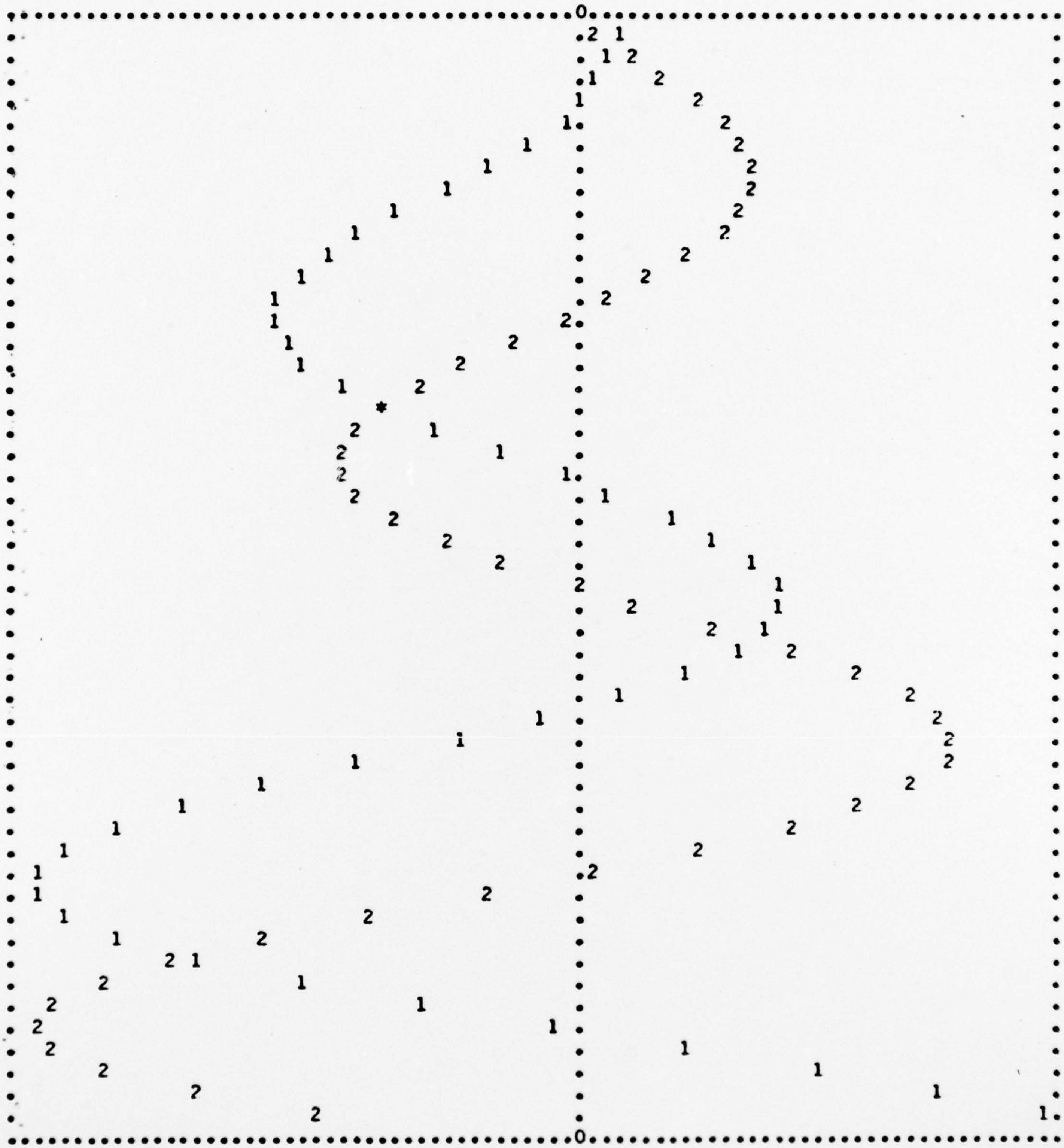


TABLE Vd

```
NATION 1'S IMPACT =
 -2.0000              .0
   .0               -2.0000

NATION 2'S IMPACT =
 -4.0000              .0
   .0               -4.0000

NATION 1'S TRAJ =
 -2.0000              .0
   .0               -2.0000

NATION 2'S TRAJ =
   .50000             .0
   .0                 .50000

NATION 1'S GOAL =        .0            .0
NATION 2'S GOAL =        10.000        15.000
```

CONCILIATORY BEHAVING AS MIXED

TABLE VIa

XMIN = -448.19 XMAX = 413.35
YMIN = -311.90 YMAX = 201.64
X SCALE = 11.189 Y SCALE = 10.480

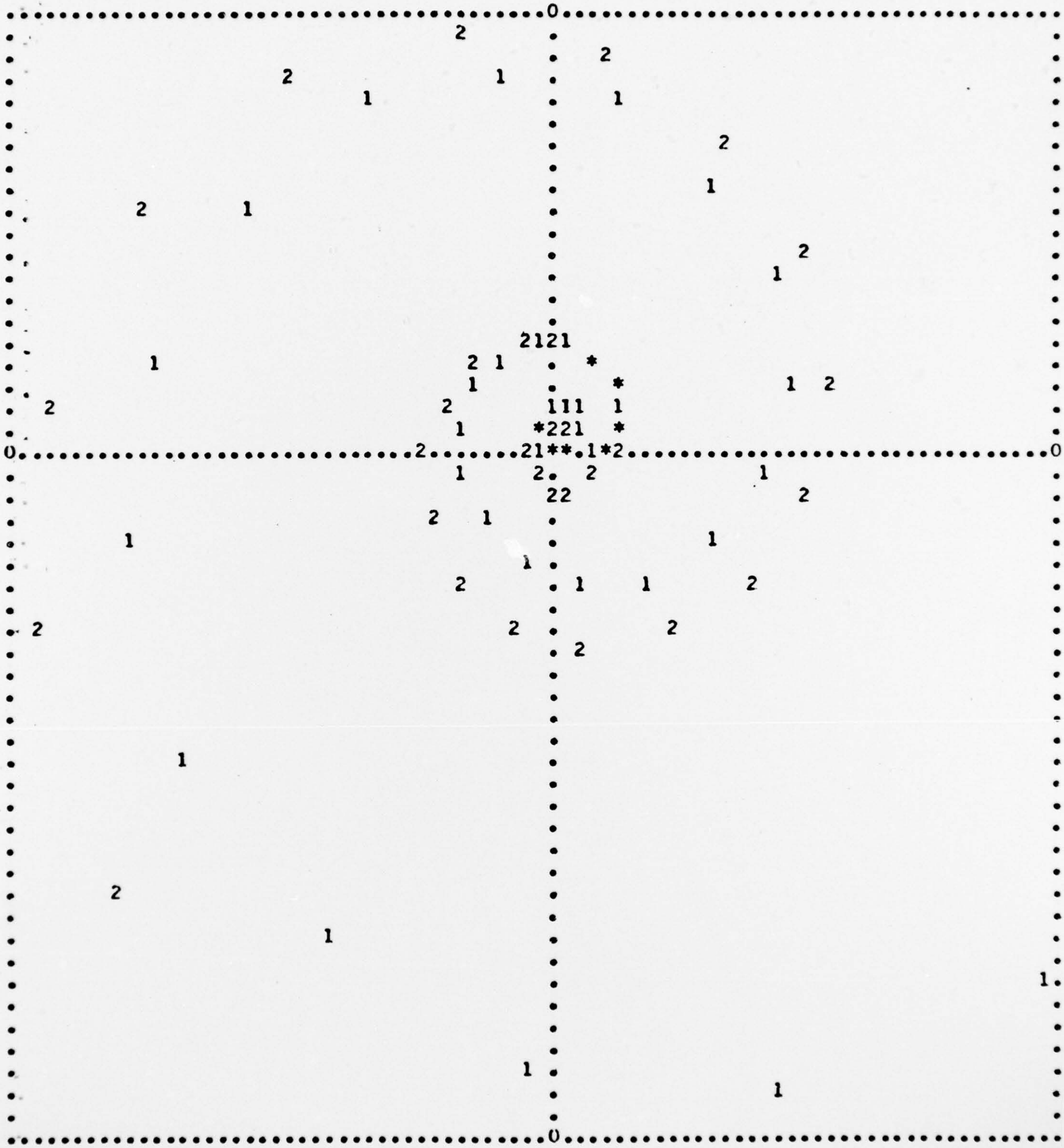


TABLE VIb

FIRST DIMENSION

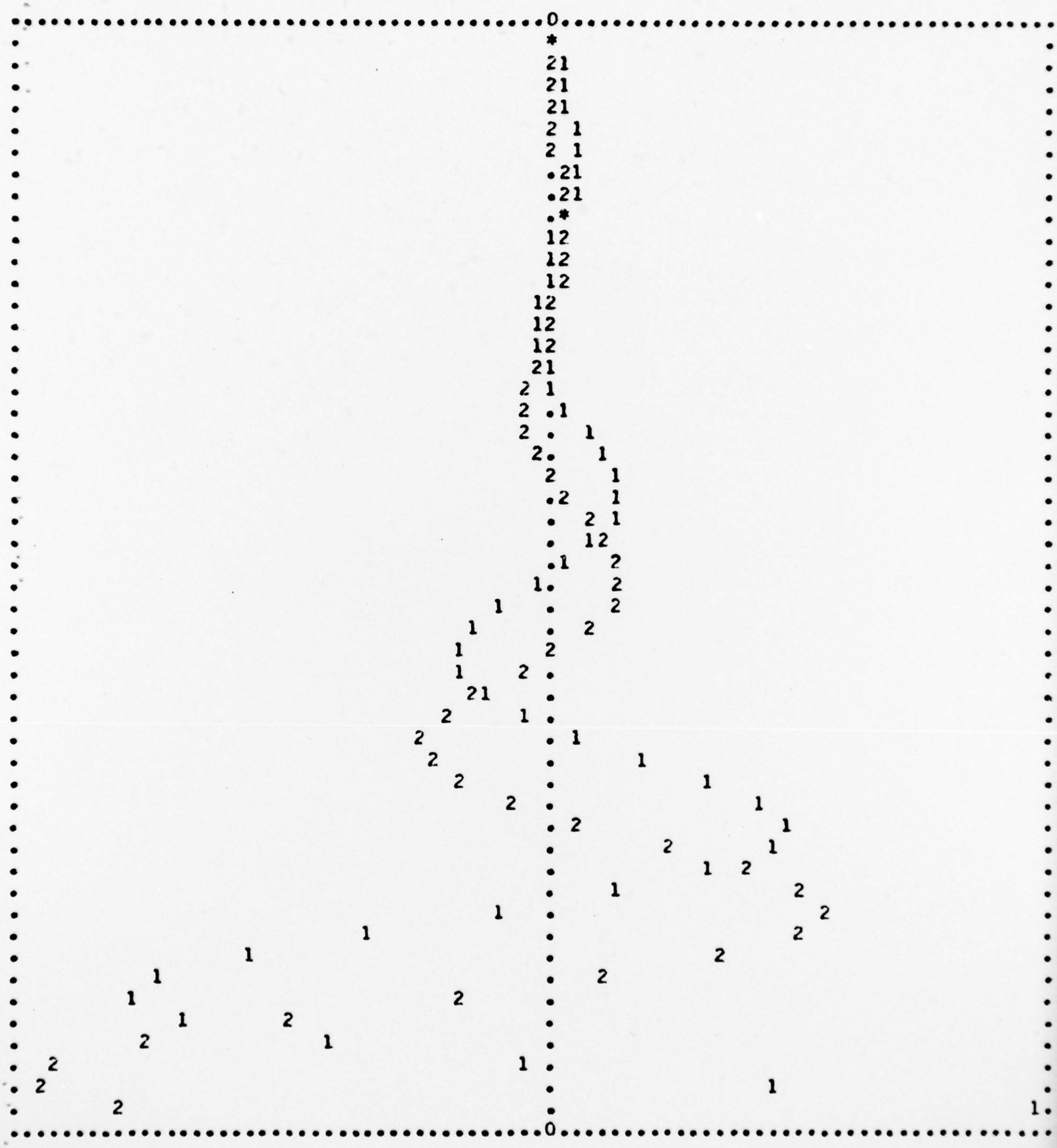


TABLE VIc

SECOND DIMENSION

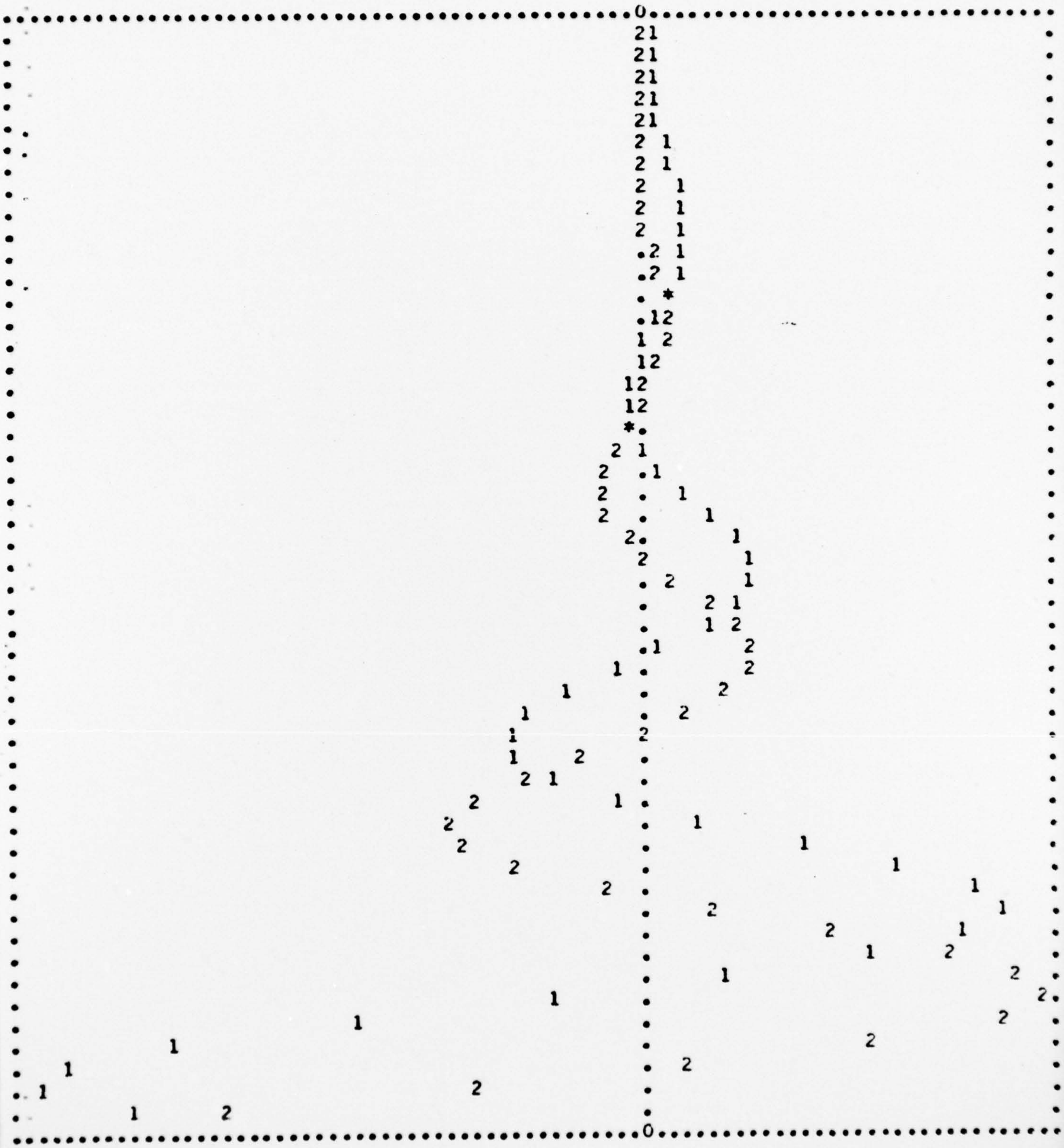


TABLE VId

```
NATION 1'S IMPACT =
 -4.0000             .0
   .0              -4.0000

NATION 2'S IMPACT =
   16.000             .0
   .0               16.000

NATION 1'S TRAJ =
 -4.0000             .0
   .0              -4.0000

NATION 2'S TRAJ =
   .50000             .0
   .0               .50000

NATION 1'S GOAL =        .0              .0
NATION 2'S GOAL =        10.000          15.000
```

MIXED BEHAVING AS FORCE

TABLE VIIa

XMIN = -.28862E+06 XMAX = .81636E+06
YMIN = -.53926E+06 YMAX = .19066E+06
X SCALE = 14350. Y SCALE = 14896.

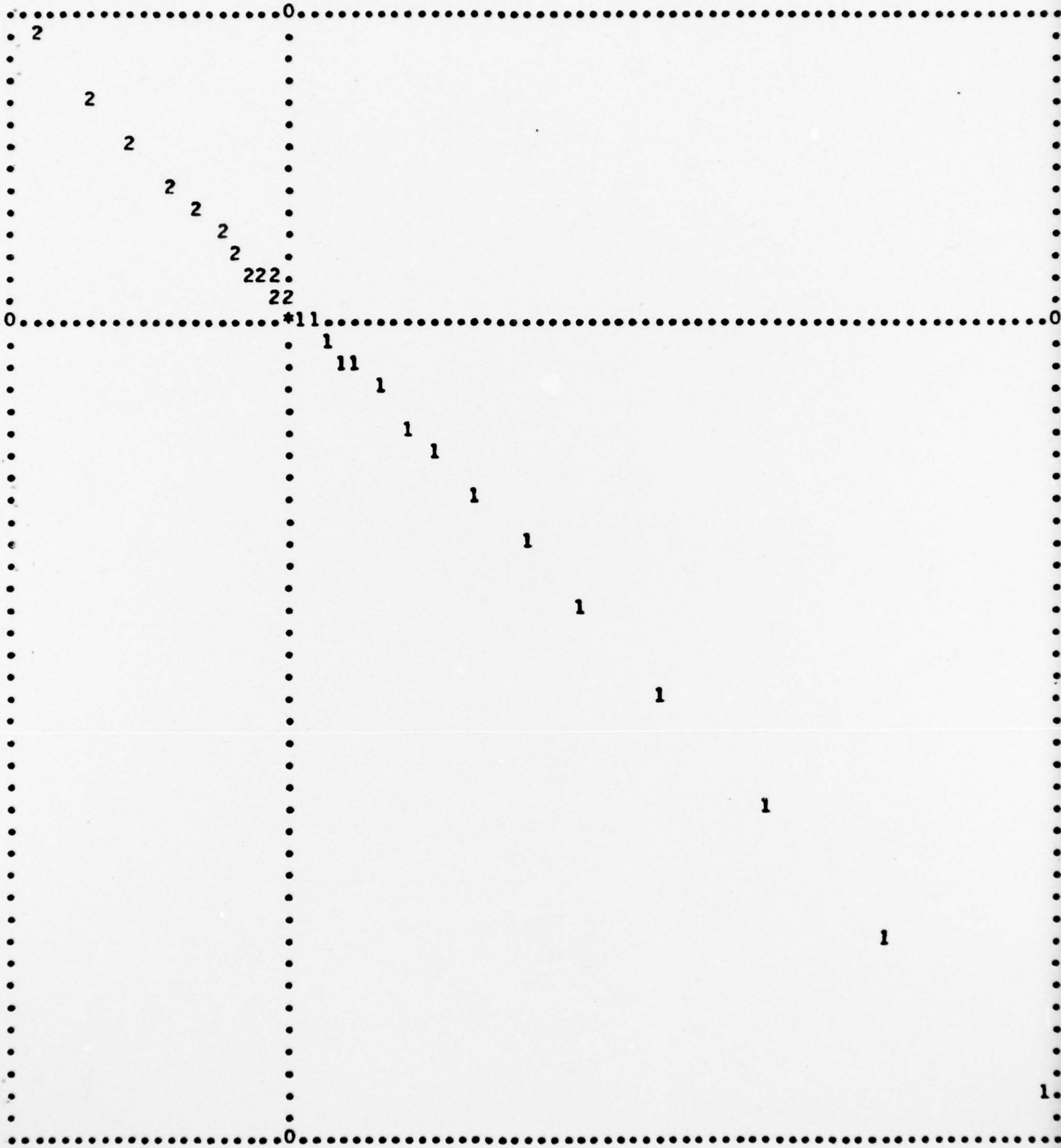


TABLE VIIb

FIRST DIMENSION

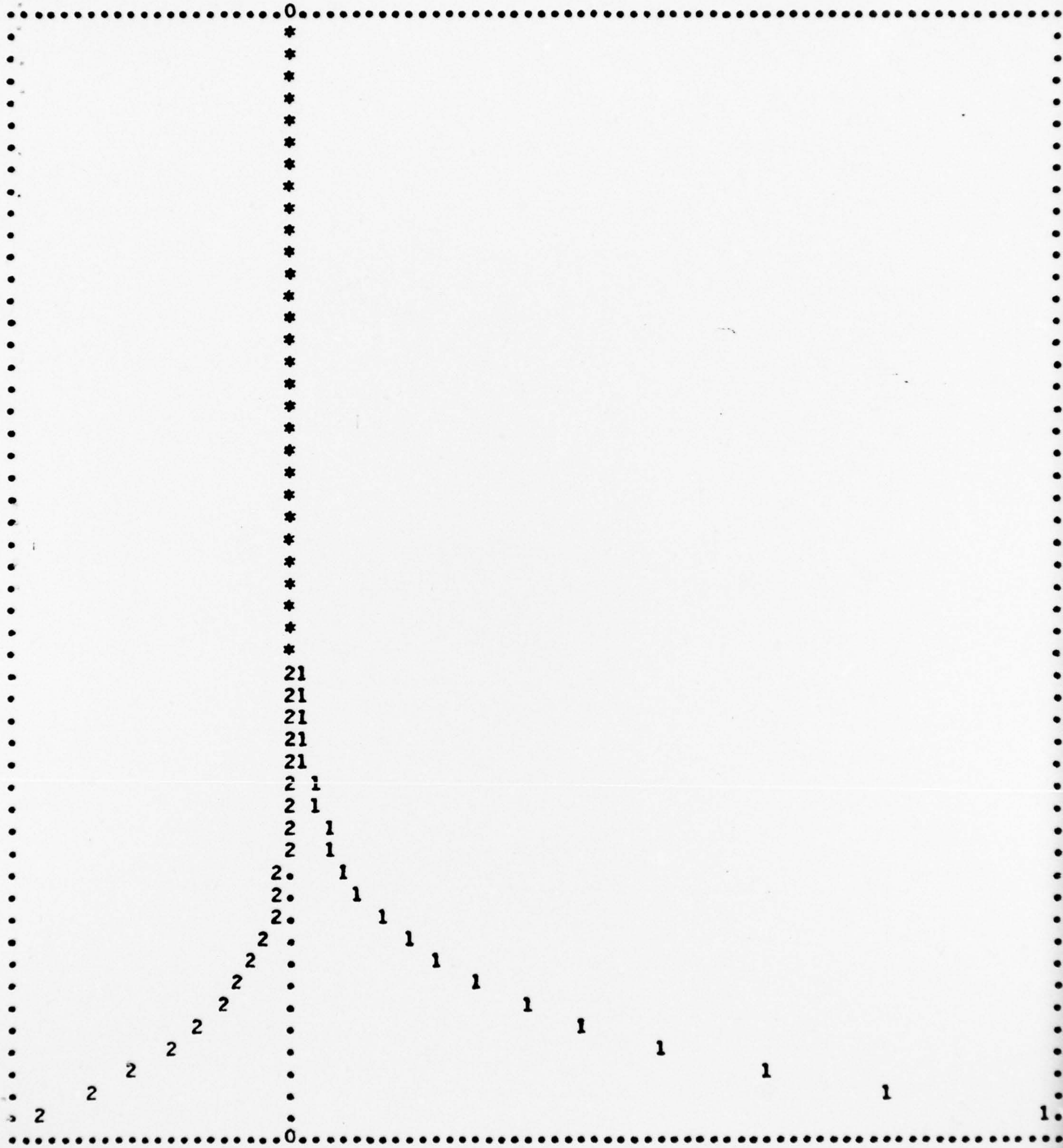


TABLE VIIc

SECOND DIMENSION

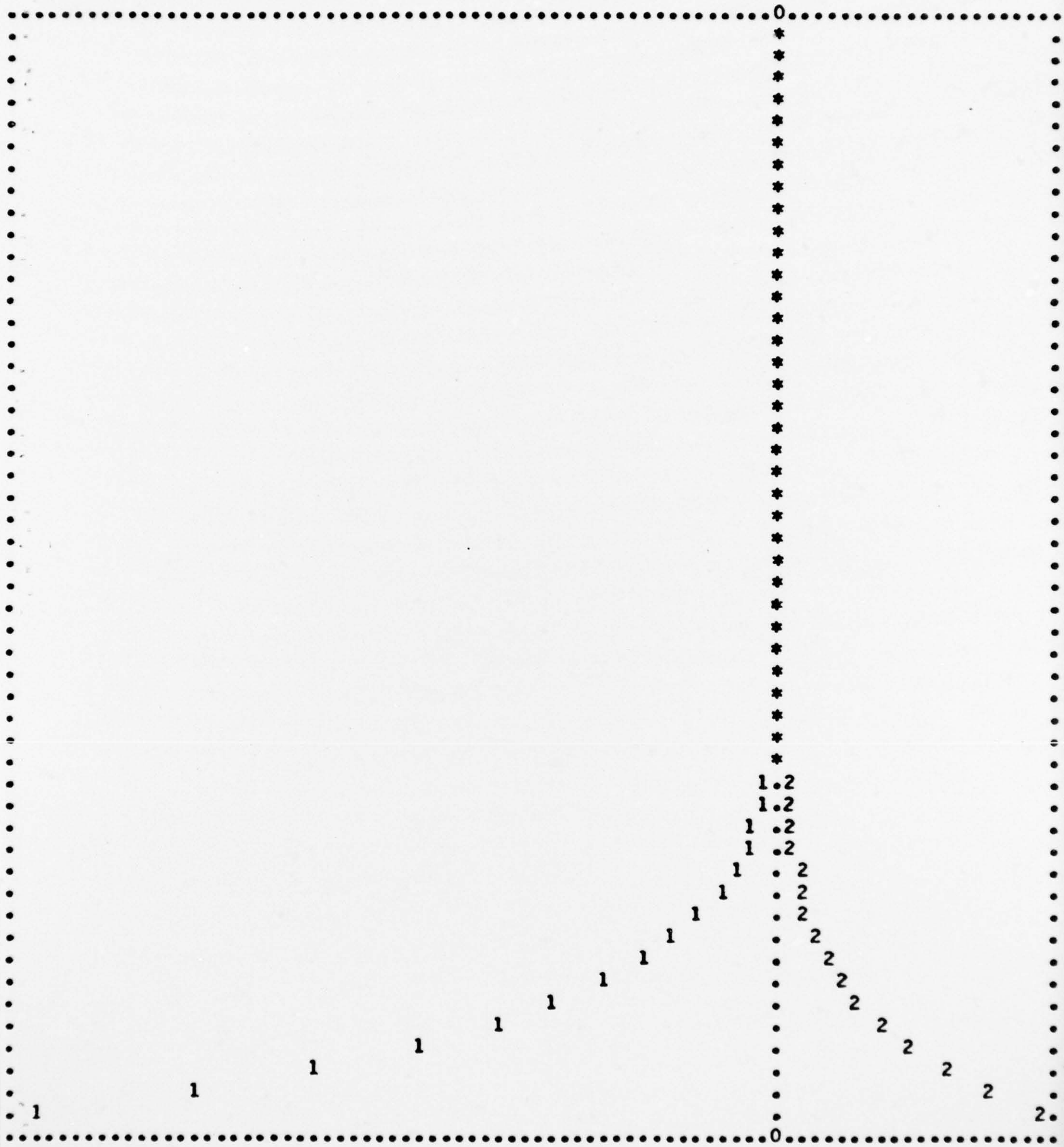


TABLE VIId

```
NATION 1'S IMPACT =
  2.0000              .0
  .0                  2.0000

NATION 2'S IMPACT =
  4.0000              .0
  .0                  4.0000

NATION 1'S TRAJ =
 -.50000              .0
  .0                 -.50000

NATION 2'S TRAJ =
 -1.0000              .0
  .0                 -1.0000

NATION 1'S GOAL =        .0              .0
NATION 2'S GOAL =        10.000          15.000
```

FORCE WITH -1 TRAJECTORY

TABLE VIIIa

XMIN = -646.25 XMAX = 115.50
YMIN = -1122.0 YMAX = 185.75
X SCALE = 9.8929 Y SCALE = 26.689

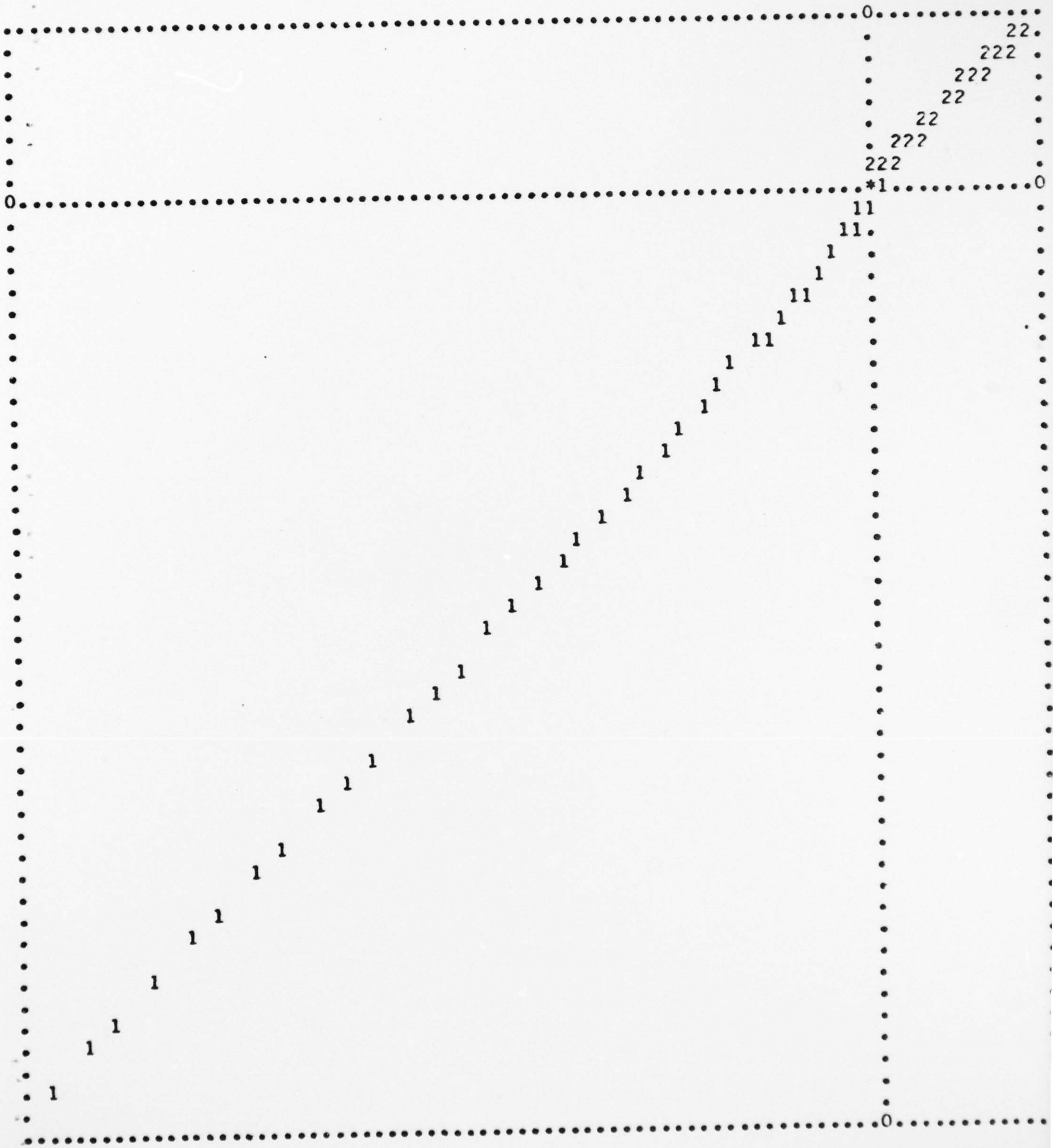


TABLE VIIIb

FIRST DIMENSION

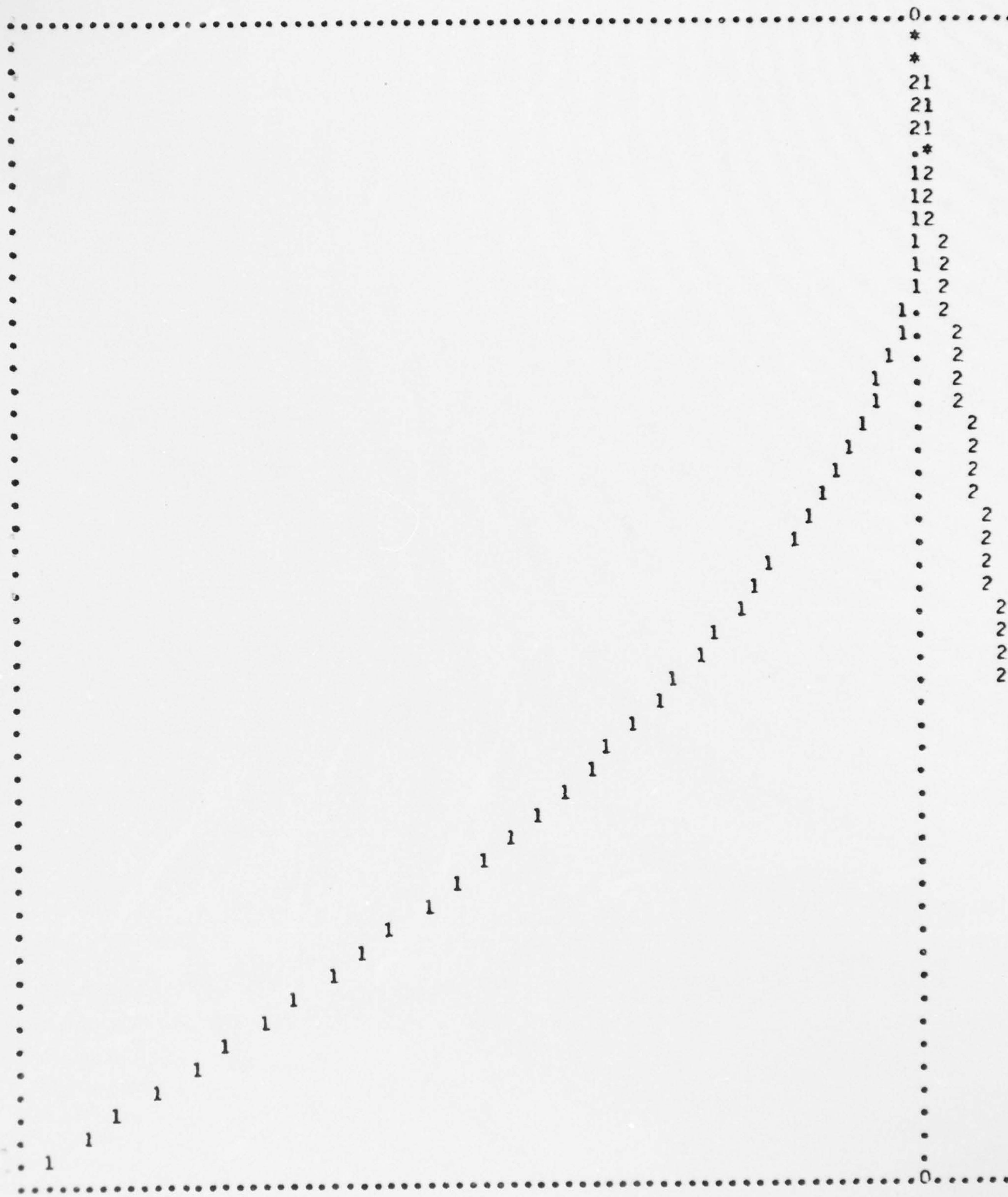


TABLE VIIIc

SECOND DIMENSION

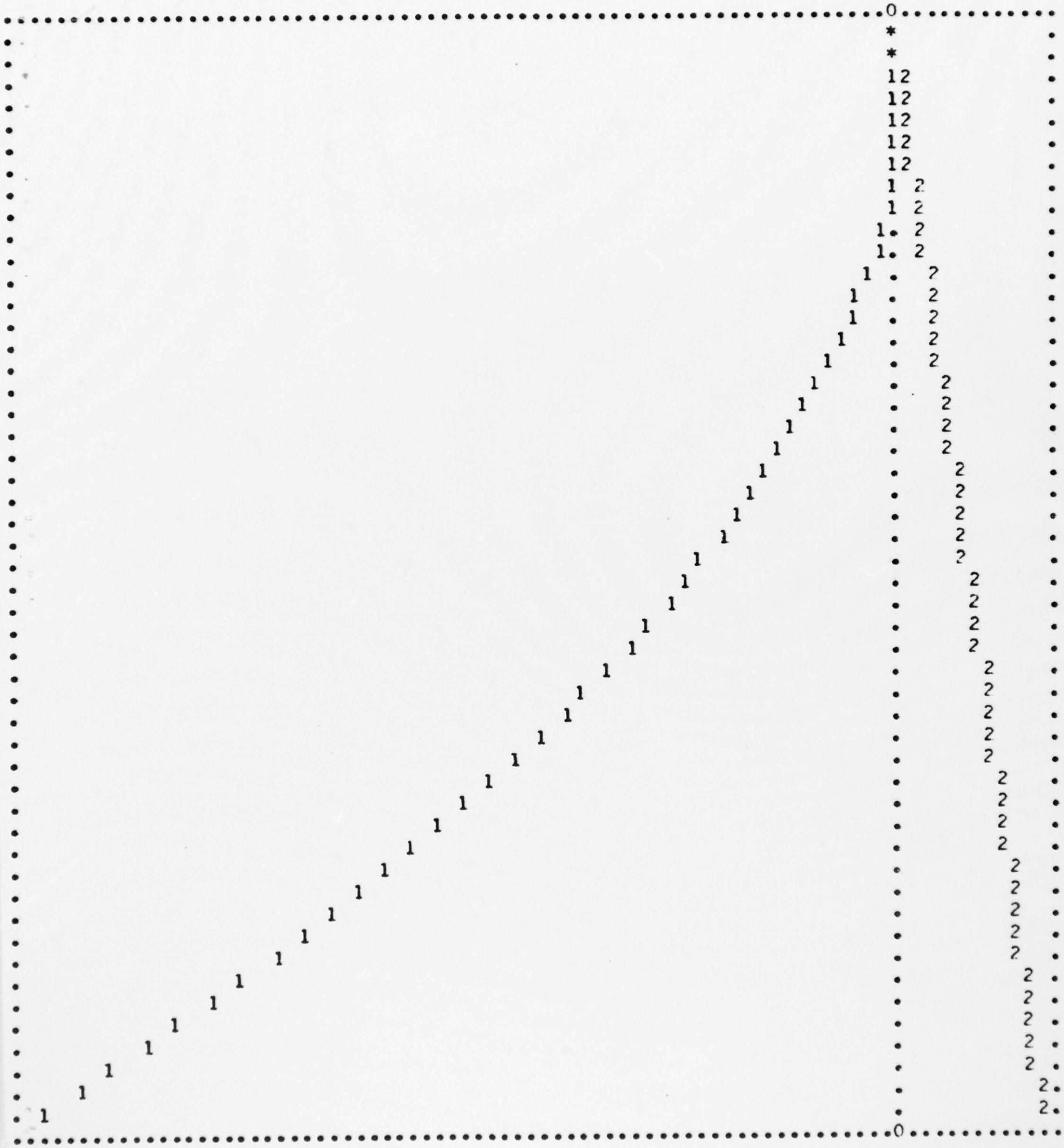


TABLE VIIId

D

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