APPLICATIONS OF MULTICONDUCTOR TRANSMISSION LINE THEORY TO THE PREDICTION OF CABLE COUPLING
Digital Computer Programs for the Analysis of Multiconductor Transmission Lines

University of Kentucky

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<td>Four digital computer programs, XTALK, XTALK2, FLATPAK, FLATPAK2, for determining the electromagnetic coupling within an (n+1) conductor, uniform transmission line are presented. Sinusoidal steady state behavior of the line as well as the TEM or &quot;quasi-TEM&quot; mode of propagation are assumed. XTALK and XTALK2 consider lines consisting of n wires (cylindrical conductors) and a reference conductor. The surrounding medium is homogeneous and lossless. (Con'd)</td>
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XTALK assumes that all (n+1) conductors are perfect conductors whereas X1ALK2 considers the conductors to be lossy. There are three choices for the reference conductor: a wire, a ground plane, an overall cylindrical shield.

FLATPAK and FLATPAK2 consider (n+1) wire ribbon (flatpack) cables in which all wires are identical and are coated with cylindrical, dielectric insulations of identical thicknesses. All wires lie in a horizontal plane and all adjacent wires are separated by identical distances. FLATPAK considers the wires to be perfect conductors and FLATPAK2 considers the wires to be lossy. The dielectric insulations are considered to be lossless.

General termination networks are provided for at the ends of the line and the programs compute the voltages (with respect to the reference conductor) at the terminals of these termination networks for sinusoidal steady state excitation of the line.
PREFACE

This effort was conducted by The University of Kentucky under the sponsorship of the Rome Air Development Center Post-Doctoral Program for RADC's Compatibility Branch. Mr. Jim Brodock of RADC was the task project engineer and provided overall technical direction and guidance.

The RADC Post-Doctoral Program is a cooperative venture between RADC and some sixty-five universities eligible to participate in the program. Syracuse University (Department of Electrical Engineering), Purdue University (School of Electrical Engineering), Georgia Institute of Technology (School of Electrical Engineering), and State University of New York at Buffalo (Department of Electrical Engineering) act as prime contractor schools with other schools participating via sub-contracts with the prime schools. The U.S. Air Force Academy (Department of Electrical Engineering), Air Force Institute of Technology (Department of Electrical Engineering), and the Naval Postgraduate School (Department of Electrical Engineering) also participate in the program.

The Post-Doctoral Program provides an opportunity for faculty at participating universities to spend up to one year full time on exploratory development and problem-solving efforts with the post-doctorals splitting their time between the customer location and their educational institutions. The program is totally customer-funded with current projects being undertaken for Rome Air Development Center (RADC), Space and Missile Systems Organization (SAMSO), Aeronautical Systems Division (ASD), Electronics Systems Division (FSD), Air Force Avionics Laboratory (AFAL), Foreign Technology Division (FTD), Air Force Weapons Laboratory (AFWL), Armament Development and Test Center (ADTC), Air Force Communications Service (AFCS), Aerospace Defense
Command (ADC), Hq USAF, Defense Communications Agency (DCA), Navy, Army, Aerospace Medical Division (AMD), and Federal Aviation Administration (FAA).

Further information about the RADC Post-Doctoral Program can be obtained from Mr. Jacob Scherer, RADC/RBC, Griffiss AFB, NY, 13441, telephone Autovon 587-2543, commercial (315) 330-2543.

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The author wishes to acknowledge the capable efforts of Ms. Donna Toon in typing this manuscript.
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I. INTRODUCTION

This report is the seventh in a seven volume series documenting the Application of Multiconductor Transmission Line Theory to the Prediction of Cable Coupling. The purpose of this report is to implement the analytical techniques described in Volume I of this series [1] in the form of digital computer programs.

Crosstalk or electromagnetic coupling between wires (cylindrical conductors) in densely packed cable bundles can be a serious contributor to the degradation in performance of modern electronic systems. A recently developed digital computer program, IEMCAP, provides a general analysis capability for determining overall electromagnetic compatibility of aircraft, ground and spacecraft systems [3]. The computer programs described in this report are intended to provide a supplement to the analysis capabilities of IEMCAP by providing a more fine-grained analysis of wire-coupled interference.

IEMCAP is intended to be used to model all recognizable coupling paths on aircraft, ground and spacecraft systems. By virtue of the large size and complexity of many of these systems, detailed modeling of the coupling paths is not feasible in a program such as IEMCAP. To avoid excessive computer run times, the models of the various coupling paths used in IEMCAP are generally quite simple and represent bounds on the coupling. Consequently, the predictions of IEMCAP are generally somewhat conservative. However, once a potential wire-coupled interference problem is pinpointed by IEMCAP, the computer programs described in this report can, in many cases, be used to determine if an actual interference situation exists and the precise level of the interference.

Four programs are described: XTALK, XTALK2, FLATPAK, and FLATPAK2.
XTALK analyzes three configurations of transmission lines: (1) \((n+1)\) bare wires, (2) \(n\) bare wires above an infinite ground plane, and (3) \(n\) wires within a cylindrical shield which is filled with a homogeneous dielectric. All conductors are considered to be perfect conductors. XTALK2 analyzes the same three structural configurations as XTALK except that the conductors are considered to be imperfect conductors. FLATPAK analyzes \((n+1)\) wire ribbon cables. All wires are assumed to be perfect conductors. FLATPAK2 analyzes the same configuration as FLATPAK except that the wires are considered to be imperfect conductors. In all of the above programs, the medium (media) surrounding the conductors is assumed to be lossless. Sinusoidal, steady-state excitation of the line is considered, i.e., the transient solution is not directly obtained. Comparison of predicted to experimental results are obtained using these programs in Volume III and Volume IV of this series [4,5].

All programs are written in FORTRAN IV Language and are double precision. Changes in the programs to convert them to single precision arithmetic will be indicated. All programs have been implemented on an IBM 370/165 computer at The University of Kentucky using the Fortran IV, G level compiler and should be easily implemented on other computers.

It is, of course, difficult if not impossible to write a general computer program which will address all types of transmission line structures which the user may wish to investigate. The four programs included in this report form an initial library of analysis capabilities for wire-coupled interference problems. Other programs which address more specific structures and structures not considered by these four programs will be documented in other volumes of this series as well as in future RADC publications as they are developed.
II. FORMULATION OF THE MULTICONDUCTOR TRANSMISSION LINE (MTL) EQUATIONS

In this chapter, the distributed parameter, multiconductor transmission line (MTL) model will be described and the programmed equations will be derived. This model is exact in the sense that interactions between all conductors in the transmission line are considered, and the distributed parameter representation (assuming the TEM mode or "quasi-TEM" mode of propagation on the line) is used. The line is assumed to be uniform in the sense that all conductors are parallel to each other and there is no variation in the cross section of the conductors or the surrounding media along the line.

2.1 The Multiconductor Transmission Line (MTL) Model

The MTL model is described in detail in Volume I of this series [1] and in reference [2]. In this section, a brief review of the MTL model will be given and the reader should consult Volume I [1] or reference [2] for further details.

If the line is immersed in a homogeneous medium, e.g., bare wires in free space, the fundamental mode of propagation is the TEM (Transverse Electro-Magnetic) mode. If the line is immersed in an inhomogeneous medium, e.g., wires with cylindrical dielectric insulations surrounded by free space, the fundamental mode of propagation is taken to be the "quasi-TEM" mode. The essential difference in these two cases is as follows. For lines in a homogeneous medium the TEM mode assumption is legitimate. For lines in an inhomogeneous medium, the TEM mode cannot exist except in the limiting case of zero frequency (DC). However, for the inhomogeneous medium case, the assumption is made that the electric and magnetic fields are almost trans-
verse to the direction of propagation, i.e., the mode of propagation is almost TEM or "quasi-TEM".

With the assumption of the TEM mode or "quasi-TEM" mode of propagation, line voltages and currents may be defined. Consider a general \((n + 1)\) conductor, uniform transmission line shown in Figure 2-1. The \((n + 1)\)st or zero-th conductor is the reference conductor for the line voltages. For sinusoidal, steady-state excitation of the line, the line voltages, \(V_1(x, t)\), (with respect to the reference, the zero-th, conductor) and line currents, \(I_1(x, t)\) are

\[
V_1(x, t) = V_1(x) e^{j\omega t} \quad (2-1a)
\]

\[
I_1(x, t) = I_1(x) e^{j\omega t} \quad (2-1b)
\]

for \(i = 1, \ldots, n\) where \(V_1(x)\) and \(I_1(x)\) are the complex, phasor line voltages and currents and \(\omega\) is the radian frequency of excitation of the line, \(\omega = 2\pi f\). The current in the reference conductor satisfies

\[
\Phi_0(x, t) = -\sum_{i=1}^{n} \Phi_i(x, t) \quad (2-2a)
\]

\[
i_0(x) = -\sum_{i=1}^{n} I_1(x) \quad (2-2b)
\]

The MTL equations can be derived from the per-unit-length equivalent circuit in Figure 2-2 and are a set of \(2n\), complex-valued, first order, ordinary differential equations

\[
\frac{d}{dx} \begin{bmatrix} V(x) \\ I(x) \end{bmatrix} = - \begin{bmatrix} 0 & Z \\ Y & 0 \end{bmatrix} \begin{bmatrix} V(x) \\ I(x) \end{bmatrix} + \begin{bmatrix} V_s(x) \\ I_s(x) \end{bmatrix} \quad (2-3)
\]

A matrix \(M\) with \(m\) rows and \(p\) columns is said to be \(m \times p\) and the element in the \(i\)-th row and \(j\)-th column is designated by \([M]_{ij}\) with \(i = 1, \ldots, m\).
Fig. 2-1(cont.). An (n+1) conductor, uniform transmission line.
Fig. 2-1. An (n+1) conductor, uniform transmission line.
Fig. 2-2. The per-unit-length equivalent circuit.
An n×1 vector is denoted with a bar, e.g., \( \bar{V} \), with the entry in the i-th row denoted by \( [\bar{V}]_i = V_i \). The matrix \( \mathbf{0}_{m \times p} \) is the m×p zero matrix with zeros in every position, i.e., \( [\mathbf{0}_{m \times p}]_{ij} = 0 \) for \( i = 1, \ldots, m \) and \( j = 1, \ldots, p \). The complex-valued phasor line voltages with respect to the reference conductor (the zero-th conductor), \( V_i(x) \), and line currents, \( I_i(x) \), are given by \( [\bar{V}(x)]_i = V_i(x) \) and \( [\bar{I}(x)]_i = I_i(x) \).

The n×n complex-valued, symmetric matrices, \( \bar{Z} \) and \( \bar{Y} \), are the per-unit-length impedance and admittance matrices of the line, respectively. Since the line is assumed to be uniform, these matrices are independent of \( x \). These per-unit-length matrices are separable as

\[
\bar{Z} = \bar{R} + j\omega \bar{L} + j\omega \bar{C} \tag{2-4a}
\]

\[
\bar{Y} = \bar{G} + j\omega \bar{C} \tag{2-4b}
\]

where the n×n real, symmetric matrices \( \bar{R} \), \( \bar{L} \), \( \bar{G} \), \( \bar{C} \) are the per-unit-length conductor resistance, conductor internal inductance, external inductance, conductance and capacitance matrices, respectively. The entries in these matrices may be straightforwardly obtained in terms of the elements of the per-unit-length equivalent circuit in Figure 2-2 as

\[
\begin{align*}
[\bar{R}]_{ii} &= r_1 + r_0 \quad [\bar{R}]_{ij} = r_0 \quad i \neq j \\
[\bar{L}]_{ii} &= \ell_1 + \ell_0 \quad [\bar{L}]_{ij} = \ell_0 \quad i \neq j \\
[\bar{L}]_{ii} &= \ell_1 + \ell_0 - 2m_{10} \quad [\bar{L}]_{ij} = \ell_0 + m_{ij} - m_{i0} - m_{j0} \quad i \neq j \\
[\bar{G}]_{ii} &= g_{10} + \sum_{j=1}^{n} g_{ij} \quad [\bar{G}]_{ij} = -g_{ij} \quad i \neq j
\end{align*}
\]
The $n\times l$ column vectors, $V_s(x)$ and $I_s(x)$ contain per-unit-length equivalent voltage and current sources, $[V_s(x)]_i = V_s(x)$ and $[I_s(x)]_i = I_s(x)$, which are included to represent the effects of the spectral components of incident electromagnetic field sources which illuminate the line. These entries are complex-valued functions of frequency and position, $x$, along the line. In this report, no external incident fields are considered and these sources are set equal to zero, i.e., $V_s(x) = 0$ and $I_s(x) = 0$.

The solution to (2-3) is

$$
\begin{bmatrix}
V(x) \\
I(x)
\end{bmatrix}
\approx
\begin{bmatrix}
\hat{V}(x_0) \\
\hat{I}(x_0)
\end{bmatrix}
+ \int_{x_0}^{x} \begin{bmatrix}
\hat{V}(x) \\
\hat{I}(x)
\end{bmatrix}
\, d\hat{x}
= \begin{bmatrix}
\hat{V}(x_0) \\
\hat{I}(x_0)
\end{bmatrix}
+ \begin{bmatrix}
\hat{V}(x) \\
\hat{I}(x)
\end{bmatrix}
$$

(2-6)

where $\hat{\phi}(x,x_0)$ is the $2n\times2n$ chain parameter matrix (or state transition matrix) and $x_0$ is some arbitrary position along the line $x \geq x_0$. The chain parameter matrix can be partitioned as

$$
\hat{\phi}(x,x_0) =
\begin{bmatrix}
\phi_{11}(x,x_0) & \phi_{12}(x,x_0) \\
\phi_{21}(x,x_0) & \phi_{22}(x,x_0)
\end{bmatrix}
$$

(2-7)

where $\phi_{ij}(x,x_0)$ are $n\times n$ for $i, j = 1, 2$. Thus (2-6) can be written as

$$
V(x) = \phi_{11}(x,x_0) \hat{V}(x_0) + \phi_{12}(x,x_0) \hat{I}(x_0) + \hat{V}_s(x)
$$

(2-8a)

$$
I(x) = \phi_{21}(x,x_0) \hat{V}(x_0) + \phi_{22}(x,x_0) \hat{I}(x_0) + \hat{I}_s(x)
$$

(2-8b)

The entries $\phi_{ij}(x,x_0)$ are given by
\[ \Phi_{11}(x,x_0) = \frac{1}{2} Y^{-1} T (e^\gamma(x-x_0) + e^{-\gamma(x-x_0)}) T^{-1} Y \] (2-9a)

\[ \Phi_{12}(x,x_0) = -\frac{1}{2} Y^{-1} T \gamma (e^\gamma(x-x_0) - e^{-\gamma(x-x_0)}) T^{-1} \] (2-9b)

\[ \Phi_{21}(x,x_0) = -\frac{1}{2} T (e^\gamma(x-x_0) - e^{-\gamma(x-x_0)}) \gamma^{-1} T^{-1} Y \] (2-9c)

\[ \Phi_{22}(x,x_0) = \frac{1}{2} T (e^\gamma(x-x_0) + e^{-\gamma(x-x_0)}) T^{-1} \] (2-9d)

where \( e^\gamma(x-x_0) \) is an \( n \times n \) diagonal matrix with [\( e^\gamma(x-x_0) \)]_{ii} = \( e^{\gamma_1(x-x_0)} \) and [\( e^\gamma(x-x_0) \)]_{ij} = 0 for \( i, j = 1, \ldots, n \) and \( i \neq j \). The matrix \( T \) is an \( n \times n \) complex-valued matrix which diagonalizes the matrix product \( YZ \) as

\[ T^{-1} Y Z T = \gamma^2 \] (2-10)

where \( \gamma^2 \) is an \( n \times n \) diagonal matrix with [\( \gamma^2 \)]_{ii} = \( \gamma^2_1 \) and [\( \gamma^2 \)]_{ij} = 0 for \( i, j = 1, \ldots, n \) and \( i \neq j \). The \( n \times n \) characteristic impedance matrix, \( Z_C \), is given by

\[ Z_C = \gamma^{-1} T Y^{-1} T = Z T Y^{-1} T^{-1} \] (2-11)

The transmission line is of length \( L \) with termination networks at \( x = 0 \) and at \( x = L \) as shown in Fig. 2-3. For generality, the termination networks are considered to be in the form of linear \( n \)-ports and are characterizable by "Generalized Thevenin Equivalents" as

\[ V(0) = V_0 - Z_0 I(0) \] (2-12a)

\[ V(L) = V_L + Z_L I(L) \] (2-12b)

where \( V_0 \) and \( V_L \) are \( n \times 1 \) complex-valued vectors of equivalent, open-circuit, port excitation voltages (with respect to the reference conductor) and \( Z_0 \)
Fig. 2-3. The termination networks.
and \( Z_s \) are \( n \times n \) symmetric, complex-valued port impedance matrices.

As an alternate characterization, (2-12) may be written as "Generalized Norton Equivalents" by multiplying (2-12a) on the left by \( Z_0^{-1} \) and (2-12b) on the left by \( Z_s^{-1} \) and rearranging as

\[
I(0) = I_0 - Y_0 V(0) \tag{2-13a}
\]
\[
I(\tau) = -I_s + Y_s V(\tau) \tag{2-13b}
\]

where \( I_0 \) and \( I_s \) are equivalent, short-circuit, port excitation current sources. The \( n \times n \) port admittance matrices \( Y_0 \) and \( Y_s \) are given by \( Y_0 = Z_0^{-1} \) and \( Y_s = Z_s^{-1} \) where the inverse of an \( n \times n \) matrix \( M \) is denoted by \( M^{-1} \) and \( I_0 = Y_0 V_0 \), \( I_s = Y_s V_s \). These port admittance matrices can be found by treating the line currents \( I(0) \) or \( I(\tau) \) as independent sources and writing the node voltage equations for the termination networks. The transmission line voltages, \( V(0) \) or \( V(\tau) \), will comprise subsets of the node voltages of the termination networks. The additional node voltages can be eliminated from the node voltage equations describing the networks to yield (2-13). 

If the termination networks at \( x = 0 \) and \( x = \Lambda \) consist only of admittances between the \( i \)-th and \( j \)-th wires, \( Y_{ij} \) and \( Y_{ij}^s \), respectively, and between the \( i \)-th wire and the reference conductor, \( Y_{ii} \) and \( Y_{ii}^s \), respectively, then the entries in \( Y_0 \) and \( Y_s \) become

\[
[Y_0]_{ii} = Y_{0i} + \sum_{j=1}^{n} Y_{0ij}, \quad [Y_0]_{ij} = -Y_{ij} \quad \text{for} \quad i, j = 1, \ldots, n \quad \text{and} \quad i \neq j.
\]

With \( x = \Lambda \) and \( x_0 = 0 \) in (2-8), one can straightforwardly obtain using the "Generalized Thevenin Equivalent" characterization of the termination networks given in (2-12)\(^2\)

\(^2\)In (2-8a) with \( x = \Lambda, x_0 = 0 \) substitute (2-12a) for \( V(0) \) and (2-12b) for \( V(\tau) \). Then substitute \( I(\tau) \) from (2-8b) with \( x = \Lambda, x_0 = 0 \) into the result and rearrange into the form in (2-14a). Substitute \( V(0) \) from (2-12a) into (2-8b) and rearrange to yield (2-14b).
\[ [Z_{22} \phi_{22}(\lambda) - Z_{21} \phi_{21}(\lambda) Z_0 - \phi_{12}(\lambda) + \phi_{11}(\lambda) Z_0] I(0) = \]
\[ [\phi_{11}(\lambda) - Z_{22} \phi_{21}(\lambda)] V_0 - V_s + \hat{V}_s(\lambda) - Z_{22} \hat{I}_s(\lambda), \]
\[ I(\lambda) = \phi_{22}(\lambda) V_0 + [\phi_{22}(\lambda) - \phi_{21}(\lambda) Z_0] I(0) + \hat{I}_s(\lambda) \] (2-14b)

where \( \phi_{ij}(\lambda) = \phi_{ij}(\lambda) \). \( V(\lambda) \) and \( I(\lambda) \) can be obtained for any \( \lambda \), \( 0 \leq \lambda \leq L \), from (2-8) with \( I(0) \) from the solution of (2-14a) and \( V(0) \) determined from (2-12a). Generally, we are only interested in the terminal voltages and currents, \( V(0), V(\lambda), I(0), I(\lambda) \). The terminal currents, \( I(0) \) and \( I(\lambda) \), can be obtained from (2-14) and the terminal voltages, \( V(0) \) and \( V(\lambda) \), can be obtained from (2-12). Here one only needs to solve \( n \) equations in \( n \) unknowns (equation 2-14a).

The \( \phi_{ij} \) submatrices of the chain parameter matrix in (2-7) satisfy certain fundamental identities, \([1,2]\). These identities can be used to formulate (2-14a) in an alternate form \([1,2]\):

\[ [\phi_{21}(\lambda) Z_{22} - \phi_{22}(\lambda)] \{ \phi_{21}(\lambda) Z_0 - \phi_{22}(\lambda) \} - \frac{1}{n} I(0) = \]
\[ \phi_{21}(\lambda) V_0 + [\phi_{21}(\lambda) Z_{22} - \phi_{22}(\lambda)] \phi_{21}(\lambda) V_0 - \phi_{21}(\lambda) \]
\[ \hat{V}_s(\lambda) - Z_{22} \hat{I}_s(\lambda) \] (2-15)

where \( \frac{1}{n} \) is the \( n \times n \) identity matrix with \( \frac{1}{n} \) \( ii = 1 \) and \( \frac{1}{n} \) \( ij = 0 \) for \( i, j = 1, \ldots, n \) and \( i \neq j \). Note that the formulations in (2-15) and (2-14b) require computation of only two of the four chain parameter submatrices, \( \phi_{21}(\lambda) \) and \( \phi_{22}(\lambda) \).

As an alternate formulation, the above equations can be written in terms
of the "Generalized Norton Equivalent" representation of the termination networks given in (2-13). Rather than rederiving the above equations it is much simpler to note the direct similarity of the Norton equivalent representation in (2-13) and the Thevenin equivalent representation in (2-12). By noting the analogous variables in (2-13) and (2-12) and observing the form of (2-8), we may simply make certain substitutions of those analogous variables in (2-14) and (2-15) as shown in Table 1. The result is

\[
[\begin{bmatrix}
Y_{11}(x) - Y_{12}(x) \\
Y_{21}(x) - Y_{22}(x)
\end{bmatrix} Y_0 - \begin{bmatrix}
\phi_{11}(x) \\
\phi_{12}(x)
\end{bmatrix} Y_0] V(0) =
\]

\[
[\begin{bmatrix}
\phi_{22}(x) - \phi_{21}(x) \\
\phi_{21}(x) - \phi_{22}(x)
\end{bmatrix} I_0 + \begin{bmatrix}
I_0 \\
I_0
\end{bmatrix}
\begin{bmatrix}
Y_{11}(x) - Y_{12}(x) \\
Y_{21}(x) - Y_{22}(x)
\end{bmatrix} Y_0 - \begin{bmatrix}
\phi_{11}(x) \\
\phi_{12}(x)
\end{bmatrix} Y_0]
\]

\[
V(x) = \phi_{12}(x) I_0 + [\phi_{11}(x) - \phi_{12}(x) Y_0] V(0) + \hat{V}_s(x)
\]

\[
[\begin{bmatrix}
\phi_{12}(x) Y_{11}(x) - \phi_{11}(x) Y_{12}(x) \\
\phi_{12}(x) Y_{21}(x) - \phi_{11}(x) Y_{22}(x)
\end{bmatrix} Y_0 - \begin{bmatrix}
\phi_{11}(x) \\
\phi_{12}(x)
\end{bmatrix} Y_0] V(0) =
\]

\[
- \phi_{12}(x) [\begin{bmatrix}
\hat{I}_s(x) \\
\hat{I}_s(x)
\end{bmatrix} - \begin{bmatrix}
Y_{11}(x) - Y_{12}(x) \\
Y_{21}(x) - Y_{22}(x)
\end{bmatrix} \phi_{11}(x) I_0]
\]

2.2 The Equations to Be Programmed

The equations for \( I(x) \) and \( V(x) \) are given in (2-14b) and (2-16b), respectively. Either (2-14a) or (2-15) could be used for determining \( I(0) \) and either (2-16a) or (2-16c) could be used for determining \( V(0) \). However, (2-14a) and (2-16a) will be selected for determining \( I(0) \) and \( V(0) \), respectively. Since no external incident fields are considered, \( \hat{V}_s(x) \) and \( \hat{I}_s(x) \) in (2-14), (2-15) and (2-16) will be zero, i.e., \( \hat{V}_s(x) = \hat{I}_s(x) = 0 \).

Certain modifications to these equations will be made to produce the final equations. The matrix chain parameters given in (2-9) for a line of
TABLE 1

Analogous variables in the Generalized Thevenin Equivalent (2-12) and Generalized Norton Equivalent (2-13) representation of the termination networks. The analogous variables are substituted in equations (2-14) and (2-15) to obtain equations (2-16).

<table>
<thead>
<tr>
<th>Generalized Thevenin Equivalent (2-12)</th>
<th>Generalized Norton Equivalent (2-13)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I(0)$</td>
<td>$V(0)$</td>
</tr>
<tr>
<td>$I(\phi)$</td>
<td>$V(\phi)$</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>$\bar{V}_0$</td>
</tr>
<tr>
<td>$Z_\phi$</td>
<td>$\bar{V}_\phi$</td>
</tr>
<tr>
<td>$V(0)$</td>
<td>$I(0)$</td>
</tr>
<tr>
<td>$V(\phi)$</td>
<td>$-I(\phi)$</td>
</tr>
<tr>
<td>$\phi_{11}(\phi)$</td>
<td>$\phi_{22}(\phi)$</td>
</tr>
<tr>
<td>$\phi_{12}(\phi)$</td>
<td>$\phi_{21}(\phi)$</td>
</tr>
<tr>
<td>$\phi_{21}(\phi)$</td>
<td>$\phi_{12}(\phi)$</td>
</tr>
<tr>
<td>$\phi_{22}(\phi)$</td>
<td>$\phi_{11}(\phi)$</td>
</tr>
<tr>
<td>$V_0(\phi)$</td>
<td>$I_0(\phi)$</td>
</tr>
<tr>
<td>$V_{-S}(\phi)$</td>
<td>$I_{-S}(\phi)$</td>
</tr>
<tr>
<td>$I_{-S}(\phi)$</td>
<td>$V_{-S}(\phi)$</td>
</tr>
</tbody>
</table>
total length \( L(x_0 = 0, x = \mathcal{X}) \) become

\[
\phi_{11}(\mathcal{X}) = Y^{-1} \hat{T} E^+ \hat{T}^{-1} Y
\]
\[
\phi_{12}(\mathcal{X}) = -Y^{-1} \hat{T} Y E^+ \hat{T}^{-1}
\]
\[
\phi_{21}(\mathcal{X}) = -T E^- Y^{-1} \hat{T}^{-1} Y
\]
\[
\phi_{22}(\mathcal{X}) = T Y^+ \hat{T}^{-1}
\]

where the \( n \times n \) diagonal matrices \( E^+ \) and \( E^- \) are given by

\[
E^+ = \frac{1}{2} (e_2 \hat{T} + e_1 \hat{T}^+) \tag{2-18a}
\]
\[
E^- = \frac{1}{2} (e_2 \hat{T} - e_1 \hat{T}^-) \tag{2-18b}
\]

Substituting (2-17) into (2-14a) and (2-14b) yields, for the Thevenin Equivalent representation of the termination networks

\[
\begin{align*}
[Z \hat{T} E^+ \hat{T}^{-1} + Z \hat{T} E^- \hat{T}^{-1} Y^{-1} \hat{T}^{-1} Y Z_0 \\
+ Y^{-1} \hat{T} Y E^- \hat{T}^{-1} + Y^{-1} \hat{T} E^+ \hat{T}^{-1} Y Z_0 ] I(0) \\
= [Y^{-1} \hat{T} E^+ \hat{T}^{-1} Y + Z \hat{T} E^- Y^{-1} \hat{T}^{-1} Y] V_0 - V \mathcal{X}
\end{align*}
\]

\[
I(\mathcal{X}) = -T E^- Y^{-1} \hat{T}^{-1} Y V_0 
+ [T E^+ \hat{T}^{-1} + T E^- \hat{T}^{-1} Y Z_0 ] I(0)
\]

Similarly, substituting (2-17) into (2-16a) and (2-16b) yields, for the Norton Equivalent representation of the termination networks,

\[
\begin{align*}
[Y \hat{T} E^+ \hat{T}^{-1} Y + Y \hat{T} E^- \hat{T}^{-1} Y \hat{T} E^+ \hat{T}^{-1} Y Z_0 \\
+ T E^- \hat{T}^{-1} Y + T E^+ \hat{T}^{-1} Y Z_0 ] V(0) \\
= [T E^+ \hat{T}^{-1} Y \hat{T} E^- \hat{T}^{-1} Y \hat{T} E^+ \hat{T}^{-1} Y Z_0 ] I_0 + I \mathcal{X}
\end{align*}
\]

\[
V(\mathcal{X}) = -Y^{-1} \hat{T} Y E^- \hat{T}^{-1} L_0 + [Y^{-1} \hat{T} E^+ \hat{T}^{-1} Y + Y^{-1} \hat{T} Y E^- \hat{T}^{-1} Y Z_0 ] V(0)
\]
The medium surrounding all conductors is assumed throughout this report to be lossless. Therefore the per-unit-length conductance matrix, $G$, which represents these losses in (2-4b) is zero, i.e., $G = 0$. Therefore the per-unit-length admittance matrix becomes

$$Y = j \omega C$$

(2-21)

The per-unit-length impedance matrix is

$$Z = R + j \omega L + j \omega C$$

(2-22)

where $R$ and $L$ are zero matrices, i.e., $0_{n \times n}$, when perfect conductors are assumed.

To reduce the number of matrix multiplications, the above equations will be placed in an alternate form. For the Norton Equivalent representation in (2-20), define

$$Y_0^* = T^{-1} Y_0 C^{-1} T$$

(2-23a)

$$Y^* = T^{-1} Y C^{-1} T$$

(2-23b)

$$V^*(\tau) = T^{-1} C V(\tau)$$

(2-23c)

$$V^*(0) = T^{-1} C V(0)$$

(2-23d)

$$I^* = T^{-1} I$$

(2-23e)

$$I_0^* = T^{-1} I_0$$

(2-23f)

$$Y^* = j \omega C$$

(2-23g)

Equations (2-20) can then be written as

$$[Y_0^* E^* + Y^* A E^* - Y^* - A^{-1} + E^* Y_0^*] V^*(0)$$

$$= [E^* + Y^* A E^*] L_0^* + I^*$$

(2-24a)
\[ V^*(z) = - \Delta e^{- \frac{1}{2} z} + [\Delta e^{- \frac{1}{2} z} \mathcal{Z}_0^* \mathcal{Z}_0^*] V^*(0) \quad (2-24b) \]

and the actual termination voltages can be determined by solving (2-24) for \( V^*(0) \) and \( V^*(z) \) and using (2-23c) and (2-23d) to obtain

\[ V(0) = c^{-1} z V^*(0) \quad (2-25a) \]
\[ V(z) = c^{-1} z V^*(z) \quad (2-25b) \]

These equations are summarized in Table 2.

Similarly, equations (2-19) for the Thevenin Equivalent representation of the terminal networks can be reduced to an equivalent form by defining

\[ Z^* = \frac{1}{T} C Z T \quad (2-26a) \]
\[ Z^*_0 = \frac{1}{T} C Z_0 T \quad (2-26b) \]
\[ \mathcal{I}^*(z) = \frac{1}{T} \mathcal{I}(z) \quad (2-26c) \]
\[ \mathcal{I}^*(0) = \frac{1}{T} \mathcal{I}(0) \quad (2-26d) \]
\[ V^*_z = \frac{1}{T} C V z \quad (2-26e) \]
\[ V^*_0 = \frac{1}{T} C V 0 \quad (2-26f) \]
\[ \gamma = j \omega \Delta \quad (2-26g) \]

Equations (2-19) can then be written as

\[ [Z^*_z e^+ + Z^*_z e^- \Delta^{-1} Z^*_0 + \Delta e^+ e^+ Z^*_0] I^*(z) \quad (2-27a) \]
\[ = [e^+ + Z^*_z e^- \Delta^{-1}] V^*_0 - I^*_z \]
\[ I^*(z) = -e^- \Delta^{-1} V^*_0 + [e^+ + e^- \Delta^{-1} Z^*_0] I^*(0) \quad (2-27b) \]

and the actual termination currents can be obtained by solving (2-27) for \( I^*(0) \) and \( I^*(z) \) and using (2-26c) and (2-26d) to obtain
\[ I(0) = T \tilde{I}^*(0) \]  
\[ I(\ell) = T \tilde{I}^*(\ell) \]  

(2-28a)  
(2-28b)

These equations are summarized in Table 3.

There are two reasons for using the equivalent representations in Table 2 and Table 3 rather than the representations in (2-20) and (2-19). First of all, note the direct similarity of the equations in Table 2 and Table 3. The only differences (other than symbols) between equations (1) and (2) in Table 2 and the corresponding equations (1) and (2) in Table 3 is that \( \tilde{\Lambda} \) used in Table 2 corresponds to \( \tilde{\Lambda}^{-1} \) in Table 3, and \( \tilde{I} \) in Table 2 corresponds to \( -\tilde{V}^* \) in Table 3. (Note that since \( \tilde{\Lambda} \), \( \tilde{\Lambda}^{-1} \) and \( \tilde{E} \) are diagonal, \( \tilde{\Lambda}^{-1} \tilde{\Lambda}^{-1} = \tilde{\Lambda}^{-1} \tilde{E} \) and \( \tilde{\Lambda} \tilde{\Lambda} = \tilde{\Lambda} \tilde{E} \).) Therefore we may form the Norton Equivalent equations in the programs and not need to write a duplicate set for the Thevenin Equivalent representations.

The second reason for using the representations in Table 2 and Table 3 is that if the termination networks are purely resistive, i.e., \( Z_0 \), \( Z_L \), \( Y_0 \), and \( Y_L \) are real, and the transformation matrix, \( T \), is frequency independent, i.e., perfect conductors are assumed (as in XTALK and FLATPAK), then the matrix multiplications as well as the inversion of \( T \) to form \( T^{-1} \) needed to obtain \( \tilde{Y}_0^*, \tilde{Y}_L^*, \tilde{Z}_0^*, \tilde{Z}_L^* \) need only be performed once and need not be changed as the frequency is changed. Only equations (1) and (2) in Table 2 and Table 3 need be reformulated for each frequency. This can represent a significant savings in computation time when the line response for many frequencies is desired (as it usually is) since \( n^3 \) operations (multiplications or divisions) are required to multiply two "full" n\times n matrices which is the minimum number of operations required to obtain the inverse of a general n\times n matrix [1].
TABLE 2
Programmed Equations for the Generalized Norton Equivalent Representation

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>[ Y^<em>_L E^</em>_L + Y^<em>_L \Lambda E^</em>_L Y^<em>_0 + E^- \Lambda^{-1} + E^+ Y^</em>_0 I V^<em>(0) = [E^+ + Y^</em>_L \Lambda E^-] I_0^* + I^*_L ]</td>
</tr>
<tr>
<td>(2)</td>
<td>[ T^{-1} YZT = T^{-1}(j\omega C_{R_L} + j\omega L_{0}) T ]</td>
</tr>
<tr>
<td>(3)</td>
<td>[ \gamma = j\omega \Lambda ]</td>
</tr>
<tr>
<td>(4)</td>
<td>[ I(0) = I_0 - Y_0 V(0) \quad , \quad I(\lambda) = -I_L + Y_L V(\lambda) ]</td>
</tr>
<tr>
<td>(5)</td>
<td>[ Y^<em>_0 = T^{-1} Y_0 C^{-1} T \quad , \quad Y^</em>_L = T^{-1} Y_L C^{-1} T ]</td>
</tr>
<tr>
<td>(6)</td>
<td>[ I^<em>_0 = T^{-1} I_0 \quad , \quad I^</em>_L = T^{-1} I_L ]</td>
</tr>
<tr>
<td>(7)</td>
<td>[ E^+ = \frac{1}{2} (e^y_L + e^- L) \quad , \quad E^- = \frac{1}{2} (e^y_L - e^- L) ]</td>
</tr>
<tr>
<td>(8)</td>
<td>[ V(0) = C^{-1} T V^<em>(0) \quad , \quad V(\lambda) = C^{-1} T V^</em>(\lambda) ]</td>
</tr>
</tbody>
</table>
TABLE 3

Programmed Equations for the Generalized Thevenin Equivalent Representation

\[
\begin{align*}
\text{(1)} & \quad [Z^*_{E} E^+ + Z^*_{E} E^- \Lambda^{-1} Z^*_0 + \Lambda E^- + E^+ Z^*_0] I^*(0) \\
& \quad = [E^+ + Z^*_{E} E^- \Lambda^{-1}] V_0^* - V_2^*
\end{align*}
\]

\[
\begin{align*}
\text{(2)} & \quad I^*_{(0)} = -E^- \Lambda^{-1} V_0^* + [E^+ + E^- \Lambda^{-1} Z^*_0] I^*(0)
\end{align*}
\]

\[
\begin{align*}
\text{(3)} & \quad T^{-1} Y Z I = T^{-1} \{j\omega C\{R_{c} + j\omega L_{c} + j\omega L\}\} T = Y
\end{align*}
\]

\[
\begin{align*}
\text{(4)} & \quad \gamma = j\omega \Lambda
\end{align*}
\]

\[
\begin{align*}
\text{(5)} & \quad V(0) = V_0 - Z_0 I(0) \quad , \quad V^*(0) = V_{zE} + Z_{zE} I^*(0)
\end{align*}
\]

\[
\begin{align*}
\text{(6)} & \quad Z_0^* = T^{-1} C Z_0 T \quad , \quad Z_{zE}^* = T^{-1} C Z_{zE} T
\end{align*}
\]

\[
\begin{align*}
\text{(7)} & \quad V_0^* = T^{-1} C V_0 \quad , \quad V_{zE}^* = T^{-1} C V_{zE}
\end{align*}
\]

\[
\begin{align*}
\text{(8)} & \quad E^+ = \frac{1}{2} (e^{\gamma T} + e^{-\gamma T}) \quad , \quad E^- = \frac{1}{2} (e^{\gamma T} - e^{-\gamma T})
\end{align*}
\]

\[
\begin{align*}
\text{(9)} & \quad I(0) = T I^*(0) \quad , \quad I^*(0) = T I_{zE}^*(0)
\end{align*}
\]

Note: \[
\begin{align*}
V^*(0) = V_0^* - Z_0^* I(0) \quad , \quad V^*(0) = V_{zE}^* + Z_{zE} I^*(0)
\end{align*}
\]

where: \[
\begin{align*}
V^*(0) = T^{-1} C V(0) \quad , \quad V^*(0) = T^{-1} C V(0)
\end{align*}
\]
2.3 Formulation of the Terminal Network Equations

The previous formulation requires that one determine the entries in the \( n \times n \) matrices \( Z_0, Z, Y_0 \) and \( Y \), and the \( n \times 1 \) vectors, \( V_0, V, I_0 \) and \( I \), in the Thevenin and Norton Equivalent representations of the terminal networks in (2-12) and (2-13), respectively. In this section, some examples will be given to aid in determining these quantities.

To illustrate this, four examples will be used. The first example, Example 1, is shown in Figure 2-4a. In this example, there is no cross-coupling between the port terminals within the termination networks, i.e., at each end of the line, each endpoint of a wire is terminated directly to the reference conductor and is not physically connected to the endpoints of other wires at the same end of the line. Writing the following equations:

\[
V_1(0) = 1 - 1 I_1(0) \quad (2-29a)
\]

\[
V_2(0) = -10 I_2(0) \quad (2-29b)
\]

\[
V_1(x) = 10^3 I_1(x) \quad (2-29c)
\]

\[
V_2(x) = 10^4 I_2(x) + 1 \quad (2-29d)
\]

and comparing these equations to the Thevenin Equivalent representation

\[
V(0) = V_0 - Z_0 I(0) \quad (2-30a)
\]

\[
V(x) = V + Z - I(x) \quad (2-30b)
\]

where

\[
V(0) = \begin{bmatrix} V_1(0) \\ V_2(0) \end{bmatrix} \quad V(x) = \begin{bmatrix} V_1(x) \\ V_2(x) \end{bmatrix}
\]

\[
I(0) = \begin{bmatrix} I_1(0) \\ I_2(0) \end{bmatrix} \quad I(x) = \begin{bmatrix} I_1(x) \\ I_2(x) \end{bmatrix}
\]

(2-31)
Fig. 2-4. Example termination networks. (No cross-coupling)
one can readily identify

\[
\begin{bmatrix}
1 \\
0
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
1 & 0 \\
0 & 10
\end{bmatrix}
\]

(2-32)

\[
\begin{bmatrix}
0 \\
1
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
10^3 & 0 \\
0 & 10^4
\end{bmatrix}
\]

Similarly, one can convert the termination networks to a Norton equivalent representation in Figure 2-4b and obtain (Example 2)

\[
I_2(0) = 1 - 1 \cdot V_1(0)
\]

(2-33a)

\[
I_2(0) = -10^{-1} \cdot V_2(0)
\]

(2-33b)

\[
I_1(t) = 10^{-3} \cdot V_1(t)
\]

(2-33c)

\[
I_2(t) = -10^{-4} + 10^{-4} \cdot V_2(t)
\]

(2-33d)

Comparing these equations to the Norton Equivalent representation

\[
I(0) = I_0 - Y_0 \cdot V(0)
\]

(2-34a)

\[
I(t) = -I_0 + Y_0 \cdot V(t)
\]

(2-34b)

where \( I(0), I(t), V(0), V(t) \) are given in (2-31), one can readily identify for Example 2

\[
\begin{bmatrix}
1 \\
0
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
1 & 0 \\
0 & 10^{-1}
\end{bmatrix}
\]

(2-35)

\[
\begin{bmatrix}
0 \\
10^{-4}
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
10^{-3} & 0 \\
0 & 10^{-4}
\end{bmatrix}
\]

Note that

\[
I_0 = Z_0^{-1} \cdot V_0
\]

(2-36a)
Note also that as far as the network terminal characteristics are concerned, the termination networks in Figure 2-4a are the same as those in Figure 2-4b.

The third and fourth examples, Example 3 and Example 4, are shown in Figure 2-5. As far as terminal characteristics are concerned, the terminations in Figure 2-5a and in Figure 2-5b are the same as shown by the following. First, write the Norton Equivalent characterization for the terminations in Figure 2-5b as (treat the terminal currents as independent sources and write the node-voltage circuit equations of the networks)

\[
\begin{align*}
\begin{bmatrix}
I_1(0) \\
I_2(0) \\
I(0)
\end{bmatrix} &=
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} -
\begin{bmatrix}
0.6 & -0.4 \\
-0.4 & 0.6
\end{bmatrix}
\begin{bmatrix}
V_1(0) \\
V_2(0) \\
V_0
\end{bmatrix} \\
(2-37a)
\end{align*}
\]

Similarly, from Figure 2-5a write the Thevenin Equivalent characterization as (treat the terminal voltages as independent sources and write the loop current circuit equations of the networks)

\[
\begin{align*}
\begin{bmatrix}
I_1(x) \\
I_2(x) \\
I(x)
\end{bmatrix} &=
\begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix} +
\begin{bmatrix}
0.6 & -0.2 \\
-0.2 & 0.4
\end{bmatrix}
\begin{bmatrix}
V_1(x) \\
V_2(x) \\
V_0
\end{bmatrix} \\
(2-37b)
\end{align*}
\]
(a) Example 3

(b) Example 4

Fig. 2-5. Example termination networks. (cross-coupling)
\[
\begin{bmatrix}
V_1(x) \\
V_2(x) \\
V(z)
\end{bmatrix} =
\begin{bmatrix}
1 \\
3
\end{bmatrix} +
\begin{bmatrix}
2 & 1 \\
1 & 3
\end{bmatrix}
\begin{bmatrix}
I_1(x) \\
I_2(x)
\end{bmatrix}
\] (2-38b)

Note that

\[
Y_0 = Z_0^{-1} \quad (2-39a)
\]
\[
Y = Z^{-1} \quad (2-39b)
\]
\[
I_0 = V_0 \quad (2-39c)
\]
\[
I = V_0 \quad (2-39d)
\]

and as far as the terminal characteristics of the networks are concerned, the termination networks in Figure 2-5a are the same as those in Figure 2-5b.

The above examples will serve a dual purpose. Each of the computer programs will be run for each of the above four examples for the same transmission line structure. Typical solution printouts will be shown for these results. This will serve as a partial check on the proper functioning of the programs since the corresponding terminal voltages \((V_1(0), V_2(0), V_1(x), V_2(x))\) for Example 1 should equal those for Example 2. Similarly the corresponding terminal voltages for Example 3 should equal those for Example 4.

As can be seen from the above examples, if there is no cross-coupling within the termination networks, then formulation of the entries in \(V_0, V, Z_0, Z, I_0, I, Y_0, Y\) is particularly simple. The situation in which there is no cross-coupling within the termination networks is generally the problem of interest in wire-coupled interference calculations.
However, it was felt that the more general case of allowing cross-coupling within the terminal networks be included in the capabilities of the programs.

To save computer time, one has four options for inputting the terminal data: OPTIONS 11, 12, 21, or 22. The first digit in each number indicates to each program that the terminal characterization chosen is either the Thevenin Equivalent (1) or Norton Equivalent (2). The second digit indicates to the program whether the admittance ($Y_0$ and $Y_L$) or impedance ($Z_0$ and $Z_L$) matrices are diagonal (1), i.e., no cross-coupling, or full (2), i.e., cross-coupling. For example, OPTION 11 indicates Thevenin Equivalent, diagonal impedance matrices; OPTION 22 indicates Norton Equivalent, full admittance matrices; OPTION 12 indicates Thevenin Equivalent, full impedance matrices, and OPTION 21 indicates Norton Equivalent, diagonal admittance matrices.

This saves computer time and user effort in inputting the data. For example, in cases where $Z_0$ (or $Z_L$, or $Y_0$ or $Y_L$) must be multiplied by another $n \times n$ matrix such as in $T \cdot Z_0$, if $Z_0$ is diagonal one only needs $n^2$ multiplications to form this product whereas if $Z_0$ is full, $n^3$ multiplications are needed to form the product. The programs are written to take advantage of this. In addition, if the terminal admittance or impedance matrices are in fact diagonal, then the user need only input the entries on the main diagonal and is saved the drudgery of inputting the remaining zero entries. The specific details for inputting this termination network data will be given in Chapter IV, the User's Manual.

2.4 Common Impedance Coupling and the Calculation of Conductor Self Impedances

Programs XTALK and FLATPAK assume that all conductors are perfect conductors. Programs XTALK2 and FLATPAK2, however, do not assume perfect conductors and these programs include the per-unit-length conductor resistance and internal inductance, the items $r_c$ and $l_i$, respectively, in Figure 2-2 and (2-5) as well as the reference conductor resistance, $r_{c0}$.
The reason for writing two separate programs to consider the same transmission line structure such as XTALK and XTALK2 is that the inclusion of conductor losses in the transmission line solution requires a longer computer run time and more array storage than when perfect conductors are assumed. This can be seen in Tables 2 and 3 in that the transformation matrix $T$ will be frequency dependent (and complex) when losses are included, whereas $T$ will be frequency independent (and real) when perfect conductors are assumed, i.e., $R_c = L_c = 0$. Therefore when perfect conductors are assumed (in XTALK and FLATPAK), one need only compute $T$ once per problem and the same $T$ can be used throughout the frequency iteration. When lossy conductors are considered (in XTALK2 and FLATPAK2), one must recompute $T$ at each frequency in addition to reforming at each frequency those matrix products involving $T$ in Table 2 and Table 3.

The primary effect of imperfect conductors is to introduce common impedance coupling. Consider a transmission line in which there is no cross-coupling within the termination networks. In this case, clearly the voltages induced via electromagnetic field coupling at the ends of a "receptor" circuit consisting of one conductor (wire) and the reference conductor due to a "generator" circuit consisting of another wire and the reference conductor will approach zero as the frequency of excitation is reduced to zero. However, the reference conductor impedance can couple a signal into the receptor circuit even at D-C and this is usually termed common impedance coupling.

To illustrate this, consider Figure 2-6. In Figure 2-6a, a three-conductor transmission line is shown. The reference conductor has a certain total impedance, $Z_0$, which may be considerably smaller in magnitude than
Fig. 2-6. Illustration of common impedance coupling.
\[ Z_{OR} \text{ or } Z_R. \text{ Consequently, the current in the generator wire at frequencies approaching D-C may be determined as} \]
\[ I_G = \frac{V_G}{Z_{0G} + Z_G} \]  
(2-40)

The major portion of this current will pass through the reference conductor producing a voltage drop across \( Z_0 \). This results in received voltages

\[ V_{ZR} = \left( \frac{Z_R}{Z_R + Z_{0R}} \right) Z_0 I_G \]  
(2-41a)

\[ V_{OR} = \left( \frac{Z_{0R}}{Z_R + Z_{0R}} \right) Z_0 I_G \]  
(2-41b)

Although this portion of the total received voltage may be "small" it may nevertheless be larger than the contribution due to electromagnetic field coupling as shown in Figure 2-6b. Consequently, this common impedance coupling generates a "floor" of induced voltage where a solution assuming perfect conductors would indicate a perhaps negligibly small received voltage at the lower frequencies.

The frequency at which this common impedance coupling becomes significant depends on many factors some of which are line geometry (which affects the level of the electromagnetic portion of the coupling) and type of reference conductor. Reference conductors consisting of a \#36 gauge wire or a large, thick ground plane would certainly not produce the same level of common impedance coupling.

The above separation and superposition of the two coupling mechanisms is only correct when one dominates the other by a considerable amount. To obtain a quantitatively correct answer, one must include the conductor self impedances directly in the transmission line solution and this is done in
XTALK2 and FLATPAK2.

The transmission lines considered by all programs in this report consist of \( n \) wires (cylindrical conductors) and a reference conductor. In XTALK2, there are three choices for the reference conductor; (1) a wire, (2) a finite ground plane and (3) an overall cylindrical shield surrounding the \( n \) wires. When the reference conductor is a finite ground plane, the user simply inputs the per-unit-length resistance and self inductance of the ground plane. Thus there are two cases remaining to be considered.

The per-unit-length self impedance of a solid cylinder of radius \( r_w \) shown in Figure 2-7a is given by the following. Define

\[
\delta = \frac{1}{\sqrt{\pi f \mu_v \sigma}} \quad (2-42a)
\]

\[
= \frac{1}{2\pi \sqrt{f \sigma} \times 10^{-7}}
\]

\[
r_0 = \frac{1}{\pi \sigma r_w^2} \quad (2-42b)
\]

\[
\xi_0 = \frac{\mu_v}{8\pi} = 0.5 \times 10^{-7} \quad (2-42c)
\]

Where \( \sigma \) is the conductor conductivity, \( f \) is the frequency and \( \mu_v \) is the permeability of the metal which is assumed to be that of free space (\( \mu_v = 4\pi \times 10^{-7} \)). The quantity \( \delta \) is the conventional skin depth factor. The equations for the per-unit-length self impedance of a solid cylindrical conductor including skin effect are obtained from [6]. The equations used in the computer programs approximate the actual equations given in reference [6], pp. 78-80. The programmed equations are
Fig. 2-7. Conductor dimensions for calculating common impedance.
These equations are used to generate the per-unit-length self impedances of the transmission line wires \((z_i = r + j\omega l)\) and the reference conductor when the reference conductor is also a wire \((z_0 = r + j\omega l)\). They are stored within the program codes for XTALK2 and FLATPAK2 and the user needs to input only the physical dimensions of the wires and their conductivity.

For the purposes of computing these wire self impedances, the wires are considered to be stranded. The user inputs the radius of each strand (in mils) and the number of strands in each wire. The program then computes the per-unit-length self impedance of each strand and determines the net wire self impedance by dividing this result by the number of strands (the net resistance of the wire is considered to be the result of all strands of the wire in parallel). (All strands in a wire are considered to be identical)

The equations for the per-unit-length self impedance of the reference conductor when the reference conductor is a thin walled, overall, cylindrical...
shield shown in Figure 2-7b are taken from reference [7], pp. 301-303 and include skin effect. The equations used in the computer programs are approximations of the actual equations. The skin depth, \( \delta \), is given in (2-42a). Denote the interior radius of the cylinder by \( r_s \) and its wall thickness by \( t \). The equations become [7]

\[
r_0 = \frac{1}{\pi \sigma t (2r_s + t)}
\]

(2-46)

(I) \( t \leq 0.5\delta \)

\[
r = r_0 \quad \text{ohms/meter}
\]

(2-47a)

\[
\omega l = 0.4\left(\frac{t}{\delta}\right) r_0 \quad \text{ohms/meter}
\]

(2-47b)

(II) \( t > 3\delta \)

\[
r = \frac{1}{2\pi r_s \sigma \delta} \quad \text{ohms/meter}
\]

(2-48a)

\[
\omega l = r \quad \text{ohms/meter}
\]

(2-48b)

(III) \( 0.5\delta < t < 3\delta \)

\[
r = \frac{1}{2\pi r_s \sigma \delta} \left[ \frac{\sinh\left(\frac{2t}{\delta}\right) + \sin\left(\frac{2t}{\delta}\right)}{\cosh\left(\frac{2t}{\delta}\right) - \cos\left(\frac{2t}{\delta}\right)} \right] \quad \text{ohms/meter}
\]

(2-49a)

\[
\omega l = \frac{1}{2\pi r_s \sigma \delta} \left[ \frac{\sinh\left(\frac{2t}{\delta}\right) - \sin\left(\frac{2t}{\delta}\right)}{\cosh\left(\frac{2t}{\delta}\right) - \cos\left(\frac{2t}{\delta}\right)} \right] \quad \text{ohms/meter}
\]

(2-49b)

The per-unit-length self impedance of the shield is given by \( z_0 = r + j\omega l \).

These equations are stored in the XTALK2 program code. The user only needs to input the shield interior radius, the shield thickness and the conductivity of the shield.
2.5 Computation of the Per-Unit-Length Inductance and Capacitance Matrices

All of the formulations shown in Tables 2 and 3 require the computation of the nxn, real, symmetric, per-unit-length transmission line inductance and capacitance matrices, L and C, respectively. The computation of these matrices will be discussed in this section.

2.5.1 Transmission Lines Consisting of Perfect Conductors in a Lossless, Homogeneous Medium, XTALK

This section considers (n+1) conductor transmission lines consisting of (n+1) perfect conductors in a lossless, homogeneous medium. The lines consist of n wires and three choices of reference conductor (the zero-th conductor) cross sections of which are shown in Figure 2-8. Computer program XTALK considers these cases.

The per-unit-length inductance and capacitance matrices for lines in a homogeneous medium are related by [1]

\[ L \approx C = \frac{\mu L}{\varepsilon_n} \]  
(2-50)

where \( \mu \) and \( \varepsilon \) are the permeability and permittivity of the surrounding homogeneous medium. The per-unit-length capacitance matrix can be found from a knowledge of the per-unit-length inductance matrix from (2-50) as

\[ C = \frac{\mu L}{\varepsilon_n} \]  
(2-51)

A logical choice for the surrounding medium in Figure 2-8(a) and 2-8(b) would be free space with permeability \( \mu_v = 4\pi \times 10^{-7} \) henrys/meter and permittivity \( \varepsilon_v = (1/36\pi) \times 10^{-9} \) farads/meter. However, for all structure types, the homogeneous medium may be characterized, for generality, by a relative dielectric constant (permittivity) of \( \varepsilon_r \) and a relative permeability of \( \mu_r \).

(Although the permeability of typical dielectrics is that of free space, \( \mu_v \),
Fig. 2-8. Lines in a homogeneous medium.
the programs XTALK and XTALK2 allow for the more general case.)

For the case of lossless conductors in a lossless, homogeneous medium, the nxn characteristic impedance matrix, $Z_C$, in (2-11) is related to the per-unit-length inductance matrix by [1]

$$Z_C = \nu L = \frac{\nu_0}{\sqrt{\varepsilon_r \mu_r}} L$$

(2-52)

where $\nu = 1/\sqrt{\varepsilon_r \mu_r}$ is the velocity of light in the surrounding medium. The velocity of light in free space, $\nu_0$, to 7 digits is $2.997925 \times 10^8$ meters/second.

The equations used in the programs for the entries in the per-unit-length transmission line matrix are derived in Volume I of this series [1] and are valid for "large" conductor separations. Generally this means that the smallest ratio of wire separation to wire radius should be no smaller than approximately 5. A more complete discussion of this is found in Volume I.

When the reference conductor is a wire as shown in Figure 2-8(a), the entries in the per-unit-length transmission line matrix are given by [1]

$$[L]_{ii} = \frac{\mu \nu_r}{2\pi} \ln \left( \frac{d_{i0}^2}{r_{wi}r_{w0}} \right)$$

(2-53a)

$$[L]_{ij} = \frac{\mu \nu_r}{2\pi} \ln \left( \frac{d_{i0}d_{j0}}{r_{w0}d_{ij}} \right)$$

(2-53b)

for $i, j=1, \ldots, n$ and $i \neq j$ where $d_{i0}$ is the center-to-center separation between the $i$-th wire and the reference conductor, $d_{ij}$ is the center-to-center separation between the $i$-th and $j$-th wires, and $r_{wi}$ and $r_{w0}$ are the radii of the $i$-th and reference wires, respectively.

When the reference conductor is an infinite ground plane as shown in Figure 2-8(b), the entries in the per-unit-length inductance matrix are
given by [1]

$$[L]_{1i} = \frac{\mu_0 u_r}{2\pi} \ln \left( \frac{2h_i}{r_{wi}} \right) \quad (2-54a)$$

$$[L]_{ij} = \frac{\mu_0 u_r}{2\pi} \ln \left( \frac{\sqrt{d_{ij}^2 + 4h_i h_j}}{d_{ij}} \right) \quad (2-54b)$$

for $i, j=1, \ldots, n$ and $i \neq j$ where $h_i$ is the height of the $i$-th wire above the ground plane.

When the reference conductor is an overall cylindrical shield as shown in Figure 2-8(c), the entries in the per-unit-length inductance matrix are given by [1]

$$[L]_{1i} = \frac{\mu_0 u_r}{2\pi} \ln \left( \frac{r_s^2 - r_i^2}{r_i r_s} \right) \quad (2-55a)$$

$$[L]_{ij} = \frac{\mu_0 u_r}{2\pi} \ln \left( \frac{\left( \frac{r_i}{r_s} \right)^2 + \frac{r_j^4}{r_s^4} - 2r_i r_j r_s^2 \cos \theta_{ij}}{\sqrt{\left( \frac{r_i}{r_s} \right)^2 + \frac{r_j^4}{r_s^4} - 2r_i r_j^3 \cos \theta_{ij}}} \right) \quad (2-55b)$$

for $i, j=1, \ldots, n$ and $i \neq j$ where $r_s$ is the interior radius of the shield, $r_i$ is the separation of the $i$-th wire from the center of the shield and $\theta_{ij}$ is the angular separation between the $i$-th and $j$-th wires.

For the case of lossless conductors in a lossless, homogeneous medium, the equations for the terminal voltages and currents in Table 2 and Table 3 can be further simplified. Obviously, the transformation matrix, $T$, which diagonalizes the matrix product $Y Z$ can be taken to be simply the identity matrix, i.e., $T = \mathbf{1}_n$, as is clear from the fact that for this case $Z = j\omega L$, $Y = j\omega C$ and

$$YZ = -\omega^2 L C \quad (2-56)$$

$$= -\omega^2 \mathbf{1}_n$$
Also, the \( n \times n \) diagonal matrix, \( A \), in Tables 2 and 3 becomes

\[
A = \frac{1}{\nu} I_n
\]  

(2-57)

Therefore the equations for the terminal voltages, \( V(0) \) and \( V(\mathcal{L}) \), for the Norton Equivalent representation of the terminal networks in Table 2 simplify to [1] (see equations (2-19) and (2-20)).

\[
\begin{align*}
V(\mathcal{L}) &= \cos(\beta \mathcal{L}) \left\{ Z_C + Z_0 \right\} + j \sin(\beta \mathcal{L}) \left\{ Z_C Z_C Z_0 + Z_C^{-1} \right\} V(0) \\
&= \cos(\beta \mathcal{L}) I_n + j \sin(\beta \mathcal{L}) Z_C I_0 + I_\mathcal{L} \\
V(0) &= -j \sin(\beta \mathcal{L}) Z_C I_0 + \cos(\beta \mathcal{L}) I_n + j \sin(\beta \mathcal{L}) Z_C Y_0 V(0)
\end{align*}
\]  

(2-58a)

(2-58b)

where \( \beta \) is the phase constant

\[
\beta = \omega / \nu
\]  

(2-59)

and the characteristic impedance matrix \( Z_C \) is given in (2-52).

Similarly the equations for the terminal currents, \( I(0) \) and \( I(\mathcal{L}) \), for the Thevenin Equivalent representation in Table 3 simplify to

\[
\begin{align*}
I(\mathcal{L}) &= -j \sin(\beta \mathcal{L}) Z_C^{-1} V_0 + \cos(\beta \mathcal{L}) I_n + j \sin(\beta \mathcal{L}) Z_C Z_0^{-1} V_0 I(0) \\
I(0) &= \cos(\beta \mathcal{L}) \left\{ Z_C + Z_0 \right\} + j \sin(\beta \mathcal{L}) \left\{ Z_C Z_C^{-1} Z_0 + Z_C^{-1} \right\} I(0)
\end{align*}
\]  

(2-60a)

(2-60b)

The terminal voltages can be obtained from the solution of (2-60) for the terminal currents, \( I(0) \) and \( I(\mathcal{L}) \), with the equations for the terminal networks

\[
V(0) = V_0 - Z_0 I(0)
\]  

(2-61a)
\[ V(\omega) = V_\omega + Z_\omega I(\omega) \quad (2-61b) \]

2.5.2 Transmission Lines Consisting of Imperfect (Lossy) Conductors in a Lossless, Homogeneous Medium, XTALK2

This section considers the \((n+1)\) conductor transmission lines considered in the previous section and shown in Figure 2-8. However, the transmission line conductors are considered to be lossy. Computer program XTALK2 considers these cases.

The per-unit-length inductance and capacitance matrices are computed as in the previous section and satisfy the relation in (2-50). The entries in \(L\) are given in (2-53), (2-54) and (2-55). The per-unit-length admittance matrix is given by

\[ Y = j\omega C = j \frac{\omega}{\sqrt{2}} L^{-1} \quad (2-62) \]

The per-unit-length impedance matrix is given by

\[ Z = R_{nc} + j\omega L_{nc} + j\omega L \quad (2-63) \]

where the entries in \(R_c\) and \(L_c\) are due to imperfect conductors. The entries in \(R_{nc}\) and \(L_{nc}\) are given in (2-5) and these matrices can be separated as \([1]\)

\[ R_{nc} + j\omega L_{nc} = (r_{c0} + j\omega \frac{c_0}{\epsilon_0}) U_n + Z_D \quad (2-64) \]

where \(U_n\) is the \(n\times n\) unit matrix with one's in every position, i.e., \([U_n]_{ij} = 1\), and \(Z_D\) is a diagonal matrix with

\[ [Z_D]_{ii} = r_{ci} + j\omega c_i \quad (2-65) \]

and \([Z_D]_{ij} = 0\) for \(i, j = 1, \ldots, n\) and \(i \neq j\). The calculation of the wire
self impedances, \( r + j\omega L \), and the reference conductor self impedance, \( r_c + j\omega L_c \), is discussed in section 2.4.

2.5.3 Transmission Lines Consisting of Perfect Conductors in a Lossless, Inhomogeneous Medium, FLATPAK

This section considers \((n+1)\) conductor transmission lines consisting of \((n+1)\) perfect conductors in a lossless, inhomogeneous medium. For example, dielectric insulations surrounding wires result in an inhomogeneous medium (dielectric insulation and the surrounding free space). The computer program FLATPAK considers a specific case of flatpack or ribbon cable. A ribbon cable consists of \((n+1)\) identical wires with identical cylindrical dielectric insulations bonded together in a linear array as shown in Figure 2-9.

In this case, the relationship in (2-50) relating the per-unit-length inductance and capacitance matrices no longer holds. Clearly the surrounding medium does not influence the per-unit-length inductance matrix since the surrounding medium is considered to be homogeneous in its permeability characteristic, \( \mu_v \). Therefore, one may compute the per-unit-length capacitance matrix with the wire dielectric insulations removed, denoted by \( C_0 \), and determine \( L \) through (2-50) as

\[
L = \frac{\mu_v}{\varepsilon_v} C_0^{-1} \tag{2-66}
\]

Therefore, one needs to compute the per-unit-length capacitance matrix with and without the wire dielectric insulations present. A digital computer program, GETCAP, has been written to compute the per-unit-length capacitance matrices of ribbon cables. This program is described in detail in Volume II of this series [8].
Fig. 2-9. An (n+1) wire ribbon (flatpack) cable.
The per-unit-length impedance and admittance matrices become

\[
Z = j\omega L \quad (2-67a)
\]
\[
Y = j\omega C \quad (2-67b)
\]

The transformation matrix, \( T \), which diagonalizes the matrix product \( YZ \) must therefore diagonalize the product \( CL \) as

\[
T^{-1} Y Z T = -\omega^2 T^{-1} C L T \quad (2-68)
\]
\[= -\omega^2 A^2 \]

In addition, it can be shown that \([1]\)

\[
T^{-1} = T^t C^{-1} \quad (2-69)
\]

where \( T^t \) is the transpose of \( T \). A digital computer subroutine NROOT (which uses subroutine EIGEN) is used to accomplish this reduction and is discussed in a later section.

2.5.4 Transmission Lines Consisting of Imperfect (Lossy) Conductors in a Lossless, Inhomogeneous Medium, FLATPAK2

This section considers \((n+1)\) conductor transmission lines consisting of \((n+1)\) lossy conductors in a lossless, inhomogeneous medium. The program FLATPAK2 considers a particular case of flatpack or ribbon cables discussed in the previous section.

The per-unit-length capacitance and inductance matrices are computed assuming perfect conductors and can be obtained with GEICAP as described in the previous section.

The self impedances of the wires are identical since the wires in the ribbon cable are typically identical. Therefore the pre-unit-length
impedance and admittance matrices become

\[ Z = z(U_n + 1_n) + j\omega L \]  \hspace{1cm} (2-70a)

\[ Y = j\omega C \]  \hspace{1cm} (2-70b)

where \( z = r + j\omega \ell \) is the self impedance of each wire.
III. PROGRAM CODE DESCRIPTIONS

In this chapter, the content of each program will be described by card. (Each program is labeled in columns 72-80 with the card number.) All programs were written in double precision arithmetic and the program listings are given in Appendix A - Appendix D. A table is provided with each listing which shows the changes which are required to convert each program to single precision arithmetic. Listings of two of the required subroutines, NROOT and EIGEN, are provided in Appendix E - Appendix F. The remaining required subroutines, LEQT1C and EIGCC, are a part of the IMSL (International Mathematical and Statistical Library) package [9]. Appropriate alternate subroutines can be substituted for LEQT1C and EIGCC if the IMSL package is not available on the user's system.

3.1 Program XTALK

A listing of XTALK is given in Appendix A.

Cards 001 through 047 contain general comments concerning the applicability of the program. This format will be followed in the other programs.

Cards 048 through 053 are comment cards pointing out that all arrays must be properly dimensioned for each problem before using the program.

Cards 054 through 059 dimension the arrays and declare variable types.

Card 060 gives the value of \( \pi \) and the speed of light in free space.

Cards 061 through 065 define the complex numbers \( 1+j0,0+j0, \) and \( 0+j1 \) as well as other constants.

Cards 071 through 118 read and print an initial portion of the input data.

Cards 123 through 170 read and print the line dimensions and compute the entries in the characteristic impedance matrix. The entries in the characteristic impedance matrix, \( Z_C \), are related to the per-unit-length inductance matrix for the three structure types given in (2-53), (2-54) and (2-55) by

\[
Z_C = \nu L.
\]

Cards 132 through 139 compute the main diagonal entries of \( Z_C \).

Cards 141 through 170 compute the off-diagonal entries. The n\( \times \)1 complex
arrays V1 and V2 are used to temporarily store the \( Z_i \) and \( Y_i \) coordinates or the \( r_i \) and \( \theta_i \) coordinates of the wires in the real parts of the arrays (see Figure 4-1, Figure 4-2, Figure 4-3). The \( n \times n \) complex array M1 is used to temporarily store the characteristic impedance matrix in the real parts. Although the actual quantities stored are real, it was decided to use the real parts of these complex arrays to store these quantities rather than define additional real arrays. V1, V2 and M1 will be needed (as complex arrays) later.

Cards 175 through 181 compute the inverse of the characteristic impedance matrix which is temporarily stored in the real part of the \( n \times n \) complex array M2. M2 will be needed (as a complex array) later. The matrix inverse is computed with subroutine LEQT1C which is described in section 3.5.

Cards 190 through 226 read and print the entries in the terminal impedance characterizations. These matrix characterizations are given in (2-30) for the Thevenin Equivalent characterization and in (2-34) for the Norton Equivalent characterization. The \( n \times 1 \) complex arrays I0 and IL store the entries in \( I(0) \) and \( I(t) \), respectively, for the Norton Equivalent in (2-34) or \( V(0) \) and \( V(t) \), respectively, for the Thevenin Equivalent in (2-30). The \( n \times n \) complex arrays Y0 and YL store the entries in \( Y_0 \) and \( Y_\infty \), respectively, for the Norton Equivalent in (2-34) or \( Z_0 \) and \( Z_\infty \), respectively, for the Thevenin Equivalent in (2-30).

Cards 231 through 291 contain certain matrix and vector multiplications which are independent of frequency. If one requests the analysis to be done at more than one frequency (such as in computing the frequency response of the line), then these time-consuming multiplications need be computed only for the first frequency and need not be recomputed for the additional fre-
To explain these cards, consider the similarity of the forms of the equations for the Norton Equivalent characterization given in (2-58) and the Thevenin Equivalent characterization given in (2-60). The analogous variables in these two equations are summarized as:

<table>
<thead>
<tr>
<th></th>
<th>(2-58)</th>
<th>(2-60)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_C$</td>
<td>$Z_C$</td>
<td>$Z_C$</td>
</tr>
<tr>
<td>$V_0$</td>
<td>$Z_0$</td>
<td>$Z_0$</td>
</tr>
<tr>
<td>$Z_C^{-1}$</td>
<td>$Z_C$</td>
<td>$Z_C$</td>
</tr>
<tr>
<td>$Z_C^{-1}$</td>
<td>$Z_C$</td>
<td>$Z_C$</td>
</tr>
<tr>
<td>$I_0$</td>
<td>$V_0$</td>
<td>$V_0$</td>
</tr>
<tr>
<td>$I_C$</td>
<td>$-V_C$</td>
<td>$-V_C$</td>
</tr>
<tr>
<td>$V(0)$</td>
<td>$I(0)$</td>
<td>$I(0)$</td>
</tr>
<tr>
<td>$V(t)$</td>
<td>$I(t)$</td>
<td>$I(t)$</td>
</tr>
</tbody>
</table>

Therefore equations (2-58) can be programmed and used for both cases if analogous variables are substituted. Cards 231 through 240 swap the entries in $M_1$ and $M_2$ if the Thevenin Equivalent characterization is chosen. Cards 251 through 291 form the quantities in (2-58)

\[
\begin{align*}
   M_1 & : \begin{bmatrix} Z_C & V_0 \\ Y_C & Z_C^{-1} \end{bmatrix} \\
   M_2 & : \begin{bmatrix} Z_C^{-1} & 0 \\ Y_C & Z_C^{-1} \end{bmatrix}
\end{align*}
\]

for the Norton Equivalent characterization or the quantities in (2-60)

\[
\begin{align*}
   M_1 & : \begin{bmatrix} Z^{-1}_C & Z_0 \\ Y_C^{-1} & Z_C^{-1} \end{bmatrix} \\
   M_2 & : \begin{bmatrix} Z_C^{-1} & Z_0 \\ Y_C^{-1} & Z_C^{-1} \end{bmatrix}
\end{align*}
\]
for the Thevenin Equivalent characterization.

Cards 295 through 300 read the frequency and form

\[
\beta f = \frac{2\pi f}{V}
\]  
(3-3a)

\[
\sin(\beta f) 
\]  
(3-3b)

\[
\cos(\beta f) 
\]  
(3-3c)

Cards 306 through 316 form equation (2-58a) for the Norton Equivalent characterization or (2-60a) for the Thevenin Equivalent characterization. These equations are solved with subroutine LEQT1C in card 320. The solutions (\(V(0)\) for (2-58a) or \(I(0)\) for (2-60a)) are stored in the array \(B\).

Cards 332 through 336 form equation (2-58b) or (2-60b) and the entries in \(V(\xi)\) for (2-58b) or \(I(\xi)\) for (2-60b) are stored in the array \(WA\).

Cards 337 through 365 print the terminal voltages \(V(0)\) and \(V(\xi)\). Cards 337 through 352 form the terminal voltages, if the Thevenin Equivalent characterization is chosen, from

\[
V(0) = V_0 - Z_0 I(0) \quad (3-4a)
\]

\[
V(\xi) = V_0 + Z_\xi I(\xi) \quad (3-4b)
\]

since the elements of the arrays \(B\) and \(WA\) are the following:

<table>
<thead>
<tr>
<th>Array</th>
<th>Norton</th>
<th>Thevenin</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B)</td>
<td>(V(0))</td>
<td>(I(0))</td>
</tr>
<tr>
<td>(WA)</td>
<td>(V(\xi))</td>
<td>(I(\xi))</td>
</tr>
</tbody>
</table>

Cards 353 through 365 print the resulting terminal voltages, \(V(0)\) and \(V(\xi)\).
3.2 Program XTALK2

A listing of XTALK2 is given in Appendix B.

Cards 001 through 190 have the same purpose and are of the same general structure as cards 001 through 181 in XTALK. The slight exceptions are that instead of computing the characteristic impedance matrix and its inverse as is done in XTALK, the per-unit-length capacitance matrix and its inverse are computed here. The per-unit-length inductance matrix, \( L \), and capacitance matrix, \( C \), are related by

\[
L \sim C = \frac{1}{\sqrt{\varepsilon \mu}} \frac{1}{\pi n}
\]  

(3-5a)

or

\[
\frac{1}{v} C = \frac{1}{L^{-1}}
\]

(3-5b)

where \( v \) is the velocity of light in the surrounding medium given by

\[
v = \frac{1}{\sqrt{\varepsilon \mu}} = \frac{v_0}{\sqrt{\varepsilon_r \mu_r}}
\]

(3-5c)

\( \varepsilon \) is the permittivity of the medium, \( \mu \) is the permeability of the medium, \( v_0 \) is the velocity of light in free space \( (v_0 = 3 \times 10^8 \text{ m/sec}) \) and \( \varepsilon_r \) and \( \mu_r \) are the relative permittivity and permeability, respectively. The characteristic impedance matrix is given by

\[
Z_C = \frac{v}{L}
\]

(3-6)

Therefore \( L^{-1} = v Z_C \). \( C \) is stored in array C and \( C^{-1} \) is stored in array CI.

Cards 195 through 223 read and print the characteristics of the reference conductor and the \( n \) wires to be using in calculating their self impedances.

Cards 233 through 269 read and print the termination network character-
istics and are identical to the corresponding cards in XTALK.

Cards 275 through 332 perform certain frequency independent matrix multiplications for reasons similar to those given in 3.1 for the analogous group of cards. These cards form, for the Norton or Thevenin Equivalent characterizations, certain quantities in Tables 2 and 3:

<table>
<thead>
<tr>
<th>Array</th>
<th>Norton</th>
<th>Thevenin</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>( Y_0 C^{-1} )</td>
<td>( C Z_0 )</td>
</tr>
<tr>
<td>M2</td>
<td>( Y C^{-1} )</td>
<td>( C Z )</td>
</tr>
<tr>
<td>V1</td>
<td>( I_0 )</td>
<td>( C V_0 )</td>
</tr>
<tr>
<td>V2</td>
<td>( I )</td>
<td>( C V )</td>
</tr>
</tbody>
</table>

Cards 328 through 332 form the sums of entries in each row of \( C \) and are stored in the array \( V_3 \).

Cards 336 through 340 read the frequency and form the quantities \( \omega=2\pi f \) and \( j\omega \).

Cards 346 through 385 form the self impedances of the wires and the reference conductor. The equations for these self impedance terms are given in (2-42) through (2-49) in section 2.4. The self impedance of the reference conductor is stored as the complex variable \( Z_0 \) and the self impedances of the \( n \) wires are temporarily stored in the array \( B \).

Cards 391 through 398 compute the eigenvalues and eigenvectors of the product of the per-unit-length admittance and impedance matrices, \( YZ \). The per-unit-length admittance matrix is given by

\[
Y = j \omega C \quad (3-7)
\]

and the per-unit-length impedance matrix is given by

\[
Z = Z_0 + Z_D + j \omega L \quad (3-8)
\]
where \( U \) is an \( n \times n \) unit matrix with ones in every position, \( z_0 \) is the self impedance of the reference conductor and \( Z_D \) is an \( n \times n \) diagonal matrix with the self impedance of the \( i \)-th wire in the \( i \)-th row and \( i \)-th column. The matrix product becomes (with the relation in (3-5a))

\[
YZ = j\omega C \left[ z_0 U_n + Z_D + j\omega L \right] = j\omega z_0 C U_n + j\omega C Z_D - \omega_0^2 \frac{1}{2} \Omega
\]  

(3-9)

Note that \( C U \) is simply an \( n \times n \) matrix with the sum of all elements in the \( i \)-th row of \( C \) in each of the entries in the \( i \)-th row of \( C U \). These quantities were previously stored in the array \( V^3 \). The subroutine EIGCC computes the \( n \times 1 \) eigenvectors of \( YZ, T_i, \) and their associated eigenvectors, \( \gamma_i^2 \). The matrix \( T = [T_1, T_2, \ldots, T_n] \) will diagonalize \( YZ \) as \([1]\)

\[
T^{-1} Y Z T = \gamma_i^2
\]  

(3-10)

where \( \gamma_i^2 \) is an \( n \times n \) diagonal matrix with \( \gamma_i^2 \) in the \( i \)-th position on the main diagonal. This is required in Tables 2 and 3. \( T \) is stored in array \( T \) and the \( n \) entries on the main diagonal of \( \gamma_i^2 \) are temporarily stored in the array \( B \).

Cards 403 through 410 compute the inverse of \( T \) which is stored in array \( T^{-1} \).

Cards 416 through 448 compute certain other quantities in Tables 2 and 3. These are

<table>
<thead>
<tr>
<th>Array</th>
<th>Norton</th>
<th>Thevenin</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_0 )</td>
<td>( Y_0^* = T^{-1} Y_0 C^{-1} T )</td>
<td>( Z_0^* = T^{-1} C Z_0 T )</td>
</tr>
<tr>
<td>( Y_L )</td>
<td>( Y_L^* = T^{-1} Y_L C^{-1} T )</td>
<td>( Z_L^* = T^{-1} C Z_L T )</td>
</tr>
<tr>
<td>( I_0 )</td>
<td>( I_0^* = T^{-1} I_0 )</td>
<td>( V_0^* = T^{-1} C V_0 )</td>
</tr>
<tr>
<td>( I_L )</td>
<td>( I_L^* = T^{-1} I_L )</td>
<td>( V_L^* = T^{-1} C V_L )</td>
</tr>
</tbody>
</table>

-52-
Array Norton Thevenin
EP \[E^+ = \frac{1}{2}(e^{\gamma L} + e^{-\gamma L})\]
EN \[E^- = \frac{1}{2}(e^{\gamma L} - e^{-\gamma L})\]
G \[\Lambda = \frac{1}{j\omega} \gamma\]

Cards 449 through 458 form equation (1) in Tables 2 and 3. This equation is solved with subroutine LEQT1C in card 462 with the result stored in array B as:

Array Norton Thevenin
B \[V^*(0)\]
 \[I^*(0)\]

Cards 480 through 484 form equation (2) in Tables 2 and 3 with the result stored as

Array Norton Thevenin
G \[V^*(t)\]
 \[I^*(t)\]

Cards 485 through 531 form the terminal voltages \(V(0)\) and \(V(t)\) by back transforming according to equation (9) in Tables 2 and 3.

3.3 Program FLATPAK

A listing of FLATPAK is given in Appendix C.

Cards 001 through 057 are similar to corresponding cards in the previous programs.

Cards 062 through 097 read a portion of the input data describing the structure of the line. The per-unit-length capacitance matrix, \(C\), (computed with GETCAP) is stored in array \(C\). The per-unit-length capacitance matrix with the wire insulations removed, \(C_0\), (computed with GETCAP) is stored in array \(C_0\).

Cards 105 through 113 compute the eigenvectors and corresponding eigen-
values of the matrix product $C L$. Subroutine NROOT computes the matrix $K$ such that

$$K^{-1} C^{-1} C_0 K = \psi$$

(3-11)

such that $\psi$ is a diagonal matrix. $K$ is stored in array TI. The problem of interest is finding $T$ such that

$$T^{-1} C L T = \gamma^2$$

(3-12)

where

$$L = \frac{1}{\nu_o^2} C_0$$

(3-13)

Taking the inverse of both sides of (3-11) results in

$$K^{-1} C_0 K = \psi^{-1}$$

(3-14)

Taking the transpose of both sides of (3-14) results in

$$K^T C C_0^{-1} K^{-1} = \psi^{-1}$$

(3-15)

(Since $C$ and $C_0$ are symmetric, $C^T = C$ and $C_0^{-1} = C_0^{-1}$. Also $\psi$ is diagonal. Therefore $\psi^{-1} = \psi$. ) Thus comparing (3-15) to (3-12) and using (3-13) we identify

$$K = T^{-1}$$

(3-16a)

$$\frac{1}{\nu_o^2} \psi^{-1} = \gamma^2$$

(3-16b)

and $T^{-1}$ is stored in array $C$ and array $G$ contains the square roots of entries on the main diagonal of $\gamma^2$, $\gamma$.

Cards 114 through 128 compute $T$ and $\gamma^{-1}$ if the Thevenin Equivalent characterization is chosen. Thus, contained in arrays TI and $G$ are:
Cards 138 through 175 read and print the termination network characteristics and are identical to the corresponding cards in the previous programs.

Cards 182 through 220 form the following frequency independent quantities (see Tables 2 and 3)

\[
\begin{align*}
\text{Array} & & \text{Norton} & & \text{Thevenin} \\
YO & & Y^*_0 = T^{-1} Y_0 C^{-1} T & & Z^*_0 = T^{-1} C Z_0 T \\
YL & & Y^*_L = T^{-1} Y_L C^{-1} T & & Z^*_L = T^{-1} C Z_L T \\
I0 & & I^*_0 = T^{-1} I_0 & & V^*_0 = T^{-1} C V_0 \\
IL & & I^*_L = T^{-1} I_L & & -V^*_L = -T^{-1} C V_L
\end{align*}
\]

Since \( T^{-1} \) satisfies

\[
T^{-1} = T^t C^{-1}
\]

(3-17)

then

\[
T^{-1} t = C^{-1} T
\]

(3-18)

and these relations allow the entries in the arrays YO, YL, I0 and IL to be more easily generated as:

\[
\begin{align*}
\text{Array} & & \text{Norton} & & \text{Thevenin} \\
YO & & Y^*_0 = T^{-1} Y_0 T^{-1} t & & Z^*_0 = T^t Z_0 T \\
YL & & Y^*_L = T^{-1} Y_L T^{-1} t & & Z^*_L = T^t Z_L T \\
I0 & & I^*_0 = T^{-1} I_0 & & V^*_0 = T^t V_0 \\
IL & & I^*_L = T^{-1} I_L & & -V^*_L = -T^t V_L
\end{align*}
\]

-55-
Cards 224 through 227 read the frequency and compute \( \omega = 2\pi f \).

Cards 233 through 248 form equation (1) in Tables 2 and 3.

Equation (1) in Tables 2 and 3 is solved with subroutine LEQT1C in card 252.

Cards 264 through 268 form equation (2) in Tables 2 and 3. The arrays B and WA now contain, with respect to Tables 2 and 3:

<table>
<thead>
<tr>
<th>Array</th>
<th>Norton</th>
<th>Thevenin</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>( V^*(0) )</td>
<td>( I^*(0) )</td>
</tr>
<tr>
<td>WA</td>
<td>( V^*(\xi) )</td>
<td>( I^*(\xi) )</td>
</tr>
</tbody>
</table>

The terminal voltages, \( V(0) \) and \( V(\xi) \) are computed in cards 269 through 286 by back transforming \( V^*(0) \) and \( V^*(\xi) \) through (see Tables 2 and 3)

\[
\begin{align*}
V(0) &= C^{-1}T V^*(0) \\
&= \frac{1}{T} V^*(0) \\
V(\xi) &= C^{-1}T V^*(\xi) \\
&= \frac{1}{T} V^*(\xi)
\end{align*}
\]

\[
\begin{align*}
V^*(0) &= \frac{1}{Z_0} I^*(0) \\
V^*(\xi) &= \frac{1}{Z_\xi} I^*(\xi)
\end{align*}
\]

Cards 287 through 301 print the resulting terminal voltages.

3.4 Program FLATPAK2

A listing of FLATPAK2 is given in Appendix D.

Cards 001 through 106 are similar to corresponding cards (001 through 097) in FLATPAK.

Cards 112 through 133 compute the inverse of the per-unit-length
capacitance matrix which is stored in array CI. The per-unit-length inductance matrix, \( L \), is also computed from the relation

\[
L = \frac{1}{\gamma^2} C_0^{-1}
\]  
(3-19)

Cards 138 through 144 read the characteristics of the wires in the ribbon cable (all wires are assumed to be identical) for use in computing their self impedances.

Cards 154 through 191 read and print the characteristics of the termination networks and are identical to the corresponding cards in the previous programs.

Cards 197 through 262 form certain frequency independent quantities in Tables 2 and 3:

<table>
<thead>
<tr>
<th>Array</th>
<th>Norton</th>
<th>Thevenin</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>( Y_0 C^{-1} )</td>
<td>( C Z_0 )</td>
</tr>
<tr>
<td>M2</td>
<td>( Y_x C^{-1} )</td>
<td>( C Z )</td>
</tr>
<tr>
<td>V1</td>
<td>( Z_0 )</td>
<td>( C V_0 )</td>
</tr>
<tr>
<td>V2</td>
<td>( Z )</td>
<td>( C V )</td>
</tr>
</tbody>
</table>

Cards 251 through 262 form the quantities \( C L \) which is stored in array \( C_0 \) and the sums of the elements in the \( i \)-th row of \( C \) which are stored in array \( V_3 \).

Cards 266 through 270 read the frequency and form \( \omega = 2\pi f \) and \( j\omega \).

Cards 274 through 283 form the self impedances of the wires which are stored in the complex variable \( Z \) (all wires are identical).

Cards 289 through 295 compute the transformation matrix \( T \) such that

\[
T^{-1} \bar{Y} Z T = \bar{Y}^2
\]  
(3-20)
where \( \gamma^2 \) is a diagonal matrix and

\[
Y Z = j\omega C(z U + z \frac{1}{n} + j\omega L).
\]

Subroutine EIGCC computes \( T \) and stores it in array \( T \) and stores the entries on the main diagonal of \( \gamma^2 \) temporarily in Array \( B \).

The inverse of \( T \) is computed with LEQT1C in cards 300 through 307 and is stored in array \( TI \).

Cards 313 through 345 compute certain quantities in Tables 2 and 3:

<table>
<thead>
<tr>
<th>Array</th>
<th>Norton</th>
<th>Thevenin</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_0 )</td>
<td>( Y_0^* = T^{-1} Y_0 C^{-1} T )</td>
<td>( Z_0^* = T^{-1} C Z_0 T )</td>
</tr>
<tr>
<td>( Y_L )</td>
<td>( Y_L^* = T^{-1} Y_L C^{-1} T )</td>
<td>( Z_L^* = T^{-1} C Z_L T )</td>
</tr>
<tr>
<td>( I_0 )</td>
<td>( I_0^* = T^{-1} I_0 )</td>
<td>( V_0^* = T^{-1} C V_0 )</td>
</tr>
<tr>
<td>( I_L )</td>
<td>( I_L^* = T^{-1} I_L )</td>
<td>( -V_L^* = T^{-1} C V_L )</td>
</tr>
<tr>
<td>( E^+ )</td>
<td>( E^+ = \frac{1}{2}(e^{-\gamma z} + e^{-\gamma z'}) )</td>
<td>( E^+ = \frac{1}{2}(e^{-\gamma z} + e^{-\gamma z'}) )</td>
</tr>
<tr>
<td>( E^- )</td>
<td>( E^- = \frac{1}{2}(e^{-\gamma z} - e^{-\gamma z'}) )</td>
<td>( E^- = \frac{1}{2}(e^{-\gamma z} - e^{-\gamma z'}) )</td>
</tr>
<tr>
<td>( G )</td>
<td>( \Lambda = \frac{1}{j\omega} \gamma )</td>
<td>( \Lambda = \frac{1}{j\omega} \gamma )</td>
</tr>
</tbody>
</table>

Cards 346 through 355 form equation (1) in Tables 2 and 3 which is solved with subroutine LEQT1C in card 359.

Cards 376 through 380 form equation (2) in Tables 2 and 3. Thus the arrays \( B \) and \( G \) contain:

<table>
<thead>
<tr>
<th>Arrays</th>
<th>Norton</th>
<th>Thevenin</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B )</td>
<td>( V^*(0) )</td>
<td>( I^*(0) )</td>
</tr>
<tr>
<td>( G )</td>
<td>( V^*(\tau) )</td>
<td>( I^*(\tau) )</td>
</tr>
</tbody>
</table>
Cards 381 through 406 form \( V(0) \) and \( \tilde{V}(\mathcal{L}) \) by back transforming \( \tilde{V}^*(0) \) and \( \tilde{V}^*(\mathcal{L}) \) as described in FLATPAK using the relations in Table 2 and Table 3:

\[
\begin{align*}
V(0) &= C^{-1} T \tilde{V}^*(0) \\
\tilde{V}(\mathcal{L}) &= C^{-1} T \tilde{V}^*(\mathcal{L})
\end{align*}
\]

Cards 407 through 427 print the terminal voltages.

3.5 Required Subroutines

The four programs require certain subroutines: LEQT1C, EIGCC, NROOT, and EIGEN. The individual programs require:

<table>
<thead>
<tr>
<th>Program</th>
<th>Required Subroutines</th>
</tr>
</thead>
<tbody>
<tr>
<td>XTALK</td>
<td>LEQT1C</td>
</tr>
<tr>
<td>XTALK2</td>
<td>LEQT1C, EIGCC</td>
</tr>
<tr>
<td>FLATPAK</td>
<td>LEQT1C, NROOT, EIGEN</td>
</tr>
<tr>
<td>FLATPAK2</td>
<td>LEQT1C, EIGCC</td>
</tr>
</tbody>
</table>

The required subroutines must follow the main program and precede the data cards.

3.5.1 Subroutine LEQT1C

Subroutine LEQT1C is a general subroutine for solving a system of \( n \) simultaneous, complex equations. The program is a part of the IMSL (International Mathematical and Statistical Library) package [9].

The subroutine solves the system of equations

\[
A X = B \quad (3-22)
\]
where $A$ is an $n \times n$ complex matrix, $B$ is an $n \times m$ complex matrix and $X$ is an $n \times m$ complex matrix whose columns $X_i$ are solutions to

$$A X_i = B_i$$

(3-23)

where $B_i$ is the $i$-th column of $B$.

The calling statement is

```
CALL LEQTIC(A,N,N,B,N,M,WA,IER)
```

where

$$A \rightarrow A$$

$$B \rightarrow B$$

$$N \rightarrow n$$

$$M \rightarrow m$$

and WA is a complex working vector of length $n$. IER is an error parameter which is returned as

1. $IER = 128 \rightarrow$ no solution error
2. $IER = 129 \rightarrow A$ is algorithmically singular [9].

The solution $X$ is returned in array $B$ and the contents of array $A$ are destroyed.

Subroutine LEQTIC can be used to find the inverse of an $n \times n$ matrix by computing

$$A X = \mathbf{1}_n$$

(3-24)

where $\mathbf{1}_n$ is the $n \times n$ identity matrix. Thus the solution is $X = A^{-1}$. LEQTIC

---

1. The solution error parameter is printed out whenever LEQTIC is used. The printed error is $IER-128$ so that the solution error should be 0.
is used in numerous places to invert real matrices by defining the real
part of A to be the matrix and the imaginary part to be zero. Upon solution,
the real part of X is the inverse of the real matrix, A.

3.5.2 Subroutine EIGCC

Subroutine EIGCC is also a part of the IMSL subroutine package [9] and
is used to find the eigenvalues and eigenvectors of an n\times n complex matrix,
M. Denote the n\times 1 (complex) eigenvectors, T_1, T_2, \ldots, T_n and
the corresponding (complex) eigenvalues as b_1, b_2, \ldots, b_n. EIGCC computes
the n\times n matrix T = [T_1, T_2, T_3, \ldots, T_n] such that

\[ T^{-1} M T = B \]  

(3-25)

where B is an n\times n diagonal matrix with \[ B_{ii} = b_i \] and \[ B_{ij} = 0 \] for i,
j=1, \ldots, n and i\neq j.

The calling statement is

\[
\text{CALL EIGCC}(M,N,N,2,B,T,WK,IER)
\]

where WK is a real working vector of length 2n(n+1). IER is an error para-
eter which is returned as IER = 128 + J. This indicates that the routine
failed to converge on the j-th eigenvalue [9]. The precision of the eigen-
vector, eigenvalue solution is returned in the first position of array WK,
WK(1), and indicates [9]

\underline{Solution Precision}

\[
\begin{align*}
\text{WK (1)} &< 1 \rightarrow \text{Excellent} \\
1 &< \text{WK (1)} < 100 \rightarrow \text{Good} \\
\text{WK (1)} &> 100 \rightarrow \text{Poor}
\end{align*}
\]

\[\text{\underline{\text{1The solution error is printed out as IER-128. A successful solution would then be indicated by 0.}}}
\]
The matrix $T$ is stored in the $n \times n$ array $T$ and the eigenvalues, $b_i$, are stored in the $n \times 1$ array $B$ in the same order as the columns of $T$.

3.5.3 Subroutines NROOT and EIGEN

Subroutines NROOT and EIGEN are a set of subroutines from the IBM Scientific Subroutine Package (SSP) [10] which compute the eigenvectors and eigenvalues of the matrix product

$$ B^{-1} A $$

(3-26)

where $A$ and $B$ are $n \times n$ real, symmetric matrices and $B$ is positive definite.

A listing of NROOT is provided in Appendix E and a listing of EIGEN is provided in Appendix F. These subroutines are used to find the eigenvalues and eigenvectors of the product of the per-unit-length capacitance, $C$, and inductance, $L$, matrices as

$$ C L $$

(3-27)

Subroutine NROOT calls subroutine EIGEN.

NROOT computes the $n \times n$ real matrix $T$ such that

$$ T^{-1} B^{-1} A T = G $$

(3-28)

where $G$ is an $n \times n$ diagonal matrix with $[G]_{ii} = g_i$ and $[G]_{ij} = 0$ for $i, j=1, \ldots, n$ and $i \neq j$. The eigenvectors $T_i$ correspond to the eigenvalues $g_i$ and $T = [T_1, T_2, \ldots, T_n]$.

The calling statement is

```fortran
CALL NROOT(N,A,B,G,T,N*N)
```

where
The \( n \times 1 \) array \( G \) returns the eigenvalues \( g_i \) in the same sequence as the columns (corresponding eigenvectors) of \( T \).

The subroutine operates in the following manner [1,11]. NROOT first computes the \( n \times n \), real, orthogonal transformation matrix \( S \) such that

\[
S^T B S = H
\]

where \( H \) is an \( n \times n \) diagonal matrix with \( [H]_{ii} = h_i \) and \( [H]_{ij} = 0 \) for \( i, j = 1, \ldots, n \). EIGEN is called for this calculation. Since \( B \) is real, symmetric, positive definite, the eigenvalues of \( B \), \( h_i \), are real, nonzero and positive. Therefore NROOT forms the square root of \( H \), \( H^{1/2} \) and its inverse \( H^{-1/2} \).

NROOT then forms the products

\[
M = S^{1/2}
\]

and

\[
M^T A M
\]

which is real, symmetric. NROOT calls EIGEN once again to find the \( n \times n \) real, orthogonal matrix \( W \) such that

\[
W^T [M^T A M] W = G
\]
and $G$ is diagonal. The transformation matrix $T$ is given by

$$T = S H^{-1/2} W$$  \hspace{1cm} (3-33)

To show that $T$ in fact diagonalizes $B^{-1}A$, form

$$T^{-1} B^{-1} A T =$$

$$W^t H^{-1/2} S^t B^{-1} A S H^{-1/2} W =$$

$$\frac{1}{n}$$

$$W^t H^{-1/2} S^t B^{-1} S H^{-1/2} H^{-1/2} S^t A S H^{-1/2} W = G$$

$T^{-1} C L T = \gamma^2$ \hspace{1cm} (3-35)

will satisfy the identity

$$T^{-1} = T^t C^{-1}$$  \hspace{1cm} (3-36)

If the columns of $T$ (the eigenvectors) are normalized, (3-36) will no longer be true.
IV. USER'S MANUAL

This section will serve as a user's manual for the use of the programs. All input data are punched on cards which must follow the main program (and any subroutines). The format of the data input cards as well as suggestions for program usage are included. All of the programs require three groups of data input:

Group I       \{ Transmission Line
               \{ Structure Characteristics Cards \}

Group II      \{ Termination Network
               Characterization Cards \}
               \{ Group II(a) \}
               \{ Group II(b) \}

Group III     \{ F.equency Cards \}

These card groups must follow the main program (and any required subroutines) in the above order. The data entries are either in Integer (I) format, e.g., 35, or in Exponential (E) format, e.g., 12.6E-3. All data entries must be right-justified in the assigned card column block.

In all four programs, the user must appropriately dimension all arrays for each problem. Comment cards are provided at the beginning of each program to assist the user in providing proper dimensions. All arrays must be properly dimensioned by repunching the dimension statement cards in a program before using the program.

4.1 The Frequency Cards, Group III

Each frequency card contains one and only one frequency for which an analysis is desired. The format of the frequency card is shown in Table 4. The frequency in Hertz is punched in columns 1-10 of each card and must be
<table>
<thead>
<tr>
<th>frequency (Hertz)</th>
<th>card column</th>
<th>format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>E</td>
<td></td>
</tr>
</tbody>
</table>

Total number = unlimited
right justified in the card block consisting of card columns 1-10. For example, if one wished to input a frequency of 1 M Hz, one may punch

```
1. E 6
+ + + +
``` card columns 7 8 9 10

If, instead, the frequency was punched as

```
1. E 6
+ + + +
``` card columns 6 7 8 9

The program would take this to be a frequency of $10^{60}$ Hertz (zeros are added to fill out the assigned card block). This right-justification of data in an assigned card block applies to all other data entries.

More than one frequency card may be included in the frequency card group. Each program will process the data provided by Groups I and II and compute the response at the frequency on the first frequency card. It will then recompute the response at each frequency on the remaining frequency cards. The program assumes that the data on card Groups I and II are to be used for all the remaining frequencies. If this is not intended by the user, then one may only run the program for one frequency at a time. This feature, however, can be quite useful. If the termination networks are purely resistive, i.e., frequency independent, then one may use as many frequency cards as desired in this frequency card group and the program will compute the response of the line at each frequency without the necessity for the user to input the data in Groups I and II for each additional frequency. Many of the time-consuming calculations which are independent of frequency need to be computed
only once so that this mode of usage will save considerable computation time
when the response at many frequencies is desired. If, however, the termination
network characteristics (in Group II) are complex (which implies frequency
dependent), one must run the program for only one frequency at a time.

4.2 The Termination Network Characterization Cards, Group II

This group of cards conveys the terminal characteristics of the term-
ination networks at the ends of the line, \( x = 0 \) and \( x = L \). The termination
networks are characterized by either the Thevenin Equivalent or the Norton
Equivalent characterization. These characterizations are of the form

\[
\begin{align*}
\text{Thevenin} & : & \quad V(0) &= V_0 - Z_0 I(0) \\
& & \quad V(z) &= V_z + Z_z I(z) \\
\text{Norton} & : & \quad I(0) &= I_0 - Y_0 V(0) \\
& & \quad I(z) &= -I_z + Y_z V(z)
\end{align*}
\]

(4-1a)

and are discussed in detail in section 2.3. The transmission line consists
of \( n \) wires which are numbered from 1 to \( n \) and a reference conductor for the
line voltages. The reference conductor is numbered as the zero (0) con-
ductor. Thus \( V_0, V_z, I_0, I_z \) are \( n \times 1 \) vectors and \( Z_0, Z_z, Y_0, Y_z \) are \( n \times n \)
matrices which are assumed to be symmetric.

The impedance or admittance matrices \( Z_0 \) and \( Z_z \) or \( Y_0 \) and \( Y_z \), respectively,
may either be "full" in which all entries are not necessarily zero or may be
diagonal in which only the entries on the main diagonals are not necessarily
zero and the off-diagonal entries are zero. The user may select one of
four options for communicating the entries in the vectors and matrices in
(4-1). These are:

\begin{align*}
\text{OPTION} &= 11 \quad \{ \text{Thevenin Equivalent representation; diagonal impedance matrices, } Z_0 \text{ and } Z_L. \} \\
\text{OPTION} &= 12 \quad \{ \text{Thevenin Equivalent representation; full impedance matrices, } Z_0 \text{ and } Z_L. \} \\
\text{OPTION} &= 21 \quad \{ \text{Norton Equivalent representation; diagonal admittance matrices, } Y_0 \text{ and } Y_L. \} \\
\text{OPTION} &= 22 \quad \{ \text{Norton Equivalent representation; full admittance matrices, } Y_0 \text{ and } Y_L. \}
\end{align*}

The structure and ordering of the data in Group II are given in Table 5 and can be summarized in the following manner. The first group of cards in Group II, Group II(a), will describe the entries on the main diagonal in \( Y_0(Z_0) \), \( Y_{0ii}(Z_{0ii}) \), and \( Y_1(Z_1) \), \( Y_{1ii}(Z_{1ii}) \), and the entries in \( I_0(V_0) \), \( I_{0i}(V_{0i}) \), and \( I_1(V_1) \), \( I_{1i}(V_{1i}) \). These cards must be in the order from \( i = 1 \) to \( i = n \). Each of these entries is in general, complex. Therefore two card blocks are assigned for each entry; one for the real part and one for the imaginary part. For example, consider a 4 conductor line (3 wires and a reference conductor). Here \( n \) would be 3. Suppose the Thevenin Equivalent characterization is selected, with the following entries in the characterization matrices:

\[
\begin{align*}
V_0 &= \begin{bmatrix} 1 + j2 \\ 3 + j5 \\ 6 + j4 \end{bmatrix} & Z_0 &= \begin{bmatrix} 7+j8 & 0 & 0 \\ 0 & 19 & 0 \\ 0 & 0 & 10+j11 \end{bmatrix} \\
V_L &= \begin{bmatrix} 12 \\ j13 \\ 14+j15 \end{bmatrix} & Z_L &= \begin{bmatrix} 16 & 0 & 0 \\ 0 & 17+j18 & 0 \\ 0 & 0 & j19 \end{bmatrix}
\end{align*}
\]
TABLE 5 (cont.)

Format of the Termination Network Characterization Cards, Group II

Group II(a)(total = n)

<table>
<thead>
<tr>
<th>Real part</th>
<th>Imaginary part</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_{0i1}(Z_{0i1})$</td>
<td>1-10 E</td>
</tr>
<tr>
<td>$I_{0i1}(V_{0i1})$</td>
<td>11-20 E</td>
</tr>
<tr>
<td>$Y_{1i1}(Z_{1i1})$</td>
<td>21-30 E</td>
</tr>
<tr>
<td>$I_{1i1}(V_{1i1})$</td>
<td>31-40 E</td>
</tr>
<tr>
<td>$Y_{n1i}(Z_{n1i})$</td>
<td>41-50 E</td>
</tr>
<tr>
<td>$I_{n1i}(V_{n1i})$</td>
<td>51-60 E</td>
</tr>
<tr>
<td>$Y_{n1n}(Z_{n1n})$</td>
<td>61-70 E</td>
</tr>
<tr>
<td>$I_{n1n}(V_{n1n})$</td>
<td>71-80 E</td>
</tr>
</tbody>
</table>

Note: A total of n cards must be present for an n wire line and must be arranged in the order:

wire 1
wire 2
.
.
wire n
**TABLE 5**

<table>
<thead>
<tr>
<th>Group II(b)</th>
<th>Card Column</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_{ij}(Z_{0ij}) )</td>
<td>real part</td>
<td>card column 1-10</td>
</tr>
<tr>
<td></td>
<td>imaginary part</td>
<td>11-20</td>
</tr>
<tr>
<td>( Y_{ij}(Z_{ij}) )</td>
<td>real part</td>
<td>41-50</td>
</tr>
<tr>
<td></td>
<td>imaginary part</td>
<td>51-60</td>
</tr>
</tbody>
</table>

**Note:** If OPTION = 12 or 22, a total of \( n(n-1)/2 \) cards must be present and must follow Group II(a). If OPTION = 11 or 21, this card group is omitted. The cards must be arranged so as to describe the entries in the upper triangle portion of \( Y_{0}(Z_{0}) \) and \( Y_{ij}(Z_{ij}) \) by rows, i.e., the cards must contain the 12 entries, the 13 entries, ---, the \( n \) entries, the 23 entries, ---, the 2\( n \) entries, ---- etc. The ordering of the cards is therefore:

- wires 1,2
- wires 1,3
- wires 1,4
- wires 1,5
- wires 1,6
- wires 1,7
- wires 1,8
- wires 1,9
- wires 1,10
- wires 1,11
- wires 1,12
- wires (n-1), n
One would have selected OPTION 11. The n=3 cards would be arranged (in this order)

\[
\begin{bmatrix}
7,0 & 8,0 & 1,0 & 2,0 & 16,0 & 0,0 & 12,0 & 0,0 \\
10 & 20 & 30 & 40 & 50 & 60 & 70 & 80 \\
0,0 & 9,0 & 3,0 & 5,0 & 17,0 & 18,0 & 0,0 & 13,0 \\
10,0 & 11,0 & 6,0 & 4,0 & 0,0 & 19,0 & 14,0 & 15,0
\end{bmatrix}
\]

If the terminal impedance matrices were not diagonal, e.g., OPTION 12 is selected, then n(n-1)/2 additional cards, Group II(b), would follow the above n cards comprising Group II(a). These cards describe the entries in the upper triangle portion of the termination impedance or admittance matrices by rows. Suppose the networks are characterized by the same \( V_0 \) and \( V_\varphi \) vectors as above but the \( Z_0 \) and \( Z_\varphi \) matrices are

\[
Z_0 = \begin{bmatrix}
7 + j 8 & 20 + j 21 & 22 + j 23 \\
20 + j 21 & j 9 & 24 + j 25 \\
22 + j 23 & 24 + j 25 & 10 + j 11
\end{bmatrix}
\]

\[
Z_\varphi = \begin{bmatrix}
16 & 26 + j 27 & 28 \\
26 + j 27 & 17 + j 18 & j 29 \\
28 & j 29 & j 19
\end{bmatrix}
\]

The following n(n-1)/2 = 3 cards must follow the above 3 cards in the order of the 12 entries first, the 13 entries next and then the 23 entries:

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4.3 Program XTALK

XTALK considers (n+1) conductor transmission lines consisting of n wires in a lossless, homogeneous surrounding medium and a reference conductor for the line voltages. The n wires and the reference conductor are considered to be perfect (lossless) conductors. There are three choices for the reference conductor type:

TYPE = 1: The reference conductor is a wire.

TYPE = 2: The reference conductor is an infinite ground plane.

TYPE = 3: The reference conductor is an overall cylindrical shield.

Cross-sectional views of each of these three structure types are shown in Figure 4-1, 4-2 and 4-3, respectively.

For the TYPE 1 structure shown in Figure 4-1, an arbitrary rectangular coordinate system is established with the center of the coordinate system at the center of the reference conductor. The radii of all (n+1) wires, r_{wi}, as well as the Z and Y coordinates of each of the n wires serve to completely describe the structure. Negative coordinate values must be input as negative data items. For example, Z_j and Y_j in Figure 4-1 would be negative numbers.
Figure 4-1. Type 1 structure.
Figure 4-2. Type 2 structure.
Figure 4-3. Type 3 structure.
For the TYPE 2 structure shown in Figure 4-2, an arbitrary coordinate system is established with the ground plane as the Z axis. The coordinates $Y_i$ and $Y_j$ (positive quantities) define the heights of the wires above the ground plane. The necessary data are the Z and Y coordinates and the radius, $r_{wi}$, of each wire.

For the TYPE 3 structure shown in Figure 4-3, an arbitrary angular coordinate system is established with the center of the coordinate system at the center of the shield. The necessary parameters are the radii of the wires, $r_{wi}$, the angular position, $\theta_i$, and the radial position, $r_i$, of each wire and the interior radius of the shield, $r_s$.

The format of the structural characteristics cards, Group I, are shown in TABLE 6. The first card contains the structure TYPE number (1, 2, or 3), the load structure OPTION number (11, 12, 21, or 22), the number of wires, n, the relative dielectric constant of the surrounding medium (homogeneous), $\varepsilon_r$, the relative permeability of the surrounding medium (homogeneous), $\mu_r$, and the total length of the transmission line, $L$ (meters). If TYPE 1 or 3 is selected, a second card is required which contains the radius of the reference wire, $r_{w0}$ (mils) for TYPE 1 structures or the interior radius of the shield, $r_s$ (meters) for TYPE 3 structures. For TYPE 2 structures, this card is absent. These cards are followed by n cards each of which contain the radii of the wires, $r_{wi}$ (mils) and the $Z_i$ and $Y_i$ coordinates of each wire (meters) for TYPE 1 and 2 structures or the angular coordinates $r_i$ (meters) and $\theta_i$ (degrees) of the $i$-th wire for TYPE 3 structures. These n cards must be arranged in the order $i = 1, 2, \ldots, n$.

4.4 Program XTALK2

XTALK2 considers the same structure types as XTALK. The only difference between the programs is that XTALK2 considers imperfect conductors. This
### TABLE 6

**Format of the Structure Characteristics Cards, Group I, for XTALK**

<table>
<thead>
<tr>
<th>Card Group #1 (total = 1):</th>
<th>card column</th>
<th>format</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) TYPE (1,2,3)</td>
<td>10</td>
<td>I</td>
</tr>
<tr>
<td>(b) LOAD STRUCTURE OPTION (11,12,21, or 22)</td>
<td>19 - 20</td>
<td>I</td>
</tr>
<tr>
<td>(c) n (number of wires)</td>
<td>29 - 30</td>
<td>I</td>
</tr>
<tr>
<td>(d) ( \varepsilon_r ) (relative dielectric constant of the surrounding medium)</td>
<td>36 - 45</td>
<td>E</td>
</tr>
<tr>
<td>(e) ( \mu_r ) (relative permeability of the surrounding medium)</td>
<td>51 - 60</td>
<td>E</td>
</tr>
<tr>
<td>(f) ( L ) (line length in meters)</td>
<td>66 - 75</td>
<td>E</td>
</tr>
</tbody>
</table>

**Card Group #2 (total = 1 if TYPE = 1 or 3)

<table>
<thead>
<tr>
<th>Total = 0 if TYPE = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) TYPE = 1: ( r_{w0} ) (radius of reference wire in mils)</td>
</tr>
<tr>
<td>(b) TYPE = 2: absent</td>
</tr>
<tr>
<td>(c) TYPE = 3: ( r_s ) (interior radius of shield in meters)</td>
</tr>
</tbody>
</table>

**Card Group #3 (total = n)**

| (a) \( r_{wi} \) (wire radius in mils) | 6 - 15 | E |
| (b) \( Z_i \) for TYPE 1 or 2 in meters | 21 - 30 | E |
| \( r_i \) for TYPE 3 in meters |
| (c) \( Y_i \) for TYPE 1 or 2 in meters | 36 - 45 | E |
| \( \Theta_i \) for TYPE 3 in degrees |

Note: Cards in Group #3 must be arranged in the order:

wire 1
wire 2

...wire n

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requires an additional set of cards in Group I which must follow those in Table 6. The format of these cards is shown in Table 7.

4.5 Program FLATPAK

FLATPAK considers (n+1) wire flatpak or ribbon cables as shown in Figure 4-4. The (n+1) wires are considered to be perfect conductors. In addition, the surrounding media are assumed to be lossless. The required cards in the Structure Characteristics card group, Group I, are shown in Table 8.

The first card is similar to the previous programs and communicates three items to the program. The first entry on the card is the number \( n \) which is the number of wires in the cable exclusive of the reference wire. The second entry on the card is the load structure option which is to be selected from the choices 11, 12, 21, or 22 as discussed in section 3.2. The third entry on this card is the total length of the cable in meters.

Card Group 2 concerns the entries in the per-unit-length capacitance matrix, \( C \), for the ribbon cable. Since \( C \) is symmetric, it is only necessary to input the entries on the main diagonal of \( C \) and the entries in the upper (or lower) triangle of \( C \). Computer program GETCAP [8] was designed to compute these items. GETCAP has the provision for providing a punched card output of the entries in \( C \) in the form required by FLATPAK.

A few comments are in order to assist users of GETCAP. The program is documented in Volume II of this series [8]. However, some confusion as to the wire numbering sequence in GETCAP and FLATPAK may arise. The wires in the cable are numbered from left to right with numbers from 1 to \( N=n+1 \) for use in the GETCAP program with the reference wire number chosen from this sequence. In the FLATPAK program, the wires are numbered from left to
### TABLE 7 (Cont.)

Format of the Structure Characteristics Cards, Group I, for XTALK 2

<table>
<thead>
<tr>
<th>Card Group #1</th>
<th>same as XTALK (TABLE 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Card Group #2</td>
<td>same as XTALK (TABLE 6)</td>
</tr>
<tr>
<td>Card Group #3</td>
<td>same as XTALK (TABLE 6)</td>
</tr>
</tbody>
</table>

#### Card Group #4 (total = 1)

<table>
<thead>
<tr>
<th>TYPE</th>
<th>Description</th>
<th>Card Column</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a) radius of strands in reference wire (mils)</td>
<td>6 - 15</td>
<td>E</td>
</tr>
<tr>
<td></td>
<td>(b) conductivity of strands (siemens/meter)</td>
<td>21 - 30</td>
<td>E</td>
</tr>
<tr>
<td></td>
<td>(c) number of strands in reference wire</td>
<td>39 - 40</td>
<td>I</td>
</tr>
<tr>
<td>2</td>
<td>(a) per-unit-length resistance of ground plane (ohms/meter)</td>
<td>6 - 15</td>
<td>E</td>
</tr>
<tr>
<td></td>
<td>(b) per-unit-length inductance of ground plane (henrys/meter)</td>
<td>21 - 30</td>
<td>E</td>
</tr>
<tr>
<td>3</td>
<td>(a) thickness of shield (meters)</td>
<td>6 - 15</td>
<td>E</td>
</tr>
<tr>
<td></td>
<td>(b) conductivity of shield (siemens/meter)</td>
<td>21 - 30</td>
<td>E</td>
</tr>
</tbody>
</table>

#### Card Group #5 (total = n)

<table>
<thead>
<tr>
<th>Description</th>
<th>Card Column</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) radius of wire strands (mils)</td>
<td>6 - 15</td>
<td>E</td>
</tr>
<tr>
<td>(b) conductivity of wire strands</td>
<td>21 - 30</td>
<td>E</td>
</tr>
<tr>
<td>(c) number of strands in wire</td>
<td>39 - 40</td>
<td>I</td>
</tr>
</tbody>
</table>

NOTE: Cards in Group #5 must be arranged for wires from 1 to n.
Figure 4-4. Wire numbering for ribbon (flatpack) cables.
TABLE 8
Format of the Structure Characteristics Cards, Group I, for FLATPAK

<table>
<thead>
<tr>
<th>Card Group #1 (total = 1)</th>
<th>Card Column</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Number of wires (exclusive of the reference wire) (n)</td>
<td>9 - 10</td>
<td>I</td>
</tr>
<tr>
<td>(b) LOAD STRUCTURE OPTION (11,12,21,\text{ or }22)</td>
<td>19 - 20</td>
<td>I</td>
</tr>
<tr>
<td>(c) Line length (\text{meters} ) (L)</td>
<td>21 - 30</td>
<td>E</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Card Group #2 (total = (n(n+1)/2))</th>
<th>Card Column</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) (i)</td>
<td>5 - 6</td>
<td>I</td>
</tr>
<tr>
<td>(b) (j)</td>
<td>10 - 11</td>
<td>I</td>
</tr>
<tr>
<td>(c) ([C]_{ij} \text{ (farads/meter)}) (Entries in (14\text{ - }26) (E) in the per-unit-length transmission line capacitance matrix with the wire dielectric insulations in place, computed with GETCAP)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Card Group #3 (total = (n(n+1)/2))</th>
<th>Card Column</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) (i)</td>
<td>5 - 6</td>
<td>I</td>
</tr>
<tr>
<td>(b) (j)</td>
<td>10 - 11</td>
<td>I</td>
</tr>
<tr>
<td>(c) ([C_0]_{ij} \text{ (farads/meter)}) (Entries in the per-unit-length transmission line capacitance matrix with the wire dielectric insulations removed, computed with GETCAP)</td>
<td>14 - 26</td>
<td>E</td>
</tr>
</tbody>
</table>
right with numbers from 1 to \( n \) with the reference wire numbered as the zero (0) wire as shown in Figure 4-4. Whether the cross section of the cable in Figure 4-4 is at \( x = 0 \) looking to the right (increasing \( x \)) or at \( x = L \) looking to the left (decreasing \( x \)) is irrelevant so long as the user is consistent in using the same cross section for wire numbering in GETCAP and in this program when assigning the load termination entries.

The third group of cards in Group I, Card Group #3, are the elements of the per-unit-length transmission line capacitance matrix computed with the dielectric insulations removed, \( C_0 \). GETCAP may be used to compute these items and provide punched card output for direct use as data input for FLATPAK.

4.6 Program FLATPAK 2

FLATPAK 2 considers \((n+1)\) wire ribbon cables as in FLATPAK. However, FLATPAK 2 considers the \((n+1)\) wires to be imperfect (lossy) conductors.

The format of the Structure Characteristic Cards, Group I, is shown in TABLE 9. Only one additional card over those required for FLATPAK is needed. Since all wires in the cable are assumed to be identical, this card describes the characteristics of these wires for use in computing the wire self impedances.

4.7 Examples of Program Usage

In this section, some typical examples will be shown to illustrate the use of the programs. Entries on the data cards as well as typical printouts of the results will be shown.

The terminal network structures for the examples are those comprising Examples 1, 2, 3, and 4 shown in Figure 2-4 and Figure 2-5. For Examples
### TABLE 9

Format of the Structure Characteristics Cards, Group I, for FLATPAK 2

| Card Group #1 | same as FLATPAK | Card Column | Format |
| Card Group #2 | same as FLATPAK | 6 - 15 | E |
| Card Group #3 | same as FLATPAK | 21 - 30 | E |
| Card Group #4 | (total = 1) | 39 - 40 | I |

#### Card Column 6 - 15 E

(a) radius of wire strands (mils)

(b) conductivity of wire strands

(c) number of strands in each wire

(d) (siemens/meter)
and 2, the entries in the Thevenin Equivalent characterization matrices are given in (2-32) and the entries in the Norton Equivalent characterization matrices are given in (2-35). For Examples 3 and 4, the entries in the Norton Equivalent characterization matrices are given in (2-37) and the entries in the Thevenin Equivalent characterization matrices are given in (2-38).

The terminal voltages for each wire (with respect to the reference conductor) at \(x=0\) and \(x=L\) are the entries in \(V(0)\) and \(V(L)\), respectively. The magnitudes and angles of the entries in \(V(0)\) (\(V(L)\)) are denoted by \(VOM\) and \(VOA\) (\(VLM\) and \(VLA\)), respectively, on the computer printouts. Two frequencies will be considered, 10 MHz and 100 MHz.

4.7.1 Examples of the XTALK Program

The transmission line structure chosen for all examples in this section is that of two wires with another wire as the reference conductor. The wire radii (mils) are 6.3 mils (thousands of a inch) for wires \#1 and \#2 with the reference wire of radius 6.3 mils. The three wires are in a linear array with \(Z_1 = 1.27\) mm, \(Y_1 = 0\), \(Z_2 = 2.54\) mm, \(Y_2 = 0\). The line length is 5 meters and the relative dielectric constant is chosen (for the purpose of illustration) to be 3.0 with a relative permeability of 1.0.

The data cards are shown in Figure 4-5 through 4-8 and the printouts are shown in Figure 4-9 through 4-12.

4.7.2 Examples of the XTALK 2 Program

The line considered for XTALK in 4.7.1 will be used here. Each wire will be stranded, copper (\(\sigma = 5.8 \times 10^7\) ) with 7 strands in each wire. The radius of each strand is 2.5 mils.
The data cards are shown in Figure 4-13 through 4-16 and the printouts are shown in Figure 4-17 through 4-20.

4.7.3 Examples of the FLATPAK Program

A three wire ribbon cable will be considered. The wire radii are .16002 mm, the insulation thicknesses are .3479 mm and the center-to-center separations of the wires are 1.27 mm. The insulations are polyvinyl chloride and a relative dielectric constant of 3.5 is assumed. The reference wire is the middle wire in the cable. The elements in the per-unit-length capacitance matrix (with and without the dielectric insulations) were computed with GETCAP [8].

The data cards are shown in Figure 4-21 through 4-24 and the printouts are shown in Figure 4-25 through 4-28.

4.7.4 Examples of the FLATPAK 2 Program

The three wire ribbon cable considered in the previous section with the FLATPAK program will be investigated. Each wire is stranded with 7 strands (copper) and each strand is of radius 2.5 mils.

The data cards are shown in Figure 4-29 through 4-32 and the printouts are shown in Figure 4-33 through 4-36.
<table>
<thead>
<tr>
<th>1.050</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.050</td>
<td>0.960 0.960 0.950 0.950 0.954 0.951 0.951 0.951 0.951 0.951 0.951</td>
</tr>
<tr>
<td>1.050</td>
<td>0.960 0.960 0.950 0.950 0.950 0.950 0.950 0.950 0.950 0.950 0.950</td>
</tr>
<tr>
<td>1.050</td>
<td>1.051 1.051 1.051 1.051 1.051 1.051 1.051 1.051 1.051 1.051</td>
</tr>
</tbody>
</table>

**Figure 4-5.** Input Cards, XTALK, Example 1.
Figure 4-6. Input Cards, XTALK, Example 2.
Figure 4-7. Input Cards, XTALK, Example 3.
Figure 4-8. Input Cards, ZTALK, Example 4.
**Figure 4-9. Output Listing, XTALK, Example 1.**
**Figure 4-10. Output Listing, XTALx, Example 2.**
| Figure 4-11. Output Listing, XTALE, Example 3. |
Figure 4-12. Output Listing, XTALK, Example 4.
Figure 4-13. Input Cards, XTALK2, Example 1.
Figure 4.14. Input Cards, XTALK2, Example 2.
Table 4-15. Input Cards, XTALK2, Example 3.
Figure 4-16. Input Cards, XTALK2, Example 4.
Figure 4-17. Output Listing, XTALK2, Example 1.
Figure 4-18. Output Listing, XTALK2, Example 2.
**Figure 4-19. Output Listing, XTALK2, Example 3.**

<table>
<thead>
<tr>
<th>Wire</th>
<th>Wire Number</th>
<th>Wire Radius (Nils)</th>
<th>Z Coordinate (Meters)</th>
<th>V Coordinate (Meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.00000</td>
<td>6.00000</td>
<td>6.00000</td>
<td>6.00000</td>
</tr>
</tbody>
</table>

**Table: Reference Wire for Line Voltages is a Wire with Radius 6.00000 Nils**

**Table: Per-Unit-Length Capacitance Matrix Inversion Error**

<table>
<thead>
<tr>
<th>Wire</th>
<th>Wire Number</th>
<th>Wire Strand Radius (Nils)</th>
<th>Conductivity (Siemens per Meter)</th>
<th>Number of Strands</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.00000</td>
<td>6.00000</td>
<td>6.00000</td>
<td>6.00000</td>
</tr>
</tbody>
</table>

**Table: Reference Wire is a Strand with Each Strand of Radius 6.00000 Nils**

**Table: Conductivity of Reference Wire Strands 6.00000 Siemens per Meter**

<table>
<thead>
<tr>
<th>Wire</th>
<th>Wire Number</th>
<th>Wire Strand Radius (Nils)</th>
<th>Conductivity (Siemens per Meter)</th>
<th>Number of Strands</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.00000</td>
<td>6.00000</td>
<td>6.00000</td>
<td>6.00000</td>
</tr>
</tbody>
</table>

**Figure 4-19. Output Listing, XTALK2, Example 3.**

-101-
Figure 4-20. Output Listing, XTALK2, Example 4.
Figure 4-22. Input Cards, FLATPAK, Example 2.
Figure 4-26. Input Cards, FLATPAK, Example 4.
Figure 4-25. Output Listing, FLATPAK, Example 1.
Figure 4-26. Output Listing, FLATPAK, Example 2.
Figure 4-27. Output Listing, FLATPAK, Example 3.
### Frequency Response

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Solution Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wire</th>
<th>Voltage</th>
<th>Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>2</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

### Load Structure Options

<table>
<thead>
<tr>
<th>Load</th>
<th>Real</th>
<th>Imag</th>
<th>Real</th>
<th>Imag</th>
<th>Real</th>
<th>Imag</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>2</td>
<td>1.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

---

Figure 4-28. Output Listing, FLATPAK, Example 4.
| Figure 4-29. Input Cards, FLATPAK2, Example 1. |
Figure 4-33. Input Cards, FLATPAK2, Example 2.
Figure 4-31. Input Cards, FLATPAK2, Example 3.
Figure 4-32. Input Cards, FLATPAK2, Example 4.
Figure 4-33. Output Listing, FLATPAK2, Example 1.
Figure 4-34. Output Listing, FLATPAK2, Example 2.
Figure 4-35. Output Listing, FLATPAK2, Example 3.
Figure 4-36. Output Listing, FLATPAK2, Example 4.
V. SUMMARY

Four digital computer programs, XTALK, XTALK 2, FLATPAK, FLATPAK 2, for determining the electromagnetic coupling within an (n+1) conductor, uniform transmission line are presented. Sinusoidal steady state behavior of the line as well as the TEM or "quasi-TEM" mode of propagation are assumed.

XTALK and XTALK 2 consider lines consisting of n wires (cylindrical conductors) and a reference conductor. The surrounding medium is homogeneous and lossless. XTALK assumes that all (n+1) conductors are perfect conductors whereas XTALK 2 considers the conductors to be lossy. There are three choices for the reference conductor: a wire, a ground plane, an overall cylindrical shield.

FLATPAK and FLATPAK 2 consider (n+1) wire ribbon (flatpack) cables in which all wires are identical and are coated with cylindrical, dielectric insulations of identical thicknesses. All wires lie in a horizontal plane and all adjacent wires are separated by identical distances. FLATPAK considers the wires to be perfect conductors and FLATPAK 2 considers the wires to be lossy. The dielectric insulations are considered to be lossless.

General termination networks are provided for at the ends of the line and the programs compute the voltages (with respect to the reference conductor) at the terminals of these termination networks for sinusoidal steady state excitation of the line.
REFERENCES


APPENDIX A

XTALK

Program Listing
C*********************************************C
C     PROGRAM XTALK
C     (FORTRAN IV, DOUBLE PRECISION)
C     WRITTEN BY
C        CLAYTON R. PAUL
C        DEPARTMENT OF ELECTRICAL ENGINEERING
C        UNIVERSITY OF KENTUCKY
C        LEXINGTON, KENTUCKY 40506
C
C     A DIGITAL COMPUTER PROGRAM TO COMPUTE THE TERMINAL VOLTAGES
C     (WITH RESPECT TO THE REFERENCE CONDUCTOR) AT THE ENDS OF A
C     MULTICONDUCTOR TRANSMISSION LINE FOR THE TEN MODE OF
C     PROPAGATION.
C
C     THE DISTRIBUTED PARAMETER, MULTICONDUCTOR TRANSMISSION LINE
C     EQUATIONS ARE SOLVED FOR STEADY STATE, SINUSOIDAL EXCITATION
C     OF THE LINE.
C
C     THE LINE CONSISTS OF N WIRES (CYLINDRICAL CONDUCTORS) AND A
C     REFERENCE CONDUCTOR. THE REFERENCE CONDUCTOR MAY BE A WIRE
C     (TYPE=1), AN INFINITE GROUND PLANE (TYPE=2), OR AN OVERALL
C     CYLINDRICAL SHIELD (TYPE=3)
C
C     THE N WIRES ARE ASSUMED TO BE PARALLEL TO EACH OTHER AND THE
C     REFERENCE CONDUCTOR.
C
C     THE N WIRES AND THE REFERENCE CONDUCTOR ARE ASSUMED TO BE
C     PERFECT CONDUCTORS.
C
C     THE LINE IS IMMERSED IN A LINEAR, ISOTROPIC, AND HOMOGENEOUS
C     MEDIUM WITH A RELATIVE PERMITTIVITY OF epsilon_R AND A RELATIVE
C     DIELECTRIC CONSTANT OF epsilon. THE MEDIUM IS ASSUMED TO BE LOSSLESS.
C
C     LOAD STRUCTURE OPTION DEFINITIONS:
C     OPTION=11, THEVENIN EQUIVALENT LOAD STRUCTURES WITH DIAGONAL
C     IMPEDANCE MATRICES
C     OPTION=12, THEVENIN EQUIVALENT LOAD STRUCTURES WITH FULL
C     IMPEDANCE MATRICES
C     OPTION=21, NORTON EQUIVALENT LOAD STRUCTURES WITH DIAGONAL
C     ADMITTANCE MATRICES
C     OPTION=22, NORTON EQUIVALENT LOAD STRUCTURES WITH FULL
C     ADMITTANCE MATRICES
C
C     SUBROUTINES USED: LECT1C
C
C*********************************************C
C
C     ALL VECTORS AND MATRICES IN THE FOLLOWING DIMENSION STATEMENTS
C     SHOULD BE OF SIZE N WHERE N IS THE NUMBER OF WIRES (EXCLUSIVE OF
C     THE REFERENCE CONDUCTOR), L.E., IO(N), IL(N), TO(N,N), YL(N,N), B(N),
C     A(N,N), WA(N,N), N1(N,N), N2(N,N), V1(N), V2(N)
C
C     IMPLICIT REAL*8 (A-H,O-Z)
C     INTEGER TYPEOPTION
C     REAL*8 L,MU02PI,MU0R !YALK056
C     COMPLEX*16 XI,I0(2),TO(2),YL(2,2),A(2,2),B(2),
C     WA(2),N1(2,2),N2(2,2),V1(2),V2(2),SUM,SM1,SM2,YL,ZEROC,
C     2C,A1,A2,ONEC
C     DATA PI/3.141592653D0/,Y/2.997925D0/
C     DATA CHTM/2.5D-5/,MU02PI/2.D-7/,PS/5.D0/,ZER0/0.D0/,ONE/1.D0/
C
-124-
**FREQUENCY INDEPENDENT CALCULATIONS***

**READ AND PRINT INPUT DATA**

```pascal
READ(5,1) TYPE, OPTION, N, ER, HUB, L
1 FORMAT(9X,I2,2(8X,I2),3(5X,E10.3))
   IF(TYPE.GE.1.AND.TYPE.LE.3) GO TO 3
   WRITE(6,2)
2 FORMAT('STRUCTURE TYPE ERROR$/// TYPE MUST EQUAL 1,2,OR 3///')
   GO TO 82
3 IF(OPTION.EQ.11.OR.OPTION.EQ.12) GO TO 5
   IF(OPTION.EQ.21.OR.OPTION.EQ.22) GO TO 5
   WRITE(6,4)
4 FORMAT('LOAD STRUCTURE OPTION ERROR$/// OPTION MUST EQUAL 11,12,2///')
   GO TO 82
5 WRITE(6,6) N, TYPE, OPTION, L, ER, HUB
6 FORMAT(14X,I2,2 PARALLEL WIRES$///
   24X, 'TYPE OF STRUCTURE$///
   34X, 'LOAD STRUCTURE OPTION$///
   43X, 'LINE LENGTH$///
   53X, 'Dielectric CONSTANT OF THE MEDIUM$///
   63X, 'Relative PERMEABILITY OF THE MEDIUM$///
   GO TO (7,15,11), TYPE
7 READ(5,8) NW0
8 FORMAT(5X,E10.3)
9 WRITE(6,9) NW0
10 FORMAT('REFERENCE CONDUCTOR FOR LINE VOLTAGES IS A WIRE WITH RADIUS NW0$////
   NW0=RW0*CMTH
   WRITE(6,10)
11 READ(5,12) RW0
12 FORMAT(5X,E10.3)
13 WRITE(6,13) RW0
14 FORMAT('REFERENCE CONDUCTOR FOR LINE VOLTAGES IS A CYLINDRICAL SHEILD WITH INTERIOR RADIUS= RW0$////
   RW0=RW0*RS
   WRITE(6,14)
15 FORMAT('REFERENCE CONDUCTOR FOR LINE VOLTAGES IS A CYLINDRICAL SHEILD WITH INTERIOR RADIUS= RW0$////
   RW0=RW0*RS
   WRITE(6,14)
16 FORMAT('REFERENCE CONDUCTOR FOR LINE VOLTAGES IS A CYLINDRICAL SHEILD WITH INTERIOR RADIUS= RW0$////
   RW0=RW0*RS
   WRITE(6,14)
17 FORMAT('REFERENCE CONDUCTOR FOR LINE VOLTAGES IS A CYLINDRICAL SHEILD WITH INTERIOR RADIUS= RW0$////
   RW0=RW0*RS
   WRITE(6,14)
C C READ AND PRINT LINE DIMENSIONS AND COMPUTE THE CHARACTERISTIC IMPEDANCE MATRIX, ZC (STORE ZC IN ARRAY H1)
C
```

---

**END OF PROGRAM**
18 C=HGC2*Y*ONRC*V*DSQRT(MUN/EK)
DO 24 I=1, N
READ(5, 19) W*, Z, Y
19 FORMAT (3 (5X, 1E13.3))
WRITE(6, 20) I, W*, Z, Y
20 FORMAT (2X, 12, 13X, 1PE10.3, 27X, 1PE10.3, 35X, 1PE10.3/
V1(I)=ONRC*1
V2(I)=ONRC*Y
R*=W*CMHT
GO TO (21, 22, 23), TYPE
21 D12=Z*I+Y*I
M(I,J)=C*DLOG(DI2/(IN*EN0))
GO TO 24
22 M(I,J)=C*DLOG((RS2-Z*Z)/(RS*RW))
GO TO 24
23 CONTINUE
IF(N=EQ. 1) GO TO 29
K1=N-1
DO 28 I=1, K1
K2=I+1
DO 28 J=K2, N
ZJ=DREAL(V1(J))
ZI=DREAL(V1(J))
YZ=CY=DREAL(V2(J))
GO TO (25, 26, 27), TYPE
25 D12=Z*I+Y*I
D22=Z*I+Y*I
ZJ=DREAL(V1(J))
YZ=CY=DREAL(V2(J))
GO TO 28
26 ZD=Z*I+Y*I
YD=Y*I+Y*I
D12=Z*D*Y*T+YD
M(I,J)=P5*C*DLOG(DI2/DJ2/(
M(I, J)=M(I, J)
GO TO 28
27 THETA=(YI-J)*PI/80
M2=Z*I+J
M2=Z*I+J
M1(I,J)=P5*C*DLOG((BJ2/RS2)* (RI2*BJ2+RS2*THO*XI*J)*RS2*DJ2)
M1(I,J)=M(I, J)
GO TO 28
28 CONTINUE
C COMPUTE THE INVERSE OF THE CHARACTERISTIC IMPEDANCE MATRIX, ZCINT
C (STORE ZCINT IN ARRAY M2)
C
29 DO 31 L=1, N
DO 30 J=1, N
A(I,J)=M(I,J)
30 M2(I,J)=7*PMOC
M2(I,J)=M2(I,J)
CALL LQTIC(A, N, H*, M2, H*, O*, WA, KEK)
KEK=KEK-128
C READ AND PRINT ENTRIES IN LOAD ADMITTANCE(IMPEDANCE) MATRICES
C
-126-
C AND SHORT CIRCUIT CURRENT SOURCE (OPEN CIRCUIT VOLTAGE SOURCE) XALK188
C VECTORS (STORE ADMITTANCE (IMPEDANCE) MATRICES AT X=0 IN ARRAY Y0 XALK175
C AND THOSE AT X=L IN ARRAY YL. STORE SHORT CIRCUIT CURRENT SOURCE XALK186
C (OPEN CIRCUIT VOLTAGE SOURCE) VECTORS AT X=0 IN ARRAY X0 AND XALK187
C THOSE AT X=L IN ARRAY XL. XALK188
C
C IF (OPTION.EQ.11 .OR. OPTION.EQ.12) GO TO 34
WRITE(6,32)
32 FORMAT(/,1X,'ADMITTANCE AT I=0',1X,'CURRENT SOURCE AT X=0',1X,'CURRENT SOURCE AT X=L'/) XALK192
WRITE(6,33)
33 FORMAT(21X,'(SIEMENS) ',23X,'(AMPS) ',22X,'(SIEMENS) ',23X,'(AMPS)') XALK195
GO TO 37
34 WRITE(6,35)
35 FORMAT(/,1X,'IMPEDANCE AT I=0',11X,'VOLTAGE SOURCE AT X=0',11X,'VOLTAGE SOURCE AT X=L'/) XALK198
WRITE(6,36)
36 FORMAT(21X, '(OHNS) ',23X, '(VOLTS) ',24X, '(OHNS) ',23X, '(VOLTS)') XALK200
37 WRITE(6,38)
38 FORMAT(1X,' ENTRY',10X,'REAL',11X,'IMAG',11X,'REAL',11X,'IMAG') XALK203
1 REAL',11X,'IMAG',11X,'REAL',11X,'IMAG')
DO 41 I=1,N
READ(5,39) Y0(I),YL0(I),YL(I)
40 FORMAT(8(E10.3))
Y0(I,1)=Y0(I)+Y0(I,1)*Y0(I,1)*Y0(I)
Y0(I,1)=Y0(I,1)*YL(I,1)+YL(I,1)
WHITE(6,40) L1(1,1),L0(1,1),YL(I,1),YL(I,1)
WRITE(6,41)
41 FORMAT(2X,'(OHNS) ',2X,'(VOLTS) ',2X,'(OHNS) ',2X,'(VOLTS)') XALK205
42 FORMAT(2X,'ENTRY',10X,'REAL',11X,'IMAG',11X,'REAL',11X,'IMAG') XALK208
1 REAL',11X,'IMAG',11X,'REAL',11X,'IMAG')
DO 47 J=1,N
READ(5,42) Y0(J),Y0(J,1),YL(J),YL(J)
43 FORMAT(2X,'(OHNS) ',2X,'(VOLTS) ',2X,'(OHNS) ',2X,'(VOLTS)') XALK211
44 CONTINUE
C IF THEVENIN EQUIVALENT IS SPECIFIED, SWAP ENTRIES IN A1 AND A2.
C A1 WILL CONTAIN ZCINV AND A2 WILL CONTAIN ZC.
C
C 45 IF (OPTION.EQ.21 .OR. OPTION.EQ.22) GO TO 48
DO 47 I=1,N
DO 46 J=1,N
A1=R1(I,J)
A2=R2(I,J)
46 R2(J,I)=A1
47 A1=I*I
C COMPUTE THE MATRIX ZC+ZL*SCINV*Z0 FOR THE THEVENIN EQUIVALENT
C OR ZCINV+YL+ZC*YO FOR THE NORTON EQUIVALENT. STORE IN ARRAY A2.
C COMPUTE THE MATRIX ZCINV*ZO FOR THE THEVENIN EQUIVALENT OR
C
-127-
C ZC*YO FOR THE NORTON EQUIVALENT. STORE IN ARRAY H1.
C COMPUTE THE VECTOR ZL*ZC*V0 FOR THE THEVENIN EQUIVALENT OR
C T(L)*ZC*IO FOR THE NORTON EQUIVALENT. STORE IN ARRAY V2.
C COMPUTE THE VECTOR ZCINV*V0 FOR THE THEVENIN EQUIVALENT OR
C ZC*IO FOR THE NORTON EQUIVALENT. STORE IN ARRAY V1.

48 IF (OPTION.EQ.12.OR.OPTION.EQ.22) GO TO 54
   DO 50 I=1,N
   SUMO=ZERO
   DO 49 J=1,N
      A(I,J)=H1(I,J)*T0(J,J)
   49 SUM0=SUM0+A(I,J)*IO(J)
   50 V1(I)=SUM0
      DO 52 I=1,N
      DO 51 J=1,N
         V2(I)=T(L,I)*V1(I)
      51 DO 53 K=1,N
      52 V2(I)=V2(I)+V1(K)
      53 H1(I,J)=A(I,J)
      DO 54 I=1,N
      SUMO=ZERO
      DO 53 J=1,N
         SUM1=SUM1+H1(I,J)*IO(K,J)
      54 SUM0=SUMO+H1(I,J)*IO(J)
      A(I,J)=SUM1
      DO 55 I=1,N
      DO 54 J=1,N
         SUML=SUML+T(L,K)*A(K,J)
      55 SUM0=SUM0+T(L,K)*IO(J)
      56 A(I,J)=SUML
      57 V1(I)=SUM0
      DO 59 I=1,N
      DO 58 J=1,N
         SUM0=SUM0+T(L,I)*V1(J)
      59 SUM0=SUM0+T(L,I)*V1(J)
      60 V2(I)=SUM0
      DO 61 I=1,N
      DO 60 J=1,N
         H1(I,J)=A(I,J)
      61 DO 62 I=1,N
      62 IF (KER.EQ.0) GO TO 61
      BL=BL*F
      DS=DSGN(BTAL)
      DC=DCOS(BTAL)
      WRITE(6,63) KER
   63 FORMAT(1/,'CHARACTERISTI: IMPEDANCE MATRIX INVERSION ERROR='12
      1//)
   C******FREQUENCY DEPENDENT CALCULATIONS***********************
   C
   64 CONTINUE
   READ(5,65,END=82) F
   65 FORMAT(E10.3)
      BTAL=BL*F
      DS=DGSN(BTAL)
      DC=DCOS(BTAL)
      C COMPUTE THE TERMINAL VOLTAGES
      C FORM THE EQUATIONS
   C
   -128-
IF (OPTION.EQ.12.OR.OPTION.EQ.22) GO TO 68
DO 67 I=1,N
DO 66 J=1,N
66 A(I,J)=XJ*DS*M2(I,J)
        A(I,J)=DOM=(TO(I,I)+YL(I,I))*A(I,I)
67 B(I)=DC*IO(I)+XJ*DS*V2(I)+IL(I)
GO TO 71
68 DO 70 I=1,N
DO 69 J=1,N
69 A(I,J)=XJ*DS*M2(I,J)+DC*(TO(I,J)+YL(I,J))
70 B(I)=DC*IO(I)+XJ*DS*V2(I)+IL(I)

C SOLVE THE EQUATIONS
C
71 CALL LEQTIC(A,N,N,B,1,N,OA,IER)
IER=IER-128
IF (IER.NE.1) IER=0
WRITE(5,72) IER
72 FORMAT(1H1,' FREQUENCY(HERTZ)= ',1PE11.4,10X,'SOLUTION ERROR= ',
12X,12//)
WRITE(6,73)
73 FORMAT(16X,'VOLT.'8X,'VOL.'VOLS) ',3X,'VOL(DEGREES) ',6X,
1'VLA(VOLTS) ',3X,'VLA(DEGREES) '12//)
C
C COMPUTE AND PRINT THE TERMINAL VOLTAGES
C
DO 75 I=1,N
SUMO=ZEROCC
DO 74 J=1,N
74 SUMO=SUMO+M1(I,J)*B(J)
75 WA(I)=XJ*DS*(SUMO-V1(I))+DC*B(I)
DO 81 I=1,N
IF (OPTION.EQ.11.OR.OPTION.EQ.12) GO TO 76
V0=B(I)
VL=WA(I)
GO TO 79
76 IF (OPTION.EQ.12) GO TO 77
V0=IO(I)-TO(I,I)*B(I)
VL=IL(I)+YL(I,I)*WA(I)
GO TO 79
77 SUMO=ZEROCC
DO 78 J=1,N
SUMO=SUMO+TO(I,J)*B(J)
78 SUMO=SUMO+YL(I,I)*WA(I)
V0=IO(I)-SUMO
VL=IL(I)+SUML
V0M=CDABS(V0)
VLH=CDABS(VL)
V0M=DRSAL(V0)
VOI=IMAG(V0)
VLH=DRSAL(VL)
V0M=DSAL(V0)
V0M=CDABS(V0)
IF (V0M.EQ.ZERO.AND.V0M.EQ.ZERO) V0M=ONE
IF (VLH.EQ.ZERO.AND.VLH.EQ.ZERO) VLH=ONE
VOA=DATAN2(V0M,V0M)*ONE/PI
VLB=DATAN2(VLH,VLH)*ONE/PI
WRITE(6,80) I,V0M,VOA,VLH
80 FORMAT(16X,'I,Y0M,V0A,VLH ')
CONTINUE
GO TO 64
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061-062 change all D's to E's

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APPENDIX B

XTALK2

Program Listing
C** PROGRAM ITALK2 
C (FORTRAN IV, DOUBLE PRECISION) 
C WRITTEN BY 
CLAYTON R. PAUL 
DEPARTMENT OF ELECTRICAL ENGINEERING 
UNIVERSITY OF KENTUCKY 
LEXINGTON, KENTUCKY 40506 
C A DIGITAL COMPUTER PROGRAM TO COMPUTE THE TERMINAL VOLTAGES 
(WITH RESPECT TO THE REFERENCE CONDUCTOR) AT THE ENDS OF A 
MULTICONDUCTOR TRANSMISSION LINE FOR THE TEN MODE OF 
PROPAGATION. 
C THE DISTRIBUTED PARAMETER, MULTICONDUCTOR TRANSMISSION LINE 
EQUATIONS ARE SOLVED FOR STEADY STATE, SINUSOIDAL EXCITATION 
OF THE LINE. 
C THE LINE CONSISTS OF N Wires (CYLINDRICAL CONDUCTORS) AND A 
REFERENCE CONDUCTOR. THE REFERENCE CONDUCTOR MAY BE A WIRE 
(TYPE=1), AN INFINITE GROUND PLANE (TYPE=2), OR AN OVERALL 
CYLINDRICAL SHIELD (TYPE=3). 
C THE N Wires ARE ASSUMED TO BE PARALLEL TO EACH OTHER AND THE 
REFERENCE CONDUCTOR. 
C THE N Wires AND THE REFERENCE CONDUCTOR ARE CONSIDERED TO BE 
IMPERFECT conductors. THE SELF IMPEDANCES OF EACH Wire AND THE 
REFERENCE CONDUCTOR INCLUDE SKIN EFFECT. 
C THE LINE IS IMMERSION IN A LINEAR, ISOTROPIC, AND HOMOGENOUS 
MEDIUM WITH A RELATIVE PERMEABILITY OF MU AND A RELATIVE 
DIELECTRIC CONSTANT OF ER. THE MEDIUM IS ASSUMED TO BE LOSSLESS. 
C LOAD STRUCTURE OPTION DEFINITIONS: 
OPTION=11, THEVENIN EQUIVALENT LOAD STRUCTURES WITH DIAGONAL 
IMPEDANCE MATRICES 
OPTION=12, THEVENIN EQUIVALENT LOAD STRUCTURES WITH FULL 
IMPEDANCE MATRICES 
OPTION=21, NORTON EQUIVALENT LOAD STRUCTURES WITH DIAGONAL 
ADMITTANCE MATRICES 
OPTION=22, NORTON EQUIVALENT LOAD STRUCTURES WITH FULL 
ADMITTANCE MATRICES: 
C SUBROUTINES USED: LEQ, TCREIGCC 
C******************************************************************************
C******************************************************************************
C ALL VECTORS AND MATRICES IN THE FOLLOWING DIMENSION STATEMENTS 
SHOULD BE OF SIZE N WHERE N IS THE NUMBER OF Wires (EXCLUSIVE OF 
THE REFERENCE CONDUCTOR), N, NS(N), C(N,N), Z(N), Y(N), CI(N,N), 
IO(N), IL(N), IO(N,N), YL(N,N), B(N), A(N,N), P(N,N), EM(N), 
M1(N,N), M2(N,N), V1(N), V2(N), T(N,N), TX(N,N), G(N), V3(N), WA(N) 
THE VECTOR WK MUST BE OF LENGTH 2M(N+1) 
C IMPLICIT REAL*8 (A-H,O-Z) INTEGER TYPE,OPTION,NS( 2) 
REAL*8 L,LDC,LDG,C( 2, 2),Z( 2), Y( 2), CI( 2, 2), V3( 2), WK( 12), 
1MUO*PI, MUO*PI, CS*PI, CS*PI 
COMPLEX*16 XJ, SJ0,SUNL, S0, SL, V0, VL, Z0, EPP, EMM, GAM, JCRXGA
C******************************************************************************
C******************************************************************************

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C******FREQUENCY INDEPENDENT CALCULATIONS************************************

C READ AND PRINT INPUT DATA

READ(5,1) TYPE,OPTION,N,ERR,MUR,L
1 FORMAT(9X,I1,2(8X,I2),3(5X,E10.3))
IF(TYPE.GE.1.AND.TYPE.LE.3) GO TO 3
WRITE(6,2)
2 FORMAT(* STRUCTURE TYPE ERROR'///* TYPE MUST EQUAL 1,2 OR 3///)
GO TO 121
3 IF(OPTION.EQ.11.OR.OPTION.EQ.12) GO TO 5
IF(OPTION.EQ.21.OR.OPTION.EQ.22) GO TO 5
WRITE(6,4)
4 FORMAT(* LOAD STRUCTURE OPTION ERROR'///* OPTION MUST EQUAL 11,12,22///)
11,OR 22///
GO TO 121
5 WRITE(6,6) N,TYPE,OPTION,ERR,MUR
6 FORMAT(1H1,50X,*ITALK2///)
145X,I2,* PARALLEL WIRES///
243X,* TYPE OF STRUCTURE= 'I1///
341X,* LOAD STRUCTURE OPTION= '12///
439X,* LINE LENGTH= ',1PE13.6,' METERS///
532X,* DIELECTRIC CONSTANT OF THE MEDIUM= ',1PE10.3///
631X,* RELATIVE PERMEABILITY OF THE MEDIUM= ',1PE10.3///
GO TO (7,15,11),TYPE
7 READ(5,8) NW0
8 FORMAT(5X,E10.3)
WRITE(6,9) NW0
9 FORMAT(* REFERENCE CONDUCTOR FOR LINE VOLTAGES IS A LINE WITH RADIUS///)
WRITE(6,10)
10 FORMAT(* WIRE NUMBER',4X,'WIRE RADIUS (MILS) ',18X,
'X COORDINATE (METERS) ',24X,'Y COORDINATE (METERS) ',/)
GO TO 18
11 READ(5,12) RS
12 FORMAT(5X,E10.3)
WRITE(6,13) RS
13 FORMAT(* REFERENCE CONDUCTOR FOR LINE VOLTAGES IS A CYLINDRICAL OVAL///)
WRITE(6,14)
14 FORMAT(* WIRE NUMBER',2X,'WIRE RADIUS (MILS) ',2X,'SEPARATION BETWEEN///
CENTER AND CENTER OF SHIELD (METERS) ',6X,'ANGULAR COORDINATE (DEG///
2GREES) ///)
GO TO 18
15 WRITE(6,16)
16 FORMAT(* REFERENCE CONDUCTOR FOR LINE VOLTAGES IS AN INFINITE GR///
1ND FLARE'///)
WHITE(6,17)

17 FORMAT(10x,'WIRE NUMBER',4x,'WIRE RADIUS (MELS)',1x,1x,'HORIZONTAL COORDINATE (METERS)',16x,'WIRE HEIGHT (METERS')//)

READ AND PRINT LINE DIMENSIONS AND COMPUTE THE INVERSE OF THE PER-UNIT-LENGTH CAPACITANCE MATRIX,CINV

(STORE CINV IN ARRAY CI)

18 D=H*0.2*PI*VY/ZR
DO 24 I=1,N
READ(5,19) RH,Z(I),Y(I)
19 FORMAT(3(5x,E10.3'))
WHITE(6,20).1,RH,Z(I),Y(I)
20 FORMAT(2X,I2,13X,1PE10.3,27X,1PE10.3,35X,1PE10.3/) 
RR=RH*CHTR
GO TO (21,22,23),TYPE
21 DIZ2=Z(I)*Z(J)*Y(I)*Y(J)
CI(I,J)=D*LOG(DIZ2/(RH*RH))
GO TO 28
22 CI(I,J)=D*LOG(RXO*RYO(I)/RH) 
GO TO 28
23 CI(I,J)=D*LOG((RS2-Z(I)*Z(J))/(RS2*RS2)) 
CONTINUE
IF(N.EQ.1) GO TO 29
K1=1
DO 28 I=1,K1
K2=K1+1
DO 28 J=K2,N
GO TO (25,26,27),TYPE
25 DIZ2=Z(I)*Z(J)*Y(I)*Y(J)
DIZ2=C(Z(I)-Z(J))
YD=Y(I)-Y(J)
DO 30 J=1,N
CI(I,J)=PI*DIZ2/(RH*RH*DIZ2)
GO TO 28
26 ED=X(I)-Z(J)
YD=Y(I)-Y(J)
DIZ2=ED*YD*YD
CI(I,J)=D*DLOG(C/J2/(RH*RH*DI))
CONTINUE
IF(N.EQ.1) GO TO 29
K1=1
DO 28 I=1,K1
K2=K1+1
DO 28 J=K2,N
GO TO (25,26,27),TYPE
27 THETA=(Y(I)-Y(J))*PI/GHESO
RJ2=Z(I)*Z(J)
RJ22=C(Z(I)*Z(J))
CI(I,J)=PI*DLOG((RJ2/R2)*RI2*RJ22*RS2*TWO*Z(I)*Z(J)*RS2* 
1DCOS(THETA))/RI2*RJ22+RJ2-2*Z(I)*Z(J)*RJ2*DCOS(THETA))
CONTINUE
COMPUTE THE PER-UNIT-LENGTH CAPACITANCE MATRIX,C

(STORE C IN ARRAY C)

29 DO 31 I=1,N
DO 30 J=1,N
CI(I,J)=1
P(I,J)=ZERO
30 P(I,J)=ZERO
31 P(I,J)=ONE
CALL LEQ71C(A,NN,P,1,O,VA,KE,CI)
KE=KE+128

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IF (KER. NE. 1) KER=0
WRITE (6, 32) KER
32 FORMAT (1X, 'PER-UNIT-LENGTH CAPACITANCE MATRIX INVERSION ERROR=', JX, 1X, 'NORM=', JX)
1I2/
DO 33 I=1, M
DO 33 J=1, M
33 C(I,J)=REAL (PI1,J)

READ AND PRINT CHARACTERISTICS OF THE WIRES AND THE REFERENCE CONDUCTOR TO BE USED IN THE SELF IMPEDANCE CALCULATIONS

GO TO (34, 40, 37), TYPE
34 READ (5, 35), NW50, SIGO, NS
35 FORMAT (2(5X, E10.3), 8X, 12)
WRITE (6, 36) NW50, SIGO, NS
36 FORMAT (1X, 'REFERENCE WIRE IS STRANDED WITH EACH STRAND OF RADIUS=', JX, ' MILES', ' CONDUCTIVITY OF REFERENCE WIRE STRANDS=', JX, ' SIEMENS PER METER', ' NUMBER OF STRANDS=', ' IZ', '///)
NWSO=NW50*CHM
GO TO 43
37 READ (5, 38), TB, SIGO
38 FORMAT (2(5X, E10.3))
WRITE (6, 39) TB, SIGO
39 FORMAT (1X, 'SHIELD THICKNESS=', JX, ' MILES', ' SHIELD CONDUCTIVITY=', JX, ' SIEMENS PER METER', '///)
ACTIVITY=', JX, ' MILES', '///)
GO TO 43
40 READ (5, 41), RGP, LGP
41 FORMAT (2(5X, E10.3))
WRITE (6, 42) RGP, LGP
42 FORMAT (1X, 'GROUND PLANE RESISTANCE=', JX, ' MILES', ' OHMS PER METER', '///)
1/// 'GROUND PLANE INDUCTANCE=', JX, ' MILES', ' HENrys PER METER', '///)
43 WRITE (6, 44)
44 FORMAT (1X, 'WIRE NUMBER=', 4X, 'WIRE STRAND RADIUS (MILES)', ' MILES', ' CONDUCTIVITY (SIEMENS PER METER)', ' MILES', ' NUMBER OF STRANDS', '///)
DO 47 Z=1, M
READ (5, 45) Z(I), Y(I), NS(I)
45 FORMAT (2(5X, E10.3), 8X, 12)
WRITE (6, 46) Z(I), Y(I), NS(I)
46 FORMAT (1X, 'I', 12, 16X, 'IPE10.3', '35X', 'IPE10.3', '32X', 'I2', '/)
47 Z(I)=Z(I)*CHM

READ AND PRINT ENTRIES IN LOAD ADMITTANCE (IMPEDANCE) MATRICES AND SHORT CIRCUIT CURRENT SOURCE (OPEN CIRCUIT VOLTAGE SOURCE) VECTORS.

(STORE ADMITTANCE (IMPEDANCE) MATRICES AT X=0 IN ARRAY Y0 AND THOSE AT X=L IN ARRAY YL STORE SHORT CIRCUIT CURRENT SOURCE (OPEN CIRCUIT VOLTAGE SOURCE) VECTORS AT X=0 IN ARRAY Y0 AND THOSE AT X=L IN ARRAY YL)

IF (OPTION.EQ.11 OR OPTION.EQ.12) GO TO 50
WRITE (6, 48)
48 FORMAT (1X, 'IPE10.3', '10X', 'CURRENT SOURCE AT X=0', ' IPE10.3', '10X', 'CURRENT SOURCE AT X=L', '///)
WRITE (6, 49)
49 FORMAT (2I18, 'IPE10.3', '10X', 'CURRENT SOURCE AT X=0', ' IPE10.3', '10X', 'CURRENT SOURCE AT X=L', '///)
WRITE (6, 50)
50 WRITE (6, 51)
51 FORMAT (1X, 'IPE10.3', '10X', 'VOLTAGE SOURCE AT X=0', ' IPE10.3', '10X', 'VOLTAGE SOURCE AT X=L', '///)
WRITE (6, 52)
52 FORMAT (2I18, 'IPE10.3', '10X', 'VOLTAGE SOURCE AT X=0', ' IPE10.3', '10X', 'VOLTAGE SOURCE AT X=L', '///)

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53 WRITE(6,54)
DO 57 I = 1, N
READ(5,55) Y0,TO,I0(I),YL,I0(I)
55 FORMAT (8((E10.3)) )
Y0(I) = Y0 + K0*TO I = 1, N
YL(I) = YL + K1*I0*I = 1, N
WRITE(6,56) I, I0(I), I0(I), YL(I), YL(I)
56 FORMAT (1X, I2, 2X, I2, 2X, (5X, E10.3) )
57 CONTINUE

IF (OPTION.EQ.1 I OR OPTION.EQ.2) GO TO 61
IF (OPTION.EQ.1 I OR OPTION.EQ.2) GO TO 61
DO 60 I = 1, N
K2 = I + 1
DO 58 J = K2, N
READ(5,58) Y0J, TOJ, YLJ, YLJ
58 FORMAT (2((E10.3), 20X, 2(E10.3))
Y0(I,J) = Y0 + K3*TO I = 1, N
YL(I,J) = YL + K1*I0 I = 1, N
WRITE(6,59) I, J, TO(I,J), YL(I,J)
59 FORMAT (1X, I2, 2X, I2, 2X, (5X, E10.3), 30X, 2(E10.3) )
60 CONTINUE

C COMPITE THE MATRICES C*Z0, C*ZL, C*Y0, C*YL FOR THE THEVENIN
C EQUIVALENT OR TO CINV, YL*CINV, TO, YL FOR THE NORTON EQUIVALENT AND
C STORE IN ARRAYS M1, M2, V1, V2, RESPECTIVELY.

61 IF (OPTION.EQ.11) GO TO 62
IF (OPTION.EQ.12) GO TO 65
IF (OPTION.EQ.21) GO TO 69
IF (OPTION.EQ.22) GO TO 72
62 DO 64 I = 1, N
S0 = ZERO C
SL = ZERO C
DO 63 J = 1, N
N1(I,J) = C(I,J) * Y0(I,J)
N2(I,J) = C(I,J) * YL(I,J)
S0 = S0 + C(I,J) * TO(I,J)
63 SL = SL + C(I,J) * YL(I,J)
V1(I) = S0
64 V2(I) = SL
GO TO 76
65 DO 68 I = 1, N
S0 = ZERO C
SL = ZERO C
DO 67 J = 1, N
N1(I,J) = C(I,J) * Y0(I,J)
N2(I,J) = C(I,J) * YL(I,J)
S0 = S0 + C(I,J) * TO(I,J)
SL = SL + C(I,J) * YL(I,J)
67 N2(I,J) = SUNL
V1(I) = S0
68 V2(I) = SL
GO TO 76

-138-
69 DO 71 I=1,N
   DO 70 J=1,N
   H1(I,J)=Y0(I,I)*CI(I,J)
70 H2(I,J)=YL(I,I)*CI(I,J)
   V1(I)=10(I)
71 V2(I)=YL(I)
   GO TO 76
72 DO 75 I=1,N
   DO 74 J=1,N
   SUM=ZEBOC
   SUM=SUM+TO(I,K)*CI(K,J)
73 SUM=SUM*YL(I,K)*CI(K,J)
   W1(I,J)=SUM
74 H2(I,J)=SUM
   V1(I)=70(I)
75 V2(I)=10(I)
76 CONTINUE

C
   S: THE ENTRIES IN EACH ROW OF C AND STORE IN ARRAY V3

C
   DO 78 I=1,N
   S=ZERO
   DO 77 J=1,N
   S=S+C(I,J)
77 S=S+C(I,J)
78 V3(I)=S

C
   **************************************************************
   FREQUENCY DEPENDENT CALCULATIONS
   **************************************************************

C
   DO 81 CONTINUE

C
   READ(5,80,BND=121) P
80 FORMAT(10.3)
   OMEGA=2*C*PI/P
   JOMEGA=X*OMEGA

C
   COMPUTE THE WIRE AND REFERENCE CONDUCTOR SELF IMPEDANCES

C
   (STORE SELF IMPEDANCES OF EACH WIRE IN ARRAY B AND THE SELF
   IMPEDANCE OF THE REFERENCE CONDUCTOR IS STORED IN TO VARIABLE 70)

C
   LDC=N00BFI
   DO 83 I=1,N
   DELTA=ONE/(TWO*PI*DSQRT(Y(I)*F*MO04PI))
   RDC=ONE/(PI*T(I)*Z(T(I))*Z(I))
   IF (T(I).LE.DELTA) GO TO 81
   IF (T(I).GE.THREE*DELTA) GO TO 82
   B(I)=(P25*(Z(I)/DELTA*E(I))*RDC*JOMEGA*(OMEGA15-P15*Z(I))/DELTA)
1*DELTA)/NS(I)
   GO TO 83
81 B(I)=(RDC*JOMEGA*LDC)/NS(I)
   GO TO 83
82 B(I)=(P5*E(I)*RDC/DELTA*JOMEGA*TEL*DELTA*LDC/E(I))/NS(I)
83 CONTINUE
   GO TO (84,87,88) TYPE
84 DELTA=ONE/(TWO*PI*DSQRT(SIG*F*H04PI))
   RDC=ONE/(PI*SIG*E*(RNSO*RNSO))
   IF (RNSO.LE.DELTA) GO TO 85
   IF (RNSO.GE.THREE*DELTA) GO TO 86
   B0=(P25*(RNSO/DELTA*THREE)*RDC*JOMEGA*(OMEGA15-P15*RNSO/DELTA)/LDC)
1/RNSO)
   GO TO 91

C
-139-
85 \( Z_0 = \frac{(RDC+JOMEGA*LDC)}{NSO} \) GO TO 91
86 \( Z_0 = \frac{(P5+RSO*RDC/Delta+JOMEGA*2*Delta*LDC/NSO)}{NSO} \) GO TO 91
87 \( Z_0 = (BG+JOMEGA*LDC) \) GO TO 91
88 \( RDC=\text{ONE}/(\text{PI}\times\text{SIGO}\times(\text{TH}+\text{DELTA})) \)
\( \Delta = \text{ONE}/(\text{PI}\times\text{SIGO}\times(\text{TH}+\text{DELTA})) \)
IF (TH.LE.DELTA*P5) GO TO 89
IF (TH.GE.DELTA*P5) GO TO 90
\( \text{TH} = \text{TH}+\text{DELTA} \)
\( \text{SINH} = (\text{EXP}(\text{X})*\text{EXP}(-\text{X}))*P5 \)
\( \text{COSH} = (\text{EXP}(\text{X})*\text{EXP}(-\text{X}))*P5 \)
\( ZO = (\text{SINH}*\text{SIN}(\text{X}))*N5*(\text{SINH}*-\text{SIN}(\text{X}))/((\text{TH}+\text{DELTA})* \)
\( 1\times(\text{COSH} - \text{COSH}) \))
GO TO 91
89 \( Z_0 = (\text{ONE}*-XJ*P1*TU/Delta) \) RDC
GO TO 91
90 \( Z_0 = (\text{ONE}*-XJ*/\text{DELTA})*RDC \)
GO TO 91

91 \( \Omega^2 = \text{OMEGA}\times\text{OMEGA} \)
DO 93 \( I=1,N \)
DO 92 \( J=1,M \)
92 \( A(I,J) = \text{JOMEGA}*(T3(I) + \text{SO} + \text{C}(I,J) + \text{B}(J)) \)
93 \( A(I,I) = A(I,I) - \text{OMEGA}^2 + \text{BER}/\text{VY} \)
CALL TIGCC(A, M, N, 2, M, X, W, K, LER)
LER = LER - 128
IF (LER.LT.1) LER = 0

94 \( T1(I, J) = \text{ZERO} \)
95 \( T1(I, I) = \text{OMEGA} \)
CALL LEQTIC(A, M, N, T1, M, N, 0, WA, NER)
NER = NER - 128
IF (NER.NE.1) NER = 0

96 \( T0 = \text{TERM} \)
FORM THE EQUATIONS
DO 98 \( I=1,M \)
98 \( S0 = \text{ZEROC} \)
SL = ZEROC
DO 97 \( J=1,N \)
97 \( S0 = \text{ZEROC} \)
SL = ZEROC
DO 96 \( K=1,N \)
96 \( S0 = \text{SUM0} + \text{N1}(I,K) \times T(K, J) \)
98 \( S0 = \text{SUM} \times N2(I,K) \times T(K, J) \)
SL = SL + T(I,J) \times N2(J)
A(I,J) = SUM

-140-
IF (OPTION. EQ. 11. OR. OPTION. EQ. 12) G(I) = G(I)*G(I)/G(I)

90 CONTINUE
DO 100 I=1,N
DO 100 J=1,N
SUM=ZEROC
SUM=SUM+TI(I,K)*A(K,J)
100 TL(I,J)=SUHL
DO 103 I=1,N
SO=ZEROC
DO 102 J=1,N
SL=ZEROC
DO 101 K=1,N
101 SL=SL+TL(I,K)*G(K)*EN(K)*YO(K,J)
102 SO=SO+TL(I,J)*G(J)*EN(J)*IO(J)
A(I,J)=A(I,I)*EN(I,J)/G(I)
103 B(I)=SO*EP(I)*IO(I)+IL(I)
C
SOLVE THE EQUATIONS
CALL LETCIC(A,N,N,B,1,N,G,WA,IER)
IER=IER-128
IF (IER.NE.1) IER=0
WRITE(6,104) IER
104 FORMAT(' SOLVING EQUATIONS ')
WRITE(6,105) IER,PREC
105 FORMAT(' SOLUTION ERROR = ',1PE11.4,10X,'SOLUTION PRECISION = ',1PE11.4/)
1X,12/)
PREC=HK(1)
WRITE(6,106) LEB,PREC
106 FORMAT(' SOLUTION ERROR = ',1PE11.4/)
WRITE(6,107) LEB
107 FORMAT(' SOLUTION PRECISION = ',1PE11.4/)
COMPUTE AND PRINT THE TERMINAL VOLTAGES
DO 109 I=1,N
SO=ZEROC
DO 108 J=1,N
108 SO=SO+IO(I,J)*B(J)
109 G(I)=G(I)*EN(I)*IO(I)+EP(I)*B(I)*G(I)*EN(I)*SO
IF (OPTION_EQ.21. OR. OPTION_EQ.22) GO TO 111
DO 111 I=1,N
SO=ZEROC
SL=ZEROC
-141-
DO 110 J=1,N
   SO=SO+Y0(I,J) *H(J)
110   SL=SL+Y(I,J) *G(J)
   Y0(I,J)=10(I)+SO
   TL(I,J)=-IL(I) *SL
   DO 113 I=1,N
      SO=ZEROC
      SL=ZEROC
   DO 112 J=1,N
      SO=SO+T(I,J) *Y0(J,J)
112   SL=SL+T(I,J) *Y(I,J)
   P(I,J) = SO
   GO TO 117
113   A(I,J) = SL
   DO 116 I=1,N
      SO=ZEROC
      SL=ZEROC
   DO 115 J=1,N
      SO=SO+T(I,J) *B(J)
115   SL=SL+T(I,J) *G(J)
   P(I,J) = SO
116   A(I, J) = SL
117   DO 120 I=1,N
      SO=ZEROC
      SL=ZEROC
   DO 118 J=1,N
      SO=SO+C(I,J) *P(J,J)
118   SL=SL+C(I,J) *A(J,J)
   V0=SO
   VL=SL
   V0R=CDABS(V0)
   VLH=CDABS(VL)
   V0R=DREAL(V0)
   V0I=DIMAG(V0)
   VLR=DREAL(VL)
   VLI=DIMAG(VL)
   IF (V0R.EQ.ZERO.AND.V0I.EQ.ZERO) V0R=ONE
   IF (VLR.EQ.ZERO.AND. VLI.EQ.ZERO) VLR=ONE
   V0A=DATA2(V0I, V0R) *OM80/PI
   VLA=DATA2(VLI, VLR) *OM80/PI
   WRITE(6,119) I, V0R, V0A, VLR, VLA
119   FORMAT(1X,I2,6X,1PE10.3,3X,1PE10.3,10X,1PE10.3,3X,1PE10.3)
120  CONTINUE
   GO TO 79
121  STOP
END
### TABLE B-1

**Changes in XTALK2 to Convert to Single Precision Arithmetic**

Delete Card 057

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<td>REAL *8</td>
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066-068 change all D's to E's

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<td>CMPLX(0.E0,0.E0)</td>
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<td>DCMPLX(1.DO,0.DO)</td>
<td>CMPLX(1.E0,0.E0)</td>
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</table>
APPENDIX C

FLATPAK

Program Listing
A digital computer program to compute the terminal voltages (with respect to the reference wire) of an N+1 wire flatpack or ribbon cable for the 'quasi-ten' node of propagation.

The distributed parameter, multiconductor transmission line equations are solved for steady state, sinusoidal excitation of the line.

The N+1 wires are assumed to be parallel to each other.

The N+1 wires are considered to be perfect conductors.

The surrounding media are assumed to be lossless.

The per-unit-length capacitances of the cable (with and without the dielectric insulations present) are input data and may be computed with the program GETCAP.

**LOAD STRUCTURE OPTION DEFINITIONS:**

- **OPTION = 11**, TEMHEN EQUIVALENT LOAD STRUCTURES WITH DIAGONAL IMPEDANCE MATRICES
- **OPTION = 12**, TEMHEN EQUIVALENT LOAD STRUCTURES WITH FULL IMPEDANCE MATRICES
- **OPTION = 21**, NORTON EQUIVALENT LOAD STRUCTURES WITH DIAGONAL ADMITTANCE MATRICES
- **OPTION = 22**, NORTON EQUIVALENT LOAD STRUCTURES WITH FULL ADMITTANCE MATRICES

**SUBROUTINES USED:** LEQT1C, NROOT, EIGEN

**FREQUENCY INDEPENDENT CALCULATIONS**

**READ AND PRINT INPUT DATA**
C
READ(5,1) N,OPTION,L
1 FORMAT(8X,12,8X,12,E10.3)
IF (OPTION.EQ.11.OR.OPTION.EQ.12) GO TO 3
IF (OPTION.EQ.21.OR.OPTION.EQ.22) GO TO 3
WHITE(6,2)
2 FORMAT('* LOAD STRUCTURE OPTION ERROR*/// OPTION MUST EQUAL 11,12,2')
11,OB 22*///)
GO TO 60
3 NP=N+1
WHITE(6,4) NP,L,OPTION
4 FORMAT(16X,49X,*FLATPAR*///
   145X,12,* PARALLEL WIRES*///
   239X,'LINE LENGTH= ',1PE13.6,* METERS*///
   342X,'LOAD STRUCTURE OPTION= ',12///)
   FLATP076
C
READ ENTRIES IN THE PER-UNIT-LENGTH TRANSMISSION LINE
C CAPACITANCE MATRIX, C (COMPUTED WITH GETCAP)
C (STORE C IN ARRAY C)
C
DO 5 I=1,N
DO 5 J=1,N
READ(5,5) K,N,C(K,N)
5 FORMAT(4X,12,3X,12,2X,Z13.6)
6 C(N,K)=C(K,N)
C
READ ENTRIES IN THE PER-UNIT-LENGTH TRANSMISSION LINE
C CAPACITANCE MATRIX WITH THE WIRE INSULATIONS REMOVED, CO, (COMPUTED)
C WITH GETCAP
C (STORE CO IN ARRAY CO)
C
DO 8 I=1,N
DO 8 J=1,N
READ(5,7) K,N,CO(K,N)
8 FORMAT(4X,12,3X,12,2X,Z13.6)
7 CO(M,K)=CO(K,M)
C
COMPUTE THE EIGENVECTORS (COLUMNS OF THE MATRIX T) AND EIGENVALUES
C OF THE MATRIX PRODUCT (THE ARRAYS T AND G CONTAIN T AND THE INVERSE
C OF THE EIGENVALUES FOR THE THEVENIN EQUIVALENT AND THE NORTON
C EQUIVALENT, RESPECTIVELY)
C
IF(N.EQ.1) GO TO 9
CALL NROOT(N,CO,CEG,TI,N*N)
GO TO 10
9 G(1)=CO(1,1)/C(1,1)
   TI(1,1)=ONE/DSQRT(C(1,1))
   DO 12 I=1,N
   DO 12 J=1,N
11 C(I,J)=TI(I,J)
12 G(I)=ONE/(V*DSQRT(G(I)))
   IF (OPTION.EQ.21.OR.OPTION.EQ.22) GO TO 18
   DO 14 I=1,N
   DO 14 J=1,N
A(I,J)=TI(I,J)*ONE
13 P(I,J)=ZERO
   CALL LEQTLC(A,N,N,P,N,N,O,UA,KER)
   NER=KER-128
   IF(KER.NE.1) KER=0
   IF(KER.NE.1) KER=0
-147-
WRITE(6, 15) KER
15 FORMAT (/,'TRANSFORMATION MATRIX INVERSION ERROR = 12//)
DO 17 I = 1, N
DO 16 J = 1, N
16 T(J, I) = REAL(P(J, I))
17 G(I) = ONE/G(I)

C READ AND PRINT ENTRIES IN LOAD ADMITTANCE (IMPEDANCE) MATRICES
C AND SHORT CIRCUIT CURRENT SOURCE (OPEN CIRCUIT VOLTAGE SOURCE)
C VECTORS
C (STORE ADMITTANCE (IMPEDANCE) MATRICES AT X=0 IN ARRAY TO AND THOSE AT X=L IN ARRAY IL)
C
C AT X=L IN ARRAY IL. STORE SHORT CIRCUIT CURRENT SOURCE (OPEN CIRCUIT VOLTAGE SOURCE) VECTORS AT X=0 IN ARRAY TO AND THOSE AT X=L IN ARRAY IL.)
C
18 IF (OPTION .EQ. 11 .OR. OPTION .EQ. 12) GO TO 21
WRITE(6, 19)
19 FORMAT (/,'ADMITTANCE AT X=0, 10X, 'CURRENT SOURCE AT X=0, 10X, 'CURRENT SOURCE AT X=L'/)
WRITE(6, 20)
20 FORMAT (2X,'(SIEHENS)', 23X, '(AMPS)', 22X,'(SIEHENS)', 23X, '(AMPS)'/)
GO TO 24
21 WRITE(6, 22)
22 FORMAT (/,'IMPEDANCE AT X=0, 11X, 'VOLTAGE SOURCE AT X=0, 11X, 'VOLTAGE SOURCE AT X=L'/)
WRITE(6, 23)
23 FORMAT (2X,'(OHMS)', 23X, '(VOLTS)', 22X,'(OHMS)', 23X, '(VOLTS)'/)
24 WRITE(6, 25)
25 FORMAT (E10.3)
DO 28 I = 1, N
READ(5, 26) YOR, YOI, TO(I), YLB, ILI, IL(I)
26 FORMAT (2(E10.3))
Y0(I, I) = TO+X*Y0I
YL(I, I) = YLB*X*YL1
WRITE(6, 27) I, I, Y0(I, I), IO(I), TL(I, I), IL(I)
27 FORMAT (1R, 12.2X, 12, 2(SX.1P110.3), 301.2(5X1PE10.3))
28 CONTINUE
IF (OPTION .EQ. 11 .OR. OPTION .EQ. 21) GO TO 32
IF (N .EQ. 1) GO TO 32
K1 = N-1
DO 31 I = 1, K1
K2 = I+1
DO 33 J = K1, N
READ(5, 29) YOR, YOI, YLB, ILI
29 FORMAT (2(E10.3), 20X, 2(E10.3))
Y0(I, J) = TO+X*Y0I
YL(I, J) = YLB*X*YL1
Y0(J, I) = TO+X*Y0I
YL(J, I) = YLB*X*YL1
WRITE(6, 30) I, J, Y0(I, J), Y0(J, I)
30 FORMAT (1R, 12.2X, 12, 2(SX.1P110.3), 301.2(5X1PE10.3))
31 CONTINUE

C COMPUTE THE MATRICES TTRANZ0T, TTRANZL*T, TTRANZ0Z, TTRANZL*Z
C FOR THE THEVENIN EQUIVALENT OR TINVZ0T*TINVTRAN, TINVZL*TINVTRAN
C TINVZ0Z, TINVZL FOR THE NORTON EQUIVALENT AND STORE IN ARRAYS
C TO, YL, IL, IL, RESPECTIVELY.
C
32 IF (OPTION .EQ. 12 .OR. OPTION .EQ. 22) GO TO 35
DO 34 I = 1, N
SO=ZERO
SL=ZERO
DO 33 J=1,N
A(I,J)=YO(I,J)*TI(I,J)
P(I,J)=YI(I,J)*TI(I,J)
SO=SO+TI(J,I)*IO(J)
33 SL=SL+TI(J,J)*IL(J)
B(I)=SO
34 WA(I)=SL
GO TO 39
35 DO 38 I=1,N
SO=ZERO
SL=ZERO
DO 37 J=1,N
SUMO=ZERO
SUML=ZERO
DO 36 K=1,N
SUM=SUM+YO(I,K)*TI(K,J)
36 SUML=SUML+YL(I,K)*TI(K,J)
A(I,J)=SUM
P(I,J)=SUML
SO=SO+TI(J,I)*IO(J)
37 SL=SL+TI(J,J)*IL(J)
B(I)=SO
38 WA(I)=SL
39 DO 42 I=1,N
DO 41 J=1,N
SO=ZERO
SL=ZERO
DO 40 K=1,N
SL=SL+TI(K,I)*A(K,J)
40 TL(I,J)=SL
41 IL(I,J)=B(I)
42 IF (OPTION.EQ.11.OR.OPTION.EQ.12) IL(I)=-IL(I)
C******FREQUENCY DEPENDENT CALCULATIONS*******************************
C
43 CONTINUE
READ(5,44,END=60) F
44 FORMAT(E0.3)
OMEGA=TWO*PI*F
C
C COMPUTE THE TERMINAL VOLTAGES
C
C FORM THE EQUATIONS
C
DO 45 I=1,N
45 P(I,I)=X(I)*DSIN(W)
DO 48 I=1,N
48 SO=ZERO
DO 47 J=1,N
47 SL=ZERO
DO 46 K=1,N
46 SL=SL+YL(I,K)*G(K)*P(K,K)*YO(K,J)
A(I,J) = SL + YL(I,J) * CO(J,J) * YO(I,J) * CO(I,I)
47 S0 = S0 + YL(I,J) * G(J) * P(J,J) * IO(J)
A(I,I) = A(I,I) + P(I,I) / G(I)
48 B(I) = S0 + CO(I,I) * IO(I) + IL(I)

```
SOLVE THE EQUATIONS
```

CALL LEQTIC(A, M, N, B, 1, N, QA, IER)
IER = IER - 128
IF (IER .EQ. 1) IER = 0
WRITE(6,49) P, IER

```
FORMAT (I6, ' FREQUENCY (HERTZ) = ', 1PE11.4, 10X, 'SOLUTION ERROR = ', 12X, 12/)
WRITE(6, 50)
```

```
FORMAT (1X, 'WIRE', 8X, 'V0H (VOLTS)', ',3X,' 'VOA (DEGREES)', ',8X, 1\*VLR (VOLTS)', ',3X,' 'VLA (DEGREES) ')/
```

```
COMPUTE AND PRINT THE TERMINAL VOLTAGES
```

```
DO 52 I = 1, N
S0 = ZERO
DO 51 J = 1, N
51 S0 = S0 + YO(I,J) * B(J)
IF (OPTION.EQ.21 OR OPTION.EQ.22) GO TO 56
DO 54 I = 1, N
S0 = ZERO
SL = ZERO
DO 53 J = 1, N
53 S0 = S0 + YO(I,J) * B(J)
54 P(I,I) = XL(I) + S0
DO 55 I = 1, N
B(I) = A(I,I)
55 B(J) = P(I,J)
56 DO 59 I = 1, N
S0 = ZERO
SL = ZERO
DO 57 J = 1, N
57 S0 = S0 + C(I,J) * B(J)
58 V0 = S0
VL = SL
V0H = CDABS (V0)
VLH = CDABS (VL)
V0R = DREAL (V0)
VLR = DREAL (VL)
V0I = DIMAG (V0)
VLI = DIMAG (VL)
IF (VOR.EQ.ZERO. AND. V0I.EQ.ZERO) VOR = CMX
IF (VLR.EQ.ZERO. AND. VLI.EQ.ZERO) VLR = CMX
V0A = DATAN2 (V0I, V0R) * ONE80/PI
VLA = DATAN2 (V0I, VLR) * ONE80/PI
WRITE (6,58) I, V0H, V0A, V0I, VLR
58 FORMAT (1X, 'V0H, V0A, V0I, VLR')
59 CONTINUE
GO TO 43
60 STOP
```
TABLE C-1

Changes in FLATPAK to Convert
to Single Precision Arithmetic

Delete Card 048

<table>
<thead>
<tr>
<th>Card Number</th>
<th>Double</th>
<th>Single</th>
</tr>
</thead>
<tbody>
<tr>
<td>050</td>
<td>REAL *8</td>
<td>REAL</td>
</tr>
<tr>
<td>051</td>
<td>COMPLEX *16</td>
<td>COMPLEX</td>
</tr>
<tr>
<td>053</td>
<td>3.141592653D0</td>
<td>3.1415926E0</td>
</tr>
<tr>
<td>053</td>
<td>2.997925D8</td>
<td>2.997925E8</td>
</tr>
<tr>
<td>054</td>
<td>change all D's  to E's</td>
<td></td>
</tr>
<tr>
<td>055</td>
<td>DCMPLX(0.D0,0.D0)</td>
<td>CMPLX(0.E0,0.E0)</td>
</tr>
<tr>
<td>056</td>
<td>DCMPLX(1.D0,0.D0)</td>
<td>CMPLX(1.E0,0.E0)</td>
</tr>
<tr>
<td>057</td>
<td>DCMPLX(0.D0,1.D0)</td>
<td>CMPLX(0.E0,1.E0)</td>
</tr>
<tr>
<td>109</td>
<td>DSQRT</td>
<td>SQRT</td>
</tr>
<tr>
<td>113</td>
<td>DSQRT</td>
<td>SQRT</td>
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<tr>
<td>127</td>
<td>DREAL</td>
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<tr>
<td>237</td>
<td>DCOS</td>
<td>COS</td>
</tr>
<tr>
<td>238</td>
<td>DSIN</td>
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<td>289</td>
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<td>291</td>
<td>DREAL</td>
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<td>292</td>
<td>DIMAG</td>
<td>AIMAG</td>
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<td>294</td>
<td>DIMAG</td>
<td>AIMAG</td>
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<tr>
<td>297</td>
<td>DATAN2</td>
<td>ATAN2</td>
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</tbody>
</table>
APPENDIX D

FLATPAK2

Program Listing
C PROGRAM FLATPAK2
  (FORTRAN IV, DOUBLE PRECISION)
WRITTEN BY
  CLAYTON L. PAUL
  DEPARTMENT OF ELECTRICAL ENGINEERING
  UNIVERSITY OF KENTUCKY
  LEXINGTON, KENTUCKY 40506
A DIGITAL COMPUTER PROGRAM TO COMPUTE THE TERMINAL VOLTAGES
(WITH RESPECT TO THE REFERENCE WIRE) OF AN N+1 WIRE FLATPACK OR
RIBBON CABLE FOR THE 'QUASI-TEN' MODE OF PROPAGATION.

THE DISTRIBUTED PARAMETER, MULTICONDUCTOR TRANSMISSION LINE
EQUATIONS ARE SOLVED FOR STEADY STATE, SINUSOIDAL EXCITATION
OF THE LINE.

THE N+1 WIRES ARE ASSUMED TO BE PARALLEL TO EACH OTHER.

THE N+1 WIRES ARE CONSIDERED TO BE IMPERFECT CONDUCTORS. THE SELF
INDEEDANCES OF EACH WIRE INCLUDE SKIN EFFECT.

THE SURROUNDING MEDIA ARE ASSUMED TO BE LOSSLESS.

THE PER-UNIT-LENGTH CAPACITANCES OF THE CABLE (WITH AND WITHOUT
THE DIELECTRIC INSULATIONS PRESENT) ARE INPUT DATA AND MAY BE
COMPUTED WITH THE PROGRAM GETCAP.

LOAD STRUCTURE OPTION DEFINITIONS:
  OPTION=11, THEVENIN EQUIVALENT LOAD STRUCTURES WITH DIAGONAL
      IMPEDANCE MATRICES
  OPTION=12, THEVENIN EQUIVALENT LOAD STRUCTURES WITH FULL
      IMPEDANCE MATRICES
  OPTION=21, NORTON EQUIVALENT LOAD STRUCTURES WITH DIAGONAL
      ADMITTANCE MATRICES
  OPTION=22, NORTON EQUIVALENT LOAD STRUCTURES WITH FULL
      ADMITTANCE MATRICES

SUBROUTINES USED: LEQTIEIGCC

ALL VECTORS AND MATRICES IN THE FOLLOWING DIMENSION STATEMENTS
SHOULD BE OF SIZE N WHERE N IS THE NUMBER OF WIRES (EXCLUSIVE OF
THE REFERENCE WIRE), I.E., C(N,N), CO(2,2), CI(N,N),
IO(2), TL(2), O(2,2), S(N), A(N,N), E(N,N), E(N,N), P(N,N),
N1(N,N), N2(N,N), V1(N), V2(N), T(N,N), T(N,N), G(N,N), V3(N), WA(N),

THE VECTOR WK MUST BE OF LENGTH 2(N+1)

IMPLICIT REAL*8 (A-H,O-Z)
INTEGER OPTION
REAL*8 LLDC, C(2,2), CO(2,2), CI(2,2), WK(12),
MUO4PI, MUO8PI
COMPLEX*16 XJ, SUNO, SURL, S0, SL, VO, VL, ZER0C, ONEC, Z, B, GAM
1, IO(2), TL(2), O(2,2), S(2), A(2,2), WA(2), G(2),
2P(2,2), EP(2), EN(2), N1(2,2), N2(2,2), V1(2,2), V2(2,2),
3T(2,2), XI(2,2), JOMEGA
DATA PI/3.141592653585979/,
  V/2.997925586/,
  DATA CMTR/2.50D-5, ZER0/0.D0/, TWO/2.D0/, MUO8PI/5D-7/,ONE/1.D0/,
  MUO4PI/1.D-7/, THREE/3.D0/, F25/2.50D0/, OMEP15/1.150D0/, P15/1.150D0/,
2P5/. 5D0/,01E80/180.DO/ PLATP062
V=Vv*V FLATP063
ZEBOC=DCEPLX(0.DO,0.DO)
PLATP064
(0.DO,0.DO)
PLATP065
OVIC-DCNPLX(0.DO,0.DO)
PLATP066
C PLAIP067
C****REQUXlNCY
PLATP068
INDSPENDENT
CALCTIOIS****************************e***FLAT?069
PLATP070
C PLAIP071
C
PLATP072
READ AND PRINT INPUT DATA
PLATP073
C READ(5,1) N,OPTION,L
PLATP074
1 FORMAT(8X,2I2,8x,12,E13.6) GO TO 3
PLATP075
IF(OPTION.EQ.11.OR.OPTION.EQ.12) GO TO 3
PLATP076
WRITE(6,2)
PLATP077
2 FORMAT(LOAD STRUCTURE OPTION ERROR*// OPTION MUST BE 11,12,2FLATPO78
1,OR 22**//)
PLATP079
GOT0 81
PLATP080
3 N=I*1
PLATP081
WRITE(6,4) N,P,L,OPTION
PLATP082
4 FORMAT(1H1,49X,'FLATPAK20/// FLATPO83
11.03 220///A FLATPO84
GO TO 81
FLATPO85
3
PLATP086
1P-N.1 TLAIPOSO
PLATP087
VRITE(6,4)
PLATP088
NP,L,OPTION
PLATP089
FLATPO90
FLATPO91
FLATPO92
FLATPO93
FLATPO94
FLATPO95
FLATPO96
FLATPO97
C READ ENTRIES IN THE PER-UNIT-LENGTH TRANSMISSION LINE
PLATPO98
C CAPACITANCE MATRIX,C (COMPUTED WITH GETCAP)
PLATP099
C (STORE C IN ARRAY C)
PLATPO00
C DO 6 I=1,N
PLATPO01
C DO 6 J=1,N
PLATPO02
K,=P(5,5)
PLATPO03
K,=K(5,5)
PLATPO04
5 FORMAT(4I1,49X,'FLATPAK20/// FLATPO05
11.03 220///A FLATPO06
GO TO 81
FLATPO07
3
PLATPO08
1P-N.1 TLAIPOSO
PLATPO09
VRITE(6,4)
PLATPO10
NP,L,OPTION
PLATPO11
FLATPO12
C READ ENTRIES IN THE PER-UNIT-LENGTH TRANSMISSION LINE
PLATPO13
C CAPACITANCE MATRIX WITH THE WIRE INSULATIONS REMOVED,C0 (COMPUTED
PLATPO14
C WITH GETCAP)
PLATPO15
C (STORE C0 IN ARRAY C0)
PLATPO16
C DO 8 I=1,N
PLATPO17
C DO 8 J=1,N
PLATPO18
K,=P(5,7)
PLATPO19
K,=K(5,7)
PLATPO20
7 FORMAT(4I1,49X,'FLATPAK20/// FLATPO21
11.03 220///A FLATPO22
GO TO 81
FLATPO23
3
PLATPO24
1P-N.1 TLAIPOSO
PLATPO25
VRITE(6,4)
PLATPO26
NP,L,OPTION
PLATPO27
C READ ENTRIES IN THE PER-UNIT-LENGTH TRANSMISSION LINE
PLATPO28
C CAPACITANCE MATRIX WITH THE WIRE INSULATIONS REMOVED,C0 (COMPUTED
PLATPO29
C WITH GETCAP)
PLATPO30
C (STORE C0 IN ARRAY C0)
PLATPO31
C DO 8 I=1,N
PLATPO32
C DO 8 J=1,N
PLATPO33
K,=P(5,7)
PLATPO34
K,=K(5,7)
PLATPO35
7 FORMAT(4I1,49X,'FLATPAK20/// FLATPO36
11.03 220///A FLATPO37
GO TO 81
FLATPO38
3
PLATPO39
1P-N.1 TLAIPOSO
PLATPO40
VRITE(6,4)
PLATPO41
NP,L,OPTION
PLATPO42
C COMPUTE THE PER-UNIT-LENGTH TRANSMISSION LINE INDUCTANCE MATRIX,L,FLATPO43
C AND THE INVERSE OF THE CAPACITANCE MATRIX,C
PLATPO44
C (STORE C IN ARRAY CI AND L IN ARRAY CO)
PLATPO45
C DO 10 I=1,N
PLATPO46
C DO 10 J=1,N
PLATPO47
1 Format 123X,12,2X,E13.6)
PLATPO48
10 P(I,J)=ZEROC
PLATPO49
P(I,J)=ZEROC
PLATPO50
CALL LECTIC(A,N,S,P,H,M,N,0,WA,KER)
PLATPO51
CALL LECTIC(T,N,N,T,N,M,0,WA,RES)
PLATPO52
KER=KER-128
PLATPO53
-154-
IF (KER.NE.1) KER=0
WRITE (6,11) KER
WRITE (6,11) NER
11 FORMAT ('PER-UNIT-LENGTH CAPACITANCE MATRIX INVERSION ERROR= ',F6.2)
   DO 12 I=1,N
   DO 12 J=1,N
   CI (I,J)=REAL(TI (I,J))
   12 CO (I,J)=DREAL (P (I,J))

C READ AND PRINT CHARACTERISTICS OF THE WIRES TO BE USED IN THE SELF-
C IMPEDANCE CALCULATIONS
C
READ (5,13) RWS,SIG,NS
13 FORMAT (2(5X,E10.3),8X,I2)
WRITE (6,14) RWS,SIG,NS
14 FORMAT ('WIRE ARE STRANDED WITH EACH STRAND OF RADIUS= ',F8.4,' MMS',
   ' CONDUCTIVITY OF WIRE STRANDS= ',F8.4,' SIEMENS PER METER',
   ' NUMBER OF STRANDS= ',I2,'//')
RWS=RWS*CNTR

C READ AND PRINT ENTRIES IN LOAD ADMITTANCE(IMPEDANCE) MATRICES
C AND SHORT CIRCUIT CURRENT SOURCE(OPEN CIRCUIT VOLTAGE SOURCE)
C VECTORS
C
(READ ADMITTANCE(IMPEDANCE) MATRICES AT X=0 IN ARRAY X0 AND
THOSE AT X=L IN ARRAY XL. STORE SHORT CIRCUIT CURRENT SOURCE
THOSE AT X=1 IN ARRAY IL.)

IF (OPTION.NE.11.OR.OPTION.EQ.12) GO TO 17
WRITE (6,15)
15 FORMAT ('Admittance at X=0',10X,'Current source at X=0',
   ' Admittance at X=L',10X,'Current source at X=L')
WRITE (6,16)
16 FORMAT (2(IX*(SIEMENS)*,23X,(AMPS)*,22X,(SIEMENS)*,23X,(AMPS)*))
GO TO 20
17 WRITE (6,18)
18 FORMAT ('Impedance at X=0',10X,'Voltage source at X=0',
   ' Impedance at X=L',10X,'Voltage source at X=L')
WRITE (6,19)
19 FORMAT (2(23X,(OHMS)*,23X,(VOLTS)*,24X,(OHMS)*,23X,(VOLTS)*))
20 WRITE (6,21)
21 FORMAT ('Entry',10X,'Real',11X,'Imag',11X,'Real',11X,'Imag',11X,
   'Real',11X,'Imag',11X,'Real',11X,'Imag')
   DO 24 I=1,N
   READ (5,22) YOR,YOI,YLR,YLI
22 FORMAT (8(E10.3))
   YO (I,I)=YOR+X*YOI
   YL (I,I)=YLR+X*YLI
   WRITE (6,23) I,I,YO (I,I),YO (I,I),YL (I,I),YL (I,I)
23 FORMAT (1X,12,2X,12,8(5X,1PE10.3))
24 CONTINUE
IF (OPTION.NE.11.OR.OPTION.EQ.21) GO TO 28
IF (N.EQ.1) GO TO 28
K1=N-1
DO 27 I=1,K1
   K2=I+1
   DO 27 J=K2,N
   READ (5,25) YOR,YOI,YLR,YLI
27 CONTINUE

-155-
25 FORMAT (2(10.3), 20X, 2(10.3))
TO (I,J) = TOH + LJ*TOI
TL (I,J) = TLH + LJ*TLI
YO (J,I) = YOH (I,J)
TL (J,I) = TLH (I,J)
WRITE(6,26) I,J, TO (I,J), TL (I,J)
26 FORMAT (5X, 12, 2X, 12, 2 (10.3), 30X, 2 (10.3))
27 CONTINUE

C COMPUTE AND STORE THE MATRICES AND VECTORS C*Z0, C*ZL, C*V0, C*Yl
C FOR THE THVENIN EQUVALENT OR TH*V, TL*CIV, IO, IL FOR THE
C NORTON EQUIVALENT IN ARRAYS W1,W2,V1,V2, RESPECTIVELY
C
28 IF (OPTION.EQ.11) GO TO 29
IF (OPTION.EQ.12) GO TO 32
IF (OPTION.EQ.21) GO TO 36
IF (OPTION.EQ.22) GO TO 39
29 DO 31 I=1,N
SO = ZEROC
SL = ZEROC
DO 30 J=1,N
M1 (I,J) = C (I,J) * YO (J,J)
M2 (I,J) = C (I,J) * TL (J,J)
SO = SO + C (I,J) * TO (J)
30 SL = SL + C (I,J) * IL (J)
V1 (I) = SO
31 V2 (I) = SL
GO TO 43
32 DO 35 I=1,N
SO = ZEROC
SL = ZEROC
DO 34 J=1,N
SUMC = ZEROC
SUML = ZEROC
DO 33 K=1,N
SUMC = SUMC + C (I,K) * TO (K,J)
SUML = SUML + C (I,K) * TL (K,J)
SO = SO + C (I,J) * TO (J)
SL = SL + C (I,J) * TL (J)
M1 (I,J) = SUMC
33 M2 (I,J) = SUML
V1 (I) = SO
35 V2 (I) = SL
GO TO 43
36 DO 38 I=1,N
DO 37 J=1,N
M1 (I,J) = YO (I,J) * CI (I,J)
37 M2 (I,J) = TL (I,J) * CI (I,J)
V1 (I) = IO (I)
38 V2 (I) = I1 (I)
GO TO 43
39 DO 42 I=1,N
DO 41 K=1,N
SUMC = ZEROC
SUML = ZEROC
DO 40 K=1,N
SUMC = SUMC + YO (I,K) * CI (K,J)
40 SUML = SUML + TL (I,K) * CI (K,J)
M1 (I,J) = SUMC
41 M2 (I,J) = SUML
V1 (I) = IO (I)

-156-
42 \( V2(I) = TL(I) \)
43 CONTINUE

C COMPUTE THE MATRIX \( C^L \) AND STORE IN ARRAY \( C \). COMPUTE THE SUMS OF ELEMENTS IN EACH ROW OF \( C \) AND STORE IN ARRAY \( V3 \).

DO 46 I=1,N
   S=ZERO
DO 45 J=1,N
   S=S+C(I,J)
44 SS=SS+C(I,J)*C0(K,J)
   P(I,J)=SS*OMEGA
45 S=S+C(I,J)
46 \( \gamma(I,J) = \text{DEBRAZ}(P(I,J)) \)

C********FREQUENCY DEPENDENT CALCULATIONS******************************FLATP245

48 CONTINUE
READ(5,49,END=81) P
49 FORMAT(10.3)
OMEGA=THC*P+F
JOMEGA=XJ*OMEGA

C COMPUTE THE NINE SELF IMPEDANCES

LDC=MU0DE
DELTA=ONE/(TWO*PI*DSORT(SIG*F*MU0DE))
RDC=OMEGA/(PI*SIG*WNS/BNS)
IF (RNS.LT.DELTA) GO TO 50
IF (RNS.GT.ONE*DELTA) GO TO 51
Z=(P25*(BNS/DELTA+THREE)*RDC+JOMEGA*(OMEGAP-PI15-P15*RNS/DELTA)*LDC)/NS
50 S=(RDC+JOMEGA+LDC)/NS
GO TO 52
51 S=(P5*RNS/RDC+DELTA+JOMEGA+TWO*DELTA)*LDC/NS
GO TO 52

C COMPUTE THE EIGENVALUES AND THE EIGENVECTORS OF THE PRODUCT \( YE = TZ \)

OMEGA=ONE*OMEGA
DO 53 I=1,N
  DO 53 J=1,N
53 A(I,J)=JOMEGA*Z*(V3(J)+C(I,J)+OMEGA*C0(I,J)
CALL EIGCC(A,N,N,2,B,T,N,W,LEN)
LEN=LEN-128
IF (LEN.LT.1) LEN=0

C COMPUTE THE INVERSE OF THE TRANSFORMATION MATRIX, \( T \)

DO 55 I=1,N
  DO 55 J=1,N
55 TI(I,J)=OMEGA
CALL LEQT1C(A,N,N,TI,N,W,LEN,WA,WER)

-157-
MERMER
FLATP306
128FLATP308
IT(MER.II. 1)
FLATP310
l!zRzO
FLATP311
FLATP312
C
FLATP313
FLATP314
C
FLATP315
COMPOTE 283
FLATP316
TERflXNAL VOLTAGES FLATP317
C
FLATP318
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128FLATP306
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IT(MER.II. 1)
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COMPOTE 283
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FLATP366
C

-158-
91 COMPUTE AND PRINT THE TERMINAL VOLTAGES

DO 69 I = 1, N
50 = ZERO
DO 68 J = 1, N
50 = 50 + T(I, J) * B(J)
68 G(I) = G(I) * EM(I) * IO(I) * EP(I) * B(I) * G(I) * EM(I) * 50
IF (OPTION.EQ.21 OR OPTION.EQ.22) GO TO 76
70 S0 = ZERO
SL = ZERO
DO 72 J = 1, N
50 = 50 + T(I, J) * Y0(J, J)
71 T(I, J) = - IL(I) * SL
DO 73 I = 1, N
50 = ZERO
SL = ZERO
DO 77 J = 1, N
50 = 50 + T(I, J) * B(J)
75 S0 = S0 + T(I, J) * G(J)
73 A(T, I) = S0
GO TO 77
74 DO 76 J = 1, N
50 = 50 + T(I, J) * B(J)
77 S0 = S0 + CI(I, J) * P(J, J)
78 S0 = S0 + CI(I, J) * A(J, J)
76 S0 = S0 + T(I, J) * B(J)
VL = SL
Y0 = CDABS(YO)
VLH = CDABS(VL)
Y0 = DREAL(YO)
VL = DREAL(VL)
IF (Y0 .EQ. ZERO.AND. VL .EQ. ZERO) Y0 = ONE
IF (VLH .EQ. ZERO.AND. Y0 .EQ. ZERO) VLH = ONE
Y0 = DATAN2(Y0, Y0) * ONE80/PI
VLH = DATAN2(Y0, VLH) * ONE80/PI
WRITE (6, 79) I, Y0, Y0, VLH, VL
79 FORMAT (16X, 'WIRE', 8X, 'VOLTS', 3X, 'DEGREES', 8X, 'VOLTS', 3X, 'DEGREES')
80 CONTINUE
GO TO 48
81 STOP
END
<table>
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<th>Single</th>
</tr>
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<td>COMPLEX</td>
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<td>CMPLX(0.E0,0.E0)</td>
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<td>CMPLX(1.E0,0.E0)</td>
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<td>CMPLX(0.E0,1.E0)</td>
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<td>418</td>
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<td>AIMAG</td>
</tr>
<tr>
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<td>DATAN2</td>
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APPENDIX E

NROOT

Subroutine Listing
SUBROUTINE NROOT(N, A, B, XL, X, NA)
REAL*8 A(N), B(NA), XL(NA), X(NA), SY, ZER, ONE
DATA ZERO/0.D0/,ONE/1.D0/
K=1
DO 1 J=2,N
L=H*(J-1)
DO 1 I=1,J
L=I+1
K=K+1
1 B(K)=B(L)
MV=0
CALL EIGEN(B, X, B, SY, MV)
L=0
DO 2 J=1,N
L=L+J
2 XL(J)=ONE/DSQRT(DABS(B(L)))
K=0
DO 3 J=1,N
DO 3 I=1,N
K=K+1
3 B(K)=X(K)*XL(J)
DO 4 I=1,N
N2=0
DO 4 J=1,N
N1=M*(I-1)
L=H*(J-1)+I
X(L)=ZER
DO 4 K=1,N
N1=N1+1
N2=N2+1
4 X(L)=X(L)+B(N1)*A(N2)
L=0
DO 5 J=1,N
DO 5 I=1,J
N1=I-M
N2=M*(J-1)
L=L+1
A(L)=ZERO
DO 5 K=1,N
N1=N1+M
N2=N2+1
5 A(L)=A(L)*X(N1)*B(N2)
CALL EIGEN(A, X, MV, MV)
L=0
DO 6 I=1,N
L=L+1
6 XL(I)=A(I)
DO 7 I=1,N
N2=0
DO 7 J=1,N
N1=I-M
L=H*(J-1)+I
A(L)=ZER
DO 7 K=1,N
N1=N1+M
N2=N2+1
7 A(L)=A(L)+B(N1)*X(N2)
K=0
DO 8 J=1,N
DO 8 I=1,N
K=K+1
-164-
\[ X(K) = A(K) \]

RETURN

END
### TABLE E-1

**Changes in NROOT to Convert to Single Precision Arithmetic**

<table>
<thead>
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<td>change all D's to E's</td>
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<tr>
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<td>SQRT</td>
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<tr>
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<td>DABS</td>
<td>ABS</td>
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</tbody>
</table>
APPENDIX F

ETCEN

Subroutine Listing
SUBROUTINE EIGEN(A,R,N,M,H,NH)
REAL A(N), R(M), ANORM, ANRMX, THR, X, Y, SINX, SINX2, COSX, COSX2,
1 SINCS, RANGE, ZERO, ONE, P5, TWO
DATA ZERO/0.0/, ONE/1.0/, P5/.5/, TWO/.2/, THR/.0.0/BEKRIGOS
I RANGE=1.0D-12
IF(MV-1) 2,5,2
IQ=-N
DO 4 J=1,N
IQ=IQ+N
DO 4 I=1,N
IJ=IQ+I
R(IJ)=ZERO
IF(IJ-J) 4,3,4
3 R(IJ)=ONE
4 CONTINUE
5 ANORM=ZERO
DO 7 I=1,N
DO 7 J=1,N
IF(I-J) 6,7,6
6 IA=I+(J*J-J)/2
ANORM=ANORM+A(IA)*A(IA)
7 CONTINUE
IF(ANORM) 36,36,8
8 ANORM=DSQRT(TWO)*DSQRT(ANORM)
ANRMX=ANCMX*RANGE/FLOAT(N)
IND=0
THR=ANORM
9 THR=THR/FLOAT(N)
10 L=;
11 M=L+1
12 HQ=(M-M-H)/2
LQ=(L-I-L)/2
13 IF (A(ABR(A(LM))<THR) 29,14,14
14 IND=;
LL=L/1;
MM=M+M;
X=P5*(A(IL)-A(MM))
15 Y=-A(LH)/DSQRT(A(LH)*A(LH)+X*X)
TP(X) 16,17,17
16 Y=Y
17 SINX=DSQRT(TWO*(ONE+DSQRT(ONE-Y*Y)))))
SINX2=SINX*SINX
18 COSX2=DSQRT(SINX2*SINX2
SINCS =SINX*COSX
ILQ=M*(L-1)
IQ=M*(M-1)
DO 28 I=1,N
IQ=(I*1-1)/2
IF(I-L) 19,26,19
19 IF(X-I) 20,26,21
20 IM=I+Q
GO TO 22
21 IM=I+Q
22 IF(I-L) 23,24,24
23 IL=I+Q
GO TO 25
24 IL=I+Q
25 X=A(IL)*COSX-A(IM)*SINX
A(IM)=A(IL)*SINX+A(IM)*COSX

26 IF(NV-1) 27,28,27
27 ILR=ILQ+1
IRM=INQ+1
X=R(ILR) *COS(B(INR) *SINX)
R(INR)=R(IRM) *SINX+R(INR) *COSX
R(ILR)=X
28 CONTINUE
X=2*W*A(ILR) *SINC
Y=A(LL) *COSX2*A(NN) *SINX2-X
X=A(LL) *SINX2+A(NN) *COSX2+X
A(ILR)=(A(LL)-A(NN)) *SINC+A(ILR) * (COSX2-SINX2)
A(LL)=Y
A(NN)=X
29 IF(N-N) 30,31,30
30 N=N+1
GO TO 12
31 IF(L-(N-1)) 32,33,32
32 L=L+1
GO TO 11
33 IF(IND-1) 35,34,35
34 IND=0
GO TO 10
35 IF(THI-ANNX) 36,36,9
36 IQ=-N
DO 100 I=1,N
IQ=I *M
LL=I * (I* I-1)/2
JQ=I * (I-2)
DO 40 J=1,N
JQ=JQ+M
MM=J * (J* J-J)/2
IF(A(LL)-A(NN)) 37,40,40
37 X=A(LL)
A(LL)=A(NN)
A(NN)=X
IF(NV-1) 38,40,38
38 DO 39 K=1,N
ILR=IQ+K
IRM=JQ+K
X=R(ILR)
R(INR)=R(INR)
39 R(INR)=X
40 CONTINUE
RETURN
<table>
<thead>
<tr>
<th>Card Number</th>
<th>Double</th>
<th>Single</th>
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<tr>
<td>002</td>
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<tr>
<td>004</td>
<td>change all D's to E's</td>
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