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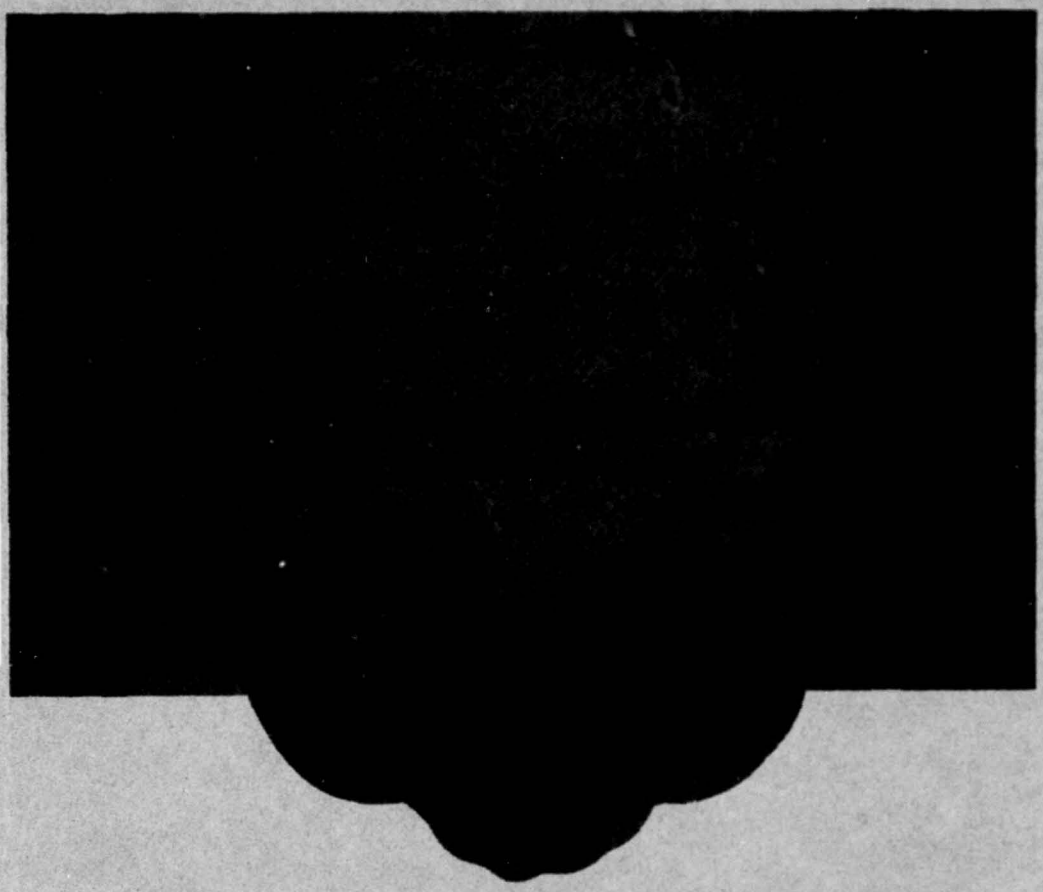

THE COVARIANCE STRUCTURE OF THE DEPARTURE PROCESS: GI/M/1 and M/G/1 Queues

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ABSTRACT

Formulae, suitable for computation, for the lag n autocorrelation of departure intervals in the GI/M/1 queues are developed for n = 1 to 6. A generating function, comparable to the one known for the M/G/1 case, is found for GI/M/1. For the $E_k/M/1$ and $M/E_k/1$ queues, expressions for arbitrary lag are given. Tables and the APL functions used to produce them are provided for $E_k/M/1$, H/M/1, and $M/E_k/1$.

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1. Introduction

Knowledge of the departure process of a queue is essential to the analysis of networks in which the output of one queue forms the input to another. Of particular interest is the covariance structure of the departures. The work in this area to date is summarized by Reynolds (1975) and Daley (1976).

The papers serving as points of departure for this report are those of Jenkins (1966) and Daley (1968). In the former, the lag one and two autocorrelations of departure intervals are derived for queues with independent Erlangian service times and Poisson arrivals. His method may be extended to general service time distributions, but only with considerable difficulty to lags of greater than two. Daley (1968) provides a generating function for the autocovariances of M/G/1 departure intervals. He also gives a rather difficult expression for the lag n covariance in the GI/M/1 case, suggesting that it can be used to derive a generating function. This derivation, the result of which has a comparatively simple form, is carried out in section 3 of this paper.

The main purpose here is to convert the GI/M/1 and M/G/1 generating functions for the departure interval covariances to explicit formulae which may be used directly to obtain results for special cases. Tables of autocorrelations are given for $E_k/M/1$, $H/M/1$, and $M/E_k/1$ queues with various traffic intensities. The special Erlang and hypergeometric distributions were chosen because they are so frequently suggested as approximations to actual arrival and/or service distributions. To match a coefficient of variation less than one, an Erlang is used; otherwise, the hypergeometric.

2. Notation

The queue assumed throughout possesses a single server and infinite waiting room. Customers arrive according to a renewal process and are served in order of arrival. Service times are i.i.d. and independent of the arrival process. Notation is as follows:

T_n = interval between arrivals of customers n and n + 1

W_n = time spent in queue (exclusive of service) by customer n

S_n = service time for customer n

D_n = interval between departures of customers n - 1 and n

Arrival intervals and service times have distribution functions A(x) and B(x), respectively, with associated Laplace-Stieltjes transforms

$$A^*(s) = E[e^{-sT_n}] \text{ and } B^*(s) = E[e^{-sS_n}] .$$

The rate of arrivals is λ, the service rate is μ, and the traffic intensity λ/μ is denoted by ρ.

E_k stands for the (special) Erlang distribution with k phases, p.d.f.

$$f(t) = \frac{(k\lambda)^k t^{k-1} e^{-k\lambda t}}{(k-1)!}, \quad t \geq 0.$$

H stands for the hypergeometric distribution with p.d.f.

$$f(t) = 2q^2\lambda e^{-2q\lambda t} + 2(1-q)^2\lambda e^{-2(1-q)\lambda t}, \quad t \geq 0,$$

where $0 < q < .5$.

3. Autocorrelations of Departure Intervals: GI/M/1.

Daley (1968) gives as his Theorem 5 the result that the departure intervals in a stationary GI/M/1 queue have

$$(3.1) \qquad \operatorname{cov}(D_o, D_n) = \left(\frac{1}{\rho} - \frac{1}{s_o}\right)\left(\frac{1}{\mu}\right)\left[E(D_n \mid W_o = 0) - E(D_o)\right],$$

where s_o is the root of smallest absolute value of

$$s = A^*(\mu[1 - s]),$$

or alternatively,

$$(3.2) \qquad \operatorname{cov}(D_o, D_n) = \left(\frac{1}{\rho} - \frac{1}{s_o}\right)\left(\frac{1}{\mu}\right)E(W_n - W_{n-1} \mid W_o = 0).$$

A method to find the expectation in (3.2) is given in Takács (1962). It is simpler, however, to use Takács' generating function for $E(W_n \mid W_o = 0)$ to derive a generating function for the autocovariances, then use the result to obtain explicit expressions for $\operatorname{cov}(D_o, D_n)$.

Making certain changes in equation (28) on page 121 of Takács (customer n here corresponds to his customer n+1), one obtains

$$(3.3) \qquad \sum_{n=1}^{\infty} E[e^{-sW_{n-1}} \mid W_o = 0]z^n = \frac{z[1 - s_1(z)]}{(1-z)\left[1 - \frac{\mu s_1(z)}{\mu + s}\right]},$$

where $s_1(z)$ is the root of smallest absolute value of

$$s = z A^*(\mu[1 - s]) .$$

Evaluating at $s = 0$ the partial derivative of each side with respect to s results in

$$\sum_{n=1}^{\infty} E(W_{n-1}|W_o = 0)z^n = \frac{z\, s_1(z)}{\mu(1 - z)[1 - s_1(z)]} ,$$

and

$$\begin{aligned}\sum_{n=1}^{\infty} E(W_n|W_o = 0)z^n &= \sum_{n=0}^{\infty} E(W_n|W_o = 0)z^n \\ &= \sum_{n=1}^{\infty} E(W_{n-1}|W_o = 0)z^{n-1} \\ &= \frac{s_1(z)}{\mu(1 - z)[1 - s_1(z)]} .\end{aligned}$$

Combining these yields

$$\sum_{n=1}^{\infty} E(W_n - W_{n-1}|W_o = 0)z^n = \frac{s_1(z)}{\mu[1 - s_1(z)]} , \quad |z| < 1 . \tag{3.4}$$

Substituting in (3.2), the desired generating function is found to be

$$\sum_{n=1}^{\infty} \mathrm{cov}(D_o, D_n)z^n = \left(\frac{1}{\rho} - \frac{1}{s_o}\right)\left(\frac{1}{\mu^2}\right)\left\{\frac{s_1(z)}{1 - s_1(z)}\right\} , \tag{3.5}$$

where s_o and $s_1(z)$ are the roots of smallest absolute value of $s = A^*(\mu[1 - s])$ and $s = zA^*(\mu[1 - s])$, respectively.

To determine the autocovariance lag n for specific n, it is first necessary to expand $s_1(z)$ in z. Takács (Lemma 1, page 113) provides the formula

$$(3.6) \qquad s_1(z) = z \sum_{j=1}^{\infty} \frac{(-\mu z)^{j-1}}{j!} \left[\frac{d^{j-1} \{A^*(s)\}^j}{d\, s^{j-1}} \right]_{s=\mu} .$$

For compactness of notation, let $\alpha_j = \left. \frac{d^j A^*(s)}{d\, s^j} \right|_{s=\mu}$.

Then

$$s_1(z) = z\alpha_0 - z^2\mu\alpha_0\alpha_1 + \frac{z^3\mu^2}{2!}\{2\alpha_0\alpha_1^2 + \alpha_0^2\alpha_2\}$$
$$- \frac{z^4\mu^3}{3!}\{6\alpha_0\alpha_1^3 + 9\alpha_0^2\alpha_1\alpha_2 + \alpha_0^3\alpha_3\}$$
$$+ \frac{z^5\mu^4}{4!}\{24\alpha_0\alpha_1^4 + 72\alpha_0^2\alpha_1^2\alpha_2 + 12\alpha_0^3\alpha_2^2 + 16\alpha_0^3\alpha_1\alpha_3 + \alpha_0^4\alpha_4\}$$
$$- \ldots \quad ;$$

$$[s_1(z)]^2 = z^2\alpha_0^2 - 2z^3\mu\alpha_0^2\alpha_1 + z^4\mu^2\{3\alpha_0^2\alpha_1^2 + \alpha_0^3\alpha_2\} \ldots ;$$

$$[s_1(z)]^3 = z^3\alpha_0^3 - 3z^4\mu\alpha_0^3\alpha_1 + 2^5\mu^2\{6\alpha_0^3\alpha_1^2 + \tfrac{3}{2}\alpha_0^4\alpha_2\} \ldots ;$$

and so on.

Carrying out these expansions to a number of terms, working out the first six terms of

$$\frac{s_1(z)}{1-s_1(z)} = \sum_{j=1}^{\infty} [s_1(z)]^j \quad ,$$

and multiplying by $(\frac{1}{\rho} - \frac{1}{s_0})(\frac{1}{\mu^2})$, one obtains the desired autocovariances. From Daley (1968),

$$(3.7) \qquad \mathrm{Var}(D_0) = \mathrm{Var}(T_0) - (\frac{1}{\rho} - \frac{1}{s_0}) \frac{2s_0}{\mu^2(1-s_0)} \quad .$$

The autocorrelations, lags 1 through 6, of departure intervals for the GI/M/1 queue are as follows:

(3.8)
$$\text{corr}(D_0,D_1) = M\alpha_0$$

$$\text{corr}(D_0,D_2) = M[\alpha_0^2 - \mu\alpha_0\alpha_1]$$

$$\text{corr}(D_0,D_3) = M[\alpha_0^3 - 2\mu\alpha_0^2\alpha_1 + \frac{\mu^2}{2!}(2\alpha_0\alpha_1^2 + \alpha_0^2\alpha_2)]$$

$$\text{corr}(D_0,D_4) = M[\alpha_0^4 - 3\mu\alpha_0^3\alpha_1 + \frac{\mu^2}{2!}(6\alpha_0^2\alpha_1^2 + 2\alpha_0^3\alpha_2) - \frac{\mu^3}{3!}(6\alpha_0\alpha_1^3 + 9\alpha_0^2\alpha_1\alpha_2 + \alpha_0^3\alpha_3)]$$

$$\text{corr}(D_0,D_5) = M[\alpha_0^5 - 4\mu\alpha_0^4\alpha_1 + \frac{\mu^2}{2!}(12\alpha_0^3\alpha_1^2 + 3\alpha_0^4\alpha_2) - \frac{\mu^3}{3!}(24\alpha_0^2\alpha_1^3 + 24\alpha_0^3\alpha_1\alpha_2 + 2\alpha_0^4\alpha_3) + \frac{\mu^4}{4!}(24\alpha_0\alpha_1^4 + 72\alpha_0^2\alpha_1^2\alpha_2 + 12\alpha_0^3\alpha_2^2 + 16\alpha_0^3\alpha_1\alpha_3 + \alpha_0^4\alpha_4)]$$

$$\text{corr}(D_0,D_6) = M[\alpha_0^6 - 5\mu\alpha_0^5\alpha_1 + \frac{\mu^2}{2!}(20\alpha_0^4\alpha_1^2 + 4\alpha_0^5\alpha_2) - \frac{\mu^3}{3!}(60\alpha_0^3\alpha_1^3 + 45\alpha_0^4\alpha_1\alpha_2 + 3\alpha_0^5\alpha_3) + \frac{\mu^4}{4!}(120\alpha_0^2\alpha_1^4 + 240\alpha_0^3\alpha_1^2\alpha_2 + 30\alpha_0^4\alpha_2^2 + 40\alpha_0^4\alpha_1\alpha_3 + 2\alpha_0^5\alpha_4) - \frac{\mu^5}{5!}(120\alpha_0\alpha_1^5 + 600\alpha_0^2\alpha_1^3\alpha_2 + 300\alpha_0^3\alpha_1\alpha_2^2 + 200\alpha_0^3\alpha_1^2\alpha_3 + 50\alpha_0^4\alpha_2\alpha_3 + 25\alpha_0^4\alpha_1\alpha_4 + \alpha_0^5\alpha_5)]$$

where $M = \frac{1}{\mu^2}\left[\text{Var } T_0\left(\frac{1}{\rho} - \frac{1}{s_0}\right)^{-1} - \frac{2s_0}{\mu^2(1-s_0)}\right]^{-1}$, with s_0 and α_j defined as above.

There apparently exists an expression for lag of arbitrary n, starting

(3.9)
$$\text{corr}(D_0,D_n) = M\{\alpha_0^n - (n-1)\mu\alpha_0^{n-1}\alpha_1 + \frac{\mu^2}{2!}\left[\frac{(n-1)!}{(n-3)!}\alpha_0^{n-2}\alpha_1^2 + (n-2)\alpha_0^{n-1}\alpha_2\right] - \frac{\mu^3}{3!}\left[\frac{(n-1)!}{(n-4)!}\alpha_0^{n-3}\alpha_1^3 + 3(n-1)(n-3)\alpha_0^{n-2}\alpha_1\alpha_2 + (n-3)\alpha_0^{n-1}\alpha_3\right] + \ldots\},$$

but its complete statement requires further work.

For the $E_k/M/1$ case,

$$\alpha_j = \frac{(k+n-1)!}{(k-1)!} \left(\frac{-1}{k\rho\mu}\right)^n \left(\frac{k\rho}{k\rho+1}\right)^{k+j} ,$$

and inspection of the results of substituting this in (3.8) reveals a general formula:

$$\text{(3.10)} \qquad \text{corr}(D_0,D_n) = M\left(\frac{k\rho}{k+1}\right)^{nk} \sum_{i=0}^{n-1} \binom{nk+i}{i} \frac{k(n-i)}{(nk+i)(\rho k)^i} \left(\frac{k\rho}{k\rho+1}\right)^i ,$$

where $M = \left[\frac{1}{k\rho^2}\left(\frac{1}{\rho} - \frac{1}{s_0}\right)^{-1} - \frac{2s_0}{1-s_0}\right]^{-1}$.

Tables for the autocorrelations of lags one through five for $E_k/M/1$ queues with various k and ρ, and for $H/M/1$ queues with various q and ρ, are given in section 5 (Table I). The last column of these tables, labelled "TOTAL", contains the values of

$$\text{(3.11)} \qquad \sum_{n=1}^{\infty} \text{corr}(D_0,D_n) = M\left(\frac{s_0}{1-s_0}\right) .$$

"CV" denotes the coefficient of variation of T_0 for the given k or q.

To find M requires determination of the root s_0. The method of "false position", which was recommended for this problem by Sahin (1970), proved quite fast, particularly when used with Aitken's method of acceleration. The algorithm and computer routine used are described in Appendix 1. The other APL functions required are displayed and discussed briefly in Appendix 2.

4. Autocorrelations of Departure Intervals: M/G/1.

For the M/G/1 case, Daley (1968) provides the generating function:

$$(4.1) \qquad \sum_{n=1}^{\infty} \operatorname{cov}(D_0, D_n) z^n = \left(\frac{1-\rho}{\lambda^2}\right)\left\{\frac{s_1(z)-z}{(1-z)[1-s_1(z)]} + \frac{z s_1'(z) - s_1(z)}{s_1(z) s_1'(z)(1-z)}\right\} \quad |z| < 1,$$

where $s_1(z)$ is the root of smallest absolute value of

$$s_1 = zB^*(\lambda[1 - s]),$$

and

$$s_1'(z) = \left.\frac{d\, s_1'(x)}{dx}\right|_{x=z} .$$

The expansion of $s_1(z)$ is the same as that given in section 3, with $\beta_j = \left.\frac{d^j B^*(s)}{d\, s^j}\right|_{s=\lambda}$ replacing α_j . Also needed is

$$s_1'(z) = \beta_0 - 2z\lambda\beta_0\beta_1 + \frac{3z^2\lambda^2}{2!}[2\beta_0\beta_1^2 + \beta_0^2\beta_2]$$
$$- \frac{4z^3\lambda^3}{3!}[6\beta_0\beta_1^3 + 9\beta_0^2\beta_1\beta_2 + \beta_0^3\beta_3] + \ldots \quad .$$

Note that s_0 is not involved here. However, in other aspects the M/G/1 case is computationally more difficult than the GI/M/1. The expression in brackets in (4.1) can be rewritten as

$$(4.2) \qquad (1 + z + z^2 + \ldots)\{[s_1(z) - z][1 + s_1(z) + s_1^2(z) + \ldots] + \frac{z}{s_1(z)} - \frac{1}{s_1'(z)}\} .$$

By methods similar to those of section 3 one obtains

$$(4.3)\qquad [s_1(z) - z][1 + s_1(z) + s_1^2(z) + \ldots] = z[-1 + \beta_0] - z^2[\lambda\beta_0\beta_1 + \beta_0 - \beta_0^2]$$
$$+ z^3[\lambda^2(\beta_0\beta_1^2 + \tfrac{1}{2}\beta_0^2\beta_2) + \lambda(\beta_0\beta_1 - 2\beta_0^2\beta_1) - \beta_0^2 + \beta_0^3]$$
$$- z^4[\lambda^3(\beta_0\beta_1^3 + \tfrac{3}{2}\beta_0^2\beta_1\beta_2 + \tfrac{1}{6}\beta_0^3\beta_3)$$
$$+ \lambda^2(\beta_0\beta_1^2 + \tfrac{1}{2}\beta_0^2\beta_2 - 3\beta_0^2\beta_1^2 - \beta_0^3\beta_2)$$
$$+ \lambda(-2\beta_0^2\beta_1 + 3\beta_0^3\beta_1) + \beta_0^3 - \beta_0^4] + \ldots \quad .$$

The remaining two terms in (4.2) must be expanded in Taylor series about zero ($s_1(z)$ and $s_1'(z)$ are non-vanishing in $|z| < 1$). The results of this tedious procedure are

$$(4.4)\qquad \frac{z}{s_1(z)} = \frac{1}{\beta_0} + z\lambda\left(\frac{\beta_1}{\beta_0}\right) - \frac{z^2\lambda^2}{2!}(\beta_2) + \frac{z^3\lambda^3}{3!}(3\beta_1\beta_2 + \beta_0\beta_3)$$
$$- \frac{z^4\lambda^4}{4!}(12\beta_1^2\beta_2 + 6\beta_0\beta_2^2 + 8\beta_0\beta_1\beta_3 + \beta_0^2\beta_4) + \ldots$$

and

$$(4.5)\qquad \frac{1}{s_1'(z)} = \frac{1}{\beta_0} + z\lambda\left(\frac{2\beta_1}{\beta_0}\right) - \frac{z^2\lambda^2}{2!}\left(3\beta_2 - \frac{2\beta_1^2}{\beta_0}\right)$$
$$+ \frac{z^3\lambda^3}{3!}(4\beta_0\beta_3) - \frac{z^4\lambda^4}{4!}(6\beta_0\beta_2^2 + 16\,\beta_0\beta_1\beta_3 + 5\beta_0^2\beta_4) + \ldots \quad .$$

Subtracting (4.5) from (4.4), adding (4.3), then multiplying by $\sum_{j=0}^{\infty} z^j$, one finally has an expansion for (4.2). Using this and

(4.6) $$\mathrm{Var}(D_0) = \mathrm{Var}(S_0) + \left(\frac{1-\rho^2}{\lambda^2}\right),$$

the autocorrelations for the M/G/1 queue are found to be

(4.7)
$$\mathrm{corr}(D_0,D_1) = M\left[-1 + \beta_0 - \frac{\lambda\beta_1}{\beta_0}\right]$$

$$\mathrm{corr}(D_0,D_2) = M\left[-1 + \beta_0^2 - \lambda\left(\frac{\beta_1}{\beta_0} + \beta_0\beta_1\right) + \lambda^2\left(\beta^2 - \frac{\beta_1^2}{\beta_0}\right)\right]$$

$$\mathrm{corr}(D_0,D_3) = M\left[-1 + \beta_0^3 - \lambda\left(\frac{\beta_1}{\beta_0} + 2\beta_0^2\beta_1\right) + \lambda^2\left(\beta_2 - \frac{\beta_1^2}{\beta_0} + \beta_0\beta_1^2 + \frac{\beta_0^2\beta_2}{2}\right)\right.$$
$$\left. - \lambda^3\left(\frac{-\beta_1\beta_2}{2} + \frac{\beta_0\beta_3}{2}\right)\right]$$

$$\mathrm{corr}(D_0,D_4) = M\left[-1 + \beta_0^4 - \lambda\left(\frac{\beta_1}{\beta_0} + \beta_0^3\beta_1\right) + \lambda^2\left(\beta_2^2 - \frac{\beta_1^2}{\beta_0} + 3\beta_0^2\beta_1^2 + \beta_0^3\beta_2\right)\right.$$
$$- \lambda^3\left(\frac{-\beta_1\beta_2}{2} + \frac{\beta_0\beta_3}{2} + \beta_0\beta_1^3 + \frac{3\beta_0^2\beta_1\beta_2}{2} + \frac{\beta_0^3\beta_3}{6}\right)$$
$$\left. + \lambda^4\left(\frac{-\beta_1^2\beta_2}{2} + \frac{\beta_0\beta_1\beta_3}{3} + \frac{\beta_0^2\beta_4}{6}\right)\right]$$

$$\mathrm{corr}(D_0,D_5) = M\left[-1 + \beta_0^5 - \lambda\left(\frac{\beta_1}{\beta_0} + 4\beta_0^4\beta_1\right) + \lambda^2\left(\beta_2 - \frac{\beta_1^2}{\beta_0} + 6\beta_0^3\beta_1^2 + \frac{3\beta_0^4\beta_2}{2}\right)\right.$$
$$- \lambda^3\left[\frac{-\beta_1\beta_2}{2} + \frac{\beta_0\beta_3}{2} + 4\beta_0^2\beta_1^3 + 4\beta_0^3\beta_1\beta_2 + \frac{\beta_0^4\beta_3}{3}\right)$$
$$+ \lambda^4\left(\frac{-\beta_1^2\beta_2}{2} + \frac{\beta_0\beta_1\beta_3}{3} + \frac{\beta_0^2\beta_4}{6} + \beta_0\beta_1^4 + 3\beta_0^2\beta_1^2\beta_2\right.$$
$$\left. + \frac{\beta_0^3\beta_2^2}{2} + \frac{2\beta_0^3\beta_1\beta_3}{3} + \frac{\beta_0^4\beta_4}{24}\right)$$
$$\left. - \lambda^5\left(\frac{-\beta_1^3\beta_2}{2} - \frac{\beta_0\beta_1\beta_2^2}{4} + \frac{\beta_0\beta_1^2\beta_3}{6} + \frac{\beta_0^2\beta_2\beta_3}{4} + \frac{7\beta_0^2\beta_1\beta_4}{24} + \frac{\beta_0^3\beta_5}{24}\right)\right]$$

$$\begin{aligned}\text{corr}(D_0,D_6) = M[&-1 + \beta_0^6 - \lambda(\frac{\beta_1}{\beta_0} + 5\beta_0^5\beta_1) + \lambda^2(\beta_2 - \frac{\beta_1^2}{\beta_0} + 10\beta_0^4\beta_1^2 \\ &+ 2\beta_0^5\beta_2) - \lambda^3(\frac{-\beta_1\beta_2}{2} + \frac{\beta_0\beta_3}{2} + 10\beta_0^3\beta_1^3 + \frac{15\beta_0^4\beta_1\beta_2}{2} + \frac{\beta_0^5\beta_3}{2}) \\ &+ \lambda^4(\frac{-\beta_1^2\beta_2}{2} + \frac{\beta_0\beta_1\beta_3}{3} + \frac{\beta_0^2\beta_4}{6} + 5\beta_0^2\beta_1^4 + 10\beta_0^3\beta_1^2\beta_2 \\ &+ \frac{5\beta_0^4\beta_2^2}{4} + \frac{5\beta_0^4\beta_1\beta_3}{3} + \frac{\beta_0^5\beta_4}{12} \\ &- \lambda^5(\frac{-\beta_1^3\beta_2}{2} - \frac{\beta_0\beta_1\beta_2^2}{4} + \frac{\beta_0\beta_1^2\beta_3}{6} + \frac{\beta_0^2\beta_2\beta_3}{4} + \frac{7\beta_0^2\beta_1\beta_4}{24} \\ &+ \frac{\beta_0^3\beta_5}{24} + \beta_0\beta_1^5 + 5\beta_0^2\beta_1^3\beta_2 + \frac{5\beta_0^3\beta_1\beta_2^2}{2} + \frac{5\beta_0^3\beta_1^2\beta_3}{3} \\ &+ \frac{5\beta_0^4\beta_2\beta_3}{12} + \frac{5\beta_0^4\beta_1\beta_4}{24} + \frac{\beta_0^5\beta_5}{120}) + \lambda^6(\frac{-\beta_1^4\beta_2}{2} - \frac{3\beta_0\beta_1^2\beta_2^2}{4} \\ &+ \frac{\beta_0^2\beta_1\beta_2\beta_3}{2} + \frac{3\beta_0^2\beta_1^2\beta_4}{8} + \frac{\beta_0^3\beta_3^2}{12} + \frac{\beta_0^3\beta_2\beta_4}{6} + \frac{7\beta_0^3\beta_1\beta_5}{60} + \frac{\beta_0^4\beta_6}{120})],\end{aligned}$$

where

$$M = [\frac{\lambda^2\,\text{Var}(S_0)}{1-\rho} + \rho + 1]^{-1},$$

with β_j as defined above.

As in the $E_k/M/1$ case, the expressions for $M/E_k/1$ autocorrelations are considerably simpler, and may be generalized to

$$\text{(4.8)}\qquad \text{corr}(D_0,D_n) = M[-1 + \sum_{i=0}^{n-1} \binom{i(k+1)}{i} + 1)(\frac{k}{i(k+1)+1})(\frac{\rho}{k})^{i+1}(\frac{k}{k+\rho})^{i(k+1)} + (\frac{k}{k+\rho})^{nk} \sum_{i=0}^{n-1} \binom{nk+i}{i} \frac{k(n-i)}{(nk+i)} (\frac{\rho}{k})^{i} (\frac{k}{k+\rho})^{i}],$$

where

$$M = \frac{k(1-\rho)}{\rho^2(1-k)+k} .$$

This leads to the same formulae as given by Jenkins (1966) for lags one and two.

Tables for autocorrelations lags one through five for various $M/E_k/1$ queues are provided in the following section (Table II). The last column of each table gives the sum

$$\sum_{n=1}^{\infty} \text{corr}(D_0, D_n) = M\left[\frac{\rho^2(k-1)}{2k(1-\rho)} \right] . \tag{4.9}$$

"CV" is the coefficient of variation of S_0. APL functions used are described in Appendix 2.

Acknowledgements

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Appendix I. Finding the Root s_0.

The APL function used here to obtain values for the root s_0 of the equation $s = A^*(\mu[1-s])$ is based on a program written by James Kitchen. Modifications were made to take advantage of the specific situation, and to allow for a vector argument. The routine uses the method of "false position", with Aitken's acceleration applied on every third iteration.

Algorithm to find the root of $f(x) = 0$ between ℓ_0 and u_0

1. Choose upper and lower values ℓ_0 and u_0 which enclose the root.
2. Initialize i(the iteration counter) and j(the acceleration flag) to 0.
3. Set $i = i + 1$.
4. Set $x_i = \dfrac{u_i f(\ell_i) - \ell_i f(u_i)}{f(\ell_i) - f(u_i)}$, and $y_i = f(x_i)$.
5. Set $j = j + 1$, and $z_j = x_i$.
6. If $|y_i| <$ tolerance, stop. If not, go on to (7).
7. If $y_i > 0$, let $u_{i+1} = x_i$, $\ell_{i+1} = \ell_i$;
   if not, let $u_{i+1} = u_i$, $\ell_{i+1} = x_i$.
8. If $j = 3$, do acceleration routine; it not, go to (3).

Acceleration routine:

1. Let $x = z_3 - \dfrac{(z_3 - z_2)^2}{z_1 - 2z_2 + z_3}$ and $y = f(x)$.
2. If $\ell_{i+1} < x < u_{i+1}$, set $x_{i+1} = x$, $y_{i+1} = y$, and $j = 0$, then go to (6).
   If not, reject the accelerated result, setting $j = 1$, $z_1 = z_3$, and returning to (3).

The APL function SØ EROOTS K takes as arguments the number of stages K for the Erlang distribution desired, and an initial guess SØ, a vector of initial upper bounds on s_0 for the n values of ρ being considered. The initial lower bounds

used are ρ^k. When $f(s_0) = s_0 - A^*(1 - s_0) < TOL$ for each ρ, the function returns the vector of roots. Setting $TOL = 5 \times 10^{-7}$, no more than three or four iterations were necessary.

Warning: this function, as written, requires a minimum of 5 values of ρ.

```
      ∇EROOTS[□]∇
      ∇ S←S0 EROOTS K;IACCEL;XHOLD;X;Y;XT;YT;N;SGN;RHOM
[1]   ⍝FINDS ROOT FOR EK/M/1: FOR EACH INTENSITY IN VECTOR RHO
[2]   ⍝USES METHOD OF FALSE POSITION WITH AITKEN'S ACCELERATION
[3]   ⍝RHO AND TOL MUST BE SUPPLIED
[4]    XHOLD←((N←⍴RHO),3)⍴0.5
[5]    RHOM←⍉(⍴⍵X←⍵(2,N)⍴((RHO*K),S0))⍴RHO
[6]    Y←X-÷(1+(1-X)÷K×RHOM)*K
[7]    IACCEL←ITER←0
[8]   MAIN:ITER←ITER+1
[9]    XT←(-/Y×ΦX)÷-/Y
[10]   YT←XT-÷(1+(1-XT)÷K×RHO)*K
[11]   XHOLD[;IACCEL←IACCEL+1]←XT
[12]  WITHIN:→FINAL×⍳TOL>⌈/|YT
[13]   X[;SGN←(YT[1]>0)+1]←XT
[14]   Y[;SGN]←YT
[15]   →MAIN×⍳IACCEL<3
[16]  ⍝ACCELERATED X NOW TRIED
[17]   XT←XHOLD[;3]-((XHOLD[;3]-XHOLD[;1])*2)÷+/((N,3)⍴ 1 ¯2 1 )×XHOLD
[18]   YT←YT-÷(1+(1-XT)÷K×RHO)*K
[19]   →WITHIN×⍳~IACCEL←(XT[5]<X[5;1])∨XT[5]>X[5;2]
[20]   XHOLD[;1]←XHOLD[;3]
[21]   →MAIN
[22]  FINAL:S←XT
      ∇
```

Appendix II. APL Functions Used

To produce the tables for $E_k/M/1$ and H/M/1 queues, an APL function which could be altered to fit any appropriate arrival distribution was devised.

ECORR takes as arguments the starting vector (R) for the root-finding routine EROOTS described in Appendix I, and a vector of values (KV) for the Erlang parameter k. KV must be arranged in increasing size, and contain at least two values. R is usually set to RHO, the vector containing the various traffic intensities for which the autocorrelations are to be computed. The value of FORMAT used for all the tables is displayed following the functions below.

HCORR takes only one argument, the vector of parameters QV. To obtain the values for QV corresponding to a vector of coefficients of variation, use the function CONVERT. The roots for H/M/1 are simply

$$s_0 = \rho + \frac{(1-\sqrt{1-4\rho(1-\rho)(1-2q)^2})}{2},$$

and are provided to HCORR by HROOTS.

To adapt either of these functions for another arrival distribution, make the following substitutions:

(a) V ← expression for $\mathrm{Var}(T_0)$

(b) M ← expression for M [see (3.8)]

(c) to (d) expressions for C1 = α_0, A1 = α_1, A2 = α_2, A3 = α_3, and A4 = α_4

(e) Where θ is the parameter to be printed, with n decimal places, the statement should be

□ ←' θ = ', (n ⍕ θ)', CV = ', 3 ⍕ (expression for c. of v.)

(f) → number of statement labelled (a).

The function for $M/E_k/1$ is designed specifically for Erlangian service times. A routine alterable for use with any M/G/1 queue would be possible, but unwieldy. CORRE takes the argument KV, the vector of values for k, and uses the pre-set variables RHO and FORMAT described above.

```
     ∇ HCORR QV;Q;D1;D2;N1;N2;I;V;N;CT;C1;C2;C3;C4;C5;A1;A2;A3;A4
[1]  ⍝TABLE OF LAG 1,2,3,4,5 AND TOTAL AUTOCORRELATIONS FOR HC/M/1
[2]  ⍝QV IS THE VECTOR OF VALUES FOR Q; RHO AND FORMAT MUST BE PROVIDED
[3]  I←1
(a)[4]  V←(¯1+÷2×Q×1-Q←QV[I])÷RHO×2
(b)[5]  M←÷(V÷(÷RHO)-÷S)-2×S÷1-S←.RHO HROOTS Q
[6]  CT←S÷1-S
(c)[7]  C1←((N1←2×RHO×Q×2)+D1←1+2×RHO×Q)+(N2←2×RHO×(1-Q)×2)+D2←1+2×RHO×1-Q
[8]  A1←-(N1÷D1×2)÷N2÷D2×2
[9]  A2←2×(N1÷D1×3)+N2÷D2×3
[10] A3←¯6×(N1÷D1×4)+N2÷D2×4
(d)[11] A4←24×(N1÷D1×5)+N2÷D2×5
[12] C3←((C1×2)×A2÷2)+(C1-A1)×C2←C1×(C1-A1)
[13] C4←((C3×C1-A1)+((C1×2)×A2×(C1÷2)-A1)-(C1×3)×A3÷6
[14] C5←C1×((C1-A1)×4)+((A4+(3⍟A2)-3×A3)×C1×3)÷24
[15] C5←C5+C1×(((2×A1×A3÷3)+(0.5×A2×2)-4×A1×A2)×C1×2)+3×C1×A2×A1×2
[16] 10 2 ρ ' '
(e)[17] D←'                    Q = ',(4▼Q),'          CV = ',2▼(¯1+÷2×Q×1-Q)×0.5
[18] '                          AUTOCORRELATION LAG N'
[19] ' RHO      N=   1          2          3          4          5          TOTAL'
[20] ' '
[21] D←FORMAT▼RHO,⍴((6,N)⍴M)×(6,N←⍴RHO)⍴C1,C2,C3,C4,C5,CT
[22] →(I=⍴QV)⍴100
[23] I←I+1
(f)[24] →4
100  ∇

     ∇ R ECORR KV;K;D;I;V;N;CT;C1;C2;C3;C4;C5;A1;A2;A3;A4
[1]  ⍝TABLE OF LAG 1,2,3,4,5 AND TOTAL AUTOCORRELATIONS FOR EK/M/1
[2]  ⍝KV IS THE VECTOR OF VALUES FOR K; RHO AND FORMAT MUST BE PROVIDED
[3]  ⍝R IS THE START FOR EROOTS; MAY BE RHO IF DESIRED
[4]  I←1
[5]  V←÷(RHO×2)×K←KV[I]
[6]  M←÷(V÷(÷RHO)-÷S)-2×S÷1-S←R EROOTS K
[7]  CT←S÷1-S
[8]  A1←-(K÷D)×C1←(K×RHO÷D←1+(K×RHO)×K
[9]  A2←-((K+2)÷D)×A2←-((K+1)÷D)×A1
[10] A4←-((K+3)÷D)×A3
[11] C3←((C1×2)×A2÷2)+(C1-A1)×C2←C1×(C1-A1)
[12] C4←(C3×C1-A1)+((C1×2)×A2×(C1÷2)-A1)-(C1×3)×A3÷6
[13] C5←C1×((C1-A1)×4)+((A4+(3⍟A2)-3×A3)×C1×3)÷24
[14] C5←C5+C1×(((2×A1×A3÷3)+(0.5×A2×2)-4×A1×A2)×C1×2)+3×C1×A2×A1×2
[15] 10 2 ρ ' '
[16] D←'                    K = ',(6▼K),'          CV =',3▼K×¯0.5
[17] '                          AUTOCORRELATION LAG N'
[18] ' RHO      N=   1          2          3          4          5          TOTAL'
[19] ' '
[20] D←FORMAT▼RHO,⍴((6,N)⍴M)×(6,N←⍴RHO)⍴C1,C2,C3,C4,C5,CT
[21] →(I=⍴KV)⍴100
[22] I←I+⍴R←S
[23] →5
100  ∇
```

```
      ∇ CORRE KV;K;I;J;N;KRO;KRHO;T;JK
[1]   ⍝TABLE OF LAG 1-5 AND TOTAL AUTOCORRELATIONS FOR M/EK/1; FOR K IN KV
[2]   ⍝RHO MUST BE SET TO THE VECTOR OF INTENSITIES.
[3]    C←((N←⍴RHO),5)⍴I←1
[4]    J←2
[5]    M←K×(1-RHO)÷((RHO*2)×1-K)+K←KV[I]
[6]    CT←(RHO*2)×(K-1)÷(1-RHO)×2×K
[7]    C[;1]←¯1+KRHO←RHO×KRO←K÷K+RHO
[8]    C[;J]←C[;J-1]+((J-1)!JK)×(K÷JK)×((RHO÷K)*J)×KRO×JK←J+K×J-1
[9]    →8×⍳(6>J←J+1)
[10]   C[;1]←C[;1]+KRO×K
[11]   C[;2]←C[;2]+(KRO*2×K)×1+KRHO
[12]   C[;3]←C[;3]+(KRO*3×K)×1+(2×KRHO)+(1+3×K)×(KRHO*2)÷2×K
[13]   C[;4]←C[;4]+(KRO*4×K)×1+(3×KRHO)+((T←1+4×K)×(KRHO*2)÷K)
[14]   C[;4]←C[;4]+(KRO*4×K)×T×(T+1)×(KRHO*3)÷6×K*2
[15]   C[;5]←C[;5]+(KRO*5×K)×1+(4×KRHO)+(3×(T←1+5×K)×(KRHO*2)÷2×K)
[16]   C[;5]←C[;5]+(KRO*5×K)×(T×(T+1)×(KRHO*3)÷3×K*2)
[17]   C[;5]←C[;5]+(KRO*5×K)×T×(T+1)×(T+2)×(KRHO*4)÷24×K*3
[18]   10 2 ⍴ '  '
[19]   ⎕← '                          K = ' ,(0⍕K), '          CV =' ,3⍕K*¯0.5
[20]   '                         AUTOCORRELATION LAG N'
[21]   ' RHO      N=    1          2          3          4          5        TOTAL' ,
[22]   '   '
[23]   ⎕←FORMAT⍕RHO,(⍪(6,N)⍴M)×C,CT
[24]   →4×((⍴KV)≥I←I+1)
       ∇
```

```
      ∇HROOTS[⎕]∇
      ∇ S←R HROOTS Q
[1]   S←R+0.5×1-(1-4×R×(1-R)×(1-2×Q)*2)*0.5
      ∇
```

```
      ∇CONVERT[⎕]∇
      ∇ Q←CONVERT CV
[1]   Q←(1-((¯1+CV*2)÷1+CV*2)*0.5)÷2
      ∇
```

```
      FORMAT
 4 2 14 6 10 6 10 6 10 6 10 6 10 6
```

I(a). $E_k/M/1$

K = 2 CV = .707

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	-.004063	-.000095	-.000003	-.000000	-.000000	-.004161
.10	-.013101	-.000970	-.000093	-.000010	-.000001	-.014176
.15	-.023690	-.003202	-.000552	-.000107	-.000022	-.027580
.20	-.033818	-.006704	-.001674	-.000467	-.000139	-.042867
.25	-.042428	-.011000	-.003550	-.001274	-.000487	-.059073
.30	-.049059	-.015523	-.006048	-.002607	-.001193	-.075568
.35	-.053597	-.019778	-.008898	-.004406	-.002310	-.091944
.40	-.056124	-.023404	-.011787	-.006503	-.003789	-.107937
.45	-.056818	-.026168	-.014429	-.008676	-.005495	-.123386
.50	-.055902	-.027951	-.016596	-.010700	-.007247	-.138197
.55	-.053602	-.028714	-.018127	-.012376	-.008855	-.152320
.60	-.050138	-.028478	-.018926	-.013553	-.010145	-.165740
.65	-.045711	-.027302	-.018952	-.014125	-.010980	-.178459
.70	-.040497	-.025264	-.018203	-.014034	-.011260	-.190494
.75	-.034653	-.022455	-.016707	-.013258	-.010927	-.201871
.80	-.028312	-.018969	-.014511	-.011805	-.009954	-.212620
.85	-.021588	-.014897	-.011677	-.009705	-.008345	-.222774
.90	-.014576	-.010326	-.008268	-.007001	-.006123	-.232367
.95	-.007358	-.005336	-.004354	-.003747	-.003325	-.241434

K = 3 CV = .577

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	-.007023	-.000056	-.000001	-.000000	-.000000	-.007080
.10	-.024163	-.000982	-.000053	-.000003	-.000000	-.025202
.15	-.044771	-.004107	-.000491	-.000066	-.000010	-.049446
.20	-.064047	-.009710	-.001890	-.000414	-.000097	-.076191
.25	-.079537	-.016994	-.004596	-.001391	-.000450	-.103204
.30	-.090415	-.024783	-.008491	-.003236	-.001314	-.129239
.35	-.096773	-.032033	-.013098	-.005924	-.002846	-.153676
.40	-.099122	-.038021	-.017820	-.009191	-.005019	-.176260
.45	-.098119	-.042348	-.022109	-.012638	-.007626	-.196950
.50	-.094417	-.044867	-.025550	-.015856	-.010357	-.215814
.55	-.088606	-.045602	-.027880	-.018494	-.012878	-.232976
.60	-.081191	-.044681	-.028974	-.020300	-.014891	-.248581
.65	-.072586	-.042285	-.028808	-.021121	-.016173	-.262778
.70	-.063127	-.038615	-.027430	-.020891	-.016578	-.275711
.75	-.053081	-.033872	-.024934	-.019610	-.016034	-.287512
.80	-.042660	-.028244	-.021438	-.017329	-.014531	-.298302
.85	-.032028	-.021902	-.017072	-.014127	-.012103	-.308189
.90	-.021312	-.014997	-.011963	-.010101	-.008814	-.317269
.95	-.010612	-.007659	-.006235	-.005358	-.004749	-.325627

K = 4 CV = .500

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	-.009630	-.000032	-.000000	-.000000	-.000000	-.009662
.10	-.034010	-.000874	-.000030	-.000001	-.000000	-.034916
.15	-.063487	-.004394	-.000401	-.000042	-.000005	-.068329
.20	-.090330	-.011357	-.001852	-.000341	-.000068	-.103966
.25	-.110786	-.020772	-.004977	-.001339	-.000386	-.138430
.30	-.123978	-.030928	-.009722	-.003411	-.001277	-.170161
.35	-.130489	-.040291	-.015478	-.006596	-.002990	-.198648
.40	-.131448	-.047853	-.021420	-.010577	-.005538	-.223905
.45	-.128055	-.053114	-.026794	-.014833	-.008679	-.246180
.50	-.121391	-.055950	-.031050	-.018816	-.012015	-.265803
.55	-.112346	-.056472	-.033862	-.022067	-.015110	-.283110
.60	-.101632	-.054917	-.035094	-.024266	-.017581	-.298415
.65	-.089794	-.051575	-.034752	-.025232	-.019146	-.311995
.70	-.077250	-.046742	-.032931	-.024902	-.019631	-.324090
.75	-.064314	-.040698	-.029779	-.023301	-.018963	-.334904
.80	-.051216	-.033696	-.025466	-.020510	-.017143	-.344612
.85	-.038129	-.025953	-.020169	-.016649	-.014233	-.353360
.90	-.025176	-.017656	-.014057	-.011852	-.010328	-.361273
.95	-.012446	-.008962	-.007288	-.006257	-.005543	-.368456

K = 5 CV = .447

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	-.012103	-.000019	-.000000	-.000000	-.000000	-.012123
.10	-.043103	-.000769	-.000019	-.000001	-.000000	-.043891
.15	-.080373	-.004482	-.000332	-.000028	-.000003	-.085219
.20	-.113348	-.012397	-.001771	-.000287	-.000050	-.127864
.25	-.137261	-.023407	-.005131	-.001266	-.000335	-.167529
.30	-.151475	-.035336	-.010441	-.003450	-.001218	-.202652
.35	-.157245	-.046246	-.016998	-.006943	-.003020	-.233064
.40	-.156355	-.054907	-.023800	-.011400	-.005794	-.259167
.45	-.150514	-.060764	-.029937	-.016208	-.009282	-.281539
.50	-.141146	-.063735	-.034755	-.020720	-.013025	-.300758
.55	-.129362	-.064016	-.037886	-.024394	-.016513	-.317341
.60	-.116002	-.061938	-.039195	-.026862	-.019300	-.331726
.65	-.101684	-.057873	-.038713	-.027925	-.021062	-.344275
.70	-.086859	-.052192	-.036573	-.027526	-.021605	-.355286
.75	-.071849	-.045229	-.032965	-.025707	-.020856	-.365002
.80	-.056884	-.037279	-.028096	-.022576	-.018830	-.373622
.85	-.042124	-.028592	-.022178	-.018279	-.015605	-.381309
.90	-.027680	-.019375	-.015407	-.012978	-.011300	-.388196
.95	-.013623	-.009798	-.007962	-.006833	-.006050	-.394396

K = 6 CV = .408

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	-.014510	-.000012	-.000000	-.000000	-.000000	-.014522
.10	-.051660	-.000682	-.000012	-.000000	-.000000	-.052355
.15	-.095788	-.004499	-.000282	-.000020	-.000002	-.100591
.20	-.133699	-.013124	-.001691	-.000247	-.000039	-.148807
.25	-.159935	-.025371	-.005194	-.001199	-.000297	-.192101
.30	-.174339	-.038673	-.010905	-.003443	-.001162	-.229173
.35	-.178909	-.050752	-.018048	-.007142	-.003011	-.260321
.40	-.176057	-.060214	-.025488	-.011935	-.005934	-.286370
.45	-.167920	-.066473	-.032187	-.017144	-.009664	-.308200
.50	-.156187	-.069495	-.037415	-.022041	-.013698	-.326600
.55	-.142119	-.069549	-.040773	-.026024	-.017471	-.342217
.60	-.126630	-.067043	-.042129	-.028689	-.020489	-.355573
.65	-.110374	-.062418	-.041537	-.029823	-.022395	-.367085
.70	-.093807	-.056095	-.039160	-.029373	-.022984	-.377078
.75	-.077247	-.048452	-.035217	-.027398	-.022180	-.385815
.80	-.060911	-.039813	-.029948	-.024024	-.020008	-.393502
.85	-.044942	-.030447	-.023586	-.019419	-.016563	-.400308
.90	-.029434	-.020576	-.016349	-.013762	-.011976	-.406367
.95	-.014443	-.010380	-.008431	-.007233	-.006403	-.411790

K = 7 CV = .378

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	-.016876	-.000008	-.000000	-.000000	-.000000	-.016884
.10	-.059795	-.000614	-.000009	-.000000	-.000000	-.060418
.15	-.109974	-.004490	-.000246	-.000015	-.000001	-.114726
.20	-.151842	-.013669	-.001620	-.000218	-.000032	-.167386
.25	-.179563	-.026905	-.005219	-.001142	-.000268	-.213186
.30	-.193623	-.041297	-.011226	-.003421	-.001115	-.251272
.35	-.196778	-.054288	-.018817	-.007265	-.002990	-.282474
.40	-.192003	-.064353	-.026747	-.012307	-.006016	-.308013
.45	-.181784	-.070896	-.033876	-.017818	-.009923	-.329037
.50	-.168004	-.073925	-.039416	-.023009	-.014175	-.346490
.55	-.152024	-.073777	-.042943	-.027229	-.018164	-.361118
.60	-.134800	-.070920	-.044331	-.030043	-.021358	-.373494
.65	-.116995	-.065849	-.043651	-.031231	-.023376	-.384061
.70	-.099062	-.059027	-.041090	-.030744	-.024000	-.393163
.75	-.081303	-.050862	-.036894	-.028651	-.023157	-.401065
.80	-.063918	-.041699	-.031322	-.025096	-.020878	-.407977
.85	-.047035	-.031823	-.024628	-.020261	-.017269	-.414064
.90	-.030731	-.021464	-.017044	-.014341	-.012475	-.419457
.95	-.015046	-.010808	-.008776	-.007528	-.006662	-.424264

K = 8 CV = .354

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	-.019209	-.000006	-.000000	-.000000	-.000000	-.019215
.10	-.067576	-.000560	-.000006	-.000000	-.000000	-.068142
.15	-.123107	-.004472	-.000219	-.000012	-.000001	-.127811
.20	-.168132	-.014098	-.001561	-.000197	-.000027	-.184019
.25	-.196719	-.028144	-.005224	-.001095	-.000247	-.231507
.30	-.210098	-.043422	-.011460	-.003393	-.001075	-.269993
.35	-.211758	-.057139	-.019404	-.007345	-.002965	-.300847
.40	-.205162	-.067672	-.027721	-.012579	-.006066	-.325654
.45	-.193076	-.074421	-.035191	-.018327	-.010108	-.345776
.50	-.177525	-.077438	-.040975	-.023748	-.014530	-.362278
.55	-.159932	-.077111	-.044633	-.028154	-.018688	-.375968
.60	-.141272	-.073963	-.046043	-.031086	-.022021	-.387454
.65	-.122207	-.068531	-.045292	-.032317	-.024126	-.397190
.70	-.103174	-.061310	-.042586	-.031801	-.024780	-.405524
.75	-.084461	-.052732	-.038190	-.029617	-.023907	-.412722
.80	-.066250	-.043158	-.032383	-.025921	-.021546	-.418988
.85	-.048652	-.032884	-.025431	-.020909	-.017811	-.424484
.90	-.031730	-.022147	-.017579	-.014786	-.012857	-.429338
.95	-.015510	-.011137	-.009041	-.007754	-.006861	-.433651

K = 9 CV = .333

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	-.021517	-.000004	-.000000	-.000000	-.000000	-.021521
.10	-.075045	-.000517	-.000005	-.000000	-.000000	-.075567
.15	-.135321	-.004453	-.000198	-.000010	-.000001	-.139983
.20	-.182850	-.014449	-.001511	-.000180	-.000023	-.199017
.25	-.211841	-.029171	-.005222	-.001056	-.000230	-.247589
.30	-.224331	-.045180	-.011638	-.003365	-.001042	-.286066
.35	-.224489	-.059489	-.019867	-.007399	-.002939	-.316340
.40	-.216200	-.070394	-.028497	-.012785	-.006097	-.340313
.45	-.202447	-.077298	-.036242	-.018723	-.010246	-.359521
.50	-.185357	-.080290	-.042223	-.024331	-.014803	-.375116
.55	-.166390	-.079808	-.045986	-.028886	-.019098	-.387947
.60	-.146525	-.076415	-.047413	-.031913	-.022542	-.398636
.65	-.126413	-.070684	-.046602	-.033179	-.024719	-.407646
.70	-.106478	-.063137	-.043778	-.032640	-.025397	-.415320
.75	-.086988	-.054224	-.039222	-.030384	-.024501	-.421920
.80	-.068109	-.044319	-.033225	-.026576	-.022075	-.427645
.85	-.049937	-.033726	-.026067	-.021422	-.018240	-.432649
.90	-.032521	-.022688	-.018002	-.015137	-.013160	-.437057
.95	-.015876	-.011396	-.009250	-.007932	-.007018	-.440962

K = 10 CV = .316

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	-.023799	-.000003	-.000000	-.000000	-.000000	-.023802
.10	-.082233	-.000482	-.000004	-.000000	-.000000	-.082718
.15	-.146722	-.004436	-.000181	-.000009	-.000000	-.151348
.20	-.196218	-.014745	-.001469	-.000167	-.000020	-.212622
.25	-.225272	-.030039	-.005215	-.001024	-.000216	-.261827
.30	-.236747	-.046662	-.011778	-.003339	-.001014	-.300022
.35	-.235441	-.061460	-.020241	-.007437	-.002916	-.329584
.40	-.225588	-.072667	-.029130	-.012947	-.006118	-.352691
.45	-.210347	-.079690	-.037102	-.019039	-.010352	-.371011
.50	-.191911	-.082653	-.043245	-.024801	-.015020	-.385761
.55	-.171761	-.082033	-.047094	-.029480	-.019426	-.397813
.60	-.150872	-.078432	-.048533	-.032586	-.022963	-.407797
.65	-.129880	-.072450	-.047673	-.033881	-.025199	-.416171
.70	-.109191	-.064632	-.044751	-.033323	-.025898	-.423275
.75	-.089057	-.055443	-.040063	-.031007	-.024984	-.429364
.80	-.069627	-.045265	-.033911	-.027108	-.022505	-.434630
.85	-.050984	-.034411	-.026585	-.021838	-.018588	-.439222
.90	-.033164	-.023127	-.018345	-.015422	-.013405	-.443256
.95	-.016173	-.011607	-.009420	-.008076	-.007146	-.446824

K = 15 CV = .258

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	-.034881	-.000001	-.000000	-.000000	-.000000	-.034882
.10	-.114628	-.000377	-.000002	-.000000	-.000000	-.115007
.15	-.194135	-.004385	-.000135	-.000005	-.000000	-.198660
.20	-.248183	-.015754	-.001332	-.000129	-.000013	-.265413
.25	-.274828	-.032961	-.005169	-.000918	-.000176	-.314096
.30	-.280796	-.051586	-.012183	-.003236	-.000922	-.349121
.35	-.273152	-.067932	-.021384	-.007521	-.002824	-.374693
.40	-.257187	-.080052	-.031095	-.013408	-.006151	-.393801
.45	-.236462	-.087392	-.039787	-.019986	-.010644	-.408430
.50	-.213272	-.090202	-.046440	-.026231	-.015655	-.419886
.55	-.189066	-.089098	-.050555	-.031303	-.020413	-.429040
.60	-.164745	-.084799	-.052029	-.034660	-.024243	-.436487
.65	-.140856	-.077999	-.051008	-.036048	-.026670	-.442640
.70	-.117724	-.069308	-.047777	-.035434	-.027436	-.447794
.75	-.095526	-.059238	-.042671	-.032934	-.026468	-.452165
.80	-.074351	-.048203	-.036033	-.028751	-.023827	-.455911
.85	-.054228	-.036532	-.028183	-.023123	-.019660	-.459152
.90	-.035150	-.024483	-.019405	-.016301	-.014160	-.461979
.95	-.017088	-.012255	-.009942	-.008521	-.007537	-.464465

K = 20 CV = .224

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	-.045453	-.000000	-.000000	-.000000	-.000000	-.045454
.10	-.142251	-.000328	-.000001	-.000000	-.000000	-.142580
.15	-.230044	-.004377	-.000114	-.000003	-.000000	-.234538
.20	-.283961	-.016369	-.001261	-.000111	-.000011	-.301713
.25	-.306633	-.034659	-.005135	-.000863	-.000156	-.347482
.30	-.307663	-.054376	-.012379	-.003172	-.000872	-.378820
.35	-.295310	-.071533	-.021969	-.007543	-.002767	-.400892
.40	-.275240	-.084104	-.032117	-.013623	-.006150	-.416956
.45	-.251064	-.091571	-.041192	-.020456	-.010773	-.429011
.50	-.225015	-.094260	-.048115	-.026957	-.015962	-.438306
.55	-.198452	-.092865	-.052368	-.032238	-.020906	-.445645
.60	-.172186	-.088171	-.053856	-.035729	-.024892	-.451557
.65	-.146690	-.080921	-.052748	-.037168	-.027421	-.456403
.70	-.122224	-.071758	-.049351	-.036526	-.028226	-.460437
.75	-.098916	-.061217	-.044025	-.033931	-.027232	-.463837
.80	-.076813	-.049729	-.037133	-.029599	-.024508	-.466739
.85	-.055910	-.037629	-.029009	-.023786	-.020213	-.469238
.90	-.036175	-.025183	-.019951	-.016754	-.014549	-.471412
.95	-.017558	-.012589	-.010210	-.008750	-.007739	-.473318

K = 25 CV = .200

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	-.055555	-.000000	-.000000	-.000000	-.000000	-.055555
.10	-.166142	-.000301	-.000001	-.000000	-.000000	-.166444
.15	-.258248	-.004388	-.000102	-.000003	-.000000	-.262741
.20	-.310127	-.016797	-.001217	-.000101	-.000009	-.328252
.25	-.328784	-.035781	-.005111	-.000828	-.000145	-.370681
.30	-.325757	-.056179	-.012494	-.003130	-.000841	-.398736
.35	-.309885	-.073831	-.022324	-.007550	-.002728	-.418023
.40	-.286915	-.086665	-.032743	-.013746	-.006143	-.431816
.45	-.260388	-.094194	-.042055	-.020735	-.010844	-.442035
.50	-.232440	-.096793	-.049144	-.027395	-.016142	-.449838
.55	-.204340	-.095206	-.053483	-.032806	-.021201	-.455951
.60	-.176826	-.090259	-.054979	-.036381	-.025284	-.460846
.65	-.150309	-.082724	-.053817	-.037851	-.027878	-.464839
.70	-.125004	-.073266	-.050317	-.037192	-.028706	-.468149
.75	-.101003	-.062433	-.044855	-.034539	-.027697	-.470931
.80	-.078323	-.050663	-.037805	-.030117	-.024924	-.473296
.85	-.056939	-.038301	-.029514	-.024191	-.020549	-.475330
.90	-.036801	-.025610	-.020284	-.017030	-.014786	-.477095
.95	-.017845	-.012792	-.010374	-.008890	-.007861	-.476639

K = 50 CV = .141

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	-.100000	-.000000	-.000000	-.000000	-.000000	-.100000
.10	-.249620	-.000256	-.000000	-.000000	-.000000	-.249876
.15	-.340405	-.004486	-.000081	-.000002	-.000000	-.344974
.20	-.377975	-.017855	-.001132	-.000082	-.000006	-.397052
.25	-.382215	-.038332	-.005060	-.000759	-.000123	-.426513
.30	-.367430	-.060140	-.012725	-.003035	-.000777	-.444396
.35	-.342429	-.078778	-.023048	-.007549	-.002643	-.456015
.40	-.312422	-.092112	-.034026	-.013978	-.006115	-.464007
.45	-.280439	-.099722	-.043829	-.021288	-.010971	-.469764
.50	-.248219	-.102096	-.051263	-.028276	-.016492	-.474068
.55	-.216739	-.100081	-.055775	-.033957	-.021788	-.477387
.60	-.186524	-.094588	-.057288	-.037707	-.026074	-.480010
.65	-.157828	-.086448	-.056009	-.039245	-.028802	-.482129
.70	-.130752	-.076370	-.052296	-.038553	-.029683	-.483870
.75	-.105298	-.064928	-.046552	-.035781	-.028644	-.485324
.80	-.081421	-.052578	-.039180	-.031174	-.025769	-.486553
.85	-.059044	-.039672	-.030543	-.025015	-.021235	-.487604
.90	-.038078	-.026480	-.020963	-.017592	-.015268	-.488512
.95	-.018428	-.013206	-.010707	-.009173	-.008111	-.489304

K = 100 CV = .100

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	-.166667	-.000000	-.000000	-.000000	-.000000	-.166667
.10	-.333017	-.000244	-.000000	-.000000	-.000000	-.333261
.15	-.403732	-.004608	-.000072	-.000001	-.000000	-.408415
.20	-.423115	-.018539	-.001093	-.000074	-.000005	-.442827
.25	-.414855	-.039807	-.005035	-.000724	-.000112	-.460554
.30	-.391591	-.062330	-.012841	-.002984	-.000744	-.470757
.35	-.360667	-.081451	-.023417	-.007540	-.002596	-.477170
.40	-.326388	-.095013	-.034684	-.014086	-.006093	-.481486
.45	-.291235	-.102638	-.044741	-.021560	-.011026	-.484547
.50	-.256610	-.104873	-.052353	-.028718	-.016661	-.486810
.55	-.223270	-.102620	-.056954	-.034540	-.022079	-.488540
.60	-.191593	-.096832	-.058473	-.038382	-.026470	-.489899
.65	-.161735	-.088371	-.057135	-.039956	-.029269	-.490990
.70	-.133723	-.077967	-.053310	-.039247	-.030178	-.491883
.75	-.107510	-.066209	-.047421	-.036414	-.029126	-.492626
.80	-.083011	-.053558	-.039882	-.031713	-.026200	-.493252
.85	-.060121	-.040373	-.031069	-.025436	-.021584	-.493787
.90	-.038730	-.026924	-.021309	-.017879	-.015513	-.494247
.95	-.018725	-.013417	-.010877	-.009318	-.008238	-.494648

K = 10000 CV = .010

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	-.490196	-.000000	-.000000	-.000000	-.000000	-.490196
.10	-.497262	-.000249	-.000000	-.000000	-.000000	-.497511
.15	-.493989	-.004828	-.000065	-.000001	-.000000	-.498883
.20	-.478856	-.019375	-.001056	-.000066	-.000004	-.499358
.25	-.452310	-.041442	-.005012	-.000689	-.000102	-.499574
.30	-.418181	-.064665	-.012958	-.002930	-.000711	-.499692
.35	-.380226	-.084246	-.023787	-.007526	-.002546	-.499763
.40	-.341109	-.098013	-.035347	-.014187	-.006066	-.499809
.45	-.302480	-.105632	-.045661	-.021826	-.011075	-.499842
.50	-.265273	-.107710	-.053452	-.029157	-.016823	-.499866
.55	-.229967	-.105203	-.058143	-.035122	-.022366	-.499884
.60	-.196765	-.099108	-.059669	-.039057	-.026864	-.499898
.65	-.165704	-.090317	-.058268	-.040668	-.029735	-.499909
.70	-.136731	-.079580	-.054331	-.039943	-.030674	-.499918
.75	-.109743	-.067499	-.048295	-.037050	-.029609	-.499926
.80	-.084612	-.054544	-.040588	-.032253	-.026631	-.499932
.85	-.061203	-.041077	-.031596	-.025857	-.021933	-.499938
.90	-.039383	-.027370	-.021656	-.018166	-.015759	-.499942
.95	-.019022	-.013628	-.011047	-.009463	-.008365	-.499946

K = 1000000 CV = .001

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	-.499900	-.000000	-.000000	-.000000	-.000000	-.499900
.10	-.499725	-.000250	-.000000	-.000000	-.000000	-.499975
.15	-.495092	-.004831	-.000065	-.000001	-.000000	-.499989
.20	-.479483	-.019385	-.001056	-.000066	-.000004	-.499994
.25	-.452715	-.041459	-.005012	-.000689	-.000102	-.499996
.30	-.418462	-.064689	-.012959	-.002930	-.000711	-.499997
.35	-.380430	-.084275	-.023791	-.007526	-.002546	-.499998
.40	-.341261	-.098044	-.035353	-.014187	-.006066	-.499998
.45	-.302595	-.105662	-.045670	-.021829	-.011075	-.499998
.50	-.265361	-.107738	-.053463	-.029161	-.016825	-.499999
.55	-.230036	-.105230	-.058155	-.035128	-.022369	-.499999
.60	-.196818	-.099131	-.059681	-.039064	-.026868	-.499999
.65	-.165745	-.090337	-.058279	-.040676	-.029740	-.499999
.70	-.136762	-.079597	-.054341	-.039950	-.030679	-.499999
.75	-.109765	-.067512	-.048304	-.037056	-.029613	-.499999
.80	-.084628	-.054554	-.040595	-.032259	-.026635	-.499999
.85	-.061214	-.041084	-.031602	-.025861	-.021937	-.499999
.90	-.039390	-.027374	-.021659	-.018169	-.015762	-.499999
.95	-.019025	-.013630	-.011048	-.009464	-.008366	-.499999

I(b). H/M/1

R = .3459 CV = 1.10

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000178	.000018	.000002	.000000	.000000	.000198
.10	.000645	.000120	.000027	.000007	.000002	.000801
.15	.001315	.000341	.000107	.000037	.000014	.001822
.20	.002119	.000683	.000264	.000112	.000050	.003275
.25	.002998	.001131	.000508	.000250	.000129	.005176
.30	.003901	.001660	.000835	.000458	.000264	.007543
.35	.004786	.002240	.001232	.000734	.000460	.010396
.40	.005614	.002840	.001677	.001070	.000715	.013755
.45	.006351	.003427	.002146	.001447	.001020	.017643
.50	.006964	.003969	.002610	.001842	.001357	.022087
.55	.007423	.004434	.003039	.002229	.001703	.027114
.60	.007701	.004790	.003401	.002577	.002031	.032756
.65	.007770	.005007	.003666	.002857	.002311	.039045
.70	.007602	.005053	.003801	.003034	.002510	.046021
.75	.007169	.004899	.003772	.003076	.002595	.053724
.80	.006444	.004514	.003548	.002947	.002528	.062201
.85	.005397	.003865	.003094	.002612	.002275	.071505
.90	.003996	.002920	.002376	.002035	.001795	.081695
.95	.002209	.001644	.001358	.001178	.001051	.092835

R = .2877 CV = 1.20

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000284	.000030	.000004	.000001	.000000	.000319
.10	.001030	.000203	.000049	.000013	.000004	.001300
.15	.002108	.000575	.000189	.000069	.000026	.002985
.20	.003410	.001150	.000464	.000205	.000096	.005420
.25	.004847	.001904	.000888	.000451	.000242	.008656
.30	.006343	.002798	.001454	.000821	.000487	.012750
.35	.007832	.003787	.002141	.001310	.000840	.017769
.40	.009254	.004820	.002917	.001902	.001298	.023783
.45	.010552	.005846	.003739	.002569	.001843	.030876
.50	.011673	.006812	.004564	.003273	.002447	.039138
.55	.012564	.007665	.005340	.003971	.003072	.048676
.60	.013173	.008351	.006016	.004613	.003673	.059608
.65	.013444	.008813	.006535	.005144	.004197	.072072
.70	.013318	.008992	.006837	.005506	.004588	.086227
.75	.012731	.008825	.006859	.005634	.004781	.102261
.80	.011613	.008240	.006531	.005457	.004706	.120391
.85	.009883	.007161	.005775	.004900	.004284	.140881
.90	.007448	.005499	.004504	.003873	.003428	.164045
.95	.004197	.003153	.002618	.002279	.002039	.190262

Q = .2468 CV = 1.30

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000344	.000039	.000005	.000001	.000000	.000390
.10	.001251	.000259	.000065	.000018	.000005	.001602
.15	.002566	.000731	.000251	.000095	.000038	.003709
.20	.004163	.001460	.000611	.000280	.000135	.006793
.25	.005939	.002417	.001164	.000610	.000337	.010951
.30	.007808	.003556	.001901	.001101	.000670	.016289
.35	.009691	.004823	.002796	.001750	.001147	.022931
.40	.011517	.006157	.003809	.002534	.001761	.031018
.45	.013220	.007499	.004891	.003418	.002492	.040710
.50	.014733	.008784	.005987	.004359	.003303	.052194
.55	.015988	.009946	.007035	.005300	.004147	.065687
.60	.016916	.010916	.007970	.006180	.004969	.081442
.65	.017437	.011618	.008717	.006929	.005701	.099762
.70	.017467	.011969	.009197	.007468	.006267	.121013
.75	.016906	.011876	.009317	.007707	.006580	.145640
.80	.015635	.011230	.008974	.007545	.006538	.174195
.85	.013513	.009901	.008042	.006859	.006021	.207375
.90	.010362	.007730	.006370	.005503	.004887	.246068
.95	.005956	.004516	.003771	.003294	.002955	.291432

Q = .2153 CV = 1.40

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000375	.000045	.000007	.000001	.000000	.000428
.10	.001366	.000295	.000077	.000022	.000007	.001771
.15	.002806	.000830	.000296	.000116	.000048	.004134
.20	.004564	.001657	.000716	.000338	.000169	.007634
.25	.006532	.002742	.001358	.000731	.000414	.012412
.30	.008619	.004037	.002213	.001313	.000817	.018629
.35	.010743	.005484	.003251	.002077	.001388	.026472
.40	.012830	.007020	.004430	.003000	.002121	.036157
.45	.014808	.008580	.005696	.004044	.002991	.047936
.50	.016605	.010094	.006990	.005159	.003958	.062105
.55	.018146	.011491	.008243	.006285	.004971	.079014
.60	.019348	.012691	.009383	.007353	.005967	.099086
.65	.020120	.013607	.010325	.008282	.006869	.122836
.70	.020355	.014141	.010975	.008983	.007589	.150905
.75	.019922	.014173	.011219	.009345	.008024	.184108
.80	.018662	.013561	.010923	.009239	.008044	.223502
.85	.016368	.012122	.009916	.008501	.007493	.270483
.90	.012768	.009619	.007977	.006921	.006167	.326948
.95	.007488	.005729	.004810	.004218	.003795	.395529

Q = .1899 CV = 1.50

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000388	.000048	.000007	.000001	.000000	.000445
.10	.001413	.000316	.000086	.000026	.000008	.001854
.15	.002907	.000889	.000326	.000132	.000056	.004356
.20	.004737	.001771	.000787	.000382	.000195	.008106
.25	.006797	.002931	.001488	.000820	.000475	.013284
.30	.008995	.004319	.002420	.001466	.000930	.020105
.35	.011251	.005875	.003553	.002311	.001572	.028819
.40	.013491	.007536	.004841	.003332	.002392	.039721
.45	.015643	.009238	.006232	.004488	.003364	.053161
.50	.017633	.010908	.007663	.005728	.004446	.069550
.55	.019384	.012474	.009066	.006990	.005584	.089385
.60	.020807	.013852	.010362	.008200	.006712	.113277
.65	.021803	.014950	.011463	.009275	.007750	.141983
.70	.022250	.015656	.012265	.010116	.008602	.176478
.75	.021998	.015836	.012642	.010601	.009152	.218039
.80	.020850	.015317	.012432	.010577	.009253	.268388
.85	.018543	.013873	.011426	.009846	.008714	.329912
.90	.014707	.011184	.009333	.008134	.007272	.406023
.95	.008802	.006794	.005736	.005050	.004555	.501780

Q = .1690 CV = 1.60

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000388	.000050	.000008	.000001	.000000	.000448
.10	.001416	.000327	.000092	.000028	.000009	.001878
.15	.002917	.000917	.000346	.000143	.000063	.004441
.20	.004762	.001827	.000832	.000413	.000216	.008322
.25	.006846	.003023	.001569	.000883	.000522	.013738
.30	.009083	.004456	.002547	.001571	.001015	.020954
.35	.011394	.006070	.003736	.002471	.001707	.030285
.40	.013709	.007801	.005092	.003556	.002588	.042102
.45	.015958	.009586	.006561	.004787	.003631	.056851
.50	.018069	.011355	.008082	.006112	.004794	.075065
.55	.019963	.013036	.009587	.007468	.006022	.097394
.60	.021553	.014545	.010998	.008783	.007248	.124640
.65	.022735	.015787	.012224	.009972	.008391	.157828
.70	.023381	.016646	.013157	.010930	.009352	.198298
.75	.023327	.016977	.013662	.011530	.010008	.247864
.80	.022349	.016587	.013561	.011603	.010198	.309056
.85	.020137	.015209	.012609	.010920	.009703	.385531
.90	.016231	.012453	.010454	.009151	.008209	.482784
.95	.009914	.007715	.006549	.005788	.005237	.609464

Q = .1515 CV = 1.70

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000381	.000050	.000008	.000001	.000000	.000442
.10	.001392	.000330	.000095	.000030	.000010	.001863
.15	.002871	.000925	.000357	.000151	.000068	.004433
.20	.004692	.001841	.000856	.000434	.000232	.008360
.25	.006757	.003046	.001611	.000923	.000556	.013896
.30	.008983	.004493	.002613	.001638	.001075	.021351
.35	.011297	.006125	.003830	.002570	.001800	.031100
.40	.013632	.007885	.005221	.003695	.002722	.043590
.45	.015920	.009710	.006733	.004971	.003812	.059364
.50	.018094	.011533	.008307	.006349	.005028	.079073
.55	.020077	.013285	.009877	.007768	.006317	.103518
.60	.021785	.014884	.011367	.009156	.007613	.133702
.65	.023114	.016236	.012686	.010428	.008834	.170916
.70	.023934	.017224	.013727	.011482	.009882	.216893
.75	.024073	.017697	.014350	.012185	.010631	.274030
.80	.023293	.017451	.014366	.012359	.010912	.345783
.85	.021245	.016188	.013506	.011754	.010485	.437338
.90	.017390	.013453	.011359	.009986	.008989	.556851
.95	.010838	.008500	.007254	.006435	.005840	.717921

Q = .1366 CV = 1.80

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000370	.000050	.000008	.000002	.000000	.000430
.10	.001351	.000328	.000097	.000031	.000011	.001824
.15	.002788	.000918	.000362	.000156	.000071	.004364
.20	.004563	.001826	.000865	.000446	.000242	.008276
.25	.006581	.003021	.001625	.000947	.000579	.013845
.30	.008764	.004458	.002633	.001675	.001115	.021421
.35	.011044	.006083	.003858	.002623	.001860	.031434
.40	.013359	.007841	.005260	.003766	.002806	.044406
.45	.015646	.009673	.006788	.005066	.003923	.060972
.50	.017840	.011518	.008386	.006472	.005171	.081901
.55	.019870	.013306	.009991	.007926	.006497	.108143
.60	.021654	.014961	.011530	.009361	.007838	.140894
.65	.023092	.016392	.012916	.010693	.009115	.181716
.70	.024056	.017483	.014042	.011821	.010230	.232736
.75	.024373	.018085	.014769	.012613	.011059	.296970
.80	.023796	.017984	.014902	.012888	.011427	.378885
.85	.021951	.016864	.014155	.012377	.011082	.485420
.90	.018235	.014217	.012070	.010656	.009624	.627959
.95	.011594	.009160	.007856	.006996	.006367	.826550

Q = .1238 CV = 1.90

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000356	.000049	.000008	.000002	.000000	.000416
.10	.001300	.000322	.000097	.000032	.000011	.001769
.15	.002685	.000901	.000362	.000159	.000074	.004253
.20	.004399	.001791	.000863	.000452	.000249	.008110
.25	.006351	.002963	.001618	.000956	.000593	.013649
.30	.008471	.004374	.002619	.001688	.001137	.021257
.35	.010695	.005973	.003836	.002639	.001893	.031415
.40	.012963	.007709	.005231	.003786	.002849	.044716
.45	.015219	.009525	.006756	.005091	.003978	.061885
.50	.017401	.011364	.008356	.006506	.005241	.083807
.55	.019445	.013162	.009974	.007976	.006586	.111574
.60	.021271	.014846	.011538	.009435	.007952	.146570
.65	.022785	.016329	.012967	.010806	.009265	.190620
.70	.023864	.017501	.014157	.011990	.010430	.246243
.75	.024337	.018213	.014972	.012857	.011325	.317086
.80	.023956	.018252	.015217	.013225	.011775	.408689
.85	.022333	.017289	.014594	.012818	.011519	.529926
.90	.018813	.014774	.012609	.011178	.010128	.695942
.95	.012198	.009703	.008363	.007475	.006822	.934816

Q = .1127 CV = 2.00

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000341	.000048	.000008	.000002	.000000	.000399
.10	.001244	.000314	.000096	.000032	.000012	.001705
.15	.002571	.000877	.000358	.000160	.000075	.004117
.20	.004215	.001743	.000852	.000453	.000253	.007890
.25	.006093	.002883	.001596	.000955	.000599	.013354
.30	.008137	.004258	.002581	.001683	.001147	.020926
.35	.010288	.005818	.003779	.002628	.001904	.031136
.40	.012492	.007516	.005154	.003766	.002860	.044645
.45	.014697	.009300	.006660	.005063	.003989	.062264
.50	.016845	.011116	.008247	.006472	.005252	.084990
.55	.018877	.012903	.009859	.007942	.006602	.114053
.60	.020718	.014594	.011430	.009410	.007979	.151017
.65	.022281	.016108	.012882	.010802	.009312	.197956
.70	.023447	.017338	.014119	.012024	.010512	.257771
.75	.024053	.018142	.015006	.012952	.011459	.334742
.80	.023854	.018309	.015353	.013406	.011983	.435516
.85	.022455	.017506	.014856	.013104	.011819	.571046
.90	.019168	.015154	.012998	.011567	.010515	.760712
.95	.012669	.010142	.008782	.007877	.007210	1.042245

Q = .0907 CV = 2.25

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000301	.000044	.000008	.000002	.000000	.000355
.10	.001099	.000287	.000091	.000032	.000012	.001529
.15	.002274	.000802	.000338	.000156	.000076	.003727
.20	.003733	.001593	.000802	.000439	.000252	.007221
.25	.005407	.002635	.001499	.000921	.000593	.012374
.30	.007239	.003893	.002420	.001616	.001127	.019668
.35	.009180	.005327	.003542	.002518	.001863	.029732
.40	.011187	.006896	.004833	.003604	.002790	.043379
.45	.013215	.008556	.006253	.004843	.003883	.061628
.50	.015220	.010263	.007760	.006195	.005109	.085741
.55	.017151	.011967	.009306	.007617	.006424	.117268
.60	.018950	.013612	.010837	.009054	.007779	.158177
.65	.020543	.015129	.012288	.010443	.009112	.211103
.70	.021831	.016430	.013574	.011705	.010346	.279829
.75	.022672	.017390	.014580	.012730	.011376	.370148
.80	.022843	.017817	.015130	.013352	.012044	.491507
.85	.021965	.017388	.014933	.013301	.012096	.660433
.90	.019321	.015499	.013443	.012072	.011055	.908543
.95	.013364	.010849	.009493	.008586	.007912	1.304606

Q = .0745 CV = 2.50

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000263	.000040	.000007	.000002	.000000	.000312
.10	.000963	.000259	.000084	.000030	.000011	.001355
.15	.001993	.000721	.000311	.000147	.000073	.003328
.20	.003276	.001431	.000737	.000412	.000242	.006508
.25	.004752	.002368	.001375	.000862	.000566	.011271
.30	.006373	.003500	.002218	.001509	.001072	.018140
.35	.008099	.004794	.003246	.002347	.001766	.027821
.40	.009894	.006214	.004430	.003357	.002638	.041257
.45	.011723	.007727	.005737	.004510	.003667	.059662
.50	.013549	.009292	.007132	.005773	.004822	.084543
.55	.015332	.010872	.008574	.007108	.006067	.117744
.60	.017024	.012418	.010019	.008471	.007358	.161572
.65	.018566	.013876	.011413	.009808	.008645	.219124
.70	.019878	.015173	.012685	.011052	.009860	.294925
.75	.020841	.016203	.013738	.012111	.010916	.396131
.80	.021265	.016802	.014421	.012842	.011676	.534871
.85	.020806	.016677	.014467	.012995	.011904	.733388
.90	.018777	.015244	.013348	.012082	.011139	1.037054
.95	.013543	.011121	.009819	.008947	.008295	1.554436

Q = .0623 CV = 2.75

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000230	.000036	.000007	.000001	.000000	.000274
.10	.000843	.000231	.000077	.000028	.000011	.001197
.15	.001745	.000644	.000283	.000136	.000069	.002959
.20	.002871	.001277	.000669	.000380	.000227	.005829
.25	.004169	.002113	.001247	.000794	.000529	.010187
.30	.005598	.003124	.002011	.001388	.001000	.016576
.35	.007126	.004283	.002942	.002156	.001643	.025760
.40	.008721	.005558	.004016	.003081	.002451	.038791
.45	.010356	.006921	.005205	.004140	.003404	.057063
.50	.012000	.008341	.006479	.005303	.004476	.082324
.55	.013621	.009783	.007804	.006537	.005634	.116684
.60	.015181	.011211	.009143	.007806	.006842	.162734
.65	.016633	.012579	.010452	.009065	.008058	.223930
.70	.017911	.013828	.011673	.010259	.009225	.305408
.75	.018921	.014872	.012727	.011311	.010270	.415503
.80	.019501	.015572	.013483	.012099	.011076	.568796
.85	.019355	.015672	.013710	.012405	.011436	.793061
.90	.017854	.014638	.012921	.011775	.010921	1.148005
.95	.013374	.011087	.009865	.009046	.008434	1.788751

Q = .0528 CV = 3.00

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000202	.000032	.000006	.000001	.000000	.000242
.10	.000739	.000206	.000069	.000026	.000010	.001058
.15	.001532	.000574	.000256	.000125	.000064	.002629
.20	.002522	.001138	.000604	.000348	.000211	.005212
.25	.003665	.001883	.001126	.000726	.000490	.009181
.30	.004926	.002785	.001814	.001267	.000923	.015084
.35	.006277	.003819	.002653	.001966	.001514	.023724
.40	.007693	.004961	.003622	.002808	.002256	.036245
.45	.009150	.006184	.004698	.003773	.003131	.054211
.50	.010624	.007464	.005854	.004835	.004116	.079603
.55	.012089	.008772	.007062	.005966	.005183	.114784
.60	.013512	.010078	.008292	.007136	.006302	.162575
.65	.014857	.011344	.009505	.008307	.007436	.226707
.70	.016073	.012523	.010657	.009434	.008540	.312797
.75	.017080	.013546	.011681	.010453	.009550	.430190
.80	.017748	.014295	.012469	.011262	.010370	.595650
.85	.017824	.014554	.012822	.011673	.010821	.842112
.90	.016751	.013845	.012304	.011279	.010515	1.243513
.95	.012982	.010847	.009714	.008957	.008392	2.006050

Q = .0453 CV = 3.25

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000178	.000028	.000006	.000001	.000000	.000213
.10	.000651	.000184	.000063	.000024	.000009	.000938
.15	.001350	.000512	.000231	.000114	.000059	.002341
.20	.002224	.001015	.000545	.000317	.000194	.004665
.25	.003233	.001679	.001014	.000660	.000450	.008272
.30	.004349	.002484	.001634	.001152	.000846	.013708
.35	.005547	.003408	.002390	.001786	.001387	.021796
.40	.006806	.004430	.003263	.002551	.002065	.033758
.45	.008106	.005528	.004234	.003427	.002865	.051314
.50	.009427	.006680	.005281	.004394	.003767	.076677
.55	.010746	.007863	.006379	.005427	.004746	.112451
.60	.012039	.009052	.007501	.006499	.005776	.161649
.65	.013275	.010215	.008619	.007581	.006826	.228193
.70	.014413	.011316	.009694	.008633	.007859	.318083
.75	.015391	.012296	.010671	.009605	.008822	.441510
.80	.016100	.013060	.011462	.010410	.009634	.617166
.85	.016328	.013425	.011898	.010889	.010142	.882696
.90	.015590	.012972	.011595	.010682	.010003	1.325703
.95	.012454	.010475	.009432	.008740	.008223	2.205913

Q = .0393 CV = 3.50

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000157	.000025	.000005	.000001	.000000	.000189
.10	.000577	.000165	.000057	.000022	.000009	.000835
.15	.001196	.000458	.000209	.000104	.000055	.002090
.20	.001970	.000907	.000492	.000289	.000178	.004185
.25	.002866	.001501	.000915	.000600	.000412	.007464
.30	.003857	.002222	.001473	.001046	.000774	.012462
.35	.004923	.003049	.002154	.001621	.001268	.020013
.40	.006045	.003965	.002942	.002315	.001887	.031398
.45	.007207	.004952	.003819	.003111	.002617	.048482
.50	.008392	.005990	.004766	.003990	.003440	.073712
.55	.009581	.007059	.005762	.004931	.004336	.109927
.60	.010753	.008139	.006785	.005912	.005281	.160291
.65	.011885	.009204	.007811	.006907	.006250	.228858
.70	.012942	.010224	.008808	.007884	.007211	.321915
.75	.013875	.011152	.009731	.008801	.008120	.450374
.80	.014594	.011908	.010506	.009586	.008910	.634611
.85	.014922	.012340	.010991	.010104	.009449	.916513
.90	.014442	.012085	.010853	.010042	.009441	1.396529
.95	.011853	.010024	.009068	.008438	.007969	2.388657

Q = .0344 CV = 3.75

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000140	.000023	.000005	.000001	.000000	.000169
.10	.000513	.000148	.000051	.000020	.000008	.000746
.15	.001065	.000411	.000189	.000095	.000050	.001874
.20	.001754	.000814	.000444	.000263	.000163	.003765
.25	.002553	.001347	.000826	.000546	.000377	.006749
.30	.003437	.001993	.001330	.000950	.000708	.011344
.35	.004389	.002736	.001945	.001473	.001159	.018383
.40	.005394	.003560	.002657	.002103	.001723	.029193
.45	.006435	.004448	.003450	.002825	.002389	.045773
.50	.007501	.005385	.004308	.003625	.003141	.070802
.55	.008573	.006353	.005213	.004482	.003960	.107354
.60	.009637	.007334	.006145	.005379	.004827	.158712
.65	.010671	.008309	.007085	.006293	.005719	.229000
.70	.011649	.009251	.008006	.007198	.006610	.324728
.75	.012529	.010122	.008872	.008058	.007463	.457418
.80	.013239	.010856	.009620	.008813	.008221	.648916
.85	.013630	.011326	.010131	.009349	.008774	.944901
.90	.013346	.011220	.010118	.009397	.008865	1.457719
.95	.011220	.009532	.008658	.008085	.007660	2.555068

Q = .0303 CV = 4.00

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000125	.000021	.000004	.000001	.000000	.000151
.10	.000459	.000133	.000047	.000018	.000007	.000670
.15	.000952	.000370	.000171	.000086	.000046	.001686
.20	.001570	.000733	.000402	.000239	.000150	.003399
.25	.002285	.001213	.000748	.000497	.000345	.006119
.30	.003077	.001795	.001204	.000865	.000648	.010346
.35	.003932	.002464	.001761	.001340	.001059	.016905
.40	.004834	.003207	.002405	.001913	.001574	.027153
.45	.005771	.004010	.003124	.002570	.002183	.043214
.50	.006732	.004857	.003903	.003298	.002870	.067999
.55	.007702	.005735	.004726	.004081	.003620	.104816
.60	.008668	.006628	.005577	.004901	.004414	.157039
.65	.009613	.007519	.006438	.005740	.005235	.228814
.70	.010516	.008388	.007288	.006576	.006060	.326818
.75	.011342	.009202	.008096	.007379	.006857	.463093
.80	.012030	.009906	.008811	.008099	.007580	.660771
.85	.012458	.010394	.009331	.008640	.008133	.968906
.90	.012320	.010399	.009411	.008768	.008296	1.510755
.95	.010582	.009025	.008226	.007705	.007321	2.706204

Q = .0241 CV = 4.50

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000102	.000017	.000003	.000001	.000000	.000123
.10	.000372	.000109	.000039	.000015	.000006	.000546
.15	.000773	.000303	.000142	.000072	.000039	.001381
.20	.001274	.000601	.000333	.000200	.000126	.002799
.25	.001856	.000994	.000619	.000415	.000291	.005074
.30	.002501	.001471	.000995	.000721	.000544	.008665
.35	.003197	.002021	.001455	.001116	.000889	.014363
.40	.003934	.002632	.001989	.001593	.001321	.023556
.45	.004702	.003292	.002585	.002141	.001831	.038574
.50	.005491	.003992	.003231	.002749	.002407	.062800
.55	.006292	.004720	.003916	.003404	.003038	.100013
.60	.007094	.005464	.004628	.004093	.003709	.153694
.65	.007887	.006212	.005354	.004802	.004405	.227918
.70	.008654	.006951	.006077	.005516	.005111	.329575
.75	.009374	.007656	.006778	.006213	.005804	.471543
.80	.010003	.008292	.007418	.006857	.006450	.679069
.85	.010455	.008779	.007927	.007379	.006981	1.006871
.90	.010508	.008925	.008123	.007607	.007233	1.597184
.95	.009356	.008029	.007358	.006927	.006613	2.967441

Q = .0196 CV = 5.00

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000084	.000014	.000003	.000001	.000000	.000102
.10	.000307	.000091	.000032	.000013	.000005	.000453
.15	.000638	.000252	.000119	.000061	.000033	.001148
.20	.001052	.000500	.000279	.000169	.000107	.002336
.25	.001533	.000827	.000518	.000349	.000246	.004258
.30	.002067	.001224	.000833	.000607	.000461	.007329
.35	.002643	.001681	.001218	.000939	.000752	.012295
.40	.003254	.002190	.001665	.001341	.001117	.020537
.45	.003892	.002741	.002164	.001802	.001548	.034548
.50	.004549	.003326	.002706	.002314	.002036	.058174
.55	.005218	.003936	.003283	.002867	.002571	.095685
.60	.005892	.004562	.003884	.003450	.003141	.150579
.65	.006562	.005196	.004499	.004054	.003735	.226744
.70	.007217	.005826	.005118	.004666	.004341	.331182
.75	.007841	.006437	.005724	.005269	.004943	.477410
.80	.008405	.007001	.006292	.005840	.005515	.692321
.85	.008846	.007464	.006769	.006326	.006008	1.035112
.90	.009000	.007681	.007020	.006601	.006299	1.663610
.95	.008246	.007110	.006543	.006183	.005924	3.181976

Q = .0163 CV = 5.50

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000070	.000012	.000002	.000001	.000000	.000085
.10	.000258	.000077	.000027	.000011	.000005	.000381
.15	.000535	.000213	.000101	.000052	.000028	.000967
.20	.000882	.000421	.000236	.000143	.000092	.001974
.25	.001286	.000697	.000438	.000297	.000211	.003614
.30	.001734	.001032	.000705	.000516	.000394	.006261
.35	.002218	.001417	.001031	.000799	.000642	.010608
.40	.002731	.001847	.001409	.001140	.000954	.018006
.45	.003268	.002312	.001833	.001532	.001322	.031066
.50	.003823	.002807	.002293	.001968	.001738	.054087
.55	.004389	.003324	.002783	.002439	.002195	.091850
.60	.004960	.003856	.003295	.002938	.002683	.147793
.65	.005531	.004397	.003821	.003455	.003194	.225539
.70	.006094	.004939	.004354	.003982	.003717	.332145
.75	.006637	.005469	.004880	.004507	.004241	.481639
.80	.007138	.005968	.005382	.005011	.004747	.702207
.85	.007552	.006396	.005819	.005456	.005197	1.056617
.90	.007758	.006644	.006093	.005746	.005499	1.715471
.95	.007273	.006293	.005810	.005507	.005291	3.358578

Q = .0137 CV = 6.00

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000060	.000010	.000002	.000000	.000000	.000073
.10	.000219	.000065	.000024	.000009	.000004	.000324
.15	.000454	.000181	.000086	.000045	.000024	.000825
.20	.000750	.000359	.000202	.000123	.000079	.001688
.25	.001092	.000594	.000375	.000255	.000182	.003100
.30	.001473	.000880	.000604	.000443	.000339	.005398
.35	.001885	.001209	.000883	.000686	.000553	.009222
.40	.002322	.001576	.001206	.000979	.000822	.015878
.45	.002780	.001973	.001569	.001316	.001138	.028050
.50	.003253	.002396	.001964	.001690	.001497	.050478
.55	.003737	.002839	.002384	.002096	.001891	.088473
.60	.004227	.003296	.002825	.002525	.002313	.145347
.65	.004718	.003762	.003279	.002973	.002755	.224404
.70	.005205	.004231	.003740	.003430	.003210	.332736
.75	.005679	.004693	.004200	.003889	.003668	.484782
.80	.006124	.005135	.004643	.004334	.004115	.709767
.85	.006506	.005525	.005040	.004737	.004523	1.073330
.90	.006733	.005783	.005317	.005026	.004821	1.756577
.95	.006432	.005580	.005165	.004907	.004725	3.504648

Q = .0101 CV = 7.00

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000045	.000008	.000002	.000000	.000000	.000054
.10	.000163	.000049	.000018	.000007	.000003	.000242
.15	.000339	.000136	.000065	.000034	.000019	.000619
.20	.000559	.000269	.000153	.000094	.000060	.001270
.25	.000815	.000446	.000283	.000193	.000138	.002345
.30	.001099	.000660	.000455	.000336	.000258	.004115
.35	.001407	.000907	.000665	.000520	.000421	.007122
.40	.001734	.001182	.000910	.000741	.000625	.012552
.45	.002077	.001482	.001183	.000996	.000866	.023144
.50	.002432	.001800	.001481	.001281	.001139	.044444
.55	.002796	.002134	.001800	.001589	.001439	.082886
.60	.003166	.002480	.002134	.001916	.001761	.141361
.65	.003540	.002835	.002481	.002258	.002100	.222462
.70	.003912	.003194	.002835	.002609	.002451	.333333
.75	.004280	.003552	.003190	.002965	.002807	.489050
.80	.004632	.003900	.003540	.003316	.003160	.720373
.85	.004950	.004221	.003865	.003645	.003493	1.097191
.90	.005179	.004466	.004121	.003909	.003762	1.816511
.95	.005087	.004431	.004116	.003924	.003791	3.728008

Q = .0078 CV = 8.00

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000034	.000006	.000001	.000000	.000000	.000042
.10	.000126	.000038	.000014	.000006	.000002	.000188
.15	.000262	.000106	.000051	.000026	.000015	.000480
.20	.000432	.000209	.000119	.000073	.000047	.000988
.25	.000630	.000346	.000220	.000151	.000108	.001830
.30	.000850	.000512	.000354	.000262	.000202	.003229
.35	.001089	.000704	.000518	.000406	.000330	.005642
.40	.001342	.000918	.000708	.000579	.000489	.010125
.45	.001608	.001150	.000921	.000778	.000678	.019374
.50	.001883	.001398	.001154	.001000	.000892	.039637
.55	.002166	.001659	.001403	.001241	.001127	.078530
.60	.002455	.001929	.001664	.001498	.001380	.138344
.65	.002747	.002206	.001936	.001766	.001647	.220950
.70	.003040	.002489	.002215	.002044	.001924	.333566
.75	.003331	.002772	.002497	.002326	.002207	.491739
.80	.003614	.003051	.002776	.002607	.002490	.727287
.85	.003877	.003315	.003043	.002877	.002763	1.113018
.90	.004086	.003533	.003268	.003108	.002998	1.857107
.95	.004092	.003574	.003328	.003180	.003079	3.886786

Q = .0061 CV = 9.00

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000027	.000005	.000001	.000000	.000000	.000033
.10	.000100	.000030	.000011	.000004	.000002	.000150
.15	.000208	.000084	.000040	.000021	.000012	.000383
.20	.000344	.000167	.000095	.000059	.000038	.000789
.25	.000501	.000276	.000176	.000121	.000087	.001466
.30	.000677	.000409	.000283	.000210	.000162	.002596
.35	.000866	.000562	.000414	.000325	.000265	.004567
.40	.001068	.000732	.000566	.000464	.000393	.008313
.45	.001280	.000918	.000737	.000623	.000544	.016425
.50	.001500	.001116	.000923	.000801	.000716	.035736
.55	.001726	.001324	.001122	.000995	.000905	.075090
.60	.001956	.001540	.001331	.001200	.001108	.136040
.65	.002191	.001763	.001550	.001417	.001323	.219785
.70	.002427	.001990	.001774	.001640	.001547	.333649
.75	.002662	.002220	.002003	.001869	.001776	.493542
.80	.002893	.002447	.002231	.002098	.002008	.732039
.85	.003112	.002666	.002451	.002322	.002233	1.124028
.90	.003297	.002856	.002646	.002520	.002435	1.885754
.95	.003348	.002929	.002732	.002615	.002536	4.002704

Q = .0050 CV = 10.00

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000022	.000004	.000001	.000000	.000000	.000027
.10	.000082	.000025	.000009	.000004	.000002	.000122
.15	.000170	.000069	.000033	.000017	.000010	.000312
.20	.000280	.000136	.000078	.000048	.000031	.000644
.25	.000408	.000225	.000144	.000099	.000071	.001199
.30	.000551	.000333	.000231	.000172	.000133	.002129
.35	.000705	.000458	.000338	.000266	.000217	.003766
.40	.000870	.000597	.000462	.000379	.000322	.006932
.45	.001042	.000748	.000602	.000510	.000446	.014080
.50	.001221	.000910	.000754	.000655	.000586	.032517
.55	.001406	.001080	.000916	.000814	.000741	.072337
.60	.001594	.001257	.001088	.000982	.000908	.134256
.65	.001786	.001439	.001267	.001160	.001085	.218882
.70	.001980	.001626	.001452	.001344	.001269	.333667
.75	.002174	.001815	.001640	.001532	.001458	.494811
.80	.002365	.002004	.001829	.001722	.001650	.735444
.85	.002549	.002187	.002013	.001909	.001838	1.131983
.90	.002711	.002351	.002181	.002080	.002012	1.906667
.95	.002781	.002436	.002276	.002181	.002117	4.089444

II. $M/E_k/1$

K = 2 CV = .707

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000566	.000054	.000006	.000001	.000000	.000626
.10	.002051	.000368	.000073	.000016	.000004	.002513
.15	.004184	.001069	.000301	.000091	.000029	.005689
.20	.006747	.002185	.000776	.000295	.000118	.010204
.25	.009558	.003684	.001548	.000694	.000326	.016129
.30	.012470	.005501	.002630	.001338	.000711	.023560
.35	.015359	.007548	.003999	.002247	.001315	.032623
.40	.018116	.009729	.005603	.003413	.002161	.043478
.45	.020645	.011941	.007371	.004796	.003239	.056328
.50	.022857	.014080	.009212	.006336	.004513	.071429
.55	.024665	.016037	.011024	.007945	.005919	.089102
.60	.025978	.017701	.012696	.009521	.007368	.109756
.65	.026697	.018950	.014100	.010941	.008746	.133914
.70	.026708	.019652	.015097	.012065	.009917	.162252
.75	.025871	.019651	.015524	.012728	.010716	.195652
.80	.024010	.018760	.015188	.012734	.010945	.235294
.85	.020889	.016738	.013849	.011840	.010362	.282779
.90	.016187	.013267	.011190	.009732	.008652	.340336
.95	.009449	.007903	.006781	.005988	.005397	.411162

K = 3 CV = .577

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000763	.000065	.000006	.000001	.000000	.000835
.10	.002798	.000452	.000084	.000017	.000004	.003356
.15	.005777	.001339	.000353	.000101	.000030	.007614
.20	.009431	.002787	.000931	.000336	.000128	.013699
.25	.013536	.004786	.001902	.000815	.000367	.021739
.30	.017904	.007280	.003308	.001615	.000827	.031915
.35	.022371	.010181	.005148	.002788	.001580	.044465
.40	.026795	.013382	.007385	.004351	.002678	.059701
.45	.031042	.016763	.009950	.006286	.004140	.078035
.50	.034985	.020193	.012746	.008538	.005949	.100000
.55	.038494	.023530	.015653	.011018	.008052	.126305
.60	.041423	.026617	.018525	.013603	.010352	.157895
.65	.043604	.029272	.021187	.016136	.012711	.196056
.70	.044824	.031276	.023425	.018412	.014942	.242574
.75	.044800	.032348	.024966	.020170	.016795	.300000
.80	.043128	.032106	.025444	.021054	.017926	.372093
.85	.039203	.029993	.024332	.020562	.017850	.464630
.90	.032060	.025137	.020821	.017923	.015824	.586957
.95	.020036	.016061	.013552	.011857	.010625	.755230

K = 4 CV = .500

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000863	.000069	.000006	.000001	.000000	.000939
.10	.003184	.000486	.000087	.000017	.000003	.003778
.15	.006615	.001452	.000369	.000102	.000030	.008582
.20	.010869	.003051	.000986	.000347	.000129	.015464
.25	.015705	.005291	.002039	.000852	.000375	.024590
.30	.020921	.008127	.003590	.001714	.000860	.036193
.35	.026341	.011480	.005657	.003002	.001672	.050585
.40	.031807	.015247	.008218	.004755	.002881	.068182
.45	.037174	.019308	.011216	.006972	.004528	.089536
.50	.042301	.023531	.014562	.009613	.006618	.115385
.55	.047041	.027765	.018138	.012602	.009112	.146726
.60	.051229	.031840	.021795	.015820	.011927	.184932
.65	.054665	.035554	.025345	.019105	.014929	.231930
.70	.057092	.038652	.028549	.022236	.017919	.290514
.75	.058145	.040794	.031085	.024911	.020618	.364865
.80	.057277	.041486	.032494	.026697	.022614	.461538
.85	.053600	.039948	.032062	.026924	.023271	.591405
.90	.045546	.034829	.028566	.024454	.021511	.773885
.95	.030032	.023504	.019653	.017113	.015287	1.047389

K = 5 CV = .447

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000924	.000071	.000006	.000001	.000000	.001002
.10	.003420	.000503	.000087	.000017	.000003	.004032
.15	.007132	.001512	.000376	.000102	.000029	.009165
.20	.011764	.003196	.001011	.000349	.000128	.016529
.25	.017070	.005575	.002108	.000867	.000376	.026316
.30	.022838	.008615	.003739	.001760	.000872	.038793
.35	.028889	.012245	.005937	.003111	.001713	.054324
.40	.035060	.016368	.008692	.004972	.002982	.073394
.45	.041201	.020871	.011959	.007357	.004736	.096659
.50	.047167	.025621	.015658	.010242	.006995	.125000
.55	.052805	.030471	.019678	.013560	.009738	.159631
.60	.057944	.035248	.023875	.017204	.012893	.202247
.65	.062377	.039748	.028063	.021016	.016338	.255287
.70	.065824	.043705	.031997	.024778	.019880	.322368
.75	.067887	.046754	.035340	.028177	.023233	.409091
.80	.067938	.048345	.037588	.030744	.025958	.524590
.85	.064908	.047566	.037919	.031717	.027339	.684834
.90	.056767	.042716	.034818	.029705	.026070	.920455
.95	.039095	.030130	.025052	.021748	.019390	1.298561

K = 6 CV = .408

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000965	.000072	.000006	.000001	.000000	.001044
.10	.003580	.000514	.000088	.000016	.000003	.004202
.15	.007483	.001549	.000379	.000102	.000029	.009554
.20	.012375	.003287	.001025	.000350	.000126	.017241
.25	.018006	.005756	.002147	.000874	.000375	.027473
.30	.024163	.008930	.003829	.001784	.000876	.040541
.35	.030662	.012747	.006112	.003174	.001734	.056845
.40	.037339	.017115	.008995	.005104	.003040	.076923
.45	.044044	.021924	.012444	.007601	.004863	.101504
.50	.050629	.027049	.016387	.010651	.007236	.131579
.55	.056944	.032344	.020722	.014198	.010148	.168524
.60	.062816	.037643	.025311	.018146	.013544	.214286
.65	.068037	.042740	.029974	.022347	.017311	.271704
.70	.072324	.047372	.034470	.026587	.021266	.345070
.75	.075264	.051168	.038461	.030557	.025130	.441176
.80	.076194	.053555	.041427	.033779	.028457	.571429
.85	.073936	.053547	.042489	.035444	.030494	.756545
.90	.066134	.049207	.039940	.033994	.029787	1.038462
.95	.047222	.036004	.029820	.025835	.023004	1.516807

K = 7 CV = .378

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.000994	.000073	.000006	.000001	.000000	.001074
.10	.003694	.000520	.000088	.000016	.000003	.004323
.15	.007736	.001574	.000381	.000101	.000029	.009832
.20	.012819	.003348	.001033	.000350	.000125	.017751
.25	.018689	.005880	.002173	.000877	.000374	.028302
.30	.025133	.009150	.003889	.001799	.000878	.041796
.35	.031966	.013100	.006230	.003214	.001746	.058659
.40	.039024	.017646	.009205	.005193	.003077	.079470
.45	.046156	.022681	.012785	.007769	.004948	.105013
.50	.053217	.028085	.016907	.010937	.007401	.136364
.55	.060057	.033717	.021476	.014654	.010438	.175024
.60	.066507	.039414	.026362	.018829	.014012	.223140
.65	.072360	.044978	.031390	.023326	.018022	.283875
.70	.077339	.050150	.036329	.027939	.022298	.362069
.75	.081029	.054562	.040845	.032369	.026568	.465517
.80	.082754	.057637	.044419	.036138	.030395	.607595
.85	.081279	.058354	.046147	.038420	.033009	.813321
.90	.074027	.054624	.044200	.037555	.032870	1.135514
.95	.054497	.041222	.034046	.029452	.026200	1.708202

K = 8 CV = .354

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.001016	.000073	.000006	.000001	.000000	.001096
.10	.003781	.000525	.000088	.000016	.000003	.004414
.15	.007928	.001591	.000382	.000101	.000028	.010041
.20	.013155	.003392	.001039	.000349	.000124	.018135
.25	.019209	.005971	.002190	.000879	.000373	.028926
.30	.025874	.009312	.003930	.001809	.000878	.042741
.35	.032965	.013362	.006315	.003242	.001753	.060028
.40	.040320	.018042	.009358	.005256	.003103	.081395
.45	.047787	.023251	.013037	.007890	.005009	.107672
.50	.055224	.028871	.017296	.011149	.007522	.140000
.55	.062482	.034765	.022046	.014994	.010652	.179983
.60	.069397	.040778	.027162	.019346	.014363	.229927
.65	.075767	.046715	.032481	.024075	.018564	.293257
.70	.081321	.052326	.037777	.028988	.023095	.375274
.75	.085651	.057251	.042725	.033792	.027696	.484615
.80	.088082	.060919	.046815	.038022	.031939	.636364
.85	.087353	.062297	.049137	.040849	.035059	.859388
.90	.080747	.059203	.047793	.040555	.035465	1.216738
.95	.061021	.045877	.037809	.032671	.029043	1.877415

K = 9 CV = .333

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.001033	.000074	.000006	.000001	.000000	.001114
.10	.003848	.000529	.000088	.000016	.000003	.004484
.15	.008078	.001604	.000382	.000100	.000028	.010204
.20	.013419	.003425	.001042	.000348	.000124	.018433
.25	.019618	.006039	.002202	.000879	.000372	.029412
.30	.026458	.009435	.003961	.001815	.000878	.043478
.35	.033756	.013564	.006378	.003262	.001758	.061097
.40	.041347	.018350	.009474	.005302	.003121	.082902
.45	.049084	.023695	.013231	.007982	.005053	.109756
.50	.056825	.029486	.017597	.011311	.007613	.142857
.55	.064423	.035592	.022490	.015258	.010817	.183891
.60	.071720	.041859	.027793	.019751	.014637	.235294
.65	.078520	.048102	.033347	.024668	.018991	.300712
.70	.084558	.054076	.038935	.029825	.023729	.385827
.75	.089436	.059433	.044245	.034940	.028604	.500000
.80	.092490	.063613	.048776	.039561	.033200	.659794
.85	.092454	.065586	.051627	.042868	.036761	.897516
.90	.086528	.063122	.050862	.043116	.037679	1.285714
.95	.066892	.050048	.041178	.035551	.031586	2.028090

K = 10 CV = .316

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.001047	.000074	.000006	.000001	.000000	.001128
.10	.003902	.000531	.000088	.000016	.000003	.004541
.15	.008199	.001614	.000382	.000100	.000028	.010334
.20	.013632	.003451	.001045	.000348	.000123	.018672
.25	.019948	.006093	.002211	.000880	.000370	.029801
.30	.026931	.009533	.003985	.001819	.000877	.044070
.35	.034396	.013725	.006428	.003277	.001761	.061956
.40	.042182	.018595	.009565	.005338	.003134	.084112
.45	.050140	.024051	.013384	.008054	.005088	.111434
.50	.058131	.029982	.017837	.011439	.007684	.145161
.55	.066013	.036260	.022848	.015469	.010948	.187049
.60	.073629	.042738	.028302	.020077	.014857	.239645
.65	.080789	.049234	.034051	.025148	.019336	.306777
.70	.087239	.055514	.039884	.030508	.024246	.394454
.75	.092592	.061239	.045499	.035885	.029350	.512658
.80	.096195	.065863	.050410	.040842	.034247	.679245
.85	.096796	.068371	.053730	.044572	.038197	.929593
.90	.091549	.066511	.053513	.045325	.039588	1.345018
.95	.072195	.053805	.044209	.038140	.033871	2.163116

K = 15 CV = .258

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.001088	.000075	.000006	.000001	.000000	.001169
.10	.004066	.000538	.000087	.000016	.000003	.004711
.15	.008565	.001642	.000382	.000099	.000027	.010725
.20	.014279	.003525	.001051	.000345	.000120	.019391
.25	.020954	.006249	.002236	.000879	.000366	.030973
.30	.028376	.009818	.004050	.001829	.000874	.045852
.35	.036363	.014198	.006569	.003317	.001768	.064546
.40	.044755	.019326	.009830	.005439	.003169	.087774
.45	.053410	.025121	.013835	.008261	.005183	.116523
.50	.062196	.031484	.018554	.011816	.007890	.152174
.55	.070982	.038302	.023925	.016097	.011334	.196702
.60	.079631	.045445	.029856	.021061	.015515	.253012
.65	.087978	.052757	.036222	.026620	.020388	.325537
.70	.095807	.060037	.042847	.032632	.025847	.421376
.75	.102791	.066997	.049475	.038873	.031703	.552632
.80	.108366	.073168	.055693	.044971	.037619	.741722
.85	.111403	.077650	.060716	.050221	.042951	1.035312
.90	.109148	.078299	.062710	.052982	.046199	1.549180
.95	.092437	.068064	.055694	.047946	.042521	2.671247

K = 20 CV = .224

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.001109	.000075	.000006	.000001	.000000	.001190
.10	.004148	.000541	.000087	.000015	.000003	.004796
.15	.008749	.001654	.000382	.000098	.000027	.010921
.20	.014606	.003560	.001053	.000344	.000119	.019751
.25	.021466	.006323	.002246	.000878	.000364	.031561
.30	.029115	.009956	.004079	.001833	.000871	.046747
.35	.037372	.014430	.006634	.003334	.001769	.065851
.40	.046081	.019688	.009956	.005484	.003183	.089623
.45	.055103	.025656	.014055	.008359	.005227	.119099
.50	.064312	.032242	.018908	.011998	.007988	.155738
.55	.073585	.039343	.024466	.016409	.011522	.201631
.60	.082796	.046840	.030647	.021558	.015844	.259878
.65	.091800	.054592	.037342	.027375	.020924	.335247
.70	.100408	.062423	.044398	.033739	.026678	.435454
.75	.108340	.070082	.051594	.040459	.032947	.573826
.80	.115111	.077166	.058570	.047214	.039446	.775510
.85	.119731	.082885	.064644	.053391	.045615	1.094261
.90	.119691	.085303	.068161	.057514	.050108	1.669197
.95	.105947	.077527	.063305	.054438	.048245	3.005697

K = 25 CV = .200

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.001121	.000075	.000006	.000001	.000000	.001203
.10	.004198	.000543	.000087	.000015	.000003	.004847
.15	.008861	.001662	.000382	.000097	.000026	.011038
.20	.014805	.003579	.001053	.000343	.000118	.019967
.25	.021776	.006366	.002251	.000876	.000362	.031915
.30	.029564	.010037	.004095	.001834	.000869	.047285
.35	.037987	.014567	.006672	.003343	.001769	.066636
.40	.046890	.019904	.010031	.005510	.003190	.090737
.45	.056139	.025977	.014185	.008416	.005251	.120655
.50	.065610	.032700	.019120	.012106	.008045	.157895
.55	.075186	.039975	.024791	.016595	.011634	.204622
.60	.084751	.047691	.031126	.021857	.016041	.264059
.65	.094170	.055718	.038026	.027833	.021248	.341184
.70	.103278	.063897	.045352	.034417	.027186	.444109
.75	.111827	.072005	.052909	.041441	.033717	.586957
.80	.119393	.079685	.060379	.048622	.040592	.796680
.85	.125105	.086244	.067159	.055418	.047318	1.131854
.90	.126699	.089939	.071764	.060508	.052689	1.748201
.95	.115581	.084256	.068711	.059048	.052310	3.242515

K = 50 CV = .141

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.001146	.000075	.000006	.000001	.000000	.001228
.10	.004297	.000546	.000086	.000015	.000003	.004948
.15	.009085	.001675	.000381	.000096	.000026	.011274
.20	.015205	.003618	.001054	.000340	.000117	.020400
.25	.022404	.006450	.002260	.000874	.000358	.032623
.30	.030473	.010196	.004126	.001835	.000865	.048366
.35	.039235	.014838	.006744	.003358	.001768	.068214
.40	.048539	.020333	.010174	.005559	.003203	.092979
.45	.058255	.026619	.014441	.008525	.005297	.123791
.50	.068270	.033620	.019540	.012318	.008156	.162252
.55	.078481	.041252	.025443	.016964	.011854	.210682
.60	.088789	.049422	.032094	.022458	.016435	.272559
.65	.099093	.058026	.039419	.028763	.021904	.353315
.70	.109275	.066943	.047315	.035809	.028225	.461908
.75	.119176	.076019	.055645	.043480	.035312	.614206
.80	.128533	.085021	.064198	.051589	.043004	.841202
.85	.136806	.093510	.072587	.059789	.050985	1.212622
.90	.142543	.100367	.079857	.067226	.058478	1.924830
.95	.139546	.100942	.082107	.070466	.062372	3.827131

K = 100 CV = .100

AUTOCORRELATION LAG N

RHO	N= 1	2	3	4	5	TOTAL
.05	.001158	.000075	.000006	.000001	.000000	.001241
.10	.004347	.000548	.000086	.000015	.000003	.004999
.15	.009198	.001682	.000380	.000096	.000026	.011391
.20	.015406	.003636	.001054	.000339	.000116	.020616
.25	.022721	.006491	.002264	.000872	.000356	.032978
.30	.030934	.010274	.004140	.001835	.000862	.048908
.35	.039869	.014972	.006778	.003365	.001766	.069006
.40	.049378	.020546	.010244	.005581	.003208	.094106
.45	.059336	.026939	.014566	.008578	.005319	.125371
.50	.069634	.034082	.019749	.012422	.008208	.164452
.55	.080176	.041898	.025769	.017147	.011962	.213750
.60	.090875	.050303	.032583	.022759	.016632	.276880
.65	.101649	.059208	.040128	.029235	.022235	.359513
.70	.112409	.068517	.048325	.036522	.028756	.471062
.75	.123051	.078115	.057069	.044538	.036138	.628350
.80	.133413	.087846	.066215	.053154	.044274	.864629
.85	.143185	.097445	.075521	.062149	.052963	1.256080
.90	.151531	.106254	.084419	.071011	.061738	2.023978
.95	.154677	.111447	.090534	.077646	.068699	4.193734

K = 10000 CV = .010

RHO	N= 1	AUTOCORRELATION LAG N 2	3	4	5	TOTAL
.05	.001171	.000076	.000006	.000001	.000000	.001253
.10	.004397	.000549	.000086	.000015	.000003	.005050
.15	.009310	.001688	.000380	.000095	.000025	.011508
.20	.015607	.003653	.001054	.000337	.000115	.020831
.25	.023037	.006530	.002267	.000870	.000354	.033330
.30	.031394	.010349	.004152	.001835	.000859	.049445
.35	.040503	.015103	.006811	.003371	.001764	.069793
.40	.050220	.020755	.010311	.005602	.003213	.095227
.45	.060422	.027256	.014689	.008629	.005338	.126943
.50	.071006	.034541	.019954	.012522	.008259	.166644
.55	.081886	.042542	.026092	.017327	.012068	.216815
.60	.092986	.051185	.033069	.023058	.016826	.281206
.65	.104244	.060398	.040839	.029706	.022565	.365738
.70	.115606	.070110	.049343	.037240	.029290	.480298
.75	.127026	.080251	.058516	.045611	.036975	.642710
.80	.138464	.090755	.068288	.054759	.045576	.888642
.85	.149884	.101560	.078585	.064611	.055026	1.301333
.90	.161250	.112600	.089332	.075085	.065246	2.130458
.95	.172491	.123791	.100432	.086078	.076126	4.623463

K = 1000000 CV = .001

RHO	N= 1	AUTOCORRELATION LAG N 2	3	4	5	TOTAL
.05	.001171	.000076	.000006	.000001	.000000	.001253
.10	.004398	.000549	.000086	.000015	.000003	.005050
.15	.009311	.001688	.000380	.000095	.000025	.011509
.20	.015609	.003653	.001054	.000337	.000115	.020833
.25	.023041	.006531	.002267	.000870	.000354	.033333
.30	.031399	.010350	.004153	.001835	.000859	.049450
.35	.040510	.015104	.006811	.003371	.001764	.069800
.40	.050229	.020758	.010312	.005602	.003213	.095238
.45	.060433	.027259	.014690	.008629	.005339	.126959
.50	.071020	.034546	.019956	.012523	.008260	.166666
.55	.081903	.042548	.026095	.017329	.012069	.216846
.60	.093007	.051194	.033074	.023061	.016828	.281250
.65	.104270	.060410	.040846	.029711	.022569	.365800
.70	.115638	.070126	.049353	.037247	.029295	.480391
.75	.127066	.080273	.058531	.045622	.036984	.642856
.80	.138516	.090785	.068309	.054776	.045589	.888886
.85	.149953	.101602	.078617	.064637	.055048	1.301797
.90	.161351	.112667	.089384	.075128	.065282	2.131568
.95	.172686	.123926	.100540	.086170	.076208	4.628158

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E sub k

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Formulae, suitable for computation, for the lag n autocorrelation of departure intervals in the GI/M/1 queues are developed for n=1 to 6. A generating function, comparable to the one known for the M/G/1 case, is found for GI/M/1. For the E_k/M/1 and M/E_k/1 queues, expressions for arbitrary lag are given. Tables and the APL functions used to produce them are provided for E_k/M/1, H/M/1, and M/E_k/1.

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