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COMPUTATION OF THE SPATIAL CORRELATION OF THE OCEAN SURFACE, VI--ETC(U)

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COMPUTATION OF THE SPATIAL CORRELATION OF THE OCEAN SURFACE, VIA THE FFT.

by

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INTRODUCTION

The statistical properties of signals reflected from the ocean surface depends on the stochastic properties of the surface. An example of this is the prediction of the average energy reflected from the surface as a function of angle of incidence. To evaluate this quantity, it is necessary to know the spatial correlation of surface heights. Measurements or assumptions of the directional wave spectrum of the ocean are first obtained. The spatial-temporal correlation of the ocean surface is expressed in terms of this directional wave spectrum. This memorandum describes a method of evaluating this expression by means of the Fast Fourier

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ADMINISTRATIVE INFORMATION

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Transform (FFT). Since the FFT method is much faster than the usual numerical integration method, it is highly desirable for experimental data. This memorandum presents the first known evaluation of surface spatial correlation by the FFT method.

DIRECTIONAL WAVE SPECTRUM

A review of the statistical properties of the ocean surface is given in References (1), (2) and (3). The review will not be repeated here. The directional wave spectrum is expressed as $A^2(\omega,\theta)$. $A^2(\omega,\theta)d\omega\, d\theta$ is the amount of surface wave energy of angular frequency ω flowing in the θ direction in the interval $d\omega\, d\theta$. The relation between the spatial-temporal correlation and the directional wave spectrum is

$$\rho(u,v,\tau) = \frac{1}{2\sigma^2}\int_{-\pi}^{\pi} d\theta \int_{0}^{\infty} d\omega\, A^2(\omega,\theta)\cos\left[\frac{\omega^2}{g}u\cos\theta + \frac{\omega^2}{g}v\sin\theta - \omega\tau\right] \qquad \#1$$

where
- g is the acceleration of gravity constant
- u is the separation of the surface points in the x direction
- v is the separation of the surface points in the y direction
- τ is a separation of time at the two points
- ρ is the correlation
- σ^2 is the mean square height of the surface

Expressing the directional wave spectrum in terms of the surface wave numbers k_x, k_y in the x and y directions, we have (see References (1), (2) and (3))

$$\rho(u,v,\tau) = \frac{1}{\sigma^2}\iint_{-\infty}^{\infty} dk_x\, dk_y\, \hat{A}^2(k_x,k_y)\cos\left[k_x u + k_y v - \sqrt{g}\,(k_x^2+k_y^2)^{1/4}\tau\right] \qquad \#2$$

The spatial correlation of interest in this study is obtained by setting $\tau=0$. Then

$$\rho(u,v,0) = \frac{1}{\sigma^2}\iint_{-\infty}^{\infty} dk_x\, dk_y\, \hat{A}^2(k_x,k_y)\cos\left[k_x u + k_y v\right] \qquad \#3$$

In order to put this in the more convenient Fourier Transform, we have

$$\rho(2\pi u', 2\pi v', 0) = \frac{1}{\sigma^2}\,\mathrm{Re}\left\{\iint_{-\infty}^{\infty} dk_x\, dk_y\, \hat{A}^2(k_x,k_y) \cdot \exp\left[i(2\pi k_x u' + 2\pi k_y v')\right]\right\} \qquad \#4$$

The expression within the brackets is the two dimensional Fourier transform of $\hat{A}^2(k_x, k_y)$. Re stands for the real part of.

Let us now consider a wind in the x direction. It is sometimes assumed that the directional wave spectrum exists in the first and fourth quadrants only and that the spectrum is symmetric about the x axis. The above assumptions are intuitively reasonable.

For these assumptions,

$$\rho(2\pi u', 2\pi v', 0) = \frac{2}{\sigma^2} \operatorname{Re}\left\{ \int_{-\infty}^{\infty} dk_y \int_0^{\infty} dk_x \, \hat{A}^2(k_x, k_y) \exp[i2\pi(k_x u' + k_y v')] \right\}$$

or

$$\rho(2\pi u', 2\pi v', 0) = \frac{2}{\sigma^2} \operatorname{Re}\left\{ \int_0^{\infty}\int_0^{\infty} dk_x \, dk_y \, \hat{A}^2(k_x, k_y) \exp[i2\pi(k_x u' + k_y v')] \right.$$

$$\left. + \int_{-\infty}^{0} dk_y \int_0^{\infty} dk_x \, \hat{A}^2(k_x, k_y) \exp[i2\pi(k_x u' + k_y v')] \right\}$$

Let us now consider the second double integral and let $k_y' = -k_y$.
Then

$$\int_{-\infty}^{0} dk_y \int_0^{\infty} dk_x \, \hat{A}^2(k_x, k_y) \exp[i2\pi(k_x u' + k_y v')]$$

$$= \int_0^{\infty}\int_0^{\infty} dk_{y'} \, dk_x \, \hat{A}^2(k_x, k_{y'}) \exp[i2\pi(k_x u' - k_{y'} v')]$$

where we have used the symmetry property of $\hat{A}^2(k_x, k_{y'})$

Let $F(u', v') = \int_0^{\infty}\int_0^{\infty} dk_x \, dk_y \, \hat{A}^2(k_x, k_y) \exp[i2\pi(k_x u' + k_y v')]$

Then

$$\rho(2\pi u', 2\pi v', 0) = \frac{2}{\sigma^2} \operatorname{Re}\left\{ F(u', v') + F(u', -v') \right\} \qquad \#5$$

Note that $\rho(2\pi u', 2\pi v', 0)$ is symmetric in v' but not in general symmetric in u'.

For many directional wave spectra, such as the Neumann-Pierson spectrum, Equation (5) can not be integrated analytically. It can by evaluated by digital methods.

DIGITAL FORM

Let us first evaluate $F(u',v')$ in digital form. The properties of the discrete Fourier transform (DFT) answer will contain $F(u',-v')$. We choose a distance D large enough so that to a good approximation,

$$F(u',v') \cong \int_0^D\int_0^D dk_x\,dk_y\,\hat{A}^2(k_x,k_y)\exp\left[i2\pi(k_x u' + k_y v')\right] \qquad \#6$$

Since Equation (6) is now defined in a finite interval, it may be approximated by a double finite sum. Consider the grid shown in Figure 1.

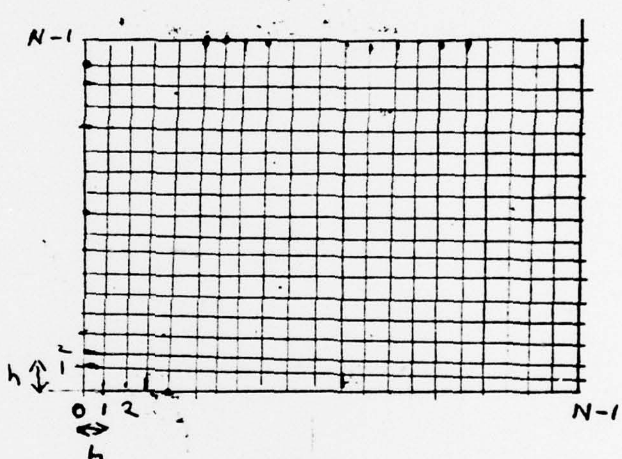

FIGURE I

We approximate Equation (6) by a sum of rectangular complex volumes. The grid size h is chosen such that there is not a significant change of the complex height in this interval. The number of grid intervals is N in each dimension, where D=Nh. Then

$$F(u',v') \cong \sum_{I=0}^{N-1}\sum_{J=0}^{N-1} h^2\hat{A}^2(hI,hJ)\exp\left[i2\pi(hIu' + hJv')\right] \qquad \#7$$

u' and v' are each periodic, with period D. Values of u' and v' may be obtained at discrete values of $\frac{1}{D} = \frac{1}{Nh}$.

Then

$$F\left(\frac{p}{D},\frac{q}{D}\right) \cong \sum_{I=0}^{N-1}\sum_{J=0}^{N-1} h^2\hat{A}^2(hI,hJ)\exp\left[i2\pi\left(\frac{hIp + hJq}{hN}\right)\right] \qquad \#8$$

where $p, q = 0, 1, 2, \ldots\ldots N-1$

Define $F_{pq} = F\left(\frac{p}{D}, \frac{q}{D}\right)$

$$\hat{A}_{IJ} = \hat{A}^2(hI, hJ)$$

In addition, for better accuracy, we introduce a weighting factor W_{IJ}, such as that given by Simpson's or the Trapezoidal rule. Then

$$F_{p,q} = \sum_{I=0}^{N-1} \sum_{J=0}^{N-1} W_{IJ} \hat{A}_{IJ} \exp\left[i \frac{2\pi}{N}(Ip + Jq)\right] \qquad \#9$$

In the same manner $F_{p,-q} \cong F(u', -v')$. In Equation (9), values of $q < N/2$ represent positive values of v', whereas values of $q > N/2$ represent negative values of v'. Then

$$C(2\pi u', 2\pi v', 0) = \frac{h^2}{6^2} \operatorname{Re}\left\{F_{p,q} + F_{p,N-q}\right\} \qquad \#10$$

where $p, q = 0, 1, 2, \ldots \ldots N/2 - 1$

NEUMANN-PIERSON SPECTRUM

For a fully aroused sea, the Neumann-Pierson Spectrum is

$$A^2(\omega, \theta) = C/\omega^6 \exp\left(-\frac{2g^2}{\omega^2 s^2}\right) \cos^2\theta, \quad \omega > 0, \; -\frac{\pi}{2} \le \theta \le \pi/2 \qquad \#11$$

$$= 0, \text{ other-wise}$$

From References (1), (2) or (3)

$$\hat{A}^2(k_x, k_y) = \frac{\sqrt{g}}{2} \frac{A^2(\omega, \theta)}{(k_x^2 + k_y^2)^{3/4}} \qquad \#12$$

where

$$\omega = \sqrt{g}\,(k_x^2 + k_y^2)^{1/4}, \quad \theta = \arg(k_x + i k_y)$$

where $\arg$ is the argument of the complex value $k_x + ik_y$. Then from Equations (11) and (12),

$$\hat{A}^2(k_x, k_y) = \frac{g^2}{2} \frac{C}{\omega^9} \exp\left(-\frac{2g^2}{\omega^2 s^2}\right) \cos^2(\theta), \quad \omega > 0, \; -\frac{\pi}{2} \le \theta \le \pi/2 \qquad \#13$$

= 0, otherwise

FFT EVALUATION

Equations (9), (10) and (13) are the main equations of this study. Letting $k_x = Ih$ and $k_y = Jh$ in Equation (12) and then defining $\hat{A}^2(Ih, Jh) = \hat{A}_{IJ}$ into Equation (9); we may compute F_{pq}. As noted earlier, F_{pq} is the DFT of $\hat{A}_{IJ}$. Thus a two dimensional FFT may be applied.

One example of the FFT will now be discussed. A wind speed of 5 knots was chosen. h was set equal to $1/64^2$. This gives a $\Delta k_x = \Delta k_y = \frac{1}{64^2}$. N was chosen as 128.

Thus the range of $k_x = k_y = \frac{128}{64^2} = \frac{1}{32}\ cm^{-1}$.

The corresponding $\Delta\omega = \sqrt{g}\left(\frac{1}{64^4}\right)^{1/4} \simeq 0.49\ sec^{-1}$

and range of $\omega = \sqrt{g}\left(\frac{1}{64^2} \times 2 \times 128 \times 128\right)^{1/4} \simeq 6.58\ sec^{-1}$

The Neumann-Pierson spectrum changes rapidly at the lower angular frequencies and decays slowly at the higher angular frequencies. The above choice of Δk_x is sufficient to sample fine enough at the lower frequencies and $N\,\Delta k_x$ is sufficiently large to obtain most of the contributing range of angular frequencies.

The computer program is shown in Appendix A. The first part of the program computes Equation (13) and fills up an array of 128 x 128 dimensions. If each computation of Equation (13) is considered as a complex quantity, the number of computer word needed is 256 x 128 = 32,768. This is within the memory capacity of the UNIVAC 1108. If N=256, then the memory capacity must equal 256 x 512 = 131,072 words, to store the complex values. This is greater than the capacity of the UNIVAC 1108. The elements of the matrix were weighted by W_{IJ}, where in this case the trapezoidal rule was used. This assigns a $W_{IJ} = .5$ on the borders of the grid and .25 on the corners of the grid and 1 for all other values. The FFT subroutine is then called. The subroutine is described in Reference (4). Following Equation (9), R_{pq} is obtained. Note that $\Delta u' = \frac{1}{N\,\Delta k_x} = 32\ cm$; and $\Delta(2\pi u') = 2\pi \cdot 32\ cm$. Thus the distance between adjacent elements of the matrix $= 2\pi(32)\ cm$.

RESULTS

The spatial correlation at the origin should be 1 . Without the trapezoidal rule we obtain a value of 1.04 and thus a 4% error. Using the trapezoidal rule, we obtain a value of $\simeq$ .944 and thus an error of .6%. The error may be further reduced by using the collapsed function technique. (see References (5) and (6)) For a 384 x 384 matrix collapsed into a 128 x 128 matrix, the maximum value = .9995 or a .05% error. The computer program is written so as to take advantage of the collapsing technique if desired.

DeBoer (Reference (7)) has devised a method of obtaining the spatial correlation for the case of the Neumann-Pierson spectrum as given in Equation (13) that is more efficient than the simple double sum. However, his method may only be used for very special cases. Noting that the Neumann-Pierson spectrum are a one parameter family, namely wind speed S , DeBoer multiplies the actual distance of the spatial correlation curve by $2g/S^2$. Following DeBoer, our normalized spatial correlation computations are presented in Figure 2 for the down wind case and in Figure 3 for the cross wind case. These curves agree with those of DeBoer and thus serve as a check on the FFT method and the computer program. The correlation for any wind speed may be obtained by dividing the normalized distance by $2g/S^2$. Note that the correlation in the cross wind direction is wider than in the down wind direction. Note that as the wind speed increases, the ocean surface becomes more correlated. As the wind speed increases, the maximum peak of the Neumann-Pierson spectrum shifts to lower frequencies. These lower frequencies result in a larger correlation distance.

The Neumann-Pierson spectrum is symmetric in u' as well as in v' and is thus a special case of Equation (5).

SUMMARY AND CONCLUSIONS

The FFT method may be applied to the directional wave spectrum to obtain the spatial correlation of the surface height. The FFT method is substantially faster than the straight forward integration method. This work is the first known FFT transform of the directional wave spectrum.

The spatial correlation values obtained by the FFT are in excellent agreement with those of DeBoer. The curves presented are dimensionless - i.e. they are valid for all wind speeds. The computer program and analysis in this memorandum should serve as a guide to future programs for the transformation of experimental directional wave spectra.

ACKNOWLEDGMENT

Discussions with Dr. A. Nuttall on the properties of the FFT were very helpful in this study.

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REFERENCES

1. Kinsman, B., "Wind Waves," Prentice Hall 1965.

2. Cartwright, D. Article in "The Sea" Volume 1, Editor Hill, N., Interscience Publishers 1962.

3. Cron, B., "A Review of the Statistical Properties of the Ocean Surface," NUSC/NL Tech Memo No. 2211-128-70.

4. Gordon, R. L, and Owsley, N. L., "A General Purpose Multi-dimensional Fast Fourier Transform for Use in Fortran V Programs," USL Tech Memo No. 2242-319-618, 6 Sept 1968.

5. Cooley, J.W., et al "Application of the Fast Fourier Transform to Computation of Fourier Integrals, Fourier Series, and Convolution Integrals," IEEE Transactions on Audio and Electroacoustics, Volume AU-15, No. 2, June 1967.

6. Nuttall, A. H., "Alternate Forms and Computational Considerations for Numerical Evaluation of Cumulative Probability Distributions Directly from Characteristic Functions," NUSC Report No. NL-3012, 12 Aug 1970.

7. DeBoer, J., "On the Correlation Functions in Time and Space of Wind-Generated Ocean Waves," Technical Report No. 160 SACLANT ASW Research Centre, 15 Dec 1969.

APPENDIX A

```
C       SPATIAL CORRELATION OF SURFACE
C       HEIGHT VIA THE FFT
        PARAMETER NS=128,NI=1,NDI=NS/NI,NDIM1=NDI-1,NIM1=NI-1
        PARAMETER NC=28,NC1=NC+1,NC2=NC+2
        IMPLICIT DOUBLE PRECISION(A-H,O-Z)
        COMPLEX A(NDI,NDI)
      DIMENSION NN(2)
      DATA NN/NDI,NDI/
      REAL X1(NC2),Y1(NC2),BUFFER(10000)
      CALL PLOTS(BUFFER,10000,6)
      CALL PLOT(0,0,100)
      CALL PLOT(0.,0.,-3)
   31 FORMAT(5D8.1,D9.4,D5.0)
   33 FORMAT(1X,2I4,2D16.8)
   34 FORMAT(1X,4(2I4,E15.8))
      CC=64.D0**2
      CCCC=CC*CC
      DK2=1.D0/CCCC
      CFM=30500.D0
      G=980.665D0
      G2=G*G
      PI=3.14159265358979324D0
   30 READ(3,31,END=32)TI,HI,TS1,HS1,HS2,SK,PL
      S=SK*185200.D0/3600.D0
      S2=S*S
      SC=S/(2.*G)
      SC2=SC*SC
      SC4=SC2*SC2
      SC5=SC4*SC
      PID2=PI/2.
      SPID2=SQRT(PID2)
      VAR=CFM*PID2*SPID2*3.D0*SC5
      SIG=SQRT(VAR)
      DF=CC/NS
      FN=2.*G/S2
      DO 1 IR=0,NDIM1
      DO 2 JS=0,NDIM1
      ASUM=0.
      DO 3 IP=0,NIM1
      DO 4 JQ=0,NIM1
      IRP=IR+IP*NDI
      ISQ=JS+JQ*NDI
      IF(IRP.EQ.0.AND.ISQ.EQ.0)SSHS=0.
      IF(IRP.EQ.0.AND.ISQ.EQ.0)GO TO 4
      II=IRP**2
      JJ=ISQ**2
```

```
      OMEG=SQRT(SQRT(II/CCCC+JJ/CCCC))*SQRT(G)
      OMEG2=OMEG**2
      SSHS=EXP(-2.*G2/(OMEG2*S2))
      SSHS=SSHS/OMEG**9
      SSHS=SSHS*G2*CFM/2.D0
      XK=IRP/CC
      YK=ISG/CC
      THET=ATAN2(YK,XK)
      SSHS=SSHS*COS(THET)**2
      ASUM=SSHS+ASUM
    4 CONTINUE
    3 CONTINUE
      IT=IR+1
      JT=JS+1
      IF(IR.EQ.0.OR.IR.EQ.NDIM1)ASUM=.5D0*ASUM
      IF(JS.EQ.0.OR.JS.EQ.NDIM1)ASUM=.5D0*ASUM
      A(IT,JT)=ASUM
    2 CONTINUE
1     CONTINUE
      CALL FOURT(A,NN,2,-1,0,0)
      DO 7 I=2,128,2
      DO 5 J=1,NC
      V=(J-1)*DF*2.*PI
      V=V*FN
      X1(J)=V
      IF(J.EQ.1)ANV=2.D0*REAL(A(I,J))
      IF(J.EQ.1)GO TO 6
      ANV=REAL(A(I,J))+REAL(A(I,NS+2-J))
6     ANV=ANV*DK2/VAR
      Y1(J)=ANV
      WRITE(4,33)I,J,V,ANV
5     CONTINUE
7     CONTINUE
      X1(NC1)=0.
      X1(NC2)=10.
      Y1(NC1)=-.6
      Y1(NC2)=.4
      CALL LINE(X1,Y1,NC,1,0,0)
      CALL AXIS(0.,0.,19HNORMALIZED DISTANCE,-19,4.,0.,X1(NC1),X1(NC2),1
     10.)
      CALL AXIS(0.,0.,11HCORRELATION,+11,4.,90.,Y1(NC1),Y1(NC2),10.)
      CALL PLOT(20.,0.,-3)
      CALL PLOT (0.,0.,999)
      STOP
32    END
```

NEUMANN-PIERSON SPECTRUM
DOWN WIND

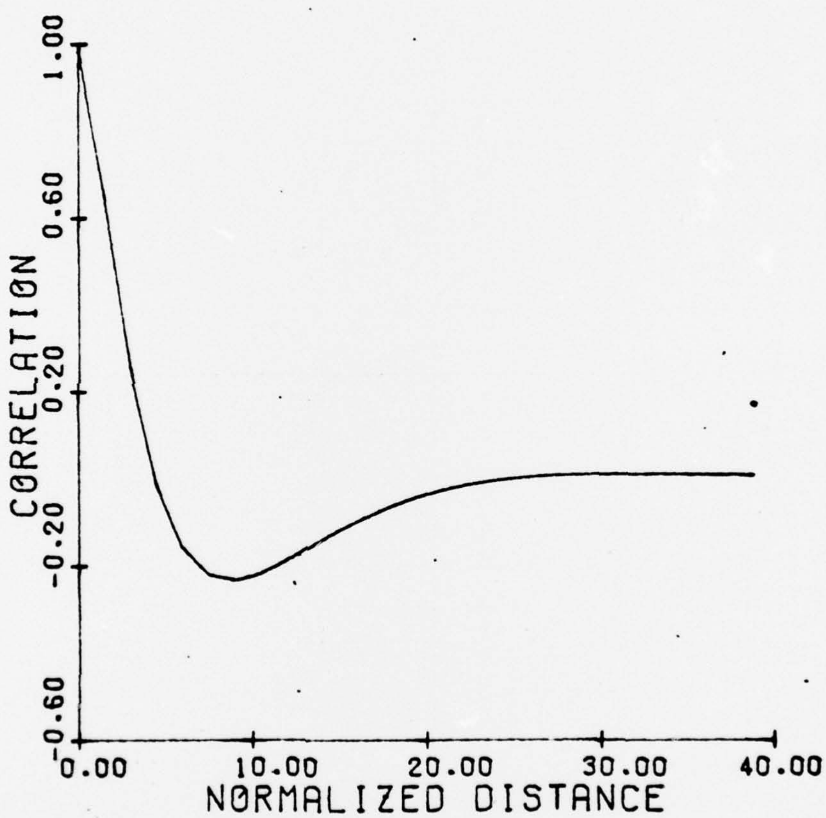

FIGURE II

NEUMANN-PIERSON SPECTRUM
CROSS WIND

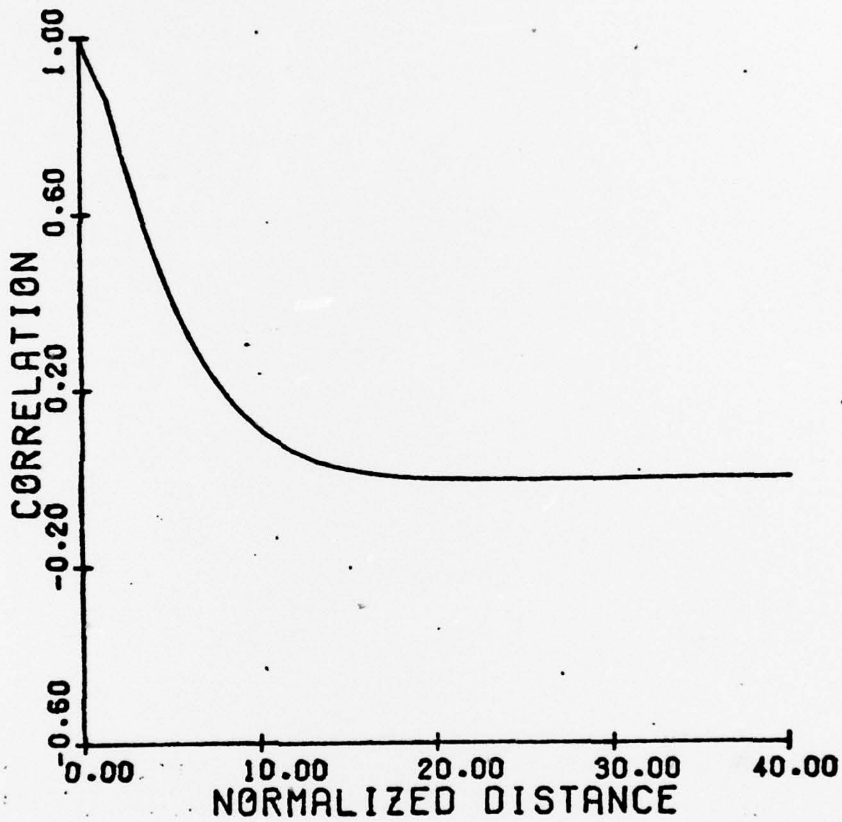

FIGURE III