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A GEOMETRIC MODEL OF HORIZONTAL SOUND
REFRACTION THROUGH A CIRCULAR EDDY.

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1974

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by

William H. Gemmill

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2pp.

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I. INTRODUCTION

Vertical sound velocity gradients in the ocean are several orders of magnitude larger than horizontal sound velocity gradients. Upon this "proposition", horizontal refraction of sound propagation is neglected in most underwater sound ray tracing models. However, in regions of sharp horizontal thermohaline gradients, such as oceanic fronts and eddies, horizontal refraction may not always be disregarded for long-range sound propagation. This note investigates the importance of horizontal refraction through cold and warm eddies.

Water masses such as the Sargasso Sea or Slope Water provide a simple setting in which to study sound propagation because of their large areal extent and weak horizontal gradients. Only vertical density stratification of these water masses affects refraction patterns of sound ray paths (neglecting bottom effects).

Recent papers by Gotthardt (1973) and Parker (1971) however, reveal that Gulf Stream rings (eddies) of different thermohaline properties may be common throughout the northwestern portion of the Sargasso Sea and in the Slope region between the Grand Banks and Cape Hatteras. The physical properties of these rings (typically 200 km in diameter) differ from the physical properties of the water mass surrounding them. Cold eddies have been shown to alter the vertical refraction sound ray paths, (Gemmill and Khedouri, 1974, Vastano and Owens, 1973), but little attempt has been made to understand whether an oceanic eddy will alter sound propagation in horizontal plane (a problem frequently referred to as bearing error). Horizontal sound speed changes across these

eddies have been calculated to be $20-30 \text{ m sec}^{-1}$ in 50 km (Khedouri and Gemmill, 1974). Although these sound velocity changes represent small gradients, the relative refractive index* of the eddy and surrounding water is about 0.98 for a warm eddy and 1.02 for a cold eddy (between air and water the relative index of refraction is 1.33, i.e.). These values are nearly unity but, at long ranges horizontal refraction of sound propagation may be important.

A model to examine the horizontal refraction of sound rays through a cylindrical eddy using geometric principles of optics was developed. In a physical sense, the model can be interpreted to portray the amount of horizontal refraction caused by an eddy on sound propagation between the ocean surface and the deep sound channel. The model results are compared with the results of the thin-lens approximation. Keep in mind that this model isolates only the horizontal plane, and in no way minimizes the importance of vertical refraction.

II. THE MODEL AND HORIZONTAL REFRACTION PATHS

This model was developed assuming an idealized cylindrical eddy. The effect of eddy currents on sound ray paths are assumed to have at least an order of magnitude less than the effect of horizontal sound speed differences. The geometry of the cylindrical eddy is shown in figure 1.

*relative refraction index of an eddy and its surrounding water is defined as V_O/V_E where V_O is the speed of sound in the surrounding water and V_E is the speed of sound in the eddy.

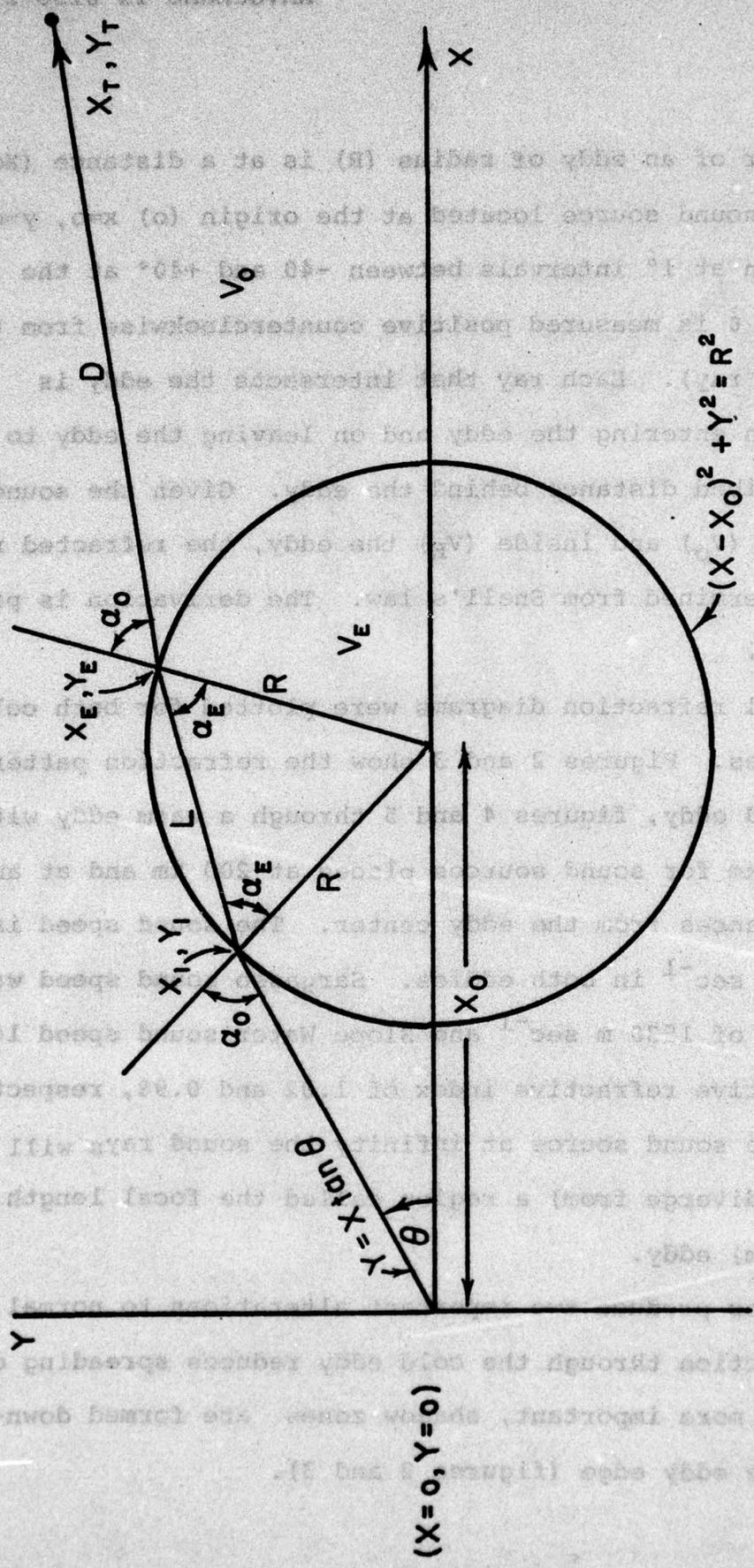


Figure 1 - GEOMETRY OF CYLINDRICAL EDDY

The center of an eddy of radius (R) is at a distance (X₀) from a point sound source located at the origin (o) x=0, y=0. Rays are drawn at 1° intervals between -40 and +40° at the source (where θ is measured positive counterclockwise from the x-axis to the ray). Each ray that intersects the eddy is refracted upon entering the eddy and on leaving the eddy to some predescribed distance behind the eddy. Given the sound speed outside (V₀) and inside (V_E) the eddy, the refracted ray paths are determined from Snell's law. The derivation is presented in Appendix A.

Horizontal refraction diagrams were plotted for both cold and warm eddies. Figures 2 and 3 show the refraction patterns through a cold eddy, figures 4 and 5 through a warm eddy with radii of 100 km, for sound sources placed at 200 km and at an infinite distances from the eddy center. The sound speed is set at 1500 m sec⁻¹ in both eddies. Sargasso sound speed was given a value of 1530 m sec⁻¹ and Slope Water sound speed 1470 m sec⁻¹ (relative refractive index of 1.02 and 0.98, respectively). By setting the sound source at infinity the sound rays will converge to (diverge from) a region called the focal length of the cold (warm) eddy.

Cold eddies produce two important alterations to normal ray paths. Refraction through the cold eddy reduces spreading of the rays, but more important, shadow zones are formed down-range from the eddy edge (figures 2 and 3).

A warm eddy acts to disperse sound rays more rapidly as they refract through the eddy (figures 4 and 5). It is possible that not all rays that intersect the warm eddy will refract. Rays intersecting the eddy near its edge may have an angle of incidence greater than the critical angle, and will be reflected.

III. COMPARISON TO THIN LENS OPTICAL MODEL

The spatial variation of the image produced by an eddy was examined from combinations of the eddy radii (R) and the eddy acoustic contrast* (AC) with sources at varying distances from the eddy center (X_o). These computations were compared to the results produced using the thin-lens equation (Jenkins and White, 1957), and some of the comparisons are presented in Table 1.

The thin-lens equation can be written:
$$\frac{1}{S} + \frac{1}{S^1} = \frac{2AC}{R}$$

where S is the distance of the source to the lens, and S¹ is the distance of the image from the lens. Note, for negative distances, the images "appear" to be in front of the eddy. For most combinations of eddy radii, and acoustic contrast and source distances, images will appear to be between the source and the eddy. For images to occur behind eddies, the source would have to be outside the focal length, or on the order of 1000 km away from the eddy. The comparison between the thin-lens image calculation and the cylindrical eddy image model calculation indicates that the thin-lens equation is a good approximation.

*acoustic contrast is defined as $(V_o - V_E)/V_E$ where V_o is the sound velocity outside the eddy and V_E is the sound velocity within the eddy.

IV. CONCLUSIONS AND RECOMMENDATIONS

Sound rays will spread horizontally more rapidly upon refraction across a warm eddy compared to totally homogeneous water mass. The reverse is true for a cold eddy, resulting in a shadow zones down range from the eddy edge.

Focal lengths and position of apparent images are approximately the same using either this cylindrical model or the physical thin-lens equation.

It appears that large horizontal gradients in the thermohaline structure of the ocean should not be neglected during sound propagation studies. Field experiments to determine the importance of horizontal refraction are planned.

where z is the distance of the source to the lens, and z' is the distance of the image from the lens. Note, for negative distances, the image "appears" to be in front of the eddy. For most combinations of eddy radii, and acoustic contrast and source distances, images will appear to be between the source and the eddy. For images to occur behind eddies, the source would have to be outside the focal length, or on the order of 1000 km away from the eddy. The comparison between the thin-lens image calculation and the cylindrical eddy image model calculation indicates that the thin-lens equation is a good approximation.

*acoustic contrast is defined as $(V_0 - V_1)/V_1$ where V_0 is the sound velocity outside the eddy and V_1 is the sound velocity within the eddy.

RAY REFRACTION DIAGRAM (Cold Eddy with source at infinite distance from eddy)

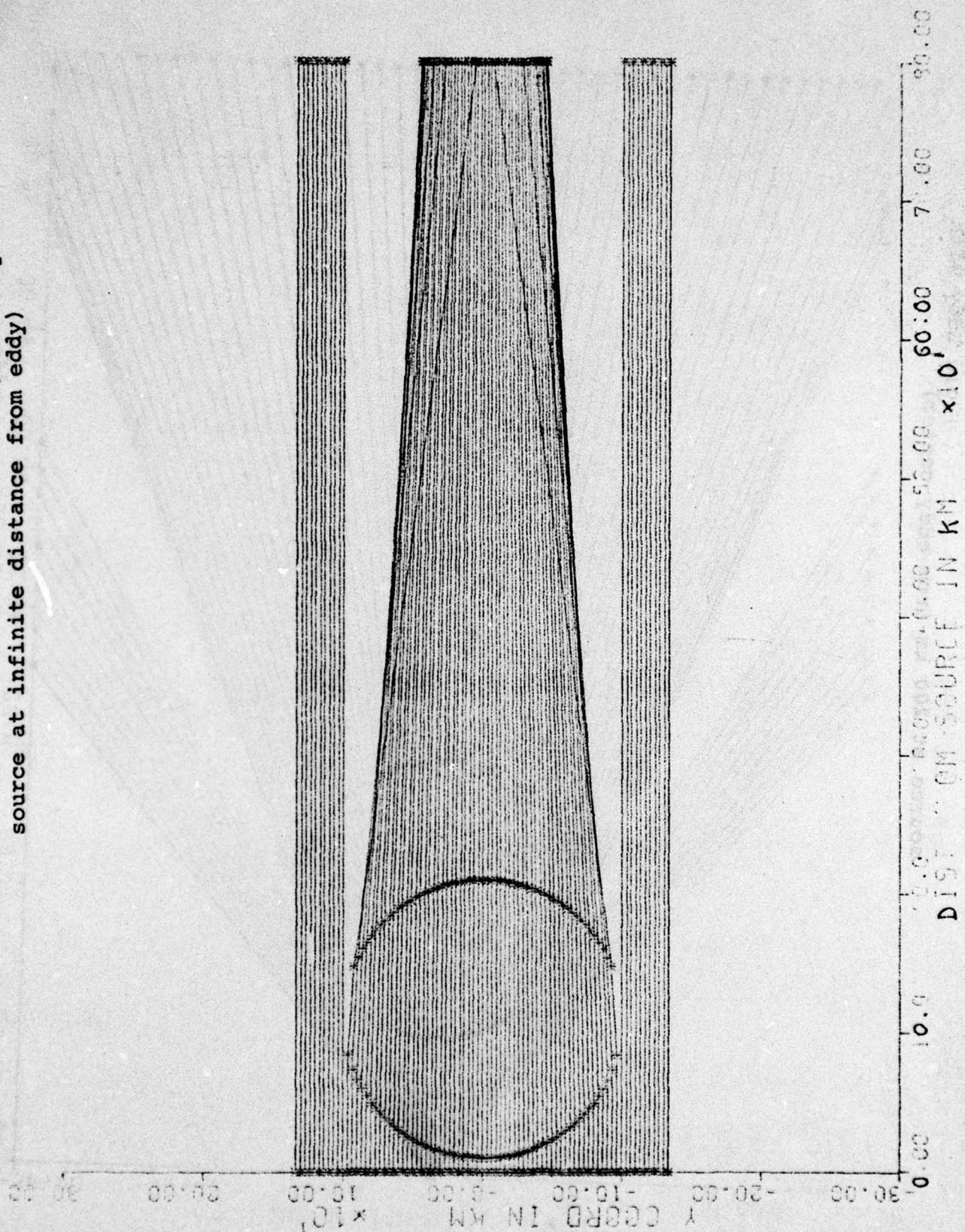


Figure 2 Eddy Radius R = 100 km

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RAY REFRACTION DIAGRAM (Cold Eddy with source at 200 km from eddy center)

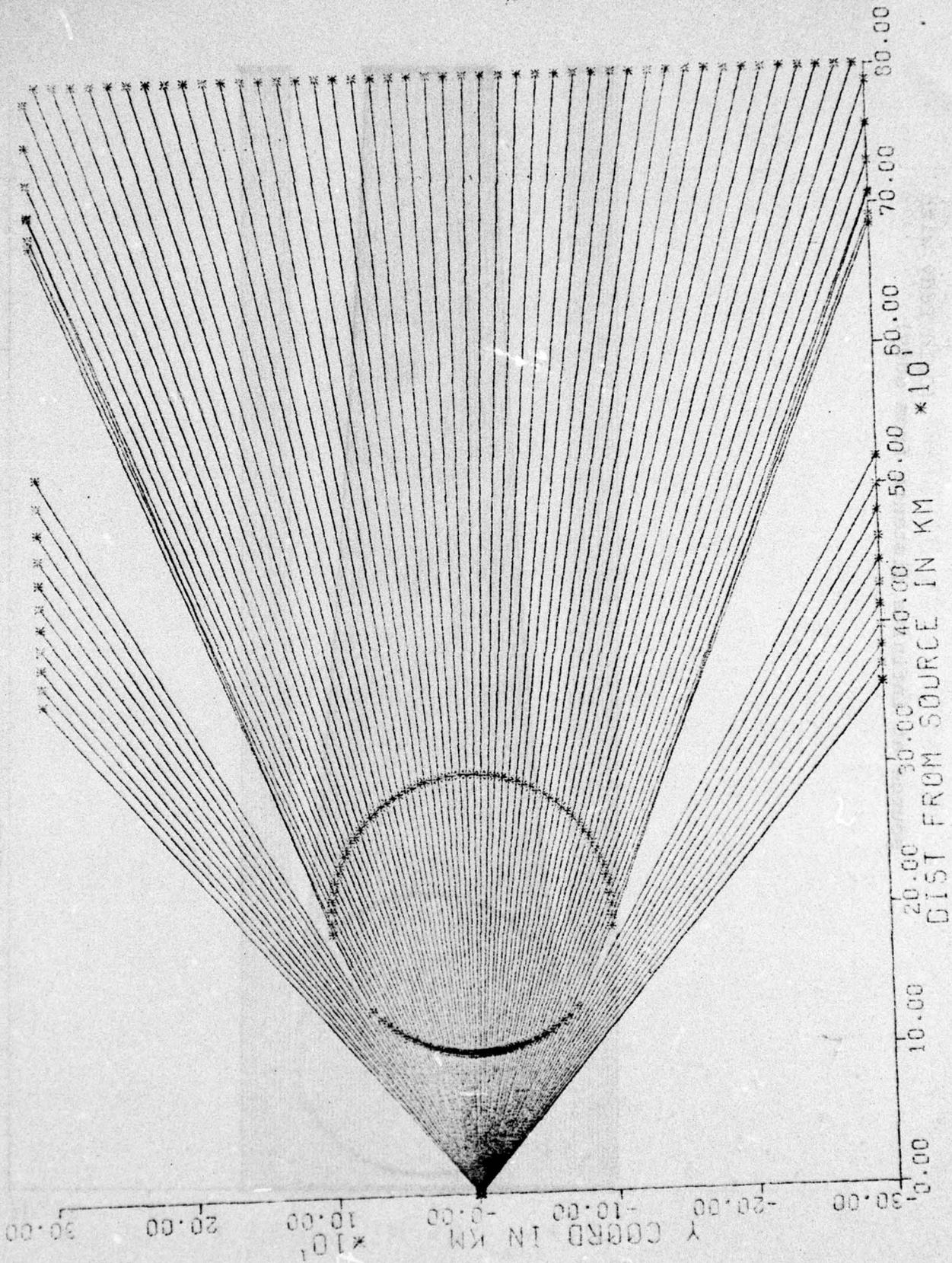


Figure 3 Eddy Radius R = 100 km Sound Velocity in Eddy = 1500 m sec⁻¹ Outside Eddy 1530 m sec⁻¹ AC = 0.02

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source at 200 km from eddy center

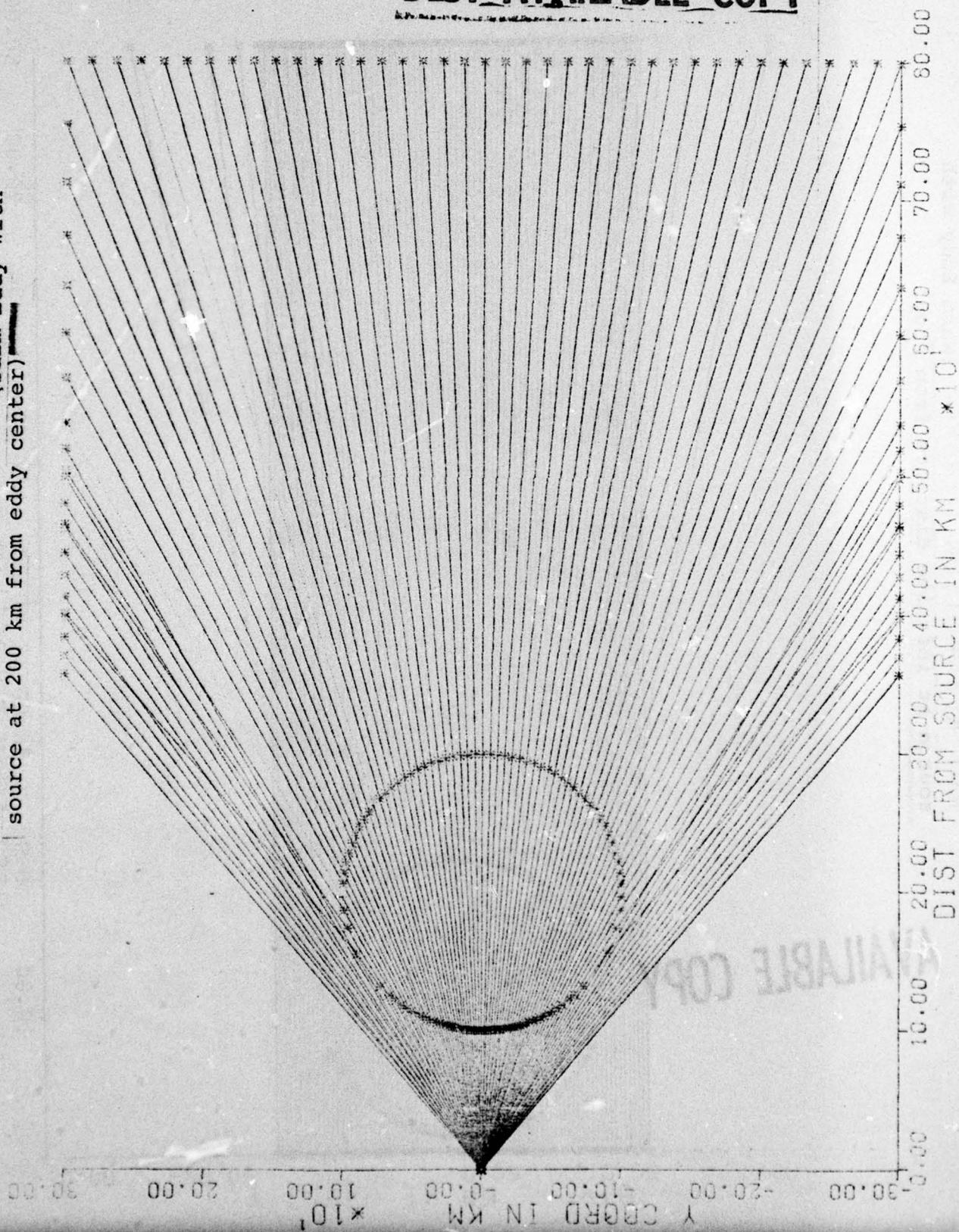


Figure 4 Eddy Radius R = 100 km
Sound Velocity in Eddy = 1500 m sec⁻¹ Outside Eddy = 1470 m sec⁻¹ AC = -0.02

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RAY REFRACTION DIAGRAM (Warm Eddy with source at infinite distance from eddy)

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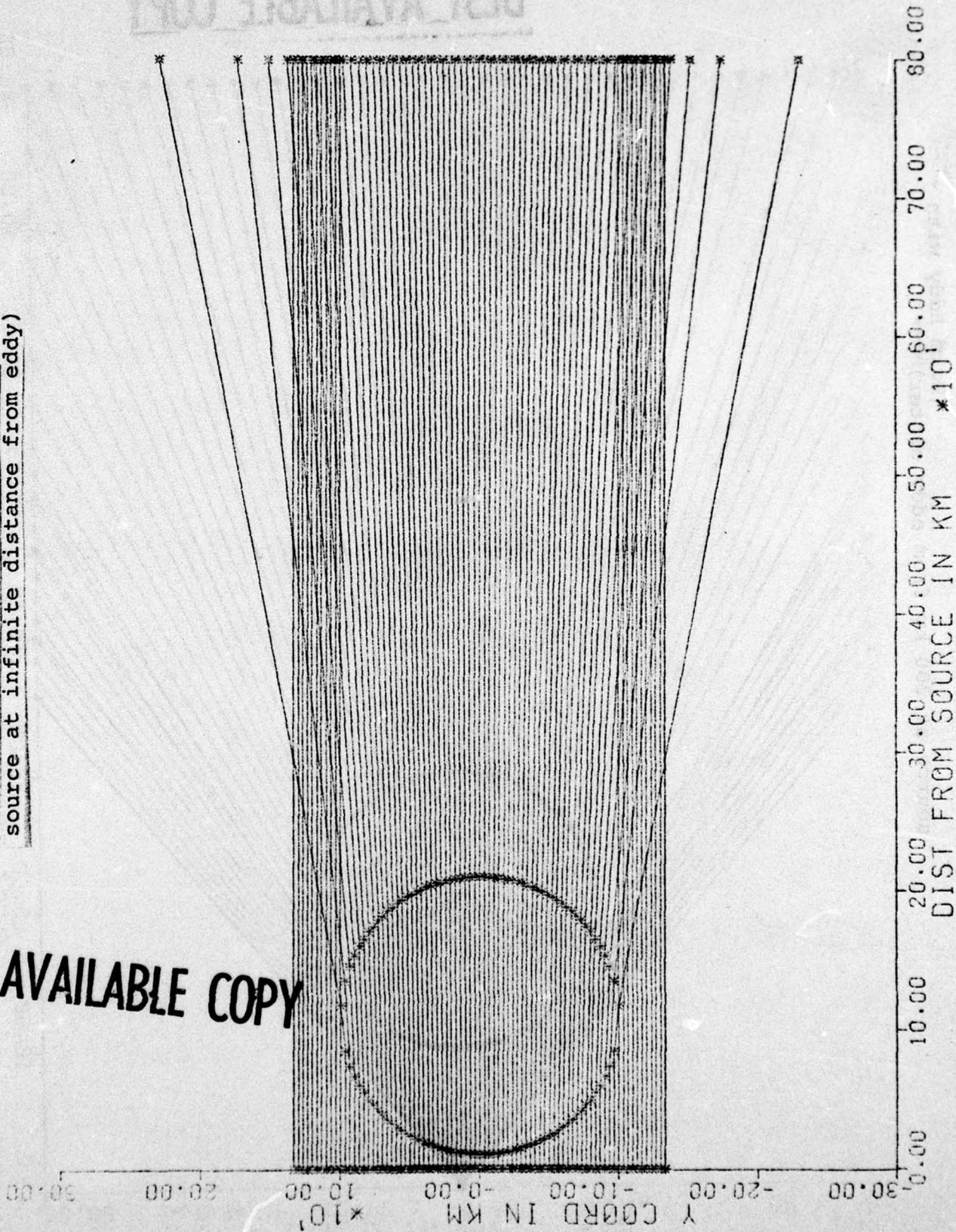


Figure 5 Eddy Radius 100 km AC = -0.02,

TABLE I
COMPARISON OF CYLINDRICAL EDDY IMAGE AND THE THIN-LENS IMAGE POSITION

XO (KM)	R (KM)	Thin Lens Focal Length (KM)	Thin Lens Image (KM)	Cylindrical Eddy Image (KM)
100000000.	75.	1875.	1875.	1912.
125.	75.	1875.	-134.	-134.
100000000.	75.	-1875.	-1875.	-1837.
125.	75.	-1875.	-117.	-117.
100000000.	75.	3750.	3750.	3787.
100000000.	75.	1875.	1875.	1912.
100000000.	75.	1250.	1250.	1287.
100000000.	75.	938.	938.	975.
100000000.	75.	750.	750.	787.
100000000.	75.	625.	625.	662.
100000000.	75.	375.	375.	412.
100.	75.	1875.	-106.	-106.
125.	75.	1875.	-134.	-134.
175.	75.	1875.	-193.	-193.
275.	75.	1875.	-322.	-321.
375.	75.	1875.	-469.	-466.
100000000.	25.	625.	625.	637.
100000000.	50.	1250.	1250.	1275.
100000000.	75.	1875.	1875.	1912.
100000000.	100.	2500.	2500.	2550.
100.	25.	625.	-119.	-119.
100.	50.	1250.	-109.	-109.
100000000.	75.	-3750.	-3750.	-3712.
100000000.	75.	-1875.	-1875.	-1837.
100000000.	75.	-1250.	-1250.	-1212.
100.	75.	-1875.	-95.	-95.
125.	75.	-1875.	-117.	-117.
175.	75.	-1875.	-160.	-160.
100.	25.	-635.	-86.	-86.
100.	50.	-1250.	-93.	-92.

APPENDIX A

THE HORIZONTAL REFRACTION MODEL

The eddy horizontal refraction model was developed assuming an idealized cylindrical eddy. Sound speed is specified as a constant within and outside the eddy. The geometry for the model was shown in Figure 1.

A point sound source is located at the origin (o) $x=0$, $y=0$, which is at a distance (X_0) from the center of an eddy of radius (R). Sound speed outside the eddy is given as V_0 , within it is V_E . For a given initial angle (θ measured positive counter-clockwise from the x - axis), the point of intersection for a ray from a source at origin to the eddy can be found by solving simultaneous equations of a straight line (from the source to the circle) and of a circle (the eddy of radius R) at distance X_0 from the source. The two equations are:

$$Y = X \tan \theta \quad \text{equation of straight line} \quad (1a)$$

$$(X-X_0)^2 + y^2 = R^2 \quad \text{equation of circle} \quad (1b)$$

By substituting 1a into 1b to eliminate Y , and rearranging the terms and returning to 1a, the sound ray will intersect the eddy at:

$$X_I = X_0 \cos^2 \theta + X_0^2 \cos^4 \theta - (X_0^2 - R^2) \cos^2 \theta \quad (2a)$$

$$Y_I = X_I \tan \theta \quad (2b)$$

At the point of intersection the ray will be refracted by the eddy according to Snell's law:

$$\frac{\sin \alpha_O}{V_O} = \frac{\sin \alpha_E}{V_E} \quad (3)$$

where V_O and α_O are the sound velocity and the incident angle at intersection with the eddy, and the E subscripted terms are the values after refraction in the eddy. The incident angle (α_O) is the angle between the incident ray and the normal to the circle. The angle of intersection to the eddy is simply computed from the slope of the normal, $n = \frac{Y_I}{X_I - X_O}$ (4a), and slope of the intersecting ray, $m_R = \frac{Y_I}{X_I}$ (4b), using the trigonometric relation:

$$\tan \alpha = \frac{n - m_R}{1 + m_R n} \quad (5)$$

The refracted angle through the eddy is computed rearranging Snell's law

$$\alpha_E = \sin^{-1} \left(\frac{V_E}{V_O} \sin \alpha_O \right) \quad (6)$$

and the distance L the refracted ray travels across the eddy is:

$$L = 2R \cos \alpha_E \quad (7)$$

The coordinates of the point of exit of the ray relative to the point of entry are:

$$\Delta X_E = L \cos (\theta - \Delta \theta) \quad (8a)$$

$$\Delta Y_E = L \sin (\theta - \Delta \theta) \quad (8b)$$

where $\Delta\theta = \alpha_0 - \alpha_E$ so that the absolute coordinates (relative to the origin are):

$$X_E = X_I + \Delta X_E \tag{9a}$$

$$Y_E = Y_I + \Delta Y_E \tag{9b}$$

The ray will be refracted again upon leaving the eddy. The coordinates at the point of ray termination relative to the point of exit from the eddy are computed from:

$$\Delta X_T = D \cos (\theta - 2\Delta\theta) \tag{10a}$$

$$\Delta Y_T = D \sin (\theta - 2\Delta\theta) \tag{10b}$$

or the termination point relative to the origin.

$$X_T = X_I + \Delta X_E + \Delta X_T \tag{11a}$$

$$Y_T = Y_I + \Delta Y_E + \Delta Y_T \tag{11b}$$

The set of equations above can be reduced for the special case of a source placed an infinite distance away. In this case, the source acts as an infinite "line source" so that all the rays are assumed parallel to the x - axis and can be placed any distance from the eddy. The governing equation can be written as:

$$(X - X_0)^2 + Y_S^2 = R^2 \tag{12}$$

For this case X_0 is the arbitrary distance from the "line source" to the vertical axis through the eddy's center and Y_S is the perpendicular distance to the x - axis. Since $Y_S = Y_I$ (parallel ray to x - axis) then X_I is solved from (12) using:

$$X_I = X_0 \pm R^2 - Y_S^2)^{1/2} \tag{13a}$$

$$Y_I = Y_S \tag{13b}$$

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Obviously (13) does not hold if $Y_s \geq R$ because rays do not intersect the eddy but pass straight on by. For $Y_s < R$, equations (13) are used in place of equations (2) to find the point of intersection of the incoming ray with the eddy. The computations following the ray paths through the eddy are calculated from (3-12) as before.

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