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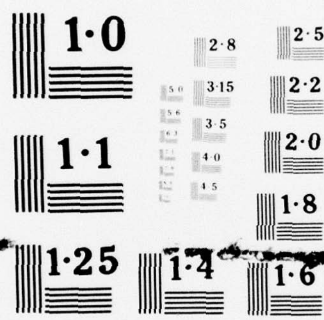

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Non-Newtonian fluid, eccentric cylinders, inertial effects, lift forces, lubrication, Poiseuille flow, second-order visco-elastic fluid, fourth-order viscoelastic fluid.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The work ~~has been concerned~~ concerns several problems with the flow of Newtonian and non-Newtonian fluids in the annular region between eccentric infinite cylinders. The following problems have been solved:

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20. Abstract (Contd.)

(i) The flow and forces resulting from the uniform longitudinal motion of one cylinder relative to the other in the case when the second or fourth-order Rivlin-Ericksen constitutive equation is applicable.

(ii) The flow and forces resulting from the application of a longitudinal pressure gradient in the case when the second or fourth-order Rivlin-Ericksen constitutive equation is applicable.

(iii) The flow and forces resulting from the rotation of one or both of the cylinders, with constant angular velocities, about their respective axes, in the case when the fluid is Newtonian, or the second-order Rivlin Ericksen constitutive equation is applicable, and the linearized inertial approximation is made.

(iv) The forces on the cylinders when the inner cylinder also executes a planetary motion about the axis of the outer cylinder.

(v) The flow and forces on a cylinder which moves transversely with constant velocity parallel to a nearby rigid wall bounding a half-space of Newtonian fluid and simultaneously rotates with constant angular velocity. (The calculations are carried out with the linearized inertial approximation.)

(vi) The flow and forces are calculated for the problem (v), in the case when the fluid is non-Newtonian and the second-order Rivlin-Ericksen constitutive equation is applicable.

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THE APPLICATION OF INVARIANCE PRINCIPLES IN CONTINUUM MECHANICS

FINAL REPORT

AUTHOR: R.S. Rivlin

DATE: November 4, 1977

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LEHIGH UNIVERSITY

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PROBLEMS STUDIED AND PRINCIPAL RESULTS

During the period of the grant, work has been concentrated mainly on the flow of Newtonian and non-Newtonian fluids in the annular region between two infinite cylinders with different radii and parallel axes.

Some of the work reported here, notably that covered by Refs.[3] and [4], was essentially completed before the reporting period. Also some of the work covered by Ref.[1] was completed before the reporting period.

Newtonian fluids

In the case of Newtonian fluids, the work has been concentrated on effects arising from inertia, when one or both of the cylinders rotate with constant angular velocities about their respective axes. Prior to this work, this problem had been discussed only in certain asymptotic cases - notably by DiPrima and Stuart in the case when the radii of the cylinders are nearly equal. An attempt had been made by Kamal to solve the problem for arbitrary geometry of the system, in the case when one cylinder is rotating, but his analysis is radically incorrect.

We have studied the problem, in the case when one or both of the cylinders are rotating with constant angular velocities and the geometry of the system is arbitrary. The analysis is carried out, as was that in the previous studies, in the framework of the linearized inertial approximation. The results have been presented in two papers [1,2].

In [1] a much more thorough study than existed previously was made of the dependence on the ratio between the angular velocities of the cylinders and on the geometry of the system of the stream-line patterns, in the case when inertia is neglected (Stokes approximation). Particular emphasis was placed on the location of stagnation points and eddy centers. An equally complete study was made of the distribution of the forces exerted by the fluid on the cylinders. Then, the perturbation of the stream-line patterns, of the force distributions, and of the resultant forces was determined on the basis of the linearized inertial approximation. In these calculations, the results were obtained in [1] in the form of quadratures which were performed numerically for a wide range of ratios between the cylinder radii, eccentricity, and ratios between the angular velocities of the cylinders. It is noteworthy that complete agreement was obtained between our more general results and the results obtained by DiPrima and Stuart in the asymptotic case which they studied.

In [2] it was found possible to perform analytically the quadratures occurring in the expressions, obtained in [1], for the lift (i.e. the transverse) forces exerted by the fluid on the cylinders, so that an explicit analytical expression (albeit a very complicated one) could be obtained for these lift forces. This enabled us to extend the already wide range of geometries for which the lift forces could be calculated, without facing difficult round-off error problems.

It was found possible to use these results, obtained in [1] and [2], as the basis for the solution of two apparently quite different flow problems. These are:

(i) the calculation of the forces exerted on the inner cylinder when it executes a planetary motion about the axis of the outer cylinder, while one, both, or neither of the cylinders may simultaneously rotate with constant angular velocities about their respective axes;

(ii) the calculation of the forces which are exerted on a cylinder which is moving with constant velocity, transversely to its length, parallel to a nearby plane rigid wall bounding a half-space of the fluid, and simultaneously rotating with constant angular velocity. The results obtained for these problems are discussed in [2].

<u>Non-Newtonian Fluids</u>

The work on non-Newtonian fluids has been concerned mainly with the flow resulting from

(i) the uniform longitudinal motion of one cylinder with respect to the other,

(ii) the application of a uniform longitudinal pressure gradient to the fluid,

(iii) the rotation of one or both of the cylinders with constant angular velocities about their respective axes.

All three problems have been analyzed for arbitrary geometry of the system.

Problems (i) and (ii), have been studied within the framework of the slow flow approximations to the constitutive equation for a viscoelastic fluid embodied in the second and fourth-order Rivlin-Ericksen constitutive equations. The results obtained in the study of problem (i) have been published in [3] and [4]. While the analysis is complete in the sense that the flow fields, force distribution on the cylinders and resultant forces on them have been calculated, the main interest lies in the result that the non-Newtonian character of the fluid gives rise to resultant transverse forces on the cylinders. Similar analyses have been carried out in the case of problem (ii), but the results obtained have not yet been fully published. Again, the fluid exerts resultant transverse forces on the cylinders, but in senses opposite to those in problem (i).

Problem (iii) has been studied in the framework of the second-order theory. The results have been published in [5] and [6]. Again, the flow field, force distribution on the cylinders, and resultant forces on them have been calculated in explicit analytical form. The stream-function in this case is the same as it is for a Newtonian fluid. However, there is a complex distribution of force over the cylinders and the cylinders are subjected to resultant transverse forces, even if inertia is neglected. The forces resulting from inertia, calculated according to the linearized inertial approximation, are the same as in a Newtonian fluid of equal viscosity.

The interesting result emerges that the transverse forces which arise from the non-Newtonian character of the fluid may oppose or reinforce those due to inertia depending on the sign of one of the coefficients in the second-order Rivlin-Ericksen constitutive equation, on the eccentricity, and on the ratio of the angular velocities of the inner and outer cylinders.

In [4] it is shown how the results obtained in [3] can be used as the basis for the solution of two apparently quite different flow problems. These are the same problems which were discussed in [2] for a Newtonian fluid - the planetary motion of the inner cylinder about the axis of the outer cylinder and the transverse motion at constant velocity of a rotating cylinder, parallel to a rigid wall bounding a half-space of the non-Newtonian fluid.

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