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A "Relativistic Mirror" Experiment with Frequency Tuning and Energy Gain

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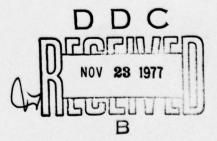
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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) READ INSTRUCTIONS BEFORE COMPLETING FORM REPORT DOCUMENTATION PAGE VRL-MR- 2. GOVT ACCESSION NO. NRL Memorandum Report 3608 OF BEROR & PERIOD COVERED TITLE (and Subtitte) Interim report, n a continuing "RELATIVISTIC MIRROR" EXPERIMENT WITH NRL problem FREQUENCY TUNING AND ENERGY GAIN. 6. PERFORMING ORG. REPORT NUMBER CONTRACT OR GRANT NUMBER(#) . Pasour*, V.L. Granatstein RK Parker FORMING ORGANIZATION NAME AND ADDRESS Naval Research Laboratory NRL Problem R08.59 Subtest RR0110941 Washington, D.C. 20375 11. CONTROLLING OFFICE NAME AND ADDRESS September 2977 13. NUMBER OF PAGES & ADDRESS(If different from Controlling Office) 15. SECURITY CLASS. (of thie report) UNCLASSIFIED DECLASSIFICATION/DOWNGRADING DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited. 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) 8. SUPPLEMENTARY NOTES *On temporary assignment at NRL. Work partially supported by the Air Force Office of Scientific Research under Contract F49620-76-C-0007. 9. KEY WORDS (Continue on reverse side if necessary and identify by block number) **Microwaves** Delta lambda/lamda Electron beam Electromagnetic Radiation 20. ABSTRACT (Continue on reverse side if necessary and identity by block number) The front of an intense relativistic electron beam has been used to reflect a counterstreaming electromagnetic wave (f_i = 9.3 GHz, P_i = 250 kW). A doubly-Dopper-shifted reflected wave with a pulse duration on the order of a nanosecond has been produced with a six-fold increase in frequency, a 13-dB power gain, and a doubling of wave energy. The output has been analyzed with a multichannel grating spectrometer and found to have a spectral width $\Delta\lambda/\lambda$ of \sim 5%. The reflected-wave frequency was tunable over a range of $\sim \pm 20\%$ by simply varying the external magnetic field, and (Continues)

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20. Abstract (Continued)

over a much broader range by simultaneously changing the beam rise time. All these observations are in good agreement with a simple theoretical model of beam-front scattering. This mechanism represents a new kind of short-pulse, high-power, tunable source of millimeter (and probably submillimeter) wavelength radiation, and it could be immediately useful as an electron beam velocity diagnostic.

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A "RELATIVISTIC MIRROR" EXPERIMENT WITH FREQUENCY TUNING AND ENERGY GAIN

INTRODUCTION

In his classic paper on electrodynamics in the framework of special relativity, 1 Einstein derived the provocative properties of electromagnetic reflection from a relativistic mirror. These properties may be listed as follows: 1) a double Doppler shift so that the ratio of reflected-wave frequency to incident-wave frequency for a backscattered plane wave is $^\omega_r/^\omega_i=(1+\beta)^2 \gamma^2$, where $\beta=v/c$, $\gamma=(1-\beta^2)^{-1/2}$, and v is the speed of the mirror; 2) a similar increase in the energy of the reflected wave at the expense of the mirror's kinetic energy; and 3) a time compression of the reflected wave so that the reflected power flux is enhanced by an even greater factor than the scattered energy: $P_r/P_i=(1+\beta)^4 \gamma^4$. In this paper we report an experimental study of electromagnetic reflection from the front of a magnetized relativistic electron beam in which all of the above features of the relativistic mirror have been observed.

The reflection of electromagnetic waves from the sharp front of a moving electron beam or plasma has been treated in a number of theoretical studies, $^{2-8}$ beginning with that of Landecker in 1952. Although these analyses predicted total reflection for a certain range of beam parameters, experimental observation of the intriguing relativistic effects was delayed by the unavailability of sufficiently dense beams of charged particles with velocity approaching that of light. This problem was circumvented in an early observation of a large Doppler shift $(^\omega_r/^\omega_i \ ^\simeq 1.2)$ by Zagorodnov et al, 9 who used the artifice of greatly reducing v_{ph} , the phase velocity in the wave-plasma interaction region, so that v/v_{ph} could approach unity. Now, however, generators of intense relativistic electron beams are available; and in the past year, fast electromagnetic waves have been reflected from the front of such beams to obtain radiation of frequency $^\omega_r$ several times

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larger than ω_i . 10,11 The present study extends these initial experiments in several important respects. First, the velocity of the beam-front electrons was made variable and the Doppler shift was found to be tunable with this velocity as predicted, thus positively identifying the reflection mechanism. Furthermore, it was found that as the beam-front velocity and the attendant Doppler shift increased, the power in the reflected wave increased even more, with a power gain as large as 20 being measured. The reflected-wave frequency spectrum was resolved using a spectrometer of special design, 12 and the spectral width (FWHM) of the reflected wave was found to be \sim 5%. Finally, the energy in the reflected wave was for the first time measured to be larger than the incident wave energy.

II. EXPERIMENT

The experimental configuration used to observe beam-front scattering is shown schematically in Fig. 1. The relativistic electron beam was generated by applying a 1-MV-peak, 60-nsec-duration voltage pulse from a pulse-charged water transmission line 13 to a cold-cathode, field-emission diode. 14 The electrons were injected through a 13-mm-diameter aperture in a graphite anode into an evacuated, 22-mm-diameter drift tube in a uniform magnetic field. A Lucite pulse-sharpening switch could be inserted into the cathode shank to decrease the cathodic voltage rise time from 15 nsec to 5 nsec and the beam current rise time from 7 nsec to \sim 3 nsec. The peak beam current was \sim 2 kA both with and without the switch. From the damage pattern on a Lucite disk placed in the drift tube, the beam profile was found to be relatively uniform over its 12-mm diameter.

The counterstreaming microwave signal was supplied by a 9.3-GHz magnetron.

The 500-kW, 500-nsec-duration pulse was transported into the drift tube through

a 90° bend in rectangular waveguide and a rectangular-to-circular transition. Consequently, the incident wave was linearly polarized and propagated in the fundamental waveguide mode (TE_{11}). The high-frequency output passed through 4-mm-diameter holes in the sidewall of the 90° bend, and a tapered transition to a Ka-band (26.5 - 40 GHz) waveguide, which led to a multichannel grating spectrometer. 12 Broad-band crystal detectors connected to 1-cm-wide rectangular horns were used to monitor the output, which was found to consist of two perpendicular components (suggesting a circularly-polarized reflected wave).

When the diode voltage and magnetron pulses were synchronized and the magnitude of the external magnetic field was properly adjusted, a strong high-frequency output pulse was observed. A typical oscilloscope trace of the output, obtained without the pulse-sharpening switch, is inset into Fig. 1. The pulse duration was only ~ 2 nsec, or more than an order of magnitude shorter than the beam pulse. With the pulse-sharpening switch, the duration decreased by ~ 20%, but the accuracy of the measurement was limited by the 1-nsec response time of the instrumentation.

The high-frequency output was found to be highly dependent on the external magnetic field. The total (i.e., spectrally integrated) high-frequency signal is plotted as a function of magnetic field in Fig. 2, both with and without the pulse-sharpening switch. The peak value of each curve has been normalized to unity, and the error bars represent the standard deviation of three data points. A resonance in output power occurred for magnetic field strengths centered at 5.4 kG without the switch and at 6.9 kG with the switch.

After the optimum value of magnetic field was found, the spectrometer was used to determine the spectral content of the output. The true spectral intensity profile was obtained from the experimental data by using Voigt-function fitting to compensate for apparatus-induced line broadening. The absolute output intensity was determined by calibrating the detectors at the observed frequencies with a water calorimeter and by measuring the attenuation of the spectrometer and of the waveguide components leading to it as a function of frequency.

Without the pulse-sharpening switch and at a magnetic field of 5.4 kG, the output was found to peak at a wavelength of 11 mm, as shown in Fig. 3. However, the attenuation of the output section leading to the spectrometer was extremely frequency dependent in this spectral region, so that the power calibration used for Fig. 3 had a potential error of nearly a factor of 3. Thus, although the intensity profile implies a total reflected power of ~ 700 kW, the actual value is only known with certainty to lie between 250 kW and 1.8 MW.

With the pulse-sharpening switch, data were taken at two values of magnetic field: 6.8 kG and 7.0 kG. From the spectral profiles shown in Fig. 4, it can be seen that the output wavelength was slightly shorter at the higher value of magnetic field. Over this wavelength range of approximately 5.5 - 6.2 mm, the attenuation of the output section was quite constant (~ 30 dB), and the various calibration data used to compute the absolute intensities plotted in Fig. 4 have a maximum cumulative uncertainty of less than \pm 2 dB. Consequently, the total integrated power, to within a factor of 1.6, is 5 MW or an order of magnitude larger than the incident power. The spectral width (FWHM) of the output is $\Delta\lambda/\lambda$ = 5%, or slightly less than the 7% width of the profile of Fig. 3.

III. DISCUSSION

A complete theoretical analysis of this experiment would have to consider the density and velocity gradients of the electrons near the front of the beam as well as the rapid temporal variation of these beam parameters. Although such an analysis has not been attempted, most of the experimental observations can be quantitatively explained by a simplified model of beam-front scattering.

First, the Doppler-shift relations given in the introduction for plane-wave reflection must be modified in the waveguide geometry of the experiment. The frequency, energy, and power shifts are given by 10

$$\omega_{\mathbf{r}} = \gamma^2 (1 + \beta^2 + 2\beta \beta_{\mathbf{q}i}) \omega_{i}$$
 (1)

$$W_{r} = \gamma^{2}(1 + \beta^{2} + 2\beta \beta_{0i})RW_{i}$$
 (2)

$$\gamma^{4}(1 + \beta^{2} + 2\beta \beta_{gi})^{2} (\beta_{gr}/\beta_{gi})R P_{i} = \frac{\beta_{gr}}{\beta_{gi}} \frac{\omega_{r}}{\omega_{i}} \frac{W_{r}}{W_{i}} P_{i}$$
 (3)

where $\beta_{gi} = v_{gi}/c$ and $\beta_{gr} = v_{gr}/c$ are the normalized group velocities of the incident and reflected waves, respectively.

The pulse duration of the reflected wave is 10 $_{\tau}$ = L(1/v - 1/v $_{gr}$), where L is the length of the reflection region. If L = 1 m and 0.5c < v < 0.75c (which follows from Eq. (1) given the experimentally-observed frequency shifts), then 1 nsec < $_{\tau}$ < 2 nsec. Although reflection probably does not occur over the full 1-m length of the drift tube, these values agree well with the observed output-pulse duration, considering the response-time limitations of the instrumentation.

According to the analysis presented by Landecker, ² reflection from a cold, uniform, semi-infinite electron beam propagating along an external field occurs

near the cyclotron resonance. In a waveguide geometry, 10 the resonance occurs at the intersection of the empty-waveguide mode (TE_{11})

$$\omega^2/c^2 - k_z^2 - b_{11}^2 = 0 (4)$$

and the fast beam-cyclotron mode

$$\omega - vk_z - \Omega_0/\gamma = 0, \qquad (5)$$

where $b_{11}=1.84/r_g$, r_g is the guide radius, and $\Omega_o=eB_o/m$ is the electron cyclotron frequency. Given a particular incident-wave frequency and external magnetic field, this condition determines the electron velocity required for reflection. Then the Doppler-shifted reflected-wave frequency is found from Eq. (1). For example, given a 9.3-GHz incident wave and magnetic fields of 5.4, 6.8, and 7.0 kG, the wavelength of the reflected wave should be 10.6, 6.3, and 6.0 mm, respectively. These values are in excellent agreement with the experimental observations, indicating convincingly that reflection does occur near this magnetic resonance. Consequently, the range of magnetic field over which reflection occurs (Fig. 2) corresponds to a frequency tunability range of $\sim \pm 20\%$ without the pulse-sharpening switch and of $\sim \pm 10\%$ with the switch.

To understand the effect of varying the rise time of the accelerating voltage on the reflected-wave frequency, the transient nature of the propagating beam front must be considered. According to a simple, free-streaming beam model in which the accelerating voltage rises linearly from zero to its peak value in a time t_R , the normalized beam-front velocity as a function of the distance z from the anode is given by

$$\beta_{f}(z) = \left\{ 1 - \left[1 + \left(\frac{\gamma_{M} - 1}{ct_{R}} z \right)^{2/3} \right]^{-1} \right\}^{1/2}$$
 (6)

where $\Upsilon_{\text{M}}\text{mc}^2$ is the energy of an electron accelerated across the peak potential. Use of such a simple model is somewhat justified by the fact that the beam current is nearly an order of magnitude less than the space-charge-limited value. However, space-charge effects could become important near the beam front where the density should become quite large. There should also be a rather large $(\pm\ 10\%)$ range of electron velocities within one incident wavelength of the front.

Equation (6) is plotted in Fig. 5 for the two different rise times used in the experiment. The beam-front velocity rises rapidly over the first portion (~ 20 cm) of the drift region and then gradually levels off. For the 15-nsec rise time, calculated values of $^{\beta}$ row vary from ~ 0.45 to ~ 0.6 over most of a 1-m-long drift tube. Experimentally, reflection is observed in this case for magnetic fields ranging from 4.7 kG to 5.9 kG, corresponding to a range of electron velocities required for reflection from 0.48 c to 0.65 c. Similarly, calculated values of $^{\beta}$ f vary from ~ 0.6 to ~ 0.75 over most of the guide when the pulse-sharpening switch is used (t_R = 5 nsec). The magnetic field for this case is required to lie between 6.4 kG and 7.4 kG, corresponding to electron velocities from 0.70 c to 0.77 c. These experimental observations suggest strongly that reflection occurs only at or very near the beam front. Although no reflection is observed at lower magnetic fields (corresponding to smaller electron velocities), the slower electrons are near the front for only a very short time and distance. Thus, even if reflection were to occur from these electrons, the reflected pulse would probably be of too short a duration to detect with our apparatus.

The fact that no reflection is observed at higher values of magnetic field, even though electrons of the corresponding velocities are present at some point

back into the beam, also indicates that the reflection mechanism depends upon the electrons' being in the immediate vicinity of the beam front. An explanation for this behavior can be formulated by considering the spatially-varying nature of the beam. It has been shown by Budden¹⁶ that, in a spatially-varying cold plasma, reflection occurs at a point where the refractive index is zero, but that absorption occurs where the index has a singularity. If the index of refraction has a zero and an infinity lying close together, reflection only occurs if the incident wave encounters the zero first. (This statement is true only if the plasma parameters change only slightly over an incident wavelength. Crescentini and Maroli¹⁷ have shown that reflection can occur at the singularity if there is a rapid variation.) When the zero is reached first, the degree of reflection depends on the separation between the two points; viz., if the separation is not large compared to a wavelength, tunneling of the incident wave between the zero and the infinity can occur, resulting in a decreased reflectivity.

To determine the effect of the spatially-varying beam-front in the present experiment, we will consider for simplicity a semi-infinite, free-streaming electron beam. The index of refraction for a right-hand-circularly-polarized wave in such a medium is

$$\eta = \left(\frac{\omega - \frac{\Omega}{0} - \frac{\omega^2}{p} / \omega}{\omega - \frac{\Omega}{0}}\right)^{1/2} \tag{7}$$

where $\omega_p = (ne^2/\varepsilon_{0}^{}m_{0}^{})^{1/2}$ is the electron plasma frequency. Equation (7) is valid in the electron rest frame. The position of an electron traveling at speed v at a time t after initiation of the accelerating voltage pulse is found from the free-streaming beam model to be

$$z(v,t) = v\left(t - \frac{\gamma - 1}{\gamma_M - 1} t_R\right). \tag{8}$$

From this relation, a Lorentz transformation gives the incident wave frequency as "seen" by the electrons as a function of position at various times t, as shown in Fig. 6. It is this frequency ω which is used in Eq. (7) to determine the local refractive index. The two values of ω at a particular position correspond to the two values of electron velocity at that point. In order that the zero in refractive index be reached before the infinity, Eq. (7) implies that ω must exceed Ω_0 as the wave approaches the singularity. Thus, for reflection, it is necessary that $\partial \omega(z)/\partial z > 0$ at the point where $\omega(z) = \Omega_0$ (such as at point B in Fig. 6). From the figure, it is evident that reflection should only occur when $\omega_0 < \Omega_0 < \omega_f(z=L)$, where ω_0 is the incident frequency in the laboratory frame and ω_f is the local frequency at the beam front. This condition on the magnetic field is equivalent to the requirement that reflection occurs only from electrons with $\beta < \beta_f(L)$. Consequently, the experimentally observed loss of reflection at higher magnetic fields (where $\Omega_0 < \omega_f(L)$ cannot be satisfied) is explained.

To quantify this model, the wave equation for the circularly polarized electric field E is obtained from the linearized, relativistic equations of motion and Maxwell's equations (under the assumption that the beam parameters vary slowly in space and time compared to the wave fields):

$$\gamma \left(\omega - \frac{\lambda}{\Omega^{0}} + i \sqrt{\frac{g^{2}}{2}} \right) \left(\frac{\omega_{S}^{2} + c^{2} \frac{g^{2}}{2}}{\omega_{S}^{2}} \right) E = \left(\omega + i \sqrt{\frac{g^{2}}{2}} \right) E.$$
 (9)

Antonsen has asymptotically solved this equation 18 using a method he has described elsewhere 19 to obtain for the reflection coefficient

$$R = e^{-\pi \rho} (1 - e^{-\pi \rho})^2, \qquad (10)$$

where $\rho = \omega_p^2 \left[\Omega_0 \frac{\partial v}{\partial z} \left(1 - \omega_p^2 / \Omega_0^2 \right) \right]^{-1}$. It is easy to show that R has a maximum value of 15%.

According to the cyclotron-resonance behavior predicted by theory and observed experimentally, only the right-hand-circularly-polarized (RHCP) component of the incident wave should be reflected. The incident power in that polarization is half the total value, or 250 kW; and from Eq. (3), the twenty-fold power increase of the 5-MW, 5.8-mm output implies a beam-frame reflection coefficient of 32%. This value lies between the maximum value of unity for a cold, uniform, sharp beam front² and the maximum value of 15% for the slowly-varying beam. The beam gradients in the experiment are probably moderately large, lying between the values of the gradients assumed in the two models.

Finally, from Eq. (2), the energy shift of the reflected wave is given by $W_r/W_i = R \ \omega_r/\omega_i$. The 5.8-mm output corresponds to a gain in energy for the RHCP component by a factor of 1.8. Thus, we have experimentally demonstrated the intriguing phenomenon of an upshift in frequency accompanied by a gain in total wave energy.

V. CONCLUSIONS

Reflection of a 9.3-GHz microwave from a magnetized relativistic electron beam has been experimentally observed. The frequency of the reflected wave was upshifted by nearly a factor of 6 with a simultaneous increase in both power (factor of 20) and energy (factor of 2). On the basis of the quantitative agreement of the results of this investigation (i.e., Doppler shift, magnetic resonance, short pulse duration, and narrow-band output) with a simple model, a convincing argument can be made for the existence of beam-front scattering.

The beam-front scattering process provides an easily-tunable source of high-peak-power millimeter waves, and it could probably be easily extended to the sub-millimeter region by appropriately choosing the incident wave frequency and beam parameters. One immediate application of this phenomenon is as a diagnostic for measuring beam-front velocity. Such a diagnostic might be useful, for example, in ion acceleration experiments of the type described by Olson. 20

V. ACKNOWLEDGMENTS

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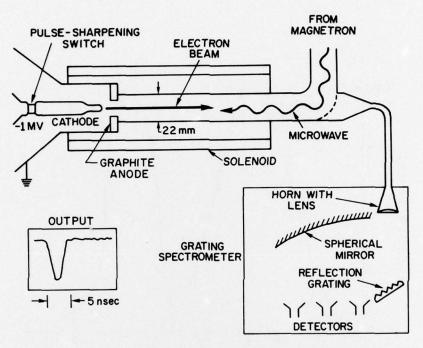


Fig. 1-Experimental configuration. Inset: oscilloscope trace of output signal

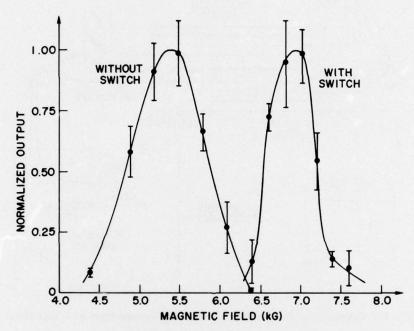


Fig. 2 — Normalized reflected-wave power vs axial magnetic field

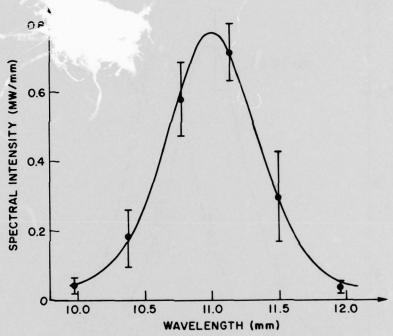
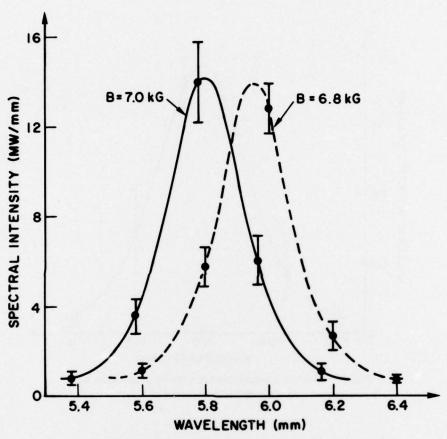


Fig. 3 — Spectral intensity profile without pulse-sharpening switch and with $B_{\rm O}$ = 5.4 kG



 ${\bf Fig.~4-Spectral~intensity~profiles~with~pulse-sharpening~switch~and~with~two~slightly~different~axial~magnetic~fields}$

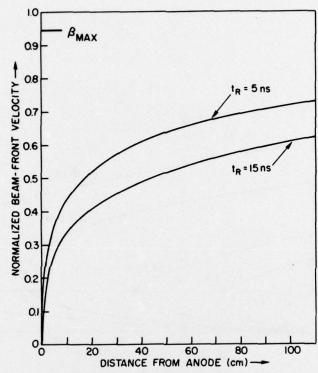
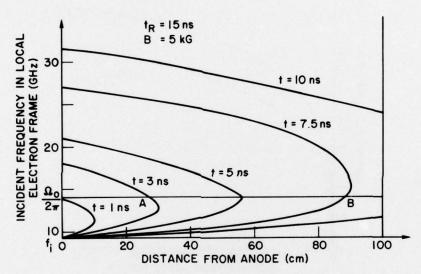


Fig. 5 — Normalized velocity of the leading electrons vs axial position of the beam front



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Fig. 6 — Incident wave frequency as viewed in local electron frame vs axial position at various times after initiation of accelerating voltage pulse. Parameters applicable to the experiment without the pulse-sharpening switch and a free-streaming beam model have been used to compute the curves. According to the model (see text), reflection should occur at point B, but not at point A.